PT phase transition in a non Hermitian gauge theory

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★ **Broad areas of the work** :

♦ Non Hermitian Parity Time reversal (PT) symmetric Quantum Physics (Extension of usual QP)

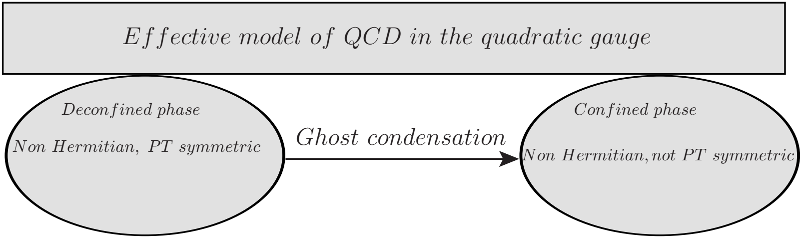
♦ Quantum Chromodynamics [QCD] (Gauge Theory of strong nuclear force)

★ **Unique Selling Propositions (USPs) of the work** :

♦ First example of the gauge theory exhibiting PT phase transition

♦ Transition between two QCD phases (Deconfined and Confined) being identified as PT phase transition

★ **Broad approach to achieving the stated objectives** :



Outlines

1. Toy model of non Hermitian complex scalars
2. Review-SU(N) QCD in the new Quadratic gauge
3. PT phase transition in the non Hermitian gauge theory
4. C-symmetry and its explicit representation in the non Abelian model
5. Concluding discussions

- The model is a mathematical preliminary for the later non Abelian non Hermitian model.
- This model is described by the following Lagrangian

$$L = \partial_\mu \phi_1^* \partial_\mu \phi_1 + \partial_\mu \phi_2^* \partial_\mu \phi_2 + [\phi_1^* \quad \phi_2^*] M^2 \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \tag{1}$$

where

$$M^2 = \begin{bmatrix} m_1^2 & \mu^2 \\ -\mu^2 & m_2^2 \end{bmatrix} \tag{2}$$

with $m_1^2, m_2^2, \mu^2 \geq 0$. We see that $(M^2)^\dagger \neq M^2$.
- Defining the doublet of two fields as

$$\Phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}. \tag{3}$$

- The parity (P) and time reversal (T) respectively are defined on the doublet as

$$\Phi \xrightarrow{\mathrm{P}} P\Phi \tag{4}$$

$$\Phi \xrightarrow{\mathrm{T}} T\Phi^*, \tag{5}$$

where complex conjugation in time reversal is due to anti-linearity.

- The parity in $\mathbb{R}^2$ suggests that

$$P = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{6}$$

- The only choice for the T for which theory is PT-invariant is $\mathrm{T} = \mathbf{1}_2$ [1].
- The eigenvalues of the mass matrix M^2 are

$$M_\pm^2 = \frac{1}{2}(m_1^2 + m_2^2) \pm \sqrt{(m_1^2 - m_2^2)^2 - 4\mu^4}. \tag{7}$$

- So, for $|m_1^2 - m_2^2| \geq 2\mu^2 \Rightarrow$ phase of unbroken PT symmetry as $M_\pm^2$ is real.

[1] Jean Alexandre, Peter Millington, Dries Seynaeve, Phys. Rev. D 96, 065027 (2017)

- When $|m_1^2 - m_2^2| < 2\mu^2 \Rightarrow$ the region of broken PT-symmetry as $M_\pm^2$ turn complex and,
- PT$\psi_\pm \neq \pm\psi_\pm$, where $\psi_\pm$ are eigenfunctions of the M^2 belonging to $M_\pm^2$.
- The charge conjugation of this system is defined as follows

$$\Phi \quad \xrightarrow{\text{C}} \quad C\Phi^*, \tag{8}$$

with C=P [1].

- The theory is CPT invariant in both PT broken and unbroken phases. In broken PT phase, the theory violates CP also but preserves CT symmetry.

★ **The quadratic gauge** : $A^a_\mu(x)A^{\mu a}(x) = f^a(x), \quad \text{for each } a;$ [2] (9)

where $f^a(x)$ is an arbitrary function of x.

- USP: (1) This gauge is Lorentz invariant. In general, algebraic gauges are not. (2) It is not Abelian Projection. (3) It has substantial non-perturbative implications such as color confinement signatures and absence of the Gribov ambiguity in this gauge.

