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The Low Redshifts-High Redshifts Tensions in Cosmological Observations and Its Possible Implications

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Plan

- Brief introduction on **current tension** between low-redshift and high-redshift observations.
- **Model independent constraint** on background evolution and dark energy evolution.
- Possible **dark energy behaviour**.
- Incorporating **Planck results from CMB** on background evolution.
- Effects on **dark energy evolution**.
- Conclusions
Universe Composition

- 73% Dark Energy
- 23% Dark Matter
- 3.6% Intergalactic Gas
- 0.4% Stars, etc.

Negative Pressure
Cosmology after Planck-2018

Concordance $\Lambda CDM$ model (Aghanim et al. ArXiv: 1807.06209):

6-parameters model

- $\Omega_b h^2 = 0.02233 \pm 0.00015$
- $\Omega_c h^2 = 0.1198 \pm 0.0012$
- $100\theta_{MC} = 1.04089 \pm 0.00031$
- $\tau = 0.0540 \pm 0.0074$
- $\ln(10^{10} A_s) = 3.043 \pm 0.014$
- $n_s = 0.9652 \pm 0.0042$

Other Parameters

- $\Omega_m = 0.3147 \pm 0.0074$
- $H_0 = 67.37 \pm 0.54 \text{ Km/sec/Mpc}$
- $\sigma_8 = 0.8101 \pm 0.0061$
- $r_{drag} = 147.26 \pm 0.29 \text{ Mpc}$
- $z_{re} = 7.64 \pm 0.74$
- $\Lambda = (2.846 \pm 0.076) \times 10^{-122} m_{pl}^2$
Planck Result 2018
Going beyond ΛCDM, evolving dark energy with

\[ w(z) = p(z)/\rho(z) = w_0 + w_a \frac{z}{1+z} \]  

(CPL Parametrisation)

\[ w(z) = -1 \text{ for } \Lambda \ (w_0 = -1, w_a = 0) \]
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Planck-2018 Result:

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$$w(z) = -1 \quad \text{for } \Lambda \quad (w_0 = -1, w_a = 0)$$


$w > -1$ non-phantom
Baryon Acoustic Oscillations

\[ c_s = \frac{1}{\sqrt{3(1 + R)}} \]

\[ R = 3 \frac{\rho_b}{\rho_r} \]

\[ r_d = \int_0^{t(z_d)} c_s (1 + z) dt \]

Eisenstein et al 2005
Tension with BAO

$D(z) = \left[ D_M^2 \frac{cz}{H(z)} \right]^{1/3}$

Planck-2018 Result:

Aghanim et al. ArXiv: 1807.06209
Tensions in $H_0$ Measurements

$H_0$ Measurements

- The Planck-2018 measurement of Hubble parameter for $\Lambda$CDM: (Aghanim et al 2018):
  \[ H_0 = 67.37 \pm 0.54 \text{ km/sec/Mpc} \]

- The local Measurement of Hubble Parameter BY HST (R18): (Riess et al. 2018, SHOES project)
  \[ H_0 = 73.45 \pm 1.66 \text{ km/s/Mpc} \]

  This is $3.5\sigma$ higher than the Planck-2018 measurement.

- Latest result by GAIA using Cepheids in Milky Way (Riess et al 2018):
  \[ H_0 = 73.52 \pm 1.62 \text{ km/s/Mpc} \]

- Independent measurement by HoliCOW using Time-Delay Strong Lensing Probe for $\Lambda$CDM:
  \[ H_0 = 71.9^{+2.4}_{-3.0} \text{ km/s/Mpc} \text{ (Bonvin et al 2017)} \]

  \[ H_0 = 72.5^{+2.3}_{-2.1} \text{ km/s/Mpc} \text{ (Birrer et al 2018)} \]
Tensions in $H_0$ measurements

![Graph showing tensions in $H_0$ measurements](image)

- Riess et al. (2018)
- BAO + Pantheon + D/H BBN
- BAO + Pantheon + D/H BBN + lensing
- BAO + Pantheon + D/H BBN + $\theta_{MC}$
- Planck TT, TE, EE + lowE

$H_0$ [km s$^{-1}$ Mpc$^{-1}$]

$\Omega_m$
Tensions in $H(z)$ measurements

Tension with Sound Horizon at Drag Epoch

Bernal, Verde, Riess, JCAP 2016

Same is also confirmed by Evslin, AAS, Ruchika, PRD 2017
Implications Of These Tensions in Dark Energy Behaviour
How to Infer about Dark Energy

