Effect of Inverse Magnetic Catalysis on Conserved Charge Fluctuations in Hadron Resonance Gas Model

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Ranjita K Mohapatra
IIT Bombay
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Motivation

• Huge magnetic field ($\sim 10^{20}$ Gauss) is produced in relativistic heavy ion collisions.

• This magnetic field survives until chemical freezeout from induced currents due to rapidly decreasing external magnetic field.

• This is important to analyze conserved charge fluctuations and correlations along the chemical freezeout curve in presence of this external magnetic field.
Hadron resonance gas model in presence of magnetic field

• The basic features of the physical system created at the time of chemical freeze-out in heavy ion collisions are well described in terms of the hadron resonance gas (HRG) model.

• The grand partition function defined for each hadron species $i$ as

$$
\ln Z_i = \pm V g_i \int \frac{d^3 p}{(2\pi)^3} \ln[1 \pm e^{-(E_i - \mu_i)/T}]
$$

• $V$ is the volume of the system, $g_i$ is the spin degeneracy factor.

• Conservation laws : net $S=0$ and $B/Q = 2.52$

• All the thermodynamic quantities like pressure, energy density and entropy density etc. can be derived from this partition function.
Now in presence of external magnetic field along z-axis

- Landau quantization of energy levels for charged particle takes place along the plane perpendicular to the magnetic field.

- Energy is given by \( E_i = \sqrt{p_z^2 + m^2 + 2|qB|(n + \frac{1}{2} - s_z)} \)

- \( n \) is the Landau level goes from 0 to \( \infty \)

- The grand partition function in presence of magnetic field is given by

\[
\ln Z_i = \pm V \sum_{s_z = -s} \sum_{n = 0}^{s + \infty} \left| qB \right| \int \frac{dp_z}{2\pi} \ln \left[ 1 \pm e^{-\left(E_i - \mu_i\right)/T} \right]
\]
Basic thermodynamic quantities

- **Pressure** \( P_i = -\frac{T}{V} \ln Z_i \)

- **Number Density** \( n_i = g_i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\left(e^{\beta(E_i-\mu)} \pm 1\right)} \)

- **Energy density** \( \varepsilon_i = g_i \int \frac{d^3 p}{(2\pi)^3} \frac{E_i}{\left(e^{\beta(E_i-\mu)} \pm 1\right)} \)

- **Entropy density** \( S = \frac{\varepsilon + P - \mu n}{T} \)

- **Conserved charge density** \( n_Q = g_i Q_i \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\left(e^{\beta(E_i-\mu)} \pm 1\right)} \)
Effect of Inverse magnetic catalysis (IMC) on the chemical freezeout curve

• The system exhibits magnetic catalysis at zero temperature where the chiral condensate increases in external magnetic field.

• Lattice QCD exhibits IMC effect at finite temperature in external magnetic field. The chiral condensate decreases and the critical temperature decreases.

• The IMC effect might be due to the decrease in interaction strength in presence of magnetic field. This decrease of interaction strength is consistent with asymptotic freedom of QCD if the relevant scale $\sqrt{eB} = \Lambda_{QCD}$

• Since the critical temperature decreases in presence of magnetic field, the chemical freezeout curve should correspond to a lower temperature in $T-\mu_B$ plane.

• The universal chemical freezeout curve is determined from the condition $E/N = \varepsilon/n \sim 1\text{ GeV}$. 
The chemical freezeout curve determined by $E/N \approx 1\text{GeV}$ with and without charge conservation. Solid line without electric charge and strangeness conservation. The dotted line is with charge conservation.

Due to IMC effect, the chemical freezeout temperature decreases in presence of nonzero magnetic field with charge conservation.

At higher $\mu_B$ and nonzero $B$, the chemical freezeout curve is pushed to higher temperature because there are more baryons, particularly more protons at nonzero $B$. (PRL 117, 102301 (2016))
Freezeout curve: \( n_B + n_{\bar{B}} = 0.12 \, fm^{-3} \)

Freezeout curve: \( \frac{S}{T^3} = 7 \)
Strange chemical potential is always large for nonzero $B$ compared to zero $B$.

At higher $\mu_B$, there are more baryons (protons and neutrons) in the system. Imposing charge conservation, i.e. $B/Q = 2.52$, one needs negative $\mu_Q$. 

**strangeness and electric charge chemical potential along the freezeout curve**
Fitted parameters for freezeout curve at zero B

Temperature \[ T = a - b\mu_B^2 - c\mu_B^4 \]

Where \( a = (0.166\pm0.002) \text{ GeV} \), \( b = (0.139\pm0.016) \text{ GeV}^{-1} \) and \( c = 0.053\pm0.021 \text{ GeV}^{-3} \)

Variation of chemical potential with collision energy:

\[
\mu_X(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}
\]

This agrees with the chemical freezeout curve at zero B.

However, at nonzero B, the chemical freezeout curve does not match with these fitted parameters.
2\textsuperscript{nd} order susceptibility of conserved charges

Susceptibilities are defined by

\[ \chi_{xy}^{ij} = \frac{\partial_x^{i+i} \left( \sum_k P_k / T^4 \right)}{\partial \left( \frac{\mu_x}{T} \right)^i \partial \left( \frac{\mu_y}{T} \right)^j} \]

Charge conservation diminishes the fluctuations along the freezeout curve.
Dominant contribution to $\chi_s$ comes from $k^+$

Dominant cont. comes from proton and neutron at zero $B$. At nonzero $B$, $\Delta^+$ cont. is very large compared to proton.
Conserved charge correlations along the freezeout curve

The most dominant cont. comes from $k^\pm, \Sigma^\pm$

Correlation is larger with charge conserv. due to the increase in freezeout temp.