★ **Effective theory in the quadratic gauge** :

$$\mathcal{L}_{eff} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{2\xi}(A^a_\mu A^{\mu a})^2 - \overline{c^a}A^{\mu a}(D_\mu c)^a, \tag{10}$$

where second, third terms are gauge fixing and ghost Lagrangians respectively and $(D_\mu c)^a = \partial_\mu c^a - gf^{abc}A^b_\mu c^c$.

[2] H. Raval and U. A. Yajnik, Phys. Rev. D 91, no. 8, 085028 (2015)

★ Two phases of QCD in the quadratic gauge

- This theory has two different phases [2]: Deconfined phase and the ghost condensed phase showing the confinement. The deconfined phase is given by Eq. (10) itself.
- **Ghost condensation**
  The ghost Lagrangian contains a term $gf^{abc}\overline{c^a}c^c A^{\mu a}A^b_\mu$ where ghost bilinears multiply the terms quadratic in gauge fields. Hence if the ghosts freeze, we have the mass matrix for gluons as follows

$$(M^2)^{ab}_{\rm dyn} = 2g \sum_{c=1}^{N^2-1} f^{abc}\langle\overline{c^a}c^c\rangle, \qquad (11)$$

  whereas diagonal components of $M^2_{\rm dyn}$ are zero since $f^{aac} = 0$.
- To obtain masses of gluons, we find eigenvalues of the matrix.

- In the state, where all ghost-anti-ghost condensates are identical i.e.,

$$\langle \overline{c^1} c^1 \rangle = ... = \langle \overline{c^1} c^{N^2-1} \rangle = ... = \langle \overline{c^{N^2-1}} c^1 \rangle = ... = \langle \overline{c^{N^2-1}} c^{N^2-1} \rangle \equiv K \tag{12}$$

the mass matrix becomes

$$(M^2)^{ab}_{\text{dyn}} = 2gK \sum_{c=1}^{N^2-1} f^{abc}. \tag{13}$$

- The mass matrix in Eq. (13) has $N(N-1)$ non-zero eigenvalues only and thus has nullity $N-1$. Because of the antisymmetry, eigenvalues occur are purely imaginary and in conjugate pairs.
- Thus, the $N(N-1)$ off-diagonal gluons acquire masses and the rest $N-1$ diagonal gluons remain massless.
- Pole of the propagator for the off-diagonal gluon is on imaginary p^2 axis, a signals the quark confinement [3].

[3] C. D. Roberts, A. G. Williams and G. Krein, Int. J. Mod. Phys. A 07, 5607 (1992)

- Massive off-diagonal gluons are inferred as evidence of Abelian dominance, which hints at quark confinement.

★ **The ghost condensation leads to confined phase whose effective theory is**

$$\mathcal{L}_{GC} = - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{2\zeta}(A^a_\mu A^{\mu a})^2 + M^2_a A^a_\mu A^{\mu a}. \quad (14)$$

- Hence for $SU(3)$, the last term of the Lagrangian in Eq. (14) would be

$$\begin{aligned} M^2_a A^a_\mu A^{\mu a} = &+ im^2_1 A^1_\mu A^{\mu 1} - im^2_1 A^2_\mu A^{\mu 2} + im^2_2 A^4_\mu A^{\mu 4} - im^2_2 A^5_\mu A^{\mu 5} \\ &+ im^2_3 A^6_\mu A^{\mu 6} - im^2_3 A^7_\mu A^{\mu 7}. \end{aligned} \quad (15)$$

where m^2_1, m^2_2, m^2_3 are positive real.

- So the gluons 1 and 2 can be considered as conjugate of each other. The same is for other pairs.

★ Both the phases are non Hermitian...

