Study of phase transition in two flavour quark matter at finite volume

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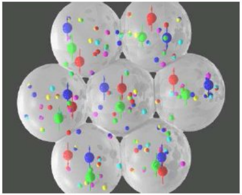
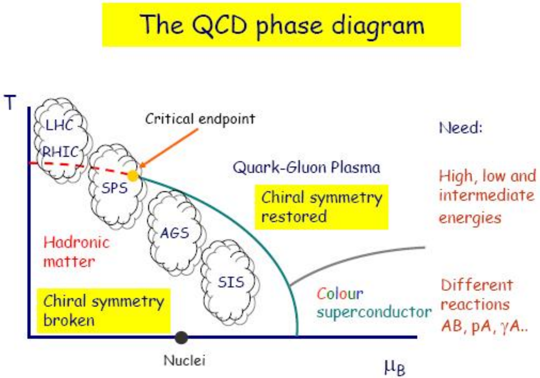
- Context of the work
- Model Set up
- Result
- Summary.

QCD Phase Diagram



Hadron



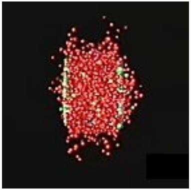
- Presence of Critical End Point (CEP)!
- CEP is important: indirect evidence of 1st order phase transition and crossover.

Where to look for the transition

Relativistic Heavy Ion Collider [RHIC]

Large Hadron Collider [LHC]

Where to look for the transition



- Only one control parameter CM energy $\sqrt{S}_{NN}$
- Important: fluctuations of the conserved quantities
- Fluctuations can be used to connect the experimental observation to theoretical predictions

Theory side

- From theory side, thermodynamics quantity of interest: susceptibility
- nth order cumulant of the fluctuations of the baryon number is related to the nth order baryon number susceptibility:

$$[B^n] \;=\; V\;T^3\;T^{n-4}\chi_B^{(n)}$$

  where V is the volume of the observed part of the fireball
- Can't employ above relation directly: V is hard to determine from experiment!!!
- Ratios are useful: can be used to compare between experimental results with that of theory

$$S\sigma \;=\; \frac{T\;\chi_B^{(3)}}{\chi_B^{(2)}},\; \kappa\sigma^2 \;=\; \frac{T^2\;\chi_B^{(4)}}{\chi_B^{(2)}},\; \frac{\kappa\;\sigma}{S} \;=\; \frac{T\chi_B^{(4)}}{\chi_B^{(3)}}$$

- Applicability of thermodynamics → See: S. Gupta et. al., Science, **332**, 2011; R.V. Gavai, S. Gupta, Phys. Lett. B, **696**, 2011

Model: MRE approximation

- The finite-size effects are taken into account by using the Multiple Reflection Expansion (MRE). (for details please see: R. Balian and C. Bloch, Ann. Phys. (N.Y.) **60**, 401 (1970))
- MRE approximation: the finite-size effects are included by modifying the density of states:

$$\rho_{MRE}(k,\kappa,R) \;=\; \frac{k^2}{2\pi^2}\left[1+\frac{6\pi^2}{kR}f_s\left(\frac{k}{\kappa}\right)+\frac{12\pi^2}{(kR)^2}f_C\left(\frac{k}{\kappa}\right)\right]$$

  where R is the radius of the sphere and the functions:

$$f_s(x) \;=\; -\frac{1}{8\pi}\left(1-\frac{2}{\pi}arctan\ x\right),$$
$$f_C(x) \;=\; \frac{1}{12\pi^2}\left[1-\frac{3}{2}x\left(\frac{\pi}{2}-arctan\ x\right)\right]$$

  the subscript "s" and "C" correspond surface and curvature contribution respectively.
- Note: curvature contribution has not been derived within the MRE framework. The form of the f_C is a proposal from J. Madsen, Phys. Rev. D **50**, 3328 (1994).

Model: PNJL model

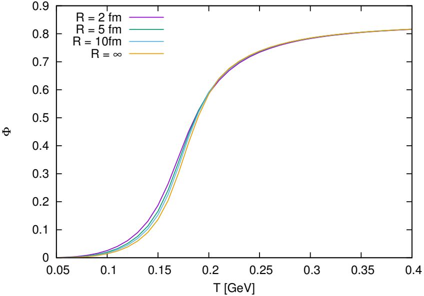
Lagrangian:

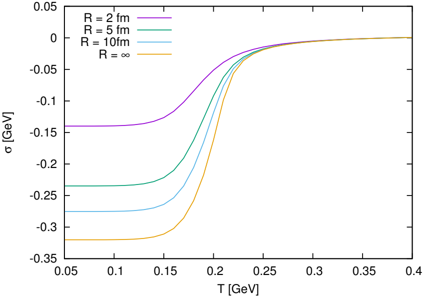
$$\mathcal{L}_{PNJL} = \bar{q}(i\gamma^{\mu} D_{\mu} - m_0 + \mu\gamma^0)q + \frac{G}{2}[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2] - V(\bar{\Phi}, \Phi)$$

