Nonlinear Effects in Singlet Quark Distribution predicted by GLR-MQ Evolution Equation

Mayuri Devee

University of Science and Technology Meghalaya

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  - Gluon-Gluon Interaction
  - Breakdown of linear DGLAP Equation
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Small-x physics

- The small-x region of structure functions for fixed Q^2 exhibits the high-energy nature of the total cross section with growing total center of mass energy squared s, since

$$s \simeq Q^2\left(\frac{1}{x} - 1\right)$$

- At very high energies, one can therefore access the region of smaller and smaller values of x.
- At small x ($\leq 10^{-2}$) the density of gluons and sea-quarks become very high.

- Study of gluon density at small x is the basic ingredient for the investigation of different high energy hadronic processes
  - ✓ Growth of total hadronic cross-sections at ultra high energy.
  - ✓ Growth of the mean transverse momentum of a parton inside the parton cascade.
  - ✓ Mini-jet production in high energy hadronic collisions.
  - ✓ Shadowing in DIS processes.

Breakdown of linear DGLAP equation

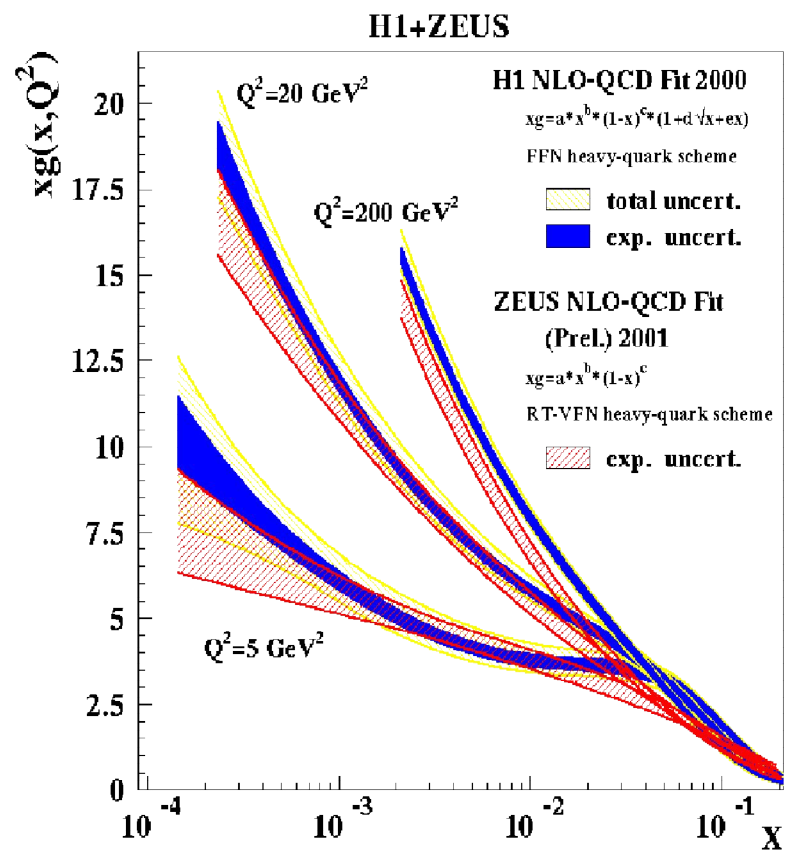
- The linear DGLAP dynamics consider the splitting processes in the partonic evolution, i.e. the processes :

$$q \rightarrow qg, \quad g \rightarrow q\bar{q} \quad g \rightarrow gg$$

- The DGLAP equations predict a steep rise of gluon densities in the small-x region ($<10^{-2}$).
- A flat gluon at small Q^2 becomes very steep after Q^2 evolution and F_2 becomes gluon dominated.

Eventually violates Froissart bound on physical cross sections.

Froissart bound : $\sigma_{tot} = \frac{\pi}{m_\pi^2}(\ln s)^2$



[F.D. Aaron et al., JHEP 01 (2010) 109]

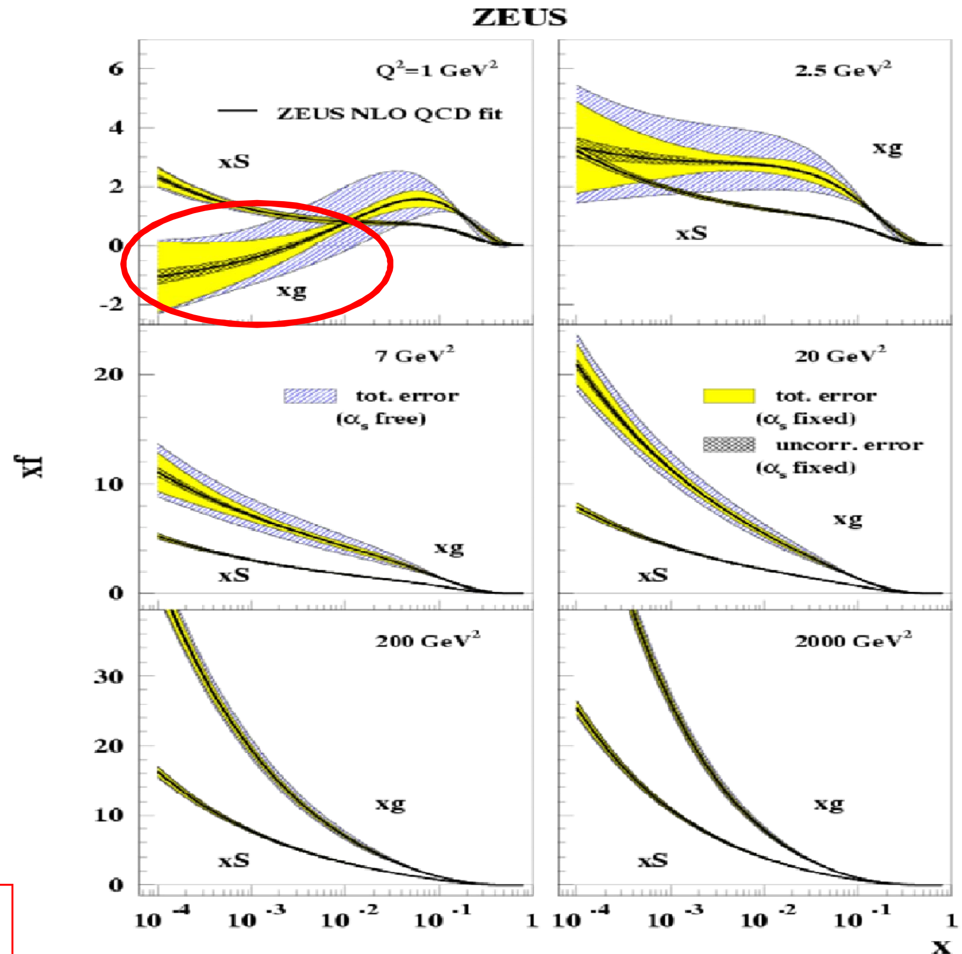
- DGLAP approach cannot provide a good description of the H1 data simultaneously in the region of large-Q^2 ($Q^2 > 4$ GeV^2) and in the region of small-Q^2 (1.5 GeV$^2 < Q^2 < 4$ GeV^2).

- NLO global fitting of MRST2001 and CTEQ6M based on leading twist DGLAP evolution leads to negative gluon distribution.

Does it mean that we have no gluons at $x < 10^{-3}$ and Q=1 GeV?

No!

Negative gluon distribution!



[Eskola et al., Nucl.Phys. B660 (2013)]

Search for a new evolution equation?