Einstein Equation:

$$3H^2 = 8\pi G (\rho_m + \rho_{de})$$

$$\rho_m \propto a^{-3} \text{ or } (1 + z)^3$$

For dark energy, two possibilities:

1) Assume: \( \rho_{de} > 0 \rightarrow \rho_{de} \propto \exp \left[ 3 \int \frac{1 + w(z)}{1 + z} dz \right] \)

2) Directly constrain \( \rho_{de} \) without assuming that it has to be > 0.
Case 1: Evidence for varying dark energy


This assumes that the late time acceleration is driven by a non-interacting minimally coupled dark energy, with $q_{de} > 0$
Case 2: Model Independent Result

Wang, Pogosian, Zhao and Zucca arXiv: 1807.03772
Case 2: Model Independent Result

Poulin, Boddy, Bird, Kaminkowski, *Phys. Rev. D* 2018

Same result also confirmed by Sahni, Shafieloo, Starobinsky *APJL*, 2014.

Also by Delubac et al. [BOSS Collaboration], *Astron. Astrophys.* 2015
Assume Brans-Dicke Lagrangian:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{F(\phi)R}{16\pi G} - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) + \mathcal{L}_m \right]
\]

This gives:

\[
G_{\mu\nu} = 8\pi G \left[ \rho_m + \rho_{de}^{\text{eff}} \right]
\]

where

\[
\rho_{de}^{\text{eff}} = F^{-1} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) - 3H \dot{F} + (1 - F)\rho_m \right]
\]

\[
p_{de}^{\text{eff}} = F^{-1} \left[ \frac{1}{2} \dot{\phi}^2 - V(\phi) + 2H \dot{F} + \ddot{F} \right]
\]
Problem with $\rho_{de} < 0$

CMB tells us that the Universe is Spatially flat

$$\Omega_m + \Omega_{DE} = 1$$

But if $\rho_{de} < 0$, and also unbounded from below, then $\Omega_m > 1$ and will grow very quickly with redshifts.
This will completely destroy the large scale structure formation in the Universe as there will be much structures in the universe than what is observed.
So we need to have a lower bound for $\rho_{de} < 0$. 
We Ask the following questions:

❖ How robust are these dark energy behaviours?

❖ To answer this, we reconstruct the evolution of the background universe using low redshift data without any assumption about the underlying model for dark energy/modified gravity.

❖ Then we study what kind of dark energy behaviour can give rise to such background evolution.

Capozziello, Ruchika, AAS, arXiv:1806.03943

The Observables

\[ D_H(z) = \frac{c}{H(z)} \]

\[ D_A(z) = \frac{c}{1 + z} \int_0^z \frac{dz'}{H(z')} \]

\[ D_V(z) = ((1 + z)D_A(z))^{2/3} (zD_H(z))^{1/3} \]

\[ D_{\Delta t}(zd, zs) = c \frac{\int_0^{zd} \frac{dz'}{H(z')}}{\int_{zd}^{zs} \frac{dz'}{H(z')}} \frac{\int_0^{zs} \frac{dz'}{H(z')}}{\int_{zd}^{zs} \frac{dz'}{H(z')}} \]

\[ D_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')} \]
The Observables

\[ D_H(z) = \frac{c}{H(z)} \]

So it is better to reconstruct the \( H(z) \) directly using existing data.

\[ D_A(z) = \frac{c}{1 + z} \int_0^z \frac{dz'}{H(z')} \]

\[ D_V(z) = \left((1 + z)D_A(z)\right)^{2/3} \left(zD_H(z)\right)^{1/3} \]

\[ D_{\Delta t}(z_d, z_s) = c \frac{\int_0^{z_d} \frac{dz'}{H(z')}}{\int_z^{z_s} \frac{dz'}{H(z')}} \int_{z_d}^{z_s} \frac{dz'}{H(z')} \]

\[ D_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')} \]
Model Independent Constraints on Dark Energy
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Model Independent Constraints on Dark Energy

