Dynamical restoration of Z_N symmetry in SU(N) Higgs theory

Minati Biswal

Institute Of Physics,India.

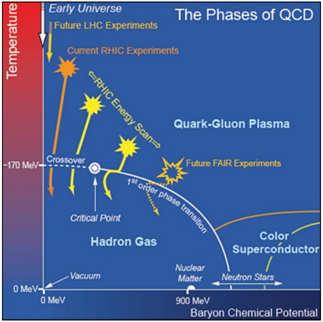
Collaborators: Sanatan Digal, P.S. Saumia

Outline

- Motivation.
- Z_N symmetry (with matter fields).
- Simulation study of Z_N symmetry in presence of Higgs.
- Results.
- Summary.

Motivation

- In QCD, Hadrons melt into quark-gluon plasma (QGP) via transition known as confinement-deconfinement (CD) transition. The transition is a cross-over for physical quark masses.



- The CD transition is present in all SU(N) [N > 1] theories.

Motivation

- In pure SU(N) gauge theories, the CD transition is described by order parameter, the average of Polyakov loop ($\langle L \rangle$) and the Z_N symmetry. Order of the transition depends on N .
- The Z_N symmetry is explicitly broken when matter fields are included into SU(N) gauge theories as a result the CD transition becomes a cross-over.
- In SU(N) Higgs theory there are very few non-perturbative studies on explicit breaking of Z_N symmetry. And also it is important to understand the similarities (differences) between bosonic and fermionic matter as to how they affect the Z_N symmetry.
- We study the Z_N symmetry in SU(N) Higgs theory using Monte Carlo simulations.

Z_N symmetry

- Partition function of a pure SU(N) gauge theory at high temperature ($T = \frac{1}{\beta}$) is

$$\mathcal{Z} = \mathrm{Tr} e^{-\beta H} = \int dA \langle A | e^{-\beta H} | A \rangle = \int_{\mathbf{bc}} DA e^{-S(A)} \tag{1}$$

$$S(A) = \int_0^\beta d\tau \int_V d^3x \left\{ \frac{1}{2} \mathrm{Tr} \left(F^{\mu\nu} F_{\mu\nu} \right) \right\} \tag{2}$$

- Where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$

- The allowed A's in the path integral are periodic in β,

$$A_\mu(\vec{x}, 0) = A_\mu(\vec{x}, \beta) \tag{3}$$

Contd...

- $S(A)$ and $\mathcal{Z}$ are invariant under the gauge transformation $V(\vec{x},\tau)$, A_μ transforms

$$A_\mu \longrightarrow VA_\mu V^{-1} - \frac{i}{g}\left(\partial_\mu V\right) V^{-1} \tag{4}$$

- $V(\vec{x},\tau)$ need not be periodic, as long as it satisfies the following eqn.

$$V(\vec{x},\tau=0) = zV(\vec{x},\tau=\beta) \tag{5}$$

Where $z \in Z_N$,with $z = \mathbb{1} exp(\frac{2\pi in}{N}),\ n = 0, 1, 2...N-1$,

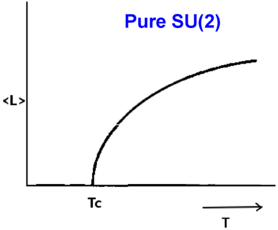
- Therefore, all the allowed gauge transformations at finite temperature are classified by Z_N group.
- Z_N is a symmetry of $\mathcal{Z}$.

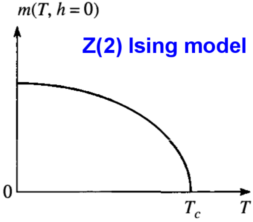
Order parameter of the theory

- The Polyakov loop transforms nontrivially under Z_N.

$$L(\vec{x}) = \frac{1}{N}\mathrm{Tr}\left\{ P e^{\left(-ig\int_0^\beta A_0(\vec{x},\tau)d\tau\right)} \right\} \quad (6)$$

Under $Z_N, L \longrightarrow zL$. $\langle L \rangle = \frac{\int dALe^{-S}}{\int dAe^{-S}}$.





- $\langle L \rangle$ is an order parameter for CD transition and it is analogous to the magnetization in a $Z(N)$ spin system.