Gluon-Gluon interaction at small-x:

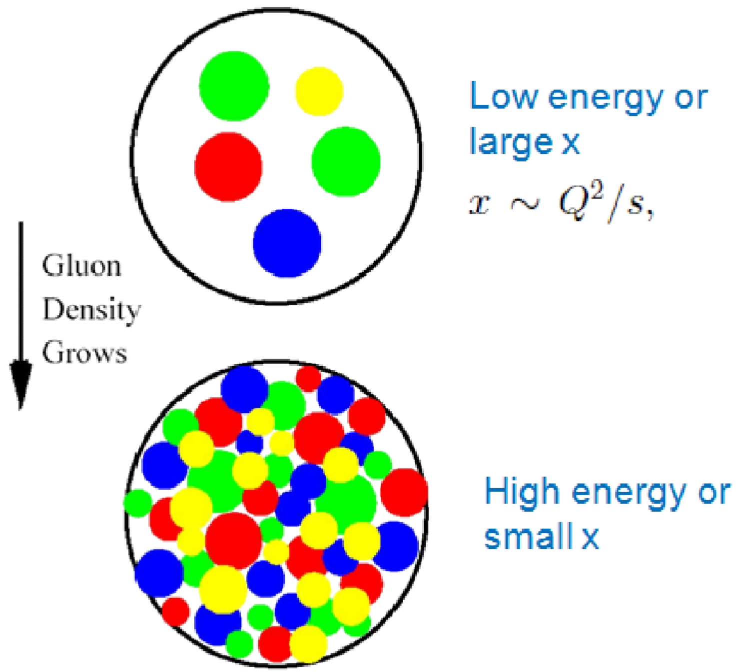
Gluon recombination is considered to be responsible for the taming of gluon density at small-x

✓At large gluon density the gluon start to spatially overlap.

✓The number of small-x gluons becomes limited by gluon recombination (gg → g or $gg \rightarrow q\bar{q}$.)

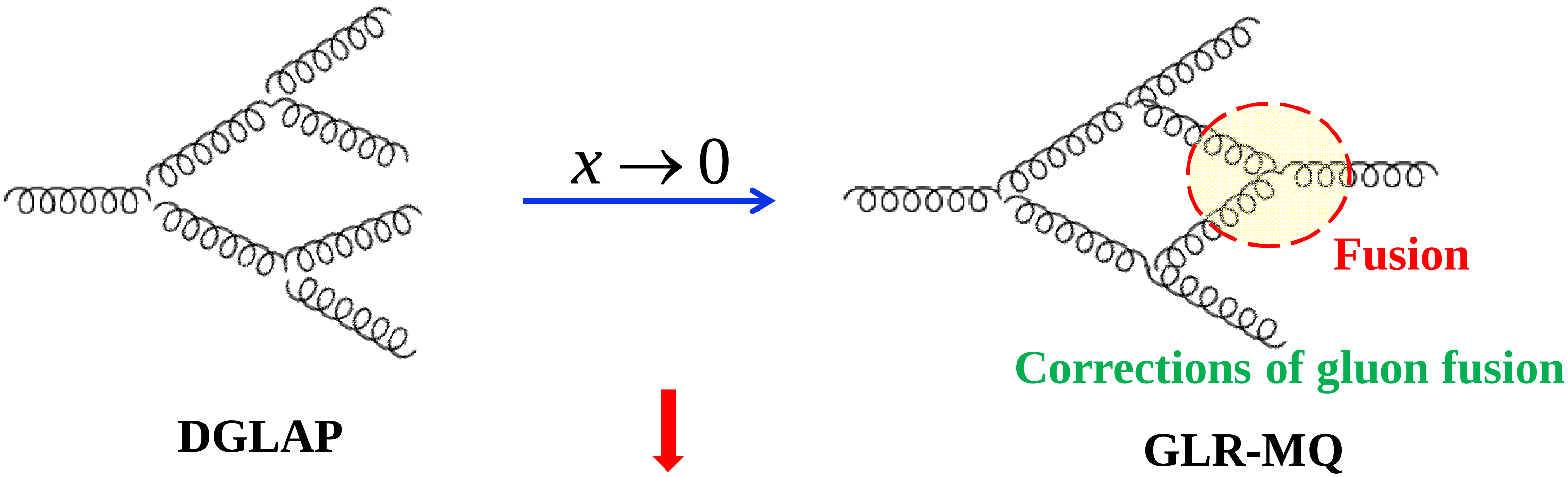
✓This eventually leads to saturation of gluon density.

✓ The corrections of the correlations among initial gluons to the elementary amplitude at small x should be considered.



Gribov-Levin-Ryskin and Mueller-Qiu Equation

- The first perturtative QCD calculations of the nonlinear effects of gluon recombination were studied by Gribov-Levin-Ryskin and Mueller-Qiu in a new equation, known as the **GLR-MQ equation**, with an additional term quadratic in gluon density .



GLR-MQ=DGLAP+gluon recombination

The GLR-MQ equation is based on:

✓The emission induced by the QCD vertex: $G \rightarrow G + G$

- emission (1→2) : probability $\propto \alpha_s \rho$

✓ The Recombination of a gluon by the same vertex: $G + G \rightarrow G$

- recombination (2→1) : $\propto \alpha_s^2 r^2 \rho^2 \propto \alpha_s^2 \frac{1}{Q^2} \rho^2$

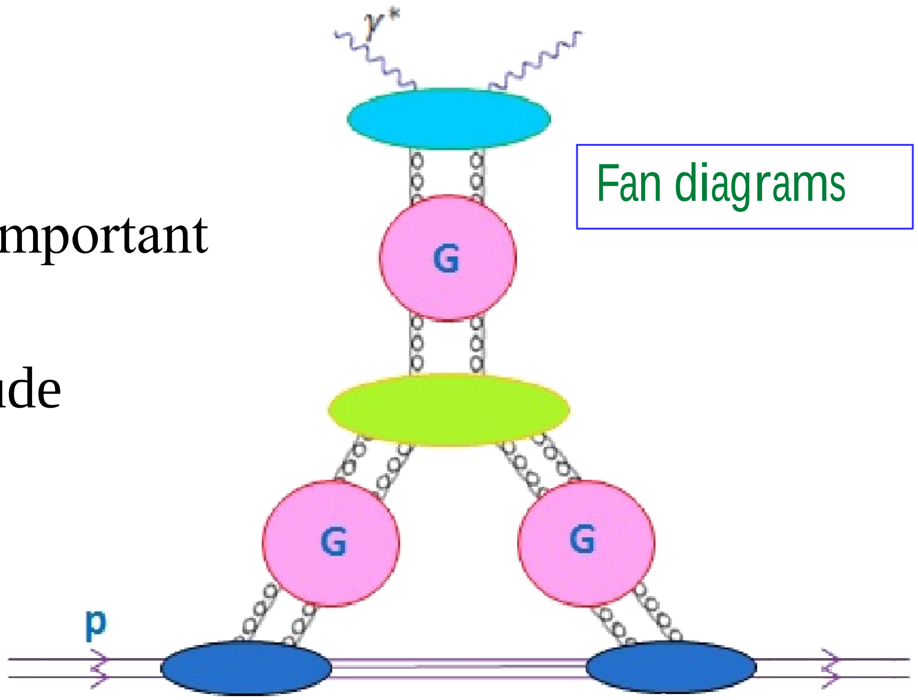
$$\rho \approx \frac{xg(x,Q^2)}{\pi R^2}$$ ⟶ number of gluons per unit area

In DIS: r ~ 1/Q

R ⟶ correlation radius

- At x~1 : emission is essential
- At x →0 : annihilation becomes important

❖The GLR-MQ equations include the fan diagrams ↴ play the key role in the restoration of unitarity



Fan diagrams

The GLR-MQ equation:

$$\frac{\partial^2 \rho}{\partial \ln(1/x)\,\partial \ln Q^2} = \frac{\alpha_S N_C}{\pi}\rho - \frac{\alpha_s^2(Q^2)\gamma}{Q^2}\rho^2$$

Nonlinear correction has "–" sign
evolution is ***slower***

$\gamma = \frac{81}{16}$, for $N_C = 3$.

