Infrared effective dual QCD at finite temperature and densities

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