The QCD Equation of state at finite density from lattice

Sayantan Sharma

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Outline

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2. Equation of state at finite μ_B
3. Critical-end point search from lattice

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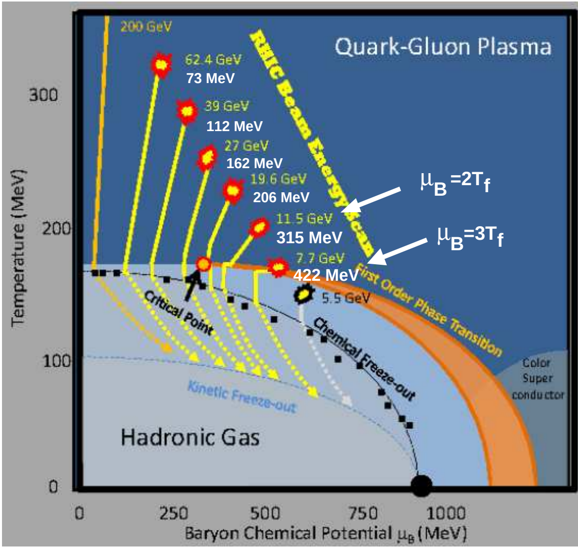

[http://www.bnl.gov/rhic/news]

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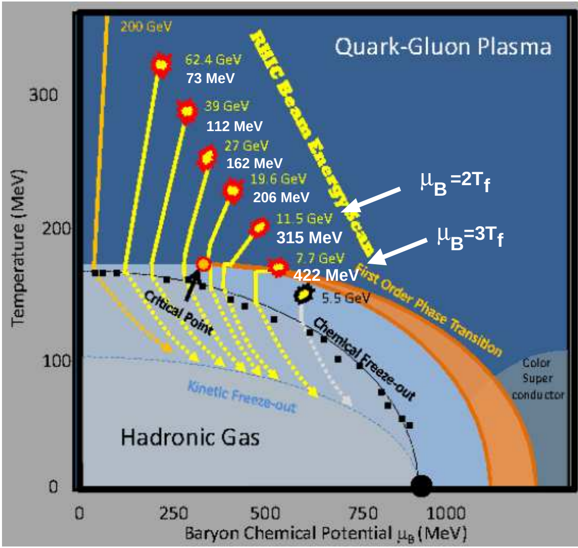

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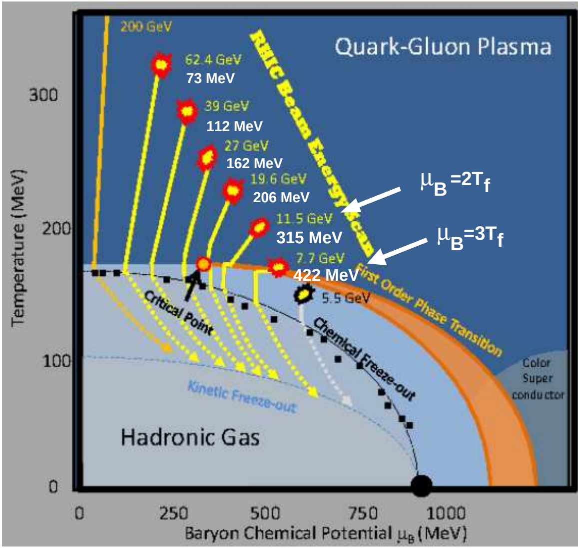