Where $D_{\mu} = \partial_{\mu} - ig\mathcal{G}_{\mu}$ and $\mathcal{G}_{\mu} = \delta_{\mu 0}\mathcal{G}_0$

- Introducing auxiliary field variables σ and $\vec{\pi}$ an $\mathcal{L}_{eff}$ is obtained, with the replacement $\exp[-\mathcal{G}_0/T] \to \Phi$.
- The mean fields $<\sigma>= G <\bar{q}q>$ and $<\vec{\pi}>= 0$ for $\mu_I < m_{\pi}$.
- Thermodynamic properties studied with $\Phi(T)$ and σ from the thermodynamic potential $\Omega[\bar{\Phi}, \Phi, \sigma, T, \mu_0, \mu_I]$.

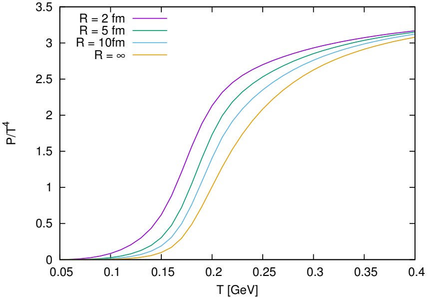
Order Parameters

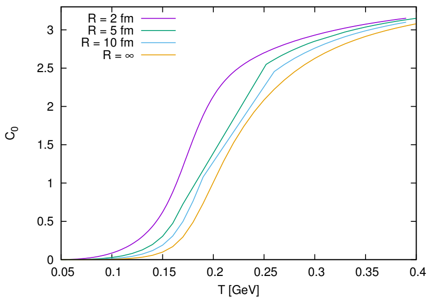




- Order Parameter for Chiral Transition is affected by finite size!
- At a critical size of the system value of Chiral Condensate will always be zero!

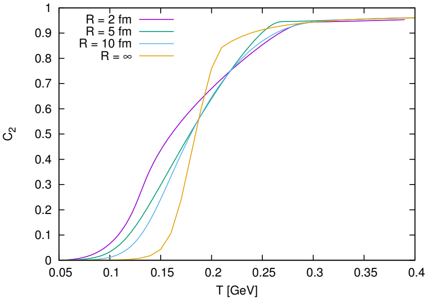
Pressure

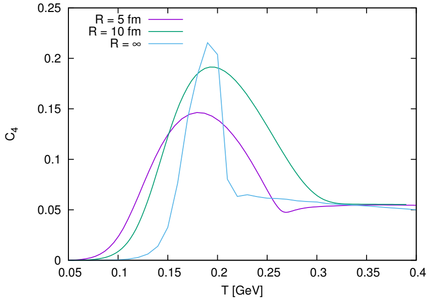




- Approaches ideal gas value at higher temperature.
- $\frac{P(T,\mu_q)}{T^4} = \sum_{n=0}^{\infty} C_n(T) \left(\frac{\mu_q}{T}\right)^n$, where $C_n(T) = \frac{1}{n!}\frac{\partial^n\left(\frac{P}{T^4}\right)}{\partial\left(\frac{\mu_q}{T}\right)^n}$

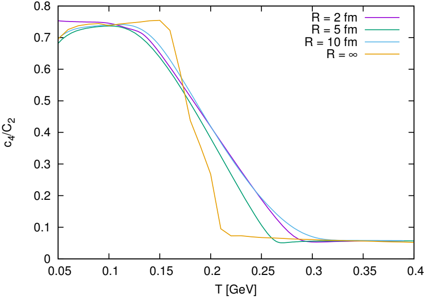
Fluctuation





- $\frac{P(T,\mu_q)}{T^4} = \sum_{n=0}^{\infty} C_n(T) \left(\frac{\mu_q}{T}\right)^n$, where $C_n(T) = \frac{1}{n!} \frac{\partial^n \left(\frac{P}{T^4}\right)}{\partial \left(\frac{\mu_q}{T}\right)^n}$
- Nature of the transition does not change with the finite size
- Peak height of C_4 decreases with the decreasing value of the radius.
- Results need to be improved around the transition temperature.

Fluctuation



- $\frac{P(T,\mu_q)}{T^4} = \sum_{n=0}^{\infty} C_n(T) \left(\frac{\mu_q}{T}\right)^n$, where $C_n(T) = \frac{1}{n!} \frac{\partial^n \left(\frac{P}{T^4}\right)}{\partial \left(\frac{\mu_q}{T}\right)^n}$
- Distinct values in two different phases!
- Effects of volume cancels out (more or less)!

Summary

- We propose to study the transition in the 2-flavour quark matter by using Multiple Reflection Expansion along with PNJL model.
- Qualitatively the model reproduces the known expectations.
- Fluctuations near the transition region needs to be addressed more carefully.
- Ongoing work: to study the fluctuations as the system approaches the critical point from crossover and first-order phase transition sides respectively.

THANK YOU!