Mesonic excitations in a magnetic field at finite temperature in the NJL model

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Outline

1. Introduction
2. Formalism
   - Propagator in external magnetic field at finite temperature
   - The current quark mass at $T \neq 0$ and $B \neq 0$
   - Mesonic modes in the NJL model
3. Results
4. Summary

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  - Lattice QCD simulations.
  - Effective-models, where QCD interactions are replaced by effective interactions.

In this work we will be dealing with the second approach and consider the Nambu-Jona Lasino (NJL) Model, which is capable of capturing some of the non-perturbative aspects of the strongly interacting matter, provides a useful frame work to probe the phase structure of QCD at arbitrary temperatures and chemical potentials.

- The interaction Lagrangian in NJL model is given by

$$\mathscr{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\left\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right\}$$

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  - * It is not a renormalizable field theory, hence a proper regularization scheme must be specified.

Introduction

Magnetic field

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\begin{aligned}
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The thumb rule is

$$m_\pi^2 \approx 0.02 \text{ GeV}^2 \text{ and } 1 \text{ GeV}^2 \approx 10^{15} \text{ Tesla}$$

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Weak field expansion of the propagator in presence of magnetic field

[1] Arghya Mukherjee *et al.* Phys. Rev. D **98**, 056024 (2018)

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The weak field expansion of the Dirac propagator for non-vanishing **anomalous magnetic moment(κ)** is given by[1]

$$S_B(p) = \hat{F}(p,m,m_1)\Delta_F(p,m_1)\Big|_{m_1=m}$$

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$$\Delta_F(p, m_1) = \frac{-1}{p^2 - m_1^2 + i\epsilon}$$

and

$$\hat{A_n} = \frac{(-1)^n}{n!}\frac{\partial^n}{\partial(m_1^2)^n}$$

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Propagator at finite temperature

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Propagator at finite temperature

To include temperature we will use the real time formalism of thermal field theory where the thermal propagator takes form of 2×2 matrix. However, it is sufficient to evaluate one of the components. Here we take the 11 component

$$S_{11}\left(p,m\right) = S_B(p) - \eta\left(p \cdot u\right)\left[S_B(p) - \gamma^0 S_B^\dagger(p)\gamma^0\right]$$

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Using $\gamma^0 \hat{F}^\dagger(p, m, m_1)\gamma^0 = \hat{F}(p, m, m_1)$, we get

$$S_{11}(p, m) = \hat{F}(p, m, m_1)\left[\Delta_F(p, m_1) - 2\pi i \eta(p \cdot u)\,\delta(p^2 - m_1^2)\right]\Big|_{m_1 = m}$$

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The Gap Equation

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Using mean field approximation and dropping the quadratic term of the fluctuation one can write

$$(\bar{\psi}\psi)^2 \approx 2\left\langle\bar{\psi}\psi\right\rangle(\bar{\psi}\psi) - \left\langle\bar{\psi}\psi\right\rangle^2$$

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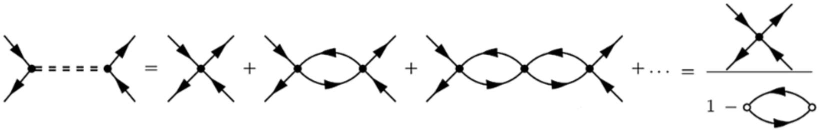
$$\begin{aligned}\left\langle\bar{\psi}\psi\right\rangle^{11} &= i\int\frac{d^4p}{(2\pi)^4}\ \mathbf{Tr}\left[\hat{F}(p,M,m_1)\right]\Delta_F(p,m_1)\Big|_{m_1=M} \\ &\quad + \int\frac{d^4p}{(2\pi)^4}\ \mathbf{Tr}\left[\hat{F}(p,M,m_1)\right]2\pi\eta\,(p\cdot u)\,\delta(p^2-m_1^2)\Big|_{m_1=M} \\ &= \left\langle\bar{\psi}\psi\right\rangle^{\text{vacuum}} + \left\langle\bar{\psi}\psi\right\rangle^{\text{medium}}.\end{aligned}$$

Outline

RPA approximation



Since mesons are the bound state of quark and anti-quark we can express meson propagators as

$$D_\sigma = \frac{2G}{1 - 2G\Pi_\sigma(k^2)} \qquad D_\pi = \frac{2G}{1 - 2G\Pi_\pi(k^2)}$$

where $\Pi_\alpha(q^2)$ is the 'polarization function' and at finite temperature and density its 11-component is given by

$$\Pi^{11}_\alpha(k^2) = \int \frac{d^4p}{(2\pi)^4}\, \mathbf{Tr}\,[S_{11}(p)\Gamma_\alpha S_{11}(p+k)\Gamma_\alpha]$$

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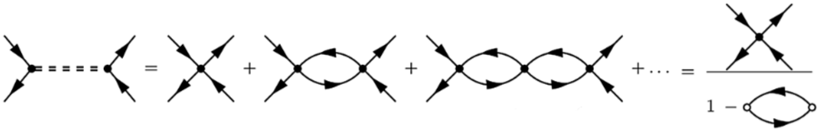

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The pole position determines the meson mass while the coupling constant can be obtained from residue at the pole.