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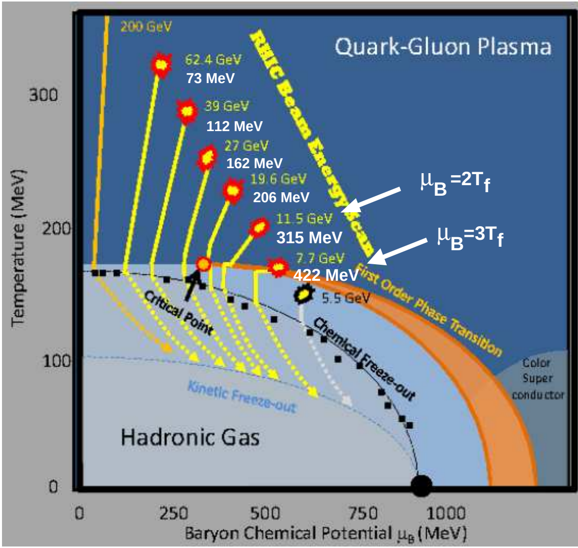
- In view of the RHIC Beam Energy Scan-II in 2019-20 it is important to have control over the Equation of State for $\mu_B/T \leq 3$.
- Measure the curvature of chiral and freezeout curves expected from QCD thermodynamics.
- Look for possible existence and bracket the position of critical end-point in the phase diagram.



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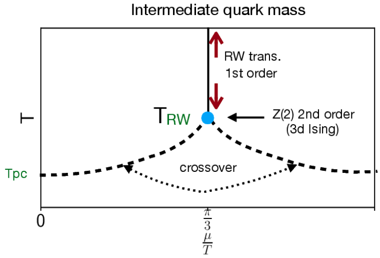

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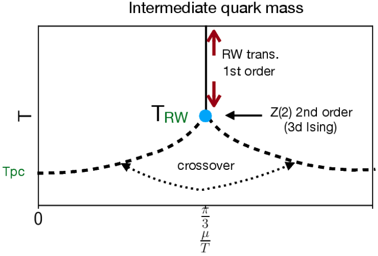

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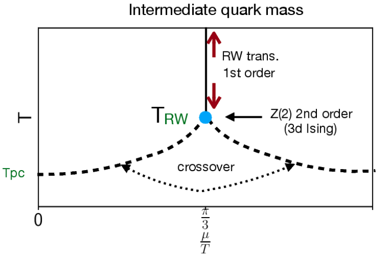

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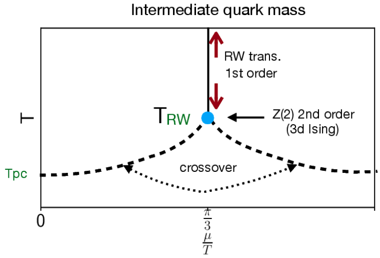

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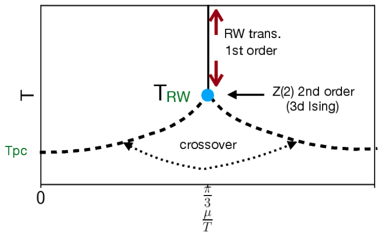
Intermediate quark mass



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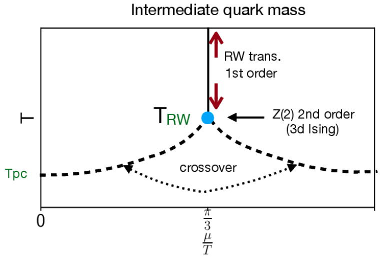

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- Limited due to discontinuities at Roberge-Weiss end-points!.

Lattice techniques at finite μ_B-II

- Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T [Bi-Swansea collaboration, 02]

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0,T)}{T^4} + \left(\frac{\mu_B}{T}\right)^2 \underbrace{\frac{\chi_2^B(0,T)}{2T^2}}_{P_2} + \left(\frac{\mu_B}{T}\right)^4 \underbrace{\frac{\chi_4^B(0)}{4!}}_{P_4} + \ldots$$

- The series for $\chi_2^B(\mu_B)$ should diverge at the critical point. On finite lattice χ_2^B peaks, ratios of Taylor coefficients equal, indep. of volume [Gavai& Gupta, 03]

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- Why extending to higher orders so difficult?
  - Matrix inversions increasing with the order
  - Delicate cancellation between a large number of terms for higher order QNS.