- Gluons must be Hermitian i.e.,

$$A_\mu^{a\dagger} = A_\mu^a. \tag{16}$$

- As the operation of conjugation in principle transforms particle to its anti particle, the natural choice of hermiticity property for ghosts is [4]

$$\begin{aligned} c^{a\dagger} &= \overline{c^a} \\ \overline{c^a}^\dagger &= c^a. \end{aligned} \tag{17}$$

- The effective theory in the deconfined phase is given in Eq. (10)

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} - \frac{1}{2\zeta}(A_\mu^a A^{\mu a})^2 - \overline{c^a} A^{\mu a}(D_\mu c)^a. \tag{18}$$

[4] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. 66, 1 (1979)

- Under Eqs. (16),(17), this phase is not Hermitian since

$$(\overline{c^a}\partial_\mu c^a)^\dagger = (\partial_\mu \overline{c^a})c^a \neq \overline{c^a}\partial_\mu c^a.$$

  Rest of the terms are Hermitian.
- The ghost condensed (confined) phase in is also not Hermitian as the mass term for gluons is purely imaginary as explained.
- Important point : non hermiticity of the theory in this confined phase is free of the hermiticity convention for ghosts and thus the non hermiticity of this phase is profound.
- thus, both the deconfined and confined phases are non Hermitian, later being profoundly.

★ Deconfined phase is PT symmetric, Confined phase is not...

- The parity and time reversal properties of the gluons are well defined but not for ghosts.
- **Parity (P)** : For gluons, parity is given as

$$
\begin{aligned}
A_i^a(\mathrm{x},t) &\xrightarrow{\mathrm{P}} -A_i^a(-\mathrm{x},t) \\
A_0^a(\mathrm{x},t) &\xrightarrow{\mathrm{P}} A_0^a(-\mathrm{x},t).
\end{aligned} \tag{19}
$$

  The rule for parity is same for all gluons as it is a linear operator.
- Theory in the deconfined phase (10) is invariant under parity if ghosts are pseudo scalars,

$$
\begin{aligned}
c^a(\mathrm{x},t) &\xrightarrow{\mathrm{P}} -c^a(-\mathrm{x},t) \\
\overline{c^a}(\mathrm{x},t) &\xrightarrow{\mathrm{P}} -\overline{c^a}(-\mathrm{x},t).
\end{aligned} \tag{20}
$$

- **Time reversal (T)** : it is an anti-linear operation. As some of the generators of $SU(N)$ are imaginary, the T is not same for all gluons.
- We explain it for $SU(3)$ group, generalization to $SU(N)$ is obvious.

- In $SU(3)$, three generators namely, 2nd, 5th and 7th are purely imaginary. Therefore, time reversal for gluons is given by

$$\begin{aligned} A_i^p(\mathrm{x},t) &\xrightarrow{\mathrm{T}} -A_i^p(\mathrm{x},-t) \\ A_0^p(\mathrm{x},t) &\xrightarrow{\mathrm{T}} A_0^p(\mathrm{x},-t), \end{aligned} \tag{21}$$

where index p is $1,3,4,6,8$ and,

$$\begin{aligned} A_i^q(\mathrm{x},t) &\xrightarrow{\mathrm{T}} A_i^q(\mathrm{x},-t) \\ A_0^q(\mathrm{x},t) &\xrightarrow{\mathrm{T}} -A_0^q(\mathrm{x},-t), \end{aligned} \tag{22}$$

where index q is $2,5,7$.

- Therefore, the field strength can utmost change up to overall negative sign i.e.,

$$F^a_{\mu\nu} \xrightarrow{\mathrm{T}} \pm F^a_{\mu\nu}. \tag{23}$$

- Thus, the deconfined phase (10) is time reversal invariant provided the T for ghosts is defined as follows,

$$
\begin{aligned}
c^p(\mathrm{x},t) &\xrightarrow{\mathrm{T}} ic^p(\mathrm{x},-t) \\
\overline{c^p}(\mathrm{x},t) &\xrightarrow{\mathrm{T}} i\overline{c^p}(\mathrm{x},-t)
\end{aligned} \tag{24}
$$

and,

$$
\begin{aligned}
c^q(\mathrm{x},t) &\xrightarrow{\mathrm{T}} c^q(\mathrm{x},-t) \\
\overline{c^q}(\mathrm{x},t) &\xrightarrow{\mathrm{T}} \overline{c^q}(\mathrm{x},-t),
\end{aligned} \tag{25}
$$

where the description of indices p and q are as above.