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- The most general way:
  \[ H(t) = \frac{1}{a} \frac{da}{dt} \]

  \[ q(t) = -\frac{1}{a} \frac{d^2a}{dt^2} \left( \frac{1}{a} \frac{da}{dt} \right)^{-2} ; j(t) = -\frac{1}{a} \frac{d^3a}{dt^3} \left( \frac{1}{a} \frac{da}{dt} \right)^{-3} \]
  \[ s(t) = -\frac{1}{a} \frac{d^4a}{dt^4} \left( \frac{1}{a} \frac{da}{dt} \right)^{-4} ; l(t) = -\frac{1}{a} \frac{d^5a}{dt^5} \left( \frac{1}{a} \frac{da}{dt} \right)^{-5} \]

  \[ H(z) = H_0 + H_{10} z + \frac{H_{20}}{2} z^2 + ... \]
Model Independent Constraints on Dark Energy

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- The most general way: \( H(t) = \frac{1}{a} \frac{da}{dt} \)

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\[ H(z) = H_0 + H_{10}z + \frac{H_{20}}{2} z^2 + \ldots \]

\[ H_1 = H_{10}/H_0 = 1 + q_0, \]
\[ H_2 = H_{20}/H_0 = -q_0^2 + j_0, \]
\[ H_3 = H_{30}/H_0 = 3q_0^2(1 + q_0) - j_0(3 + 4q_0) - s_0 \]
\[ H_4 = H_{40}/H_0 = -3q_0^2(4 + 8q_0 + 5q_0^2) + j_0(12 + 32q_0 + 25q_0^2 - 4j_0) + s_0(8 + 7q_0) + l_0 \]
Model Independent Constraints on Dark Energy

- Consider the Kinematic behaviour of expansion of Universe.

- The most general way:  
  \[ H(t) = \frac{1}{a} \frac{da}{dt} \]

  \[ q(t) = -\frac{1}{a} \frac{d^2 a}{dt^2} \left[ \frac{1}{a} \frac{da}{dt} \right]^{-2} \]
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  \[ H(z) = H_0 + H_{10} z + \frac{H_{20}}{2} z^2 + ... \]

  \[ H_1 = H_{10}/H_0 = 1 + q_0, \]
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  But with this, for \[ z > 1, \] the series does not converge.
  So taking observations from \[ z > 1, \] this is not useful.
A better way to increase the radius of convergence is Pade Approximation. Given a function $f(x)$ and two integers $n \geq 0$ and $m \geq 1$,
A better way to increase the radius of convergence is Pade Approximation. Given a function \( f(x) \) and two integers \( n \geq 0 \) and \( m \geq 1 \),

\[
R(x) = \frac{A_n(x)}{B_m(x)}
\]

\[
A_n(z) = A_0 + A_1x + A_2x^2 + \ldots + A_nx^n
\]

\[
B_m(z) = 1 + b_1x + b_2x^2 + \ldots + b_mx^m
\]
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\[
R(x) = \frac{A_n(x)}{B_m(x)} \quad \text{We call it } P_{nm}(x)
\]

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Pade Approximation

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$$R(x) = \frac{A_n(x)}{B_m(x)}$$

We call it $P_{nm}(x)$

$$A_n(z) = A_0 + A_1x + A_2x^2 + \ldots + A_nx^n$$

$$B_m(z) = 1 + b_1x + b_2x^2 + \ldots + b_mx^m$$

- such that

$$f(0) = R(0)$$

$$f'(0) = R'(0)$$

$$f''(0) = R''(0)$$

$$f^{(n+m)}(0) = R^{(n+m)}(0)$$

........................
Pade Approximation

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$$f'(0) = R'(0)$$
$$f''(0) = R''(0)$$

.........................

$$f^{(n+m)}(0) = R^{(n+m)}(0)$$

The Padé approximant often gives better approximation of the function than truncating its Taylor series, and it may still work where the Taylor series does not converge.
Pade Approximation

- We use $P_{22}(z)$ to approximate $H(z)$:
  \[
  E(z) = \frac{H(z)}{H_0} = \frac{1 + P_1 z + P_2 z^2}{1 + Q_1 z + Q_2 z^2}
  \]

- The different parameters are related to cosmographic parameters as:
  \[
  P_1 = H_1 + Q_1, \\
  P_2 = \frac{H_2}{2} + Q_1 H_1 + Q_2, \\
  Q_1 = \frac{-6H_1 H_4 + 12H_2 H_3}{24H_1 H_3 - 36H_2^2}, \\
  Q_2 = \frac{3H_2 H_4 - 4H_3^2}{24H_1 H_3 - 36H_2^2}.
  \]
To show that it fits better the $H(z)$ behaviour than Taylor expanding $H(z)$ upto 4th order, we assume a $\Lambda$CDM model and fit both the Taylor expanded $H(z)$ and the Pade Approximated $H(z)$.
Data Used