Z_N symmetry (with matter fields)

- The action in presence of fundamental Higgs field is given by,

$$S_E = \int_0^\beta d\tau \int_V d^3x \left[\frac{1}{2} \mathrm{Tr}\left(F^{\mu\nu} F_{\mu\nu}\right) + \frac{1}{2} |D_\mu \Phi|^2 + \frac{m^2}{2} \Phi^\dagger \Phi + \frac{\bar{\lambda}}{4!} (\Phi^\dagger \Phi)^2 \right. \tag{7}$$

- Being a bosonic field, $\Phi(\vec{x}, 0) = \Phi(\vec{x}, \beta)$. Under above non-periodic gauge transformations, $\Phi'(0) \neq \Phi'(\beta)$ (when $z \neq \mathbb{1}$).
- It is not clear how this Z_N explicit breaking will affect the CD transition. Fluctuations of the gauge and Higgs fields need to be considered.

Monte Carlo simulations of the CD transition

For simulations, we discretise the action on a 4D euclidean space, $\Phi(x) \to \Phi_n$, $e^{iagA_{n,\mu}} \to U_{n,\mu}$. Further we scale Φ, $\bar{\lambda}$ and m as

$$\Phi(x) \to \frac{\sqrt{\kappa}\Phi_n}{a}, \bar{\lambda} \to \frac{\lambda}{\kappa^2}, m^2 \to \frac{(1-2\lambda-8\kappa)}{\kappa a^2}$$

The discretised action is given by,

$$S(U,\Phi) = \beta_g \sum_p \mathrm{Tr}(1 - \frac{1}{2N}(U_p + U_p^\dagger)) - \kappa \sum_{\mu,n} \mathrm{Re}\left[\mathrm{Tr}(\Phi^\dagger_{n+\mu} U_{n,\mu} \Phi_n)\right] + \sum_n \left[\frac{1}{2}\mathrm{Tr}\left(\Phi_n^\dagger \Phi_n\right) + \lambda\left(\frac{1}{2}\mathrm{Tr}\left(\Phi_n^\dagger\Phi_n\right) - 1\right)^2\right]. \quad (8)$$

Contd...

- Here $\beta_g = \frac{2N}{g^2}$. Plaquette U_p is the product of links around an elementary square 'p' ($U_p = U_{n,\mu} U_{n+\mu,\nu} U^{\dagger}_{n+\nu,\mu} U^{\dagger}_{n,\nu}$).

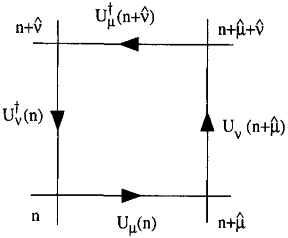


Figure: Sketch of an elementary plaquette U_P

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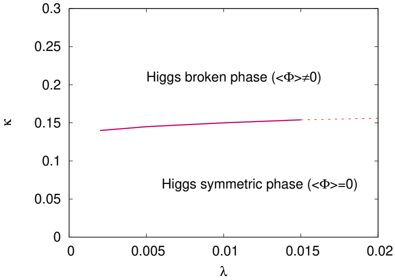
- In the Monte Carlo simulations an initial configuration of Φ_n and $U_{\mu,n}$ is repeatedly updated to generate a Monte Carlo history.
- In an update a new configuration is generated from an old one according to the Boltzmann probability factor e^{-S} taking care the principle of detailed balance.
- Boltzmann factor and principle of detailed balance are implemented using pseudo heat-bath algorithm [1] [2] for the Φ field and the standard heat-bath algorithm [3] for the link variables U_μ's.
- To reduce auto-correlation between consecutive configurations we use over-relaxation method.

[1] *B. Bunk.* **Nucl. Phys. B 42, 556 (1995)**.
[2] *A.D. Kennedy et.al* **PLB 156, 393 (1985)** .
[3] *M. Creutz.* **Phys. Rev. D 29, 306 (1984)**.

Results



- In this Higgs phase diagram, the Higgs symmetric ($\langle\Phi\rangle = 0$) and broken phase ($\langle\Phi\rangle \neq 0$) are separated by the Higgs transition line.
- We compute the Polyakov loop distribution at various points on this phase diagram to study the Z_N symmetry.
- Since the CD transition behaviour has been observed to be sensitive to N_τ, we consider larger N_τ for some values of the bare parameters.