Recent developments: A new method to introduce μ

- The staggered fermion matrix used at finite μ [Hasenfratz, Karsch, 83]

$$
\begin{aligned}
D(\mu)_{xy} &= \sum_{i=1}^{3} \eta_i(x) \left[U_i^\dagger(y)\delta_{x,y+\hat{i}} - U_i(x)\delta_{x,y-\hat{i}} \right] \\
&+ \eta_4(x) \left[\mathrm{e}^{\mu \mathrm{a}} U_4^\dagger(y)\delta_{x,y+\hat{4}} - \mathrm{e}^{-\mu \mathrm{a}} U_4(x)\delta_{x,y-\hat{4}} \right]
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\end{aligned}
$$

- One can also add μ coupled to the conserved number density as in the continuum.

$$
D(0)_{xy} - \frac{\mu a}{2} \eta_4(x) \left[U_4^\dagger(y) \delta_{x,y+\hat{4}} + U_4(x) \delta_{x,y-\hat{4}} \right] .
$$

Pros and Cons

- Linear method: $D' = \sum_{x,y} N(x,y)$, and
  $D'' = D''' = D''''... = 0$
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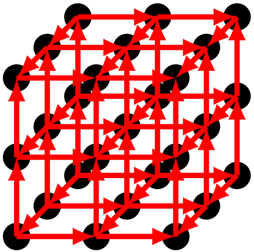
- In Exp method: counter terms already at the Lagrangian level. We use this method for χ_n^B, $n = 2, 4$.

Our Set-up



- $V = N^3 a^3$, Box size: $m_\pi \; V^{1/3} > 4$.
- $T = \frac{1}{N_\tau a}$
  We use $N_\tau = 6, 8, 12, 16$ lattices for $\chi_{2,4}$ and $N_\tau = 6, 8$ for higher order fluctuations.
- Input m_s physical and $m_\pi^G = 160$ MeV for $T > 175$ MeV and $m_\pi^G = 140$ MeV for $T <= 175$ MeV.

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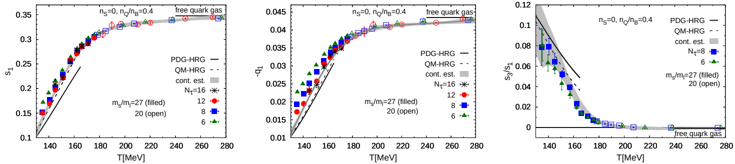
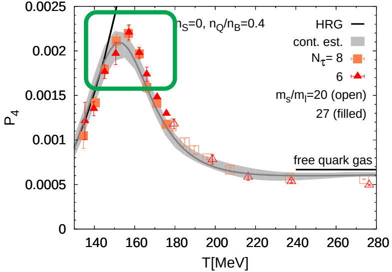
EoS in the constrained case

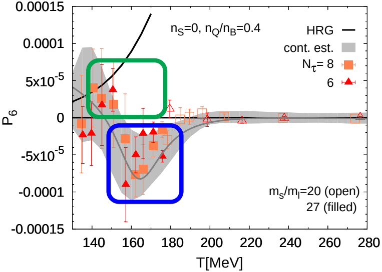
- In most central heavy-ion experiments typically:
  $n_S = 0$, Strangeness neutrality,
  $\frac{n_Q}{n_B} = \frac{n_P}{n_P+n_N} = 0.4$.
  [Bi-BNL collaboration, 1208.1220]
- For lower $\sqrt{s}$ collisions: Need to understand baryon stopping!
- Imposes non-trivial constraints on the variation of μ_S and μ_Q.
- Possible to vary them by only varying μ_B through

$$\mu_S = s_1 \mu_B + s_3 \mu_B^3 + s_5 \mu_B^5 + ...$$
$$\mu_Q = q_1 \mu_B + q_3 \mu_B^3 + q_5 \mu_B^5 + ...$$



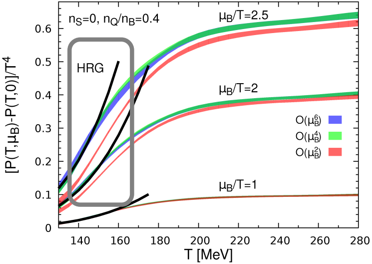
- Central values of P_4, P_6 already deviate from Hadron Resonance gas model at $T > 145$ MeV $\rightarrow$ need to analyze the errors on P_6 better.
- P_6 has characteristic structure at $T > T_c$ $\rightarrow$ remnant of the chiral symmetry due to the light quarks. Effects of $U_A(1)$ anomaly?
- Essentially non-perturbative $\rightarrow$ cannot be predicted within Hard Thermal Loop perturbation theory.