- The **SnIa data** from latest Pantheon results.
- The **H(z) data** from different Cosmic Chronometer measurements.
- Different **BAO and MegaMaser measurements**
- **Strong Lensing data** from H0LiCow experiment
- **H₀ measurement** by Riess et al (R16).
68% CL for Parameters

\( H_0 = 100 \, \text{h km/ sec/ Mpc} \)

<table>
<thead>
<tr>
<th>( Bao + Mas + SL + SN(1) )</th>
<th>( (1) + H_0 )</th>
<th>( (1) + H(z) )</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>0.7293 ± 0.031</td>
<td>0.7313 ± 0.015</td>
<td>0.7034 ± 0.024</td>
</tr>
<tr>
<td>( r_d )</td>
<td>137.06 ± 4.58</td>
<td>136.41 ± 3.82</td>
<td>148.67 ± 1.93</td>
</tr>
<tr>
<td>( q_0 )</td>
<td>−0.644 ± 0.223</td>
<td>−0.6401 ± 0.187</td>
<td>−0.930 ± 0.218</td>
</tr>
<tr>
<td>( j_0 )</td>
<td>1.961(^{+0.926}_{-0.884})</td>
<td>1.9461(^{+0.871}_{-0.816})</td>
<td>3.369(^{+1.270}_{-1.294})</td>
</tr>
</tbody>
</table>

For other two parameters:

\[ s_0 = 19.97^{+11.57}_{-10.84} \]
\[ l_0 = 121.41^{+91.94}_{-83.56} \]
Model Independent Behaviour

Hubble parameter $H(z)$
Compare with Other Model Independent Results

**Figure 2.** The reconstructed evolution history of the dark energy equation of state compared with the 2012 result and the forecasted uncertainty from future data.

The mean (white solid) and the 68% confidence level (CL) uncertainty (light blue band) of the $w(z)$ reconstructed from ALL16 compared to the ALL12 $w(z)$ reconstructed in Zhao et al. (2012) (red lines showing the mean and the 68% CL band). The red point with 68% CL error bars is the value of $w(z)$ at $z = 2$ "predicted" by the ALL12 reconstruction. The dark blue band around the ALL16 reconstruction is the forecasted 68% CL uncertainty from DESI++. The green dashed curve and the light green band show the mean and the 68% CL of $w(z)$ reconstructed from ALL16 using a different prior strength ($D = 0.4$) for which the Bayesian evidence is equal to that of Cold Dark Matter. See the text for details.

Zhou et al., 2017

Our Result
Model Independent Behaviour

Assuming Flat Universe: \[ 3H^2 = \rho_m + \rho_{de} = \rho_{m0}(1 + z)^3 + \rho_{de0}f(z) \]

\[ \frac{H^2}{H_0^2} = \Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})f(z) \]

\[ \Omega_{m0} = 0.28 \]
\[ \Omega_{m0} = 0.30 \]
\[ \Omega_{m0} = 0.32 \]
To Incorporate Matter Domination

- At higher redshifts, all models should reproduce the same matter dominated period.

- Therefore, we assume that our model independent behaviour for $H(z)$ should reproduce $H(z)$ as constrained by Planck for $\Lambda$CDM model at some appropriate higher redshifts.

- We assume three specific choices of the redshifts, $z = 4, 5, 6$. 
Results with Planck Constraint on $H(z)$
Effects of different data combinations
Results with Planck Constraint on H(z)
Results with Planck Constraint on $H(z)$
Interpretation of the result

- Without the Planck constraints on $H(z)$ for matter era, energy density of the dark energy $q(z)$ becomes negative at high redshifts.

- Putting Planck’s constraints on $H(z)$ at redshifts $z=4$ and beyond, $q(z)$ gets a negative minimum.

- Continuity equation: \[
\frac{d}{dz} \rho_{de} - 3(\rho_{de} + p_{de})/(1 + z) = 0
\]

- Minimum for $q(z) \rightarrow \rho_{de} + p_{de} = 0 \rightarrow \rho_{de} = -p_{de} \rightarrow Cosm.\text{Const.}$

- But at the minimum, $q(z) < 0$, hence there is a presence of -ve CC.