✓The negative sign in front of the nonlinear term is responsible for the gluon recombination.

✓At small-x the strong growth generated by the linear term is damped whenever gluon or sea quark density becomes large.

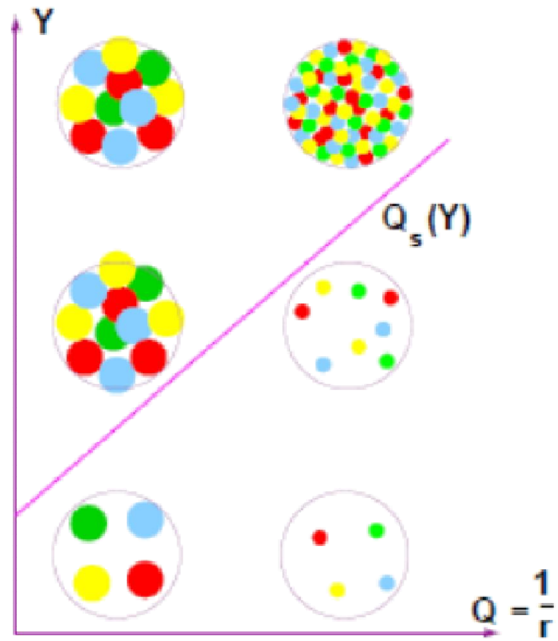
[Gribov et al., *Phys. Rep.*100 (1983) 1),
Mueller et al., *Nucl. Phys. B*268 (1986) 427]

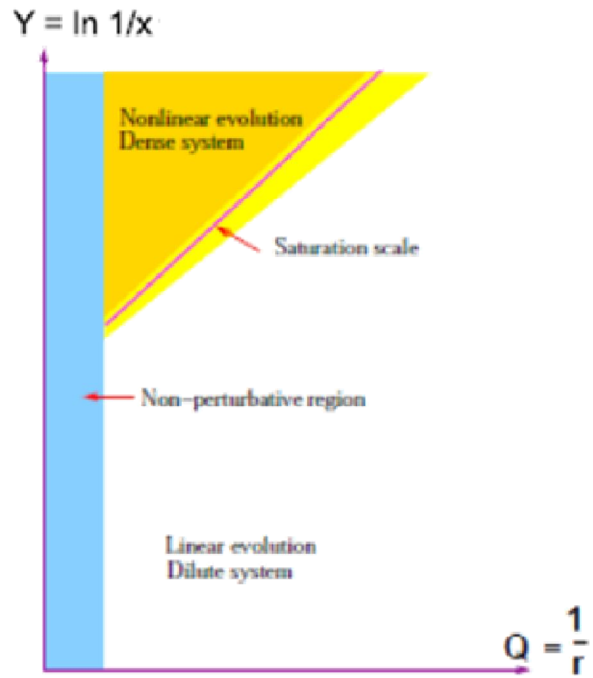
Features of GLR-MQ equation

- It predicts saturation of gluon density at very small-x ($x \longrightarrow 0$).
- It predicts a critical line separating the perturbative regime from the saturation regime.
- It is only valid in the border of this critical line.

At $Q^2 < Q_S^2$ GLR-MQ terms dominates the evolution

All non-linear terms become important





Larger the size of the parton (r ~1/Q), earlier gluons gets saturated!

GLR –MQ equation for singlet structure function

The GLR –MQ evolution equation for sea quark distribution function is

$$\frac{\partial xq(x,Q^2)}{\partial \ln Q^2} = \frac{\partial xq(x,Q^2)}{\partial \ln Q^2}\Big|_{DGLAP} - \frac{27}{160}\frac{\alpha_S^2(Q^2)}{R^2Q^2}[xg(x,Q^2)]^2$$

- The non-linear correction term should not be larger than the linear term

- In QCD improved parton model :

$$F_2(x,Q^2) = \sum_i e_i^2 xq_i(x,Q^2)$$

- The F_2 structure functions measured in DIS can be written as:

$$F_2 = \frac{5}{18}F_2^S + \frac{3}{18}F_2^{NS}$$

↓ negligible at small-x.

The GLR –MQ equation for singlet structure function:

$$\frac{\partial F_2^S(x,Q^2)}{\partial \ln Q^2} = \frac{5}{18}\frac{\partial F_2^S(x,Q^2)}{\partial \ln Q^2}\Big|_{DGLAP} - \frac{5}{18}\frac{27}{160}\frac{\alpha_S^2(Q^2)}{R^2Q^2}G^2(x,Q^2)$$

$$\begin{aligned}\frac{\partial F_2^S(x,Q^2)}{\partial \ln Q^2}\Big|_{DGLAP} = {} & \frac{\alpha_s(Q^2)}{2\pi}\Big[\frac{2}{3}\Big(3+4\ln(1-x)\Big)F_2^S(x,Q^2) \\ & + \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega}\Big\{(1+\omega^2)F_2^S\Big(\frac{x}{\omega},Q^2\Big) - 2F_2^S(x,Q^2)\Big\} \\ & + N_F\int_x^1 \Big(\omega^2+(1-\omega)^2\Big)G\Big(\frac{x}{\omega},Q^2\Big)d\omega\Big]\end{aligned}$$

Size of the nonlinear term:

The non-linear term $\propto 1/R^2$

$\propto 1/Q^2$

R and Q^2 are crucial for the magnitude of the recombination effect

The value of R depends on :

how the gluons are distributed within the proton

or how the gluon ladders couple to each other.

$R = R_h \rightarrow$ SC is negligibly small

$R << R_h \rightarrow$ SC could be large

We assume $R = 2\ GeV^{-1}$ → Hot spot

$R = 5\ GeV^{-1}$

From Regge pole model at small-x

$$F_2 \propto x^{-\lambda}$$

with $\lambda > 0$ being a constant or depending on Q^2 or x.

$$F_2^S(x, Q^2) = H(Q^2)x^{-\lambda_S}$$

λ_S is the Regge intercept for singlet structure function.

Regge pole model of gluon distribution function

$$G(x, Q^2) = H(Q^2)\, x^{-\lambda_G}$$

Regge intercept for gluon distribution function

- Successfully describes the shadowing corrections to the gluon distribution function.