Polyakov loop distribution (close to Higgs transition line)

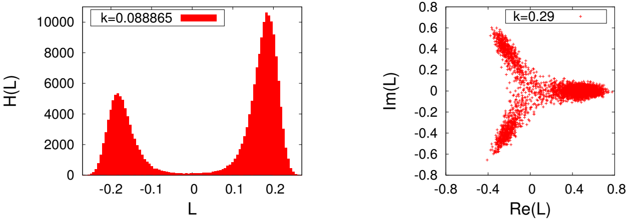


Figure: SU(2) and SU(3)

- There is no Z_2 symmetry in the distribution H(L) of the Polyakov loop for SU(2).
- Similarly for SU(3) there is no Z_3 symmetry of the Polyakov loop distribution.
- Here Z_N symmetry is explicitly broken.
- Largest peak corresponds to the stable state and others correspond to meta-stable states.

Polyakov loop distribution (Away from Higgs transition line)

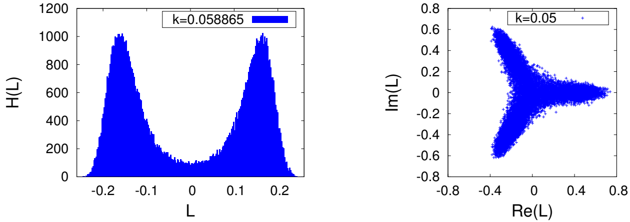
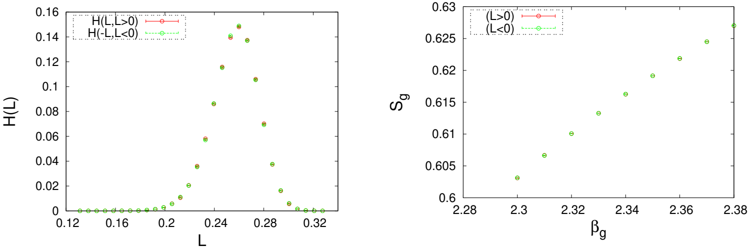


Figure: SU(2) and SU(3)

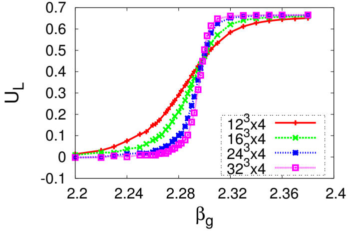
- The two peaks in case of SU(2) are related by Z_2 symmetry ($L \to -L$).
- Similarly the distribution of the Polyakov loop for SU(3) has the Z_3 symmetry.
- So away from the Higgs transition line the Z_N symmetry is restored.

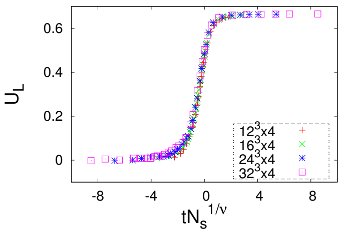
$H(L)$ and Gauge action showing Z_2 symmetry



- Within errors $H(L) = H(-L)$ and $S_g(L) = S_g(-L)$.
- This is clear evidence that there is Z_2 symmetry.
- This realization of the Z_2 symmetry makes the CD transition second order.

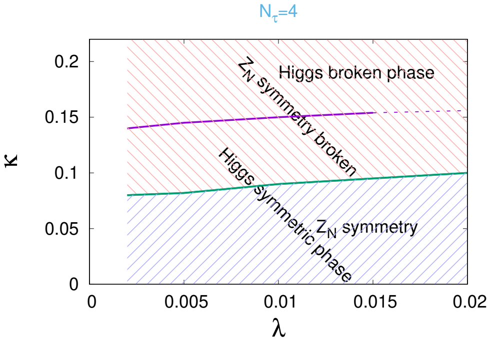
This symmetry restoration leads to critical behavior





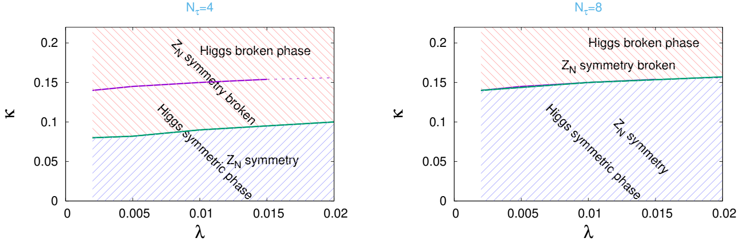
- The value of the Binder cumulant$\left(U_L = 1 - \frac{\langle L^4 \rangle}{3\langle L^2 \rangle^2}\right)$ at the crossing point for different volumes is consistent with the 3D-Ising Universality class.
- It is clearly seen that, by scaling β_g by $t = (\frac{\beta_g - \beta_{gc}}{\beta_{gc}})N_s^{\frac{1}{\nu}}$ all different volume curves collapse on one line.
- This corresponds to a second order phase transition.