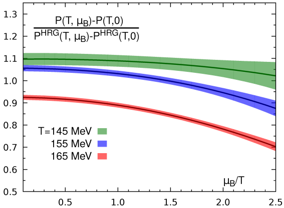




EoS in the constrained case

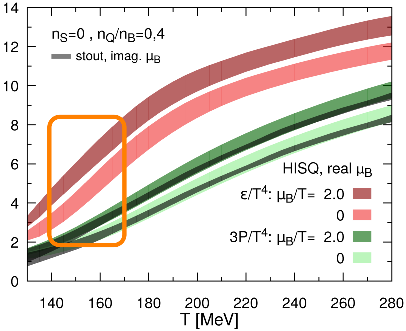
- The EoS is well under control for $\mu_B/T \sim 2.5$ with χ_6.
- Full parametric dependence for N_B on T available in arxiv: 1701.04325.
- Expanding to $\mu_B/T = 3$, need to calculate χ_8!



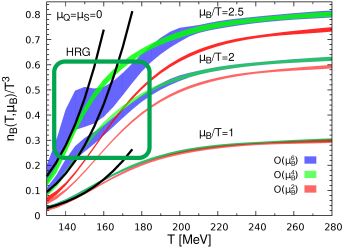


Summary for the EoS

- Continuum estimates from two different fermion discretization agree for $\mu_B/T \leq 2$.
  [Bielefeld-BNL-CCNU collaboration, 1701.04325, Borsanyi et. al, 1606.07494].
- Steeper EoS for RHIC energies compared to LHC energy.



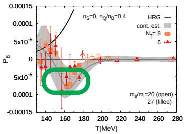
Baryon number density

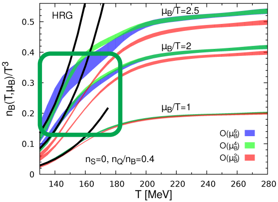


- χ_6 contribution is 30-times larger than in pressure.

$$\frac{N(\mu_B)}{T^3} = \frac{\mu_B}{T}\chi_2^B(0) + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \frac{1}{4!}\left(\frac{\mu_B}{T}\right)^6 \chi_6^B(0) + \ldots$$

- Strongly sensitive to the singular part of χ_6^B.

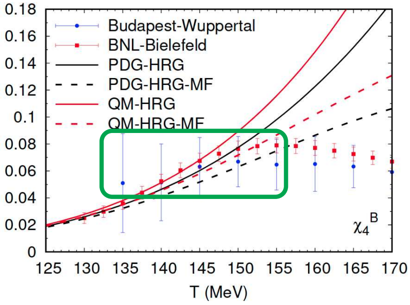




- For strangeness neutral system, effect is milder.

Can $T < T_c$ be described by Hadron Resonance Gas?

- Effects of interactions between hadrons can be mimicked by free gas of resonances+hadrons?
- Some baryon channels do not have resonances. In-medium modification of baryons?
- Lattice data for higher order baryon no. fluc. are precise enough to distinguish between diff. scenarios $\rightarrow$ support additional resonances from quark-models+interactions



[P. Huovinen, P. Petreczky, 1811.09330]

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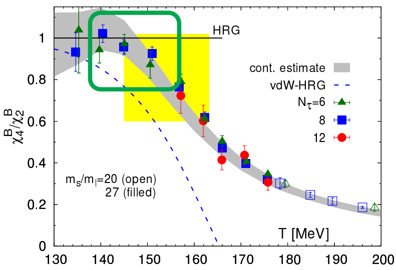

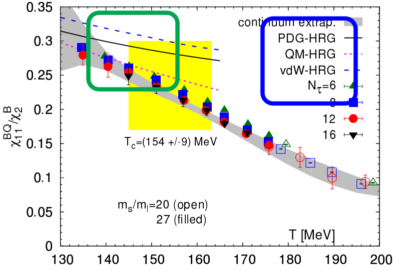

[F. Karsch, QM17 proceedings, 1706.01620]

- Including Van der Waal's interaction for baryons+non-interacting mesons+resonances, new versions of HRG has been studied $\rightarrow$ significant deviation from non-interacting HRG.