[**M.D** and J.K.S, ***Eur. Phys. J C*** **74** (2014) 2751]
[**M.D** and J.K.S, ***Nucl. Phys. B*** **885** (2014) 571]

- GLR-MQ equation for singlet structure function becomes:

$$\frac{\partial F_2^S(x,Q^2)}{\partial Q^2} = p_1(x)\frac{F_2^S(x,Q^2)}{\ln(Q^2/\Lambda^2)} - p_2(x)\frac{[F_2^S(x,Q^2)]^2}{Q^2\ln(Q^2/\Lambda^2)}$$

where,

$$p_1(x) = \frac{5}{9\beta_0}\Big[\frac{2}{3}\Big(3+4\ln(1-x)\Big)+\frac{4}{3}\int_x^1\frac{d\omega}{1-\omega}\Big(\{(1+\omega^2)\omega^{\lambda_s}-2\Big) + N_F\int_x^1\Big(\omega^2+(1-\omega)^2\Big)\omega^{\lambda_s}K\Big(\frac{x}{\omega}\Big)d\omega\Big]$$

$$p_2(x) = \frac{27}{36}\frac{\pi^2(K(x))^2}{\beta_0^2R^2}$$

The solution:

$$F_2^S(x,Q^2) = \frac{t^{p_1(x)}}{C+p_2(x)\int t^{p_1(x)-2}\exp(-t)dt}$$

C: a constant to be determined from initial boundary conditions

Physically plausible boundary conditions are:

- $F_2^S(x,Q^2) = F_2^S(x,Q_0^2)$ for some lower value $Q^2 = Q_0^2$
- $F_2^S(x,Q^2) = F_2^S(x_0,Q^2)$ at some high $x = x_0$

- **The Q^2 dependence of singlet structure function with shadowing**

$$F_2^S(x,Q^2) = \frac{t^{p_1(x)} F_2^S(x,t_0)}{t_0^{p_1(x)} + p_2(x)\left[\int t^{p_1(x)-2}\exp(-t)dt - \int t_0{}^{p_1(x)-2}\exp(-t_0)dt_0\right] F_2^S(x,t_0)}$$

- **The small-x dependence of singlet structure function with shadowing**

$$F_2^S(x,Q^2) = \frac{t^{p_1(x)} F_2^S(x_0,t)}{t^{p_1(x_0)} + \left[p_2(x)\int t^{p_1(x)-2}\exp(-t)dt - p_2(x_0)\int t^{p_1(x_0)-2}\exp(-t)dt\right] F_2^S(x_0,t)}$$

[M. Devee, ***arXiv:1808.00899v2***(2018)]

Region of validity:

- At large t ($t = \ln(Q^2/\Lambda^2)$) (t >>1):

$$F_2^S(x,Q^2) = \frac{t^{p_1(x)}}{C + p_2(x)\int t^{p_1(x)-2}\exp(-t)dt} \xrightarrow{t\gg 1} t^{p_1(x)}/C$$

- At very small-x (x<10^{-5}):

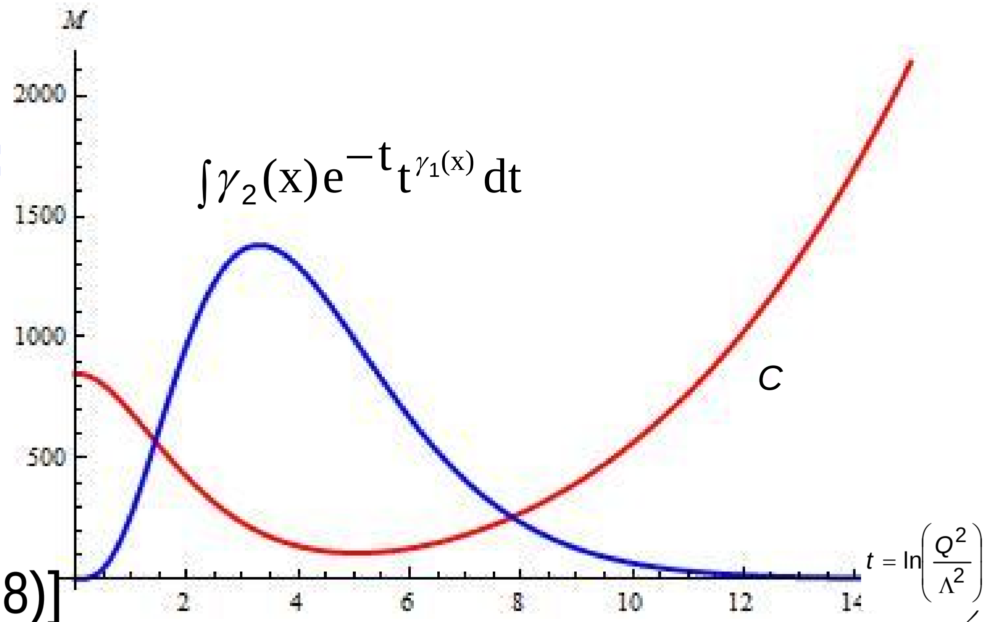
$$F_2^S(x,Q^2)_{x\to 0} = \frac{t^{p_{10}}}{C + p_{20}\int t^{p_{10}}\exp(-t)dt}$$

- At large-x (x ≈ 1):

$$F_2^S(x,Q^2) = \frac{t^{p_1(x)}}{C + p_2(x)\int t^{p_1(x)-2}\exp(-t)dt} \xrightarrow[x\approx 1]{t\gg 1} t^{p_1(x)}/C$$

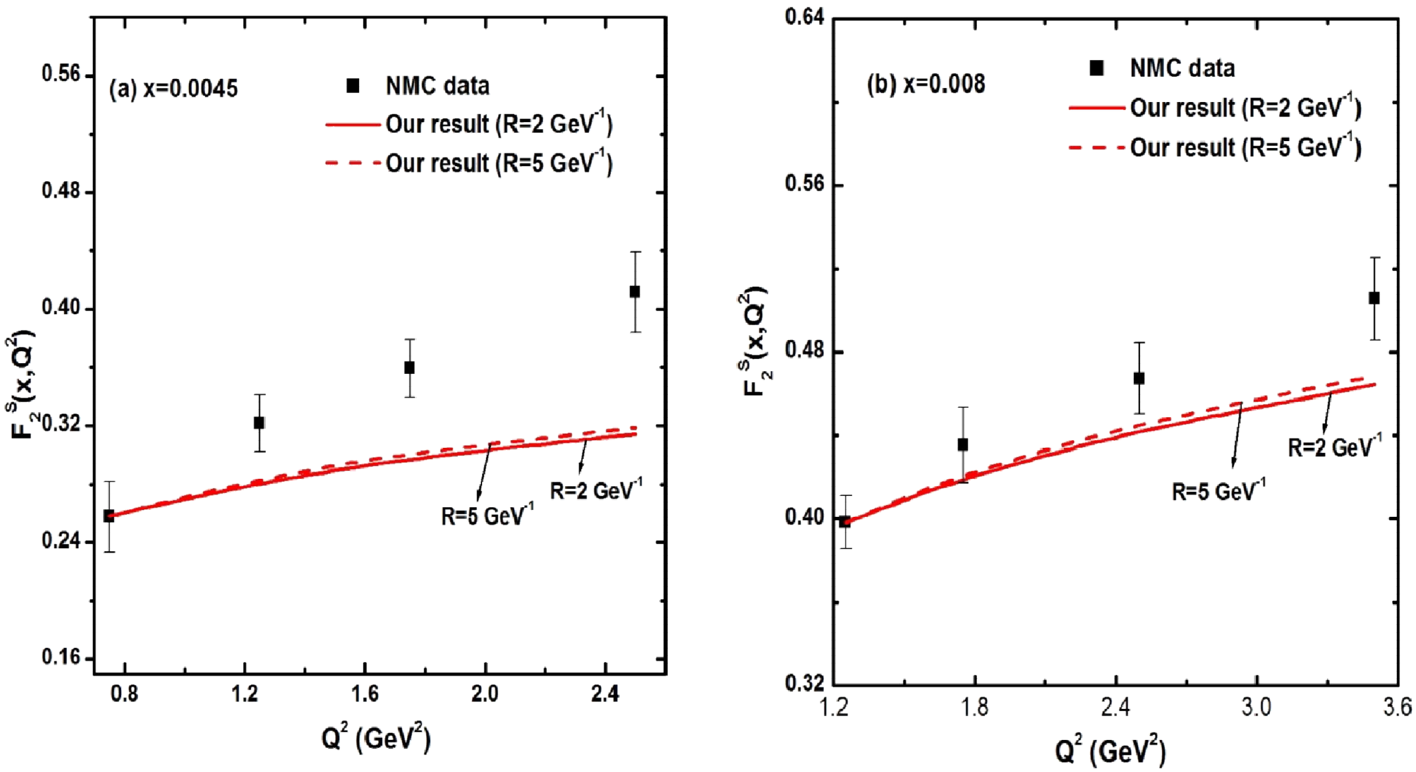
The solution is valid in the region:

$10^{-4} < x < 10^{-1}$ and
$0.6 < Q^2 < 30$ GeV^2



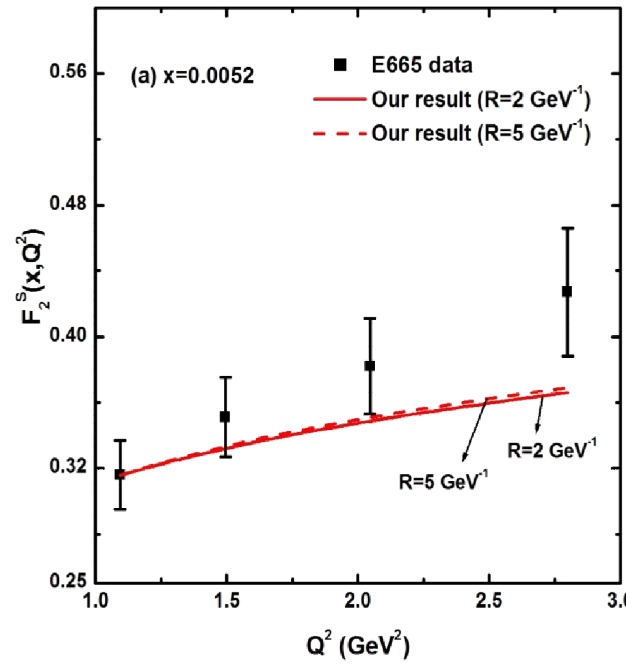
[M. Devee, ***arXiv:1808.00899v2***(2018)]

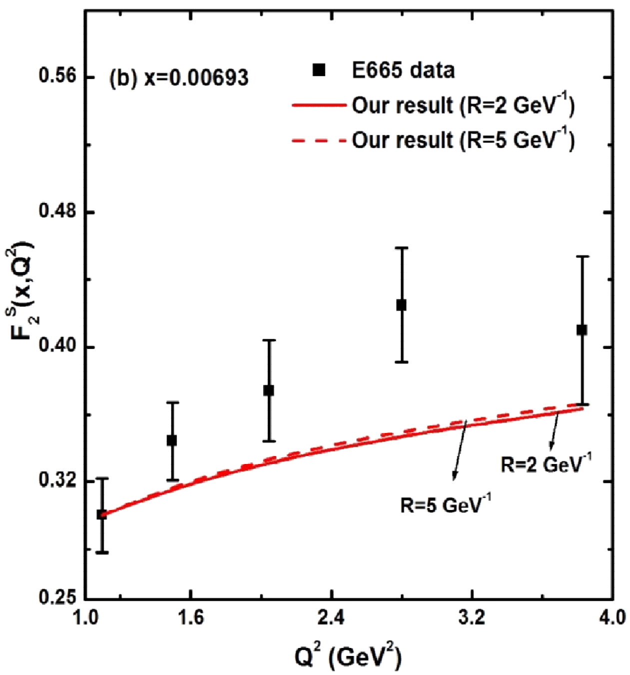
Results

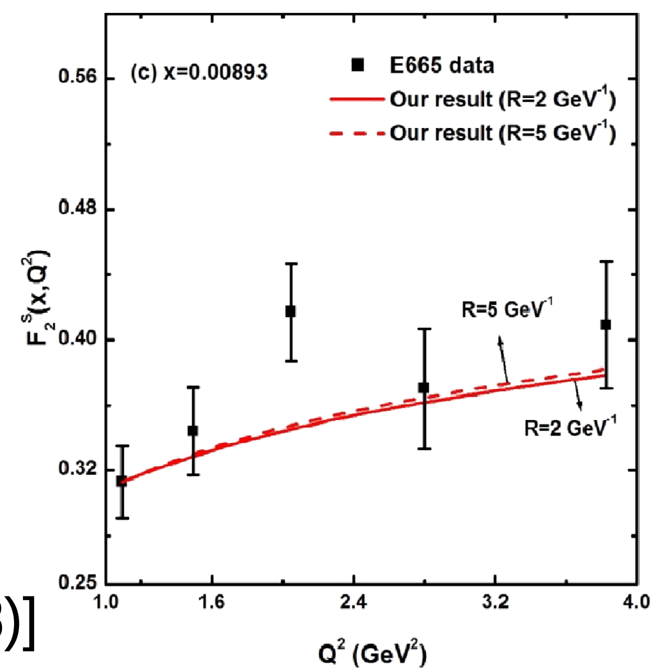


Q^2 dependence of singlet structure function with shadowing corrections for two fixed x compared to NMC data.

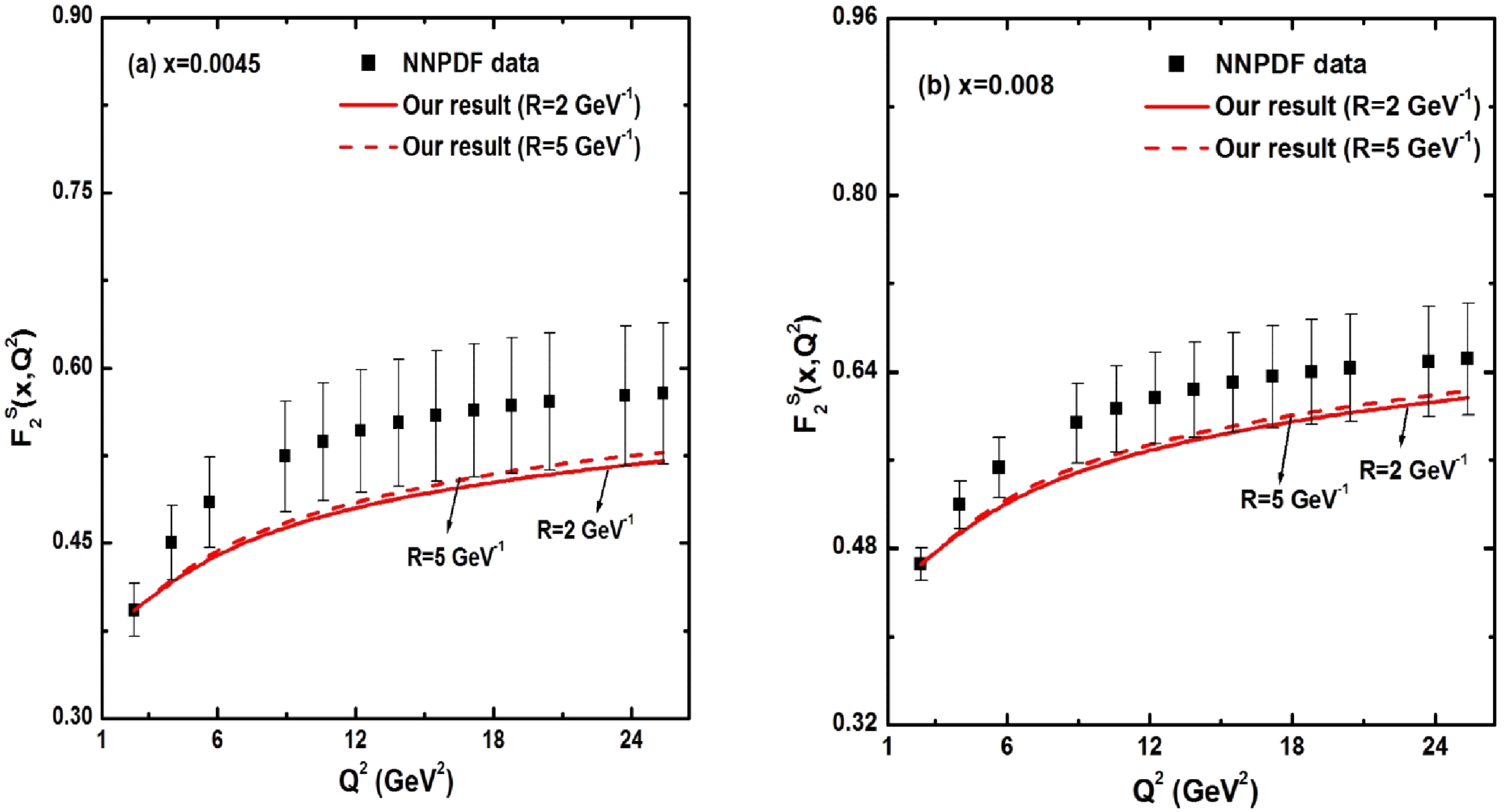
Q^2-dependence of singlet structure function with shadowing corrections for three fixed x compared to E665 data.