- Anti-linearity makes two sets of ghosts transform in a different manner.
- Thus, the theory in deconfined phase is individually both parity and time reversal invariant.
- This PT symmetry breaks down spontaneously in the confined phase as we explain now.
- The theory in the confined phase is given by Eq. (14) as

$$
\mathcal{L}_{GC} = - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{2\zeta}(A^a_\mu A^{\mu a})^2 + M_a^2 A^a_\mu A^{\mu a}.
$$

- Parity (Eq.(19)) is still a symmetry of this phase. The time reversal is broken due to mass term,

$$\begin{aligned} M_a^2 A_\mu^a A^{\mu a} = & + \quad im_1^2 A_\mu^1 A^{\mu 1} - im_1^2 A_\mu^2 A^{\mu 2} + im_2^2 A_\mu^4 A^{\mu 4} - im_2^2 A_\mu^5 A^{\mu 5} \\ & + \quad im_3^2 A_\mu^6 A^{\mu 6} - im_3^2 A_\mu^7 A^{\mu 7} \quad \xrightarrow{\text{T}} \\ & - \quad im_1^2 A_\mu^1 A^{\mu 1} + im_1^2 A_\mu^2 A^{\mu 2} - im_2^2 A_\mu^4 A^{\mu 4} + im_2^2 A_\mu^5 A^{\mu 5} \\ & - \quad im_3^2 A_\mu^6 A^{\mu 6} + im_3^2 A_\mu^7 A^{\mu 7} \\ & = \quad -M_a^2 A_\mu^a A^{\mu a}, \end{aligned} \tag{26}$$

  The first two terms of $\mathcal{L}_{GC}$ remain unaffected by the time-reversal.
- The anti symmetric nature of structure constant has led to this breaking.
- Important point : the PT symmetry violation in the confined phase is profound as it is independent of the convention for ghosts.

- Thus, we have presented a non Hermitian gauge theory in which PT phase transition is explicitly shown for the first time.
- The crucial difference : In complex scalar theory, the value of parameter $\eta \equiv \frac{2\mu^2}{|m_1^2 - m_2^2|}$ separates two phases of the PT symmetry in the theory.
- There is no such single order parameter in the non Abelian theory.
- Different ghost bilinears $\overline{c^a} c^c$ gradually condensing to the same value give rise to the PT phase transition in this model.

- The C-symmetry is inherent in all PT symmetric non Hermitian systems hence it acts as a consistency check for the theory.
- So far, C-symmetry is not known in gauge theories.
- This symmetry in QM must satisfy the following

$$[H, C]\psi = 0, \ [PT, C]\psi = 0, \ C^2 = \mathbf{1}. \tag{27}$$

- The inner automorphism (IA) provides the representation of this C-symmetry in this model as it satisfies QFT analogue of above three conditions. It is given by :

$$\begin{array}{llll} \mathfrak{T}L_1\mathfrak{T}^\dagger = L_2 & \mathfrak{T}L_4\mathfrak{T}^\dagger = L_5 & \mathfrak{T}L_6\mathfrak{T}^\dagger = L_7 & \mathfrak{T}L_3\mathfrak{T}^\dagger = L_8 \\ \mathfrak{T}L_2\mathfrak{T}^\dagger = L_1 & \mathfrak{T}L_5\mathfrak{T}^\dagger = L_4 & \mathfrak{T}L_7\mathfrak{T}^\dagger = L_6 & \mathfrak{T}L_8\mathfrak{T}^\dagger = L_3 \end{array} \tag{28}$$

with the property $\mathfrak{T}^2 = \mathfrak{T}^{\dagger 2} = 1$;
L_i are the individual terms such as $-\frac{1}{4}F^i_{\mu\nu}F^{\mu\nu i}$, $\frac{-1}{2\zeta}(A^i_\mu A^{\mu i})^2$, $im^2 A^i_\mu A^{\mu i}$.

- IA is equivalent to exchanging group indices between pairs i.e., $1 \leftrightarrow 2$, $4 \leftrightarrow 5$, $6 \leftrightarrow 7$, $3 \leftrightarrow 8$. In the adjoint representation it is given by

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

1. The deconfined phase contains sum over group index a. Hence, the Lagrangian is invariant under IA.
2. PT is a space-time symmetry and the IA is in the group space. Therefore, they commute.
3. The third of Eq. (27) has already been shown.

- The theory in both the phases is invariant under CPT.
- In the broken PT phase, the theory also violates CP symmetry but preserves the CT, in complete analogy with the scalar model described in sec. II.

Conclusions

- The first and novel example of non Hermitian gauge theory exhibiting PT phase transition.
- Transition between two QCD phases is identified as PT phase transition.
- C-symmetry and its explicit representation is identified. Hence, the present theory is consistent non Hermitian gauge theory.

Thank You for your attention...!