Results for Z_2 symmetry ($N_\tau = 4$)



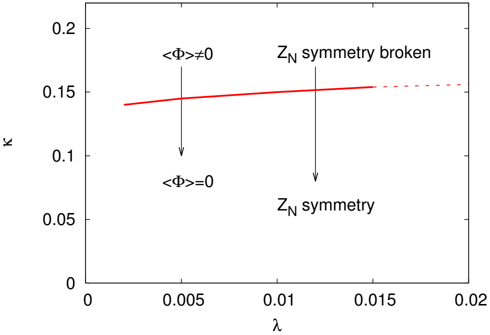
- The Z_N symmetry is explicitly broken in the Higgs broken phase and close to the Higgs transition line in the Higgs symmetric phase.
- Restoration of Z_N symmety happens in the part of Higgs symmetric phase away from Higgs transition line.

SU(N) Higgs theory N_τ dependence



- The Z_N symmetry breaking line will approach Higgs transition line for Higher N_τ.

Summary



- Z_N symmetry explicit breaking decrease with decrease in κ.
- On the other hand, Higgs condensate decreases with decrease in κ.

Summary

- Our results suggest that the Higgs condensate plays a role of symmetry breaking field like external field in the Ising model.
- We believe increase in phase space of the Higgs field is responsible for Z_2 restoration.

References

- *F. Karsch,* **Lect.Notes Phys. 583 (2002) 209-249 (arXiv:hep-lat/0106019)**.
- *M. Biswal, S. Digal and P. S. Saumia,* **Nucl. Phys. B 910, 30 (2016)**.
- *B. Bunk.* **Nucl. Phys. B 42, 556 (1995)**.
- *A.D. Kennedy et.al* **PLB 156, 393 (1985)** .
- *M. Creutz.* **Phys. Rev. D 29, 306 (1984)**.
- *N. Weiss,* **Phys. Rev. D 2667, 25 (1982)**.

Thank you

$$\left[\frac{f}{T^4}\right]_{\beta_0}^{\beta} = {N_\tau}^4 \int_{\beta_0}^{\beta} d\beta' (S_T - S_0) \tag{9}$$

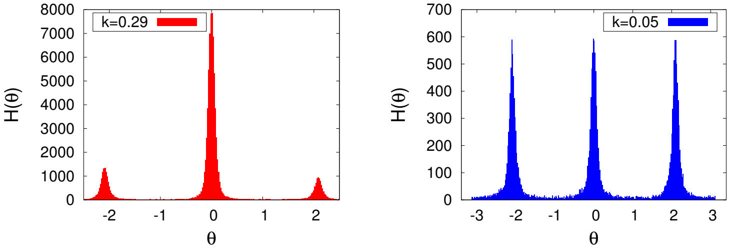
$$D_\mu \Phi = \partial_\mu \Phi + \frac{(1 - U_\mu)}{a} \Phi \tag{10}$$

$$(D_\mu \Phi)^\dagger (D_\mu \Phi) = \frac{1}{a^2} [\Phi^\dagger_{x+\mu} \Phi_{x+\mu} + \Phi^\dagger_x \Phi_x - \Phi^\dagger_x U^\dagger_{x,\mu} \Phi_{x+\mu} - \Phi^\dagger_{x+\mu} U_{x,\mu} \Phi_x] \tag{11}$$

$$S_H = \sum_x \Big[\frac{8a^2}{2} Tr(\Phi^\dagger_x \Phi_x) - a^2 Re Tr(\Phi^\dagger_{x+\mu} U_{x,\mu} \Phi_x) - \frac{m^2 a^4}{2} Tr(\Phi^\dagger_x \Phi_x) + \frac{\lambda a^4}{2} Tr(\Phi^\dagger_x \Phi_x)^2] \tag{12}$$

$$\Phi(x) \to \frac{\sqrt{k} \Phi_n}{a}, \lambda \to \frac{\lambda}{k^2}, m^2 \to \frac{(1 - 2\lambda - 8k)}{ka^2} \tag{13}$$

$$S_H = \sum_n [-\kappa Tr(\Phi^\dagger_{n+\mu} U_{n,\mu} \Phi_n) + \frac{1}{2} Tr(\Phi^\dagger_n \Phi_n) + \lambda(\frac{1}{2} Tr(\Phi^\dagger_n \Phi_n) - 1)^2] \tag{14}$$


k=0.29
H(θ)
θ
k=0.05
H(θ)
θ