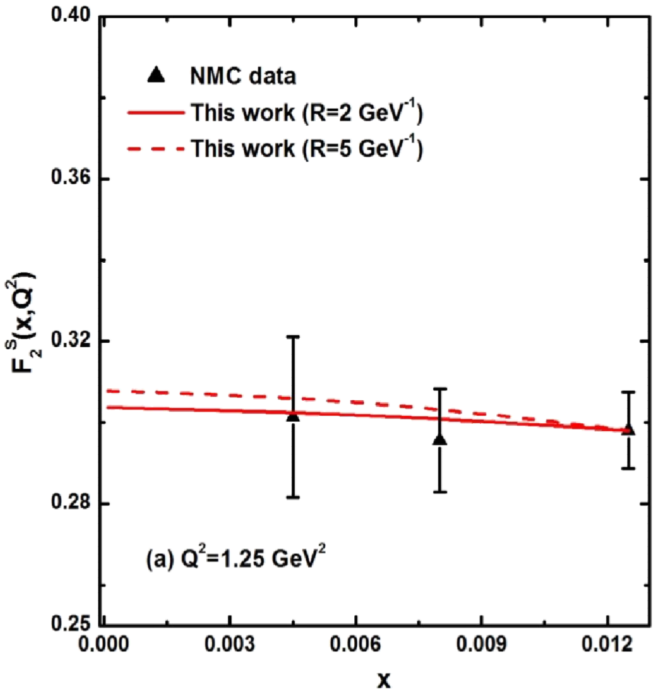


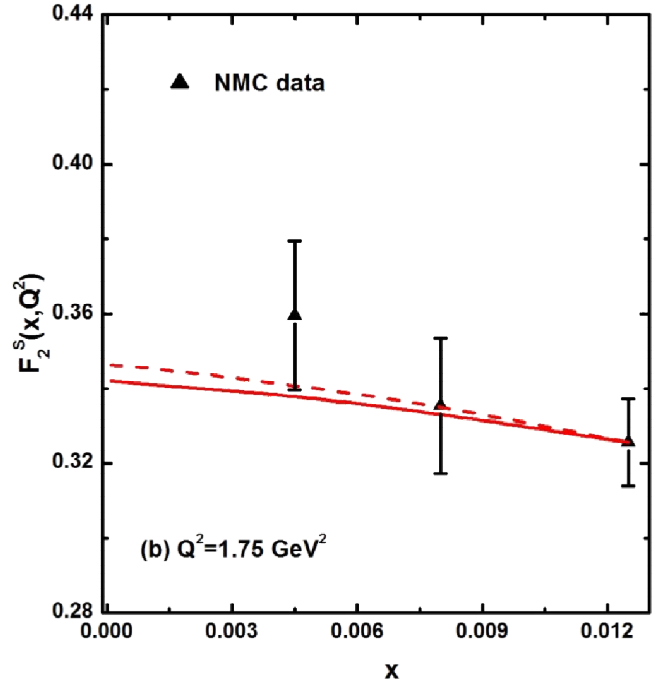
[M. Devee, ***arXiv:1808.00899v2***(2018)]



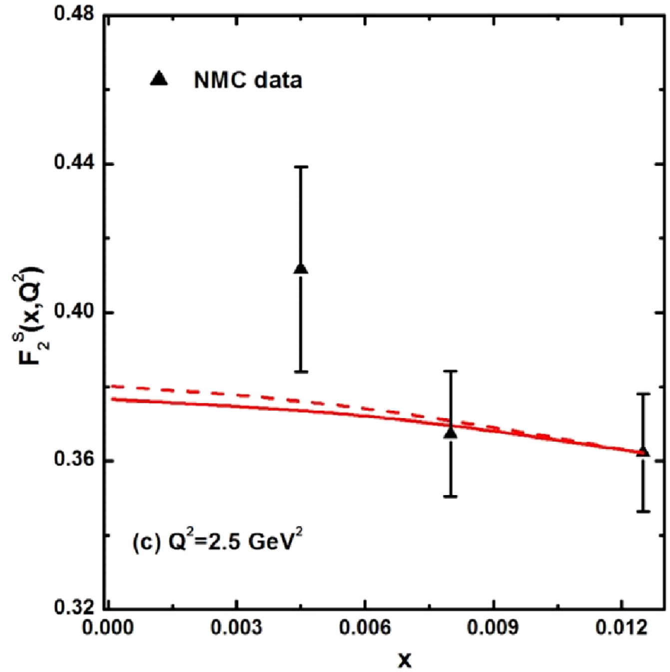
Q^2 dependence of singlet structure function with shadowing corrections for two fixed x compared to NNPDF data.

[M. Devee, ***arXiv:1808.00899v2***(2018)]