Negative $\Lambda$ !!!!
The Model

- If one takes out the matter part from the total energy density, then \( \rho_{\text{total}} - \rho_m \) can be written as

\[
\rho_{\text{de}} + (-ve \ \Lambda)
\]

This initially decreases with expansion as non-phantom field, but later increases like a phantom field to give higher \( H_0 \) at present. So it has a non-phantom to phantom transition.

This has a tiny negative value and helps to avoid the energy density going unbounded in -ve direction and also plays role to enter the matter domination at high redshifts. It does not play any role for late time acceleration.
Possible effects in Structure Formation

- Due to the presence of -ve $\Lambda$ and due to spatial flatness, the $\Omega_m > 1$ for certain redshift range. This results more growth of structures due to deeper gravitational potential and the nonlinear regime may start earlier than $\Lambda$CDM Universe and there will be more massive galaxies at high redshifts than $\Lambda$CDM.

- These are definite prediction and can be verified by upcoming large scale surveys.
Conclusions

❖ After Planck-2018 results, $\Lambda$CDM model is the simplest model that is consistent with data.

❖ But several low-redshift observations including $H_0$ measurements by Riess et al., BAO measurements using Lyman-Alpha as well as the low-redshift measurements of sound horizon at drag epoch ($r_d$) using BAO, has shown significant tensions with $\Lambda$CDM model as constrained by Planck-2018 using CMB.

❖ This opens possibility for new physics for both at early universe as well as late universe involving the dark energy behaviour.

❖ Several Model independent reconstruction for dark energy behaviour have shown either phantom-nonphantom crossing in dark energy equation of state or negative dark energy density at higher redshifts.

❖ We confirm these results in our model independent reconstruction using Pade Series Approximation for $H(z)$. 
Conclusions

- While incorporating the $H(z)$ behaviour as constrained by Planck for matter dominated era, we showed that $q_{de}$ has a small negative minimum at $z > 4$.

- One can associate this negative minimum with the existence of a tiny negative cosmological constant.

- The actual dark energy density is not positive cosmological constant, but an evolving one which evolves from a non-phantom era to a phantom one.

- The presence of tiny negative Cosmological Constant does not affect late time expansion.

- But due to its presence, the $\Omega_m$ becomes more than 1 for a certain redshift range and this has interesting implications on large scale structure formation that can be probed using near future galaxy surveys.
Thank You
Is there really any evidence for dark energy evolution?

- **Bayesian Evidence** of a model with a parameter space dimensionality $D$ is given by:

$$\mathcal{Z} = \int \mathcal{L}(\Theta)\pi(\Theta)d^D\Theta$$

- To compare two models, one uses the Jeffrey’s Scale:

<table>
<thead>
<tr>
<th>$\Delta\log(z)$</th>
<th>Conclusion</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>0----1</td>
<td>No evidence</td>
<td>Blue</td>
</tr>
<tr>
<td>1------2.5</td>
<td>Significant Evidence</td>
<td>Orange</td>
</tr>
<tr>
<td>2.5------5</td>
<td>Strong Evidence</td>
<td>Green</td>
</tr>
<tr>
<td>&gt; 5</td>
<td>Decisive Evidence</td>
<td>Red</td>
</tr>
</tbody>
</table>

- The calculations have been done using PyMultinest.
Dark Energy Models

❖ Scalar field models:

❖ Canonical Scalar field:

\[ \mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial_{\mu} \phi - V(\phi) \]

❖ Non-Canonical Scalar fields:

\[ \mathcal{L} = V(\phi) \sqrt{1 - \partial_{\mu} \phi \partial_{\mu} \phi} \]

❖ Scalar fields with higher derivative terms:

\[ \mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial_{\mu} \phi (1 + \frac{\alpha}{M^3} \square \phi) - V(\phi) \]

EOM is still second order
Dark Energy Models

- Constant equation of state $w$
- Varying $w$ as a function of redshift $z$: (Different parametrisation)
  - $w(z) = w_0 + w_a \frac{z}{1 + z}$
  - $w(z) = w_0 + w_a \left( \frac{z}{1 + z} \right)^7$
  - $w(z) = -\frac{w_0}{w_0 + (1 - w_0)(1 + z)^{3(1 + w_a)}}$
  - $w(z) = w_0 + w_a \frac{z(1 + z)}{1 + z^2}$
  - $w(z) = w_0 + w_a \frac{z}{(1 + z)^2}$