Dominant cont. comes from $p, \Sigma^\pm$ and $\Delta$ particles.

Dominant cont. comes from $\wedge$ and $\Sigma^\pm$
Different products of moments

\[
\frac{\sigma^2}{M} = \frac{\chi^2}{\chi^1}, \quad S\sigma = \frac{\chi^3}{\chi^2}, \quad k\sigma^2 = \frac{\chi^4}{\chi^2}
\]

Experimentally measured moments such as mean \( (M) \), standard deviation \( (\sigma) \), skewness \( (S) \) and kurtosis \( (k) \) of conserved charges are used to characterize the shape of charge distribution.

These ratios are independent of the volume of the system and play a crucial role for the search of possible critical point in the QCD phase diagram.
Net proton number

Net kaon

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**Conclusion**

**IMC effect is observed in HRG with charge conservation.**

At B=0, charge conservation does not play a role in the fluctuations along the freezeout curve for the conserved charges of electric charge and baryon number.

But charge conservation play an important role for strange charge at B=0. Charge conservation diminishes the fluctuations in strange charge at B=0 compared to the fluctuations without charge conservation.

For nonzero B, charge conservation play a very important role. If there is no charge conservation at nonzero B, then the fluctuations increase by a huge amount compared to zero B.

**IMC effect is clearly observed in products of net kaon moment.**
Thank you
Estimation of magnetic field in rel. heavy ion collisions

- Magnetic field is produced due to valence charges of the colliding nuclei. 
- This magnetic field decreases with time 
  \[ eB(t) = eB_0 \left[ 1 + \left( \frac{t}{t_0} \right)^2 \right]^{-3/2} \]
- Maximum magnetic field 
  \[ eB_0 = (0.05\text{GeV})^2 (1\text{fm} / b)^2 Z \sinh Y \]
  
  \( b = \text{impact parameter} \), \( Z = \text{atomic number of nuclei} \)

Beam rapidity 
  \[ \sinh Y = \sqrt{S_{NN}} / (2m_N) \]

Life time parameter 
  \[ t_0 = b / (2\sinh Y) \]

- Magnetic field decreases with time as \( t^{-2} \)
  May not play an important role in conserved charge fluctuation along the chemical freezeout curve.

- However, rapidly decreasing magnetic field produces induced current in the Plasma and magnetic field of similar magnitude can be obtained from this induced current and sustain for longer time.
$3^{rd}$ order susceptibility of conserved charges
$4^{th}$ order susceptibility of conserved charges
Net charge density for conserved charges along the freezeout curve

Net charge density is very large at $B=0.25$ GeV$^2$ without charge conservation due to more protons production.

Charge conservation diminishes net charge density at nonzero $B$.

Net strange density is zero with charge conservation at zero and nonzero $B$.

Net baryon density is very large without charge conservation at nonzero $B$ due to more production of baryons.

Using fitted parameters at zero $B$ for nonzero $B$, does not match with the results at nonzero $B$ with charge conservation.
Fluctuations and correlations in rel. heavy ion collisions

Fluctuations and correlations are important to understand the basic degrees of freedom of the system.

The fluctuations originated in the QGP phase may survive until the freezeout due to the rapid expansion of the fireball and can be exploited as a signal of the QGP formation in the early stages of relativistic heavy ion collisions.

Higher order moments of conserved charge fluctuations are more sensitive to the large correlation lengths in QGP phase and relax slowly to their equilibrium values at the freezeout.

The deviation of experimental results from HRG model predictions may conclude the presence of non hadronic constituents or non thermal physics in the primordial medium.

The fluctuations and correlations are given by the diagonal and off diagonal components of susceptibility.
Baryon number fluctuations in hadron resonance gas and QGP

In hadron resonance gas

\[
\left\langle B^n \right\rangle_c = \frac{\partial^n \ln Z}{\partial \left( \frac{\mu_B}{T} \right)^n} = \left\langle N_B \right\rangle + (-1)^n \left\langle N_B \right\rangle 
\rightarrow \text{Skellam distribution}
\]

\[
\frac{\left\langle B^{n+2} \right\rangle_c}{\left\langle B^n \right\rangle_c} = 1
\]

In QGP (neglecting Fermi-Dirac statics)

\[
\left\langle B^n \right\rangle_c = \frac{\partial^n \ln Z}{\partial \left( \frac{\mu_B}{T} \right)^n} = \frac{1}{3^n} \left[ \left\langle N_q \right\rangle + (-1)^n \left\langle N_q \right\rangle \right]
\]

\[
\frac{\left\langle B^{n+2} \right\rangle_c}{\left\langle B^n \right\rangle_c} = \frac{1}{9}
\]

Comparing fluctuations obtained in HRG with experimental data at the time of freezeout, one can say whether it is a complete hadron resonance gas or there are other basic degrees of freedom in the system whose flcts. have survived until freezeout.
nth order moment for conserved charge \( \langle Q^n \rangle = Tr(Q^n \rho) = \frac{1}{Z} \frac{\partial^n Z}{\partial \left( \frac{\mu}{T} \right)^n} \)

Density matrix \( \rho = \frac{1}{Z} e^{-\beta(H - \mu Q)} \)

1st order Cumulant = 1st order moment

2nd order cumulant = (\( \delta Q^2 \))

3rd order cumulant = (\( \delta Q^3 \))

4th order cumulant = (\( \delta Q^4 \) - 3(\( \delta Q^2 \))^2)

nth order cumulant \( \langle Q^n \rangle_c = \frac{\partial^n \ln Z}{\partial \left( \frac{\mu}{T} \right)^n} \)