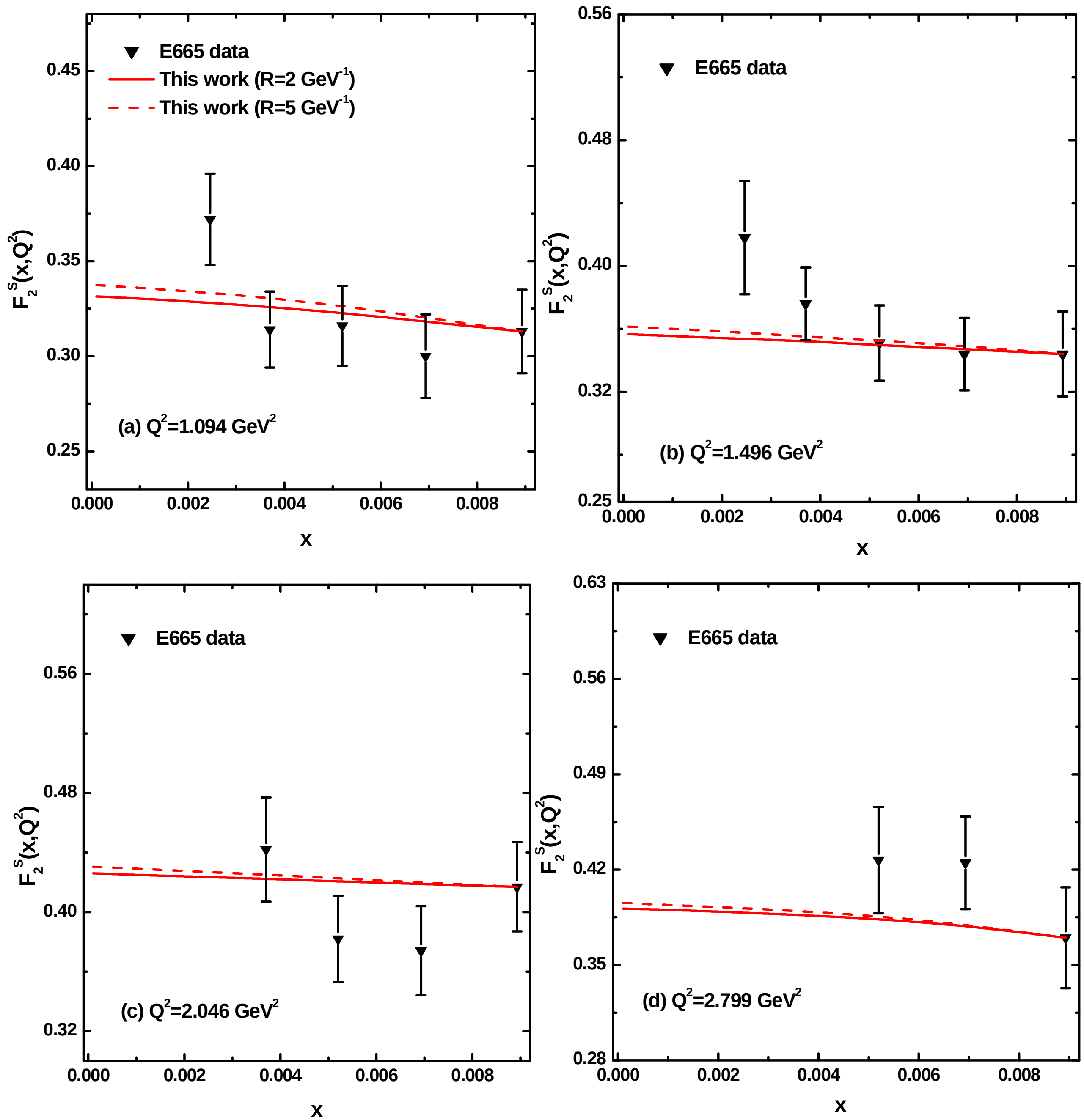




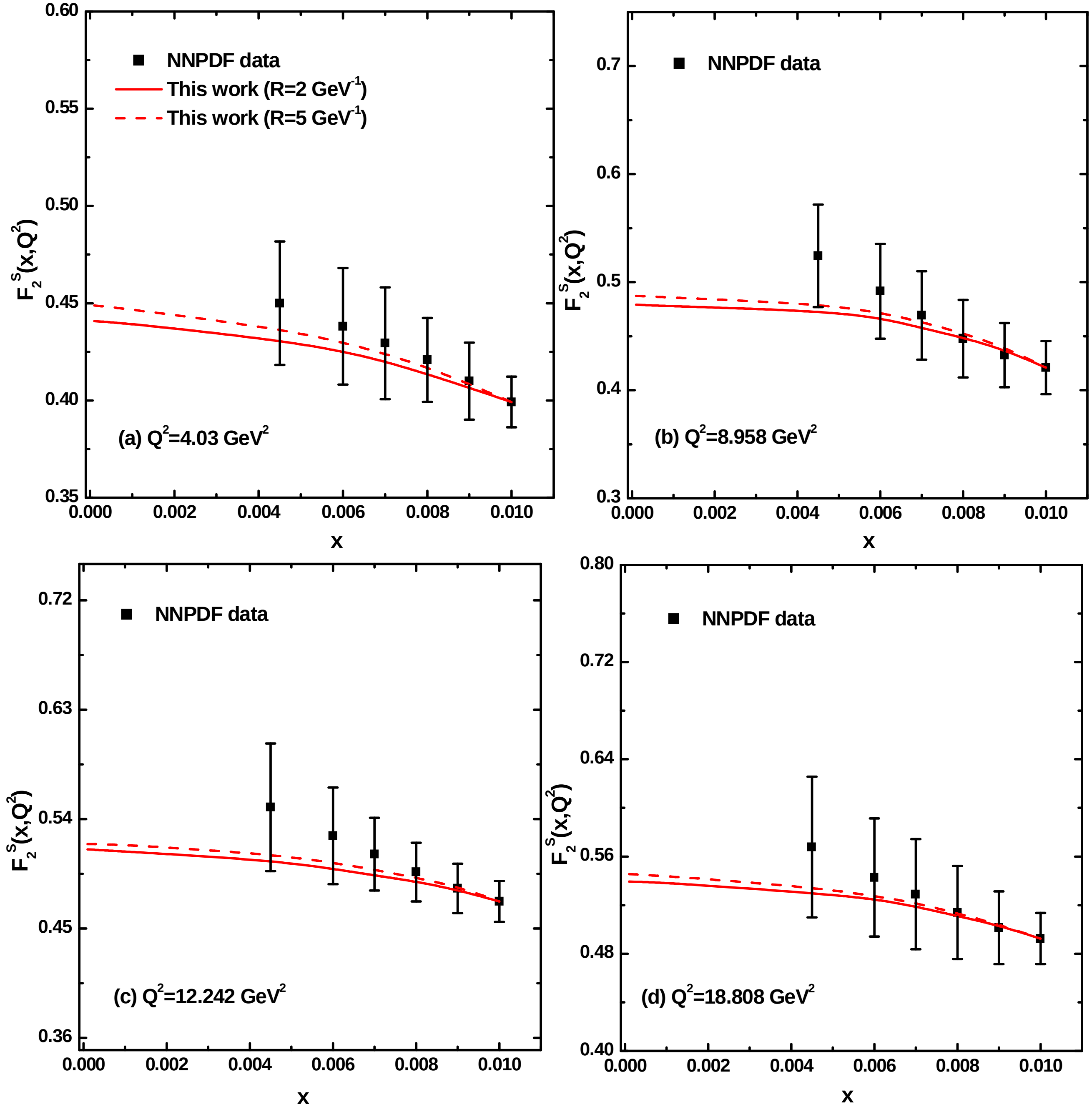
Small-x behavior of singlet structure function with shadowing corrections for three fixed Q^2 compared to NMC data