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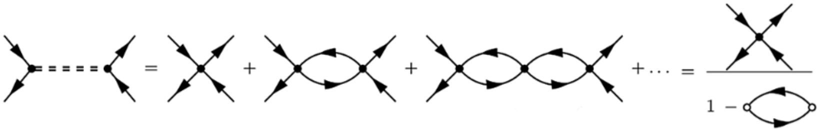

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- $\Gamma_\alpha = 1 \Rightarrow$ scalar mode (σ)
- $\Gamma_\alpha = i\gamma_5 \Rightarrow$ pseudoscalar mode (π)

Meson masses

$$\Pi^{11}_{\alpha}(k^2) = \Pi^{11}_{\alpha}(k^2)\Big|^{B\neq 0}_{\text{vacuum}} + \Pi^{11}_{\alpha}(k^2)\Big|_{\eta} + \Pi^{11}_{\alpha}(k^2)\Big|_{\eta^2}$$

$$\Pi^{11}_{\alpha}(k^2)\Big|^{B\neq 0}_{\text{vacuum}} = i\int \frac{d^4p}{(2\pi)^4}\hat{\mathcal{T}}^{\alpha}_{12}(p,p+k,M,m_1,m_2)\,\Delta_F(p,m_1)\Delta_F(p+k,m_2)\Big|_{\substack{m_1=M\\ m_2=M}}$$

$$\Pi^{11}_{\alpha}(k^2)\Big|_{\eta} = -i\int \frac{d^4p}{(2\pi)^4}\Big[\hat{\mathcal{T}}^{\alpha}_{12}(p,p+k,M,m_1,m_2)\,\Delta_F(p,m_1)2\pi i\eta\Big((p+k)\cdot u\Big)\delta\Big((p+k)^2-m_2^2\Big)$$
$$+\hat{\mathcal{T}}^{\alpha}_{12}(p,p+k,M,m_1,m_2)\,\Delta_F(p+k,m_2)2\pi i\eta\Big(p\cdot u\Big)\delta\Big(p^2-m_1^2\Big)\Big]\Big|_{\substack{m_1=M\\ m_2=M}}$$

$$\Pi^{11}_{\alpha}(k^2)\Big|_{\eta^2} = i\int \frac{d^4p}{(2\pi)^4}\hat{\mathcal{T}}^{\alpha}_{12}(p,p+k,M,m_1,m_2)\,(2\pi i)^2$$
$$\times\eta\Big(p\cdot u\Big)\eta\Big((p+k)\cdot u\Big)\delta\Big(p^2-m_1^2\Big)\delta\Big((p+k)^2-m_2^2\Big)$$

where

$$\hat{\mathcal{T}}^{\alpha}_{12}(p,p+k,M,m_1,m_2) = \mathbf{Tr}\Big[\hat{F}(p,M,m_1)\Gamma_{\alpha}\hat{F}(p+k,M,m_2)\Gamma_{\alpha}\Big]$$

Outline

Validity of weak field expansion

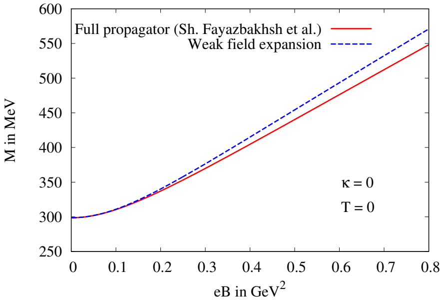

Figure: Variation of constituent quark mass with eB using full propagator and weak field expansion.

- ∗ Choice of parameters $m_0 = 0.005$ GeV, $\Lambda = 0.6643$ GeV and $G = 4.668$ GeV^{-2}.
- ∗ Up to $eB \approx 0.2$ GeV^2 the aprroximation is well in agreement with an error $< 1\%$.