- CPL (Chevallier & Polarski 2001, Linder 2003)
- 7CPL (Pantazis et al, 2016)
- GCG (Thakur, Nautiyal, Sen, Seshadri, 2012)
- BA (Barboza & Alcaniz, 2012)
- JBP (Jassal, Bagla, Padmanabhan, 2005)
BAO+TDSL+H(z)
BAO + TDSL + H(z) + SNIa

Log Evidence (Z)

ACDM, uCDM, CPL, 7CPL, BA, JB, ExpGalileon, ExpScalar, LinGalileon, LinScalar, InTachyon, SqGalileon, SqScalar, GCG track, GCG thaw

The graph shows the log evidence (Z) for various cosmological models.
$\text{BAO+TDSL+H(z)+SnIa+CMB}$

![Graph showing log evidence for various models](image)
BAO\(+\)TDSL\(+\)H\((z)\)+SnIa+CMB+Growth

\[
\begin{array}{cccccccccccc}
\text{Log Evidence}(Z) & -2.40 & 2.97 & -2.06 & -2.81 & -3.13 & 1.02 & 2.16 & 1.84 & 0.97 & 2.04 & 1.73 & 0.76 & 1.79 & 1.58 & 0.75 & 1.89 & 2.18 & 4.12 & -1.76 & 0.10 \\
\hline
\text{ACDM} & & & & & & & & & & & & & & & & & & & & \\
\text{wCDM} & & & & & & & & & & & & & & & & & & & & \\
\text{CPL} & & & & & & & & & & & & & & & & & & & & \\
\text{7CPL} & & & & & & & & & & & & & & & & & & & & \\
\text{BA} & & & & & & & & & & & & & & & & & & & & \\
\text{JBP} & & & & & & & & & & & & & & & & & & & & \\
\text{InvGalileon} & & & & & & & & & & & & & & & & & & & & \\
\text{InvScalar} & & & & & & & & & & & & & & & & & & & & \\
\text{InvTachyon} & & & & & & & & & & & & & & & & & & & & \\
\text{SqGalileon} & & & & & & & & & & & & & & & & & & & & \\
\text{SqScalar} & & & & & & & & & & & & & & & & & & & & \\
\text{SqTachyon} & & & & & & & & & & & & & & & & & & & & \\
\text{GCG} & & & & & & & & & & & & & & & & & & & & \\
\text{GCG_haw} & & & & & & & & & & & & & & & & & & & & \\
\end{array}
\]
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- Interestingly the CPL parametrisation which is a universal parametrisation used for DE constraint, is shown to be pretty inferior compared to actual scalar field models.
Thank You
Tensions in $\Lambda CDM$

$H_0$ Measurements

- The Planck-2015 measurement of Hubble parameter for $\Lambda CDM$: (Ade et al 2016):
  \[ H_0 = 66.93 \pm 0.62 \text{ Km/s/Mpc} \]

- The local Measurement of Hubble Parameter BY HST (R16): (Riess et al. 2016)
  \[ H_0 = 73.24 \pm 1.24 \text{ Km/s/Mpc} \]
  This is 3.4$\sigma$ higher than the Planck-2015 measurement.

- Latest local measurement of Hubble parameter: (Riess et al 2018):
  \[ H_0 = 73.45 \pm 1.66 \text{ Km/s/Mpc} \]
  This is 3.7$\sigma$ higher than the Planck-2015 measurement.

- Local measurements are also consistent with independent measurement by HoliCOW using Time-Delay Strong Lensing Probe for $\Lambda CDM$: (Bonvin et al 2017)
  \[ H_0 = 71.9^{+2.4}_{-3.0} \text{ Km/s/Mpc} \]
The amplitude of the cosmic shear scales as $S_8^{2.5}$ where $S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$.
Tensions in $\Lambda$CDM

- KiDS measured $S_8 = 0.745 \pm 0.039$ for $\Lambda$CDM.

- Tension with Planck-2015:

<table>
<thead>
<tr>
<th>Model</th>
<th>$T(S_8)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda$CDM</td>
<td>2.1$\sigma$</td>
</tr>
<tr>
<td>DE (const. $w$)</td>
<td>0.89$\sigma$</td>
</tr>
<tr>
<td>DE ($w_0 - w_a$)</td>
<td>0.91$\sigma$</td>
</tr>
</tbody>
</table>

Joudaki et al, arXiv:1610.04606