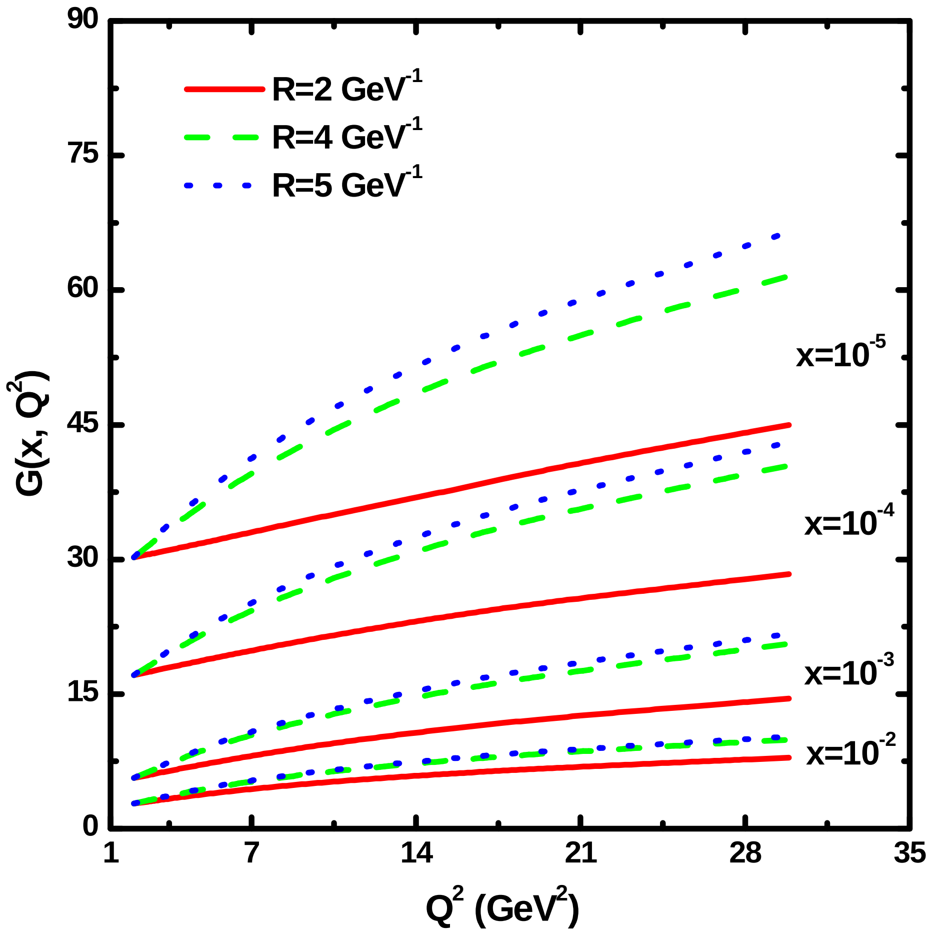
Small-x behavior of singlet structure function with shadowing corrections at four fixed Q^2 compared to E665 data



Small-x behavior of singlet structure function with shadowing corrections at four fixed Q^2 compared to NNPDF data



Sensitivity of the parameter R



- The singlet structure function is more tamed at $R = 2\ GeV^{-1}$, where gluons are supposed to be condensed in the hot-spots within the proton, compared to at $R = 4\ GeV^{-1}$ and $R = 5\ GeV^{-1}$ where gluons are almost scattered over the entire proton.
- The differences between the data as we approach from $R = 2\ GeV^{-1}$ to $R = 5\ GeV^{-1}$ increase with decreasing x.

Comparative analysis of GLR – MQ and DGLAP equations

Solution of the linear DGLAP equation with Regge ansatz of singlet structure function is $F_2^S(x,Q^2) = Dt^{p_1(x)}$

Q^2 behavior of without shadowing:

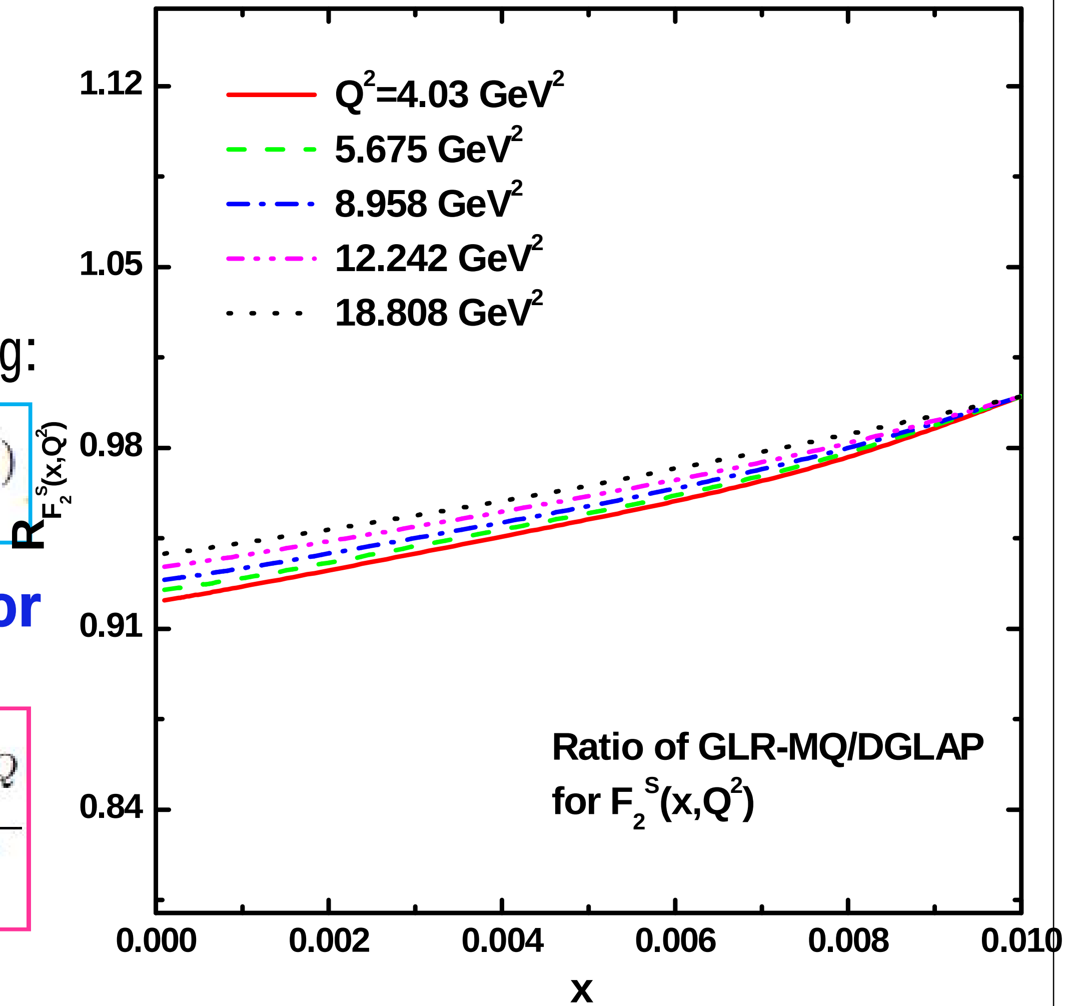
$$F_2^S(x,Q^2) = f_{10}\left(\frac{t}{t_0}\right)^{p_1(x)}$$

Small-x behavior of without shadowing:

$$F_2^S(x,Q^2) = f_{20}t^{p_1(x)-p_1(x_0)}$$

The ratio of GLR-MQ / DGLAP for singlet structure function

$$R_{F_2^S(x,Q^2)} = \frac{F_2^S(x,Q^2)^{GLR-MQ}}{F_2^S(x,Q^2)^{DGLAP}}$$



Summary

✓The effect of nonlinear or shadowing corrections to the evolution of sea quark distribution is examined on the small-x and moderate-Q^2.

✓The steep behavior of singlet structure function is slowed down towards small-x leading to a restoration of the Froissart bound.

✓It is very fascinating to observe signatures of gluon recombination in our predictions at very small-x ($10^{-4} \leq x \leq 10^{-2}$).

✓The obtained analytical results are in good agreement with different experimental data, global parametrizations as well as different models.

✓The kinematic region of validity of the semi analytical solution is

$10^{-4} < x < 10^{-1}$ and $0.6 < Q^2 < 30$ GeV^2.

The study provides an important insight into the effect of shadowing corrections in the small-x region.

Thank you

for your kind attention!