Contribution of anomalous magnetic moment

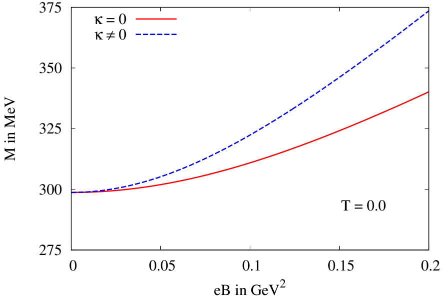

Figure: eB dependence of constituent quark mass with and without AMM.

[2]We have chosen $\kappa_u \approx 0.290$ GeV^{-1} and $\kappa_d \approx 0.360$ GeV^{-1}.

[2]Sh. Fayazbakhsh *et al.* Phys. Rev. D **90**, 105030 (2014).

Temperature dependence of constituent quark mass

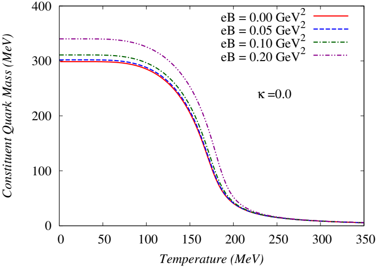

Figure: Variation of constituent quark mass with temperature.

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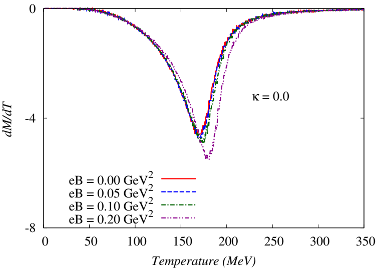

Figure: Derivative of constituent quark mass with temperature.

Outline

Summary

- We have calculated constituent quark mass in presence of external magnetic field at finite temperature in 2-flavour NJL model.
- The anomalous magnetic moments of quarks have been taken into consideration.
- The validity of the weak field approximation has been estimated by comparing with full propagator result at $\kappa = 0$.
- The effect of eB as well as κ on constituent quark mass turn out to be significant.
- The numerics related to the study of mesonic properties are in progress.

collaborators

- Dr. Snigdha Ghosh, IIT Gandhinagar, Palaj, Gandhinagar 382355, Gujarat, India
- Dr. Pradip Roy, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
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Thank You!

Constituent quark mass at finite temperature

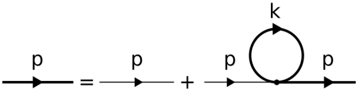

Figure: Variation of constituent quark mass with with temperature without AMM of quarks.

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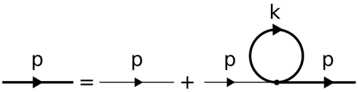

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The trace

$$\mathbf{Tr}\left[\hat{F}(p, M, m_1)\right] = 4M - 8M(qB)^2 p_\perp^2 \hat{A}_3 + 4(qB)(\kappa B)\left\{M^2 - p^2 + 4p_\parallel^2\right\}\hat{A}_2 + 4M(\kappa B)^2\left\{M^2 - p^2 + 4p_\parallel^2\right\}\hat{A}_2$$

Constituent quark mass at finite temperature

The final expressions are (only real part)

$$\langle\bar{\psi}\psi\rangle^{\text{vacuum}} = -\frac{1}{2\pi^2}\Bigg[M\Lambda\sqrt{\Lambda^2+M^2} - M^3\sinh^{-1}\left(\frac{\Lambda}{M}\right) + \frac{(qB)^2}{6M} + (qB)(\kappa B) + M(\kappa B)^2$$
$$+\Big\{(qB)(\kappa B) + M(\kappa B)^2\Big\}\left\{\sinh^{-1}\left(\frac{\Lambda}{M}\right) - \frac{\Lambda}{\sqrt{\Lambda^2+M^2}} - \frac{\Lambda^3}{3(\Lambda^2+M^2)^{3/2}}\right\}\Bigg]$$

and

$$\langle\bar{\psi}\psi\rangle^{\text{medium}} = \frac{1}{\pi^2}\int_0^\infty d|\vec{p}|\,|p|^2\Big[M\left(\mathcal{A}_0^+ + \mathcal{A}_0^-\right) + \frac{4}{3}M(qB)^2\,|\vec{p}|^2\left(\mathcal{A}_3^+ + \mathcal{A}_3^-\right)$$
$$+\left(4M^2 + \frac{8}{3}|\vec{p}|^2\right)\left\{(qB)(\kappa B) + M(\kappa B)^2\right\}\left(\mathcal{A}_2^+ + \mathcal{A}_2^-\right)\Big]$$

where Λ is the UV cut-off, introduced to regularize the diviergent integrals. In the medium part $\mathcal{A}_i^{\pm}$'s carries the temperature dependence.