  [V. Vovchenko, M. I. Gorenstein and H. Stoecker 1609.03975]

- Lattice data can constrain such models strongly! Currently none of these models are perfect to describe QCD at freezeout.

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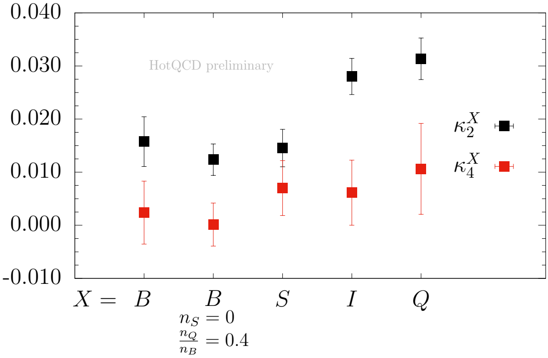
Curvature of the chiral crossover line

- $\frac{T_c(\mu_X)}{T_c(0)} = 1 - \kappa_2^X \frac{\mu_X^2}{T_c(0)^2} - \kappa_4^X \frac{\mu_X^4}{T_c(0)^4}$
- For strangess neutral system, continuum results available!
  $\kappa_2^B = 0.012(4)$, $\kappa_4^B \sim 0$ with Taylor expansions and HISQ fermions.
  [HotQCD collaboration, 1807.05607, and in prep.]



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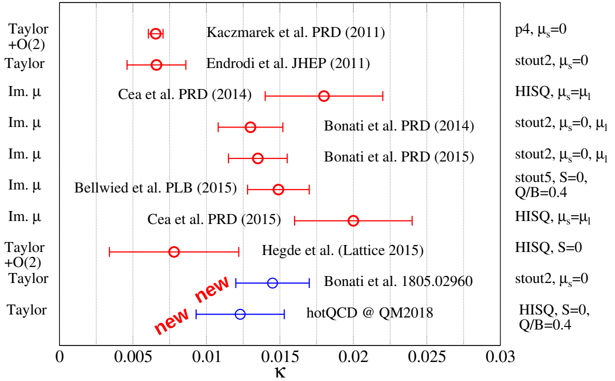
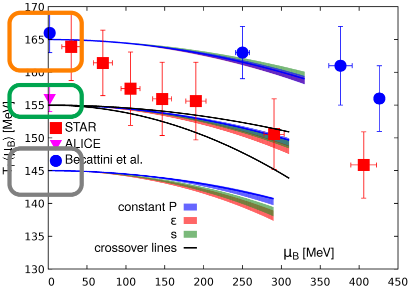
- Consistent with imaginary chemical potential method and stout fermions $\kappa_2^B = 0.0135(20)$ [C. Bonati et. al., 1805.02960]
- removes earlier tension between two methods! [courtesy M. D'Elia Quark Matter 18]



Curvature of freeze-out line vs chiral crossover line

- Different LCP's agree within 2 MeV for $\mu_B/T \leq 2$ for 3 initial choices of T_0.
- For lines $P = \text{const}$, the entropy density changes by 15% $\rightarrow$ better description of LCP for viscous medium formed in heavy-ion collisions. [HotQCD collaboration, 1701.04325].



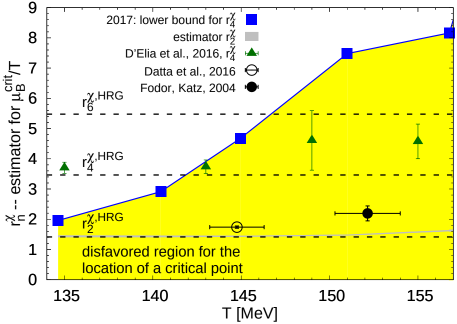
- STAR results give a steeper curvature. [arXiv:1412.0499].
- Agreement with the recent ALICE results. [arXiv:1408.6403].
- Consistent with phenomenological models. [Becattini et. al., 1605.09694].

Critical-end point search from Lattice

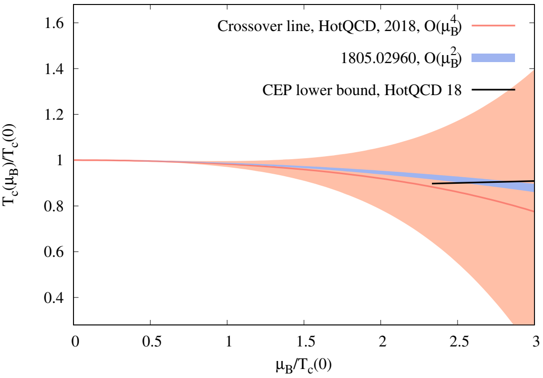
- The Taylor series for $\chi_2^B(\mu_B)$ should diverge at the critical point. On finite lattice χ_2^B peaks, ratios of Taylor coefficients equal, indep. of volume.
- The radius of convergence determines location of the critical point.
[Gavai& Gupta, 03]
- Definition: $r_{2n} \equiv \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$.
    - Strictly defined for $n \to \infty$. How large n could be on a finite lattice?
    - Signal to noise ratio deteriorates for higher order χ_n^B.

Critical-end point search from Lattice

- Current bound for CEP: $\mu_B/T > 3$ for $142 \leq T \leq 150$ MeV [HotQCD coll., 1701.04325, update 2018].
- The r_n extracted by analytic continuation of imaginary μ_B data [D'Elia et. al., 1611.08285] consistent with this bound.
- Results with a lower bound? [Datta et. al., 1612.06673, Fodor and Katz, 04] $\rightarrow$ need to understand the systematics in these studies. Ultimately all estimates will agree in the continuum limit!



Critical end-point and Chiral Crossover line: current status



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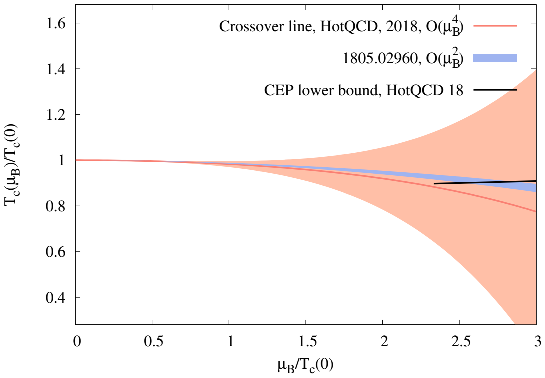

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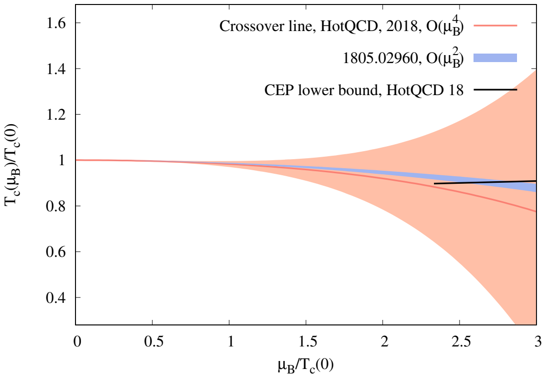

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- If $\kappa_4 \sim 0.1\kappa_2$, only significantly contributes when $\mu_B / T_{CEP} > 3$ so its precise determination is imp.

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Outlook

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- Lines of constant ϵ, p consistent with LQCD estimates of curvature of chiral crossover line.
- Higher order cumulants of baryon no. will also help in bracketing the possible CEP. Most LQCD calculations suggest $\mu_B(\mathrm{CEP})/T \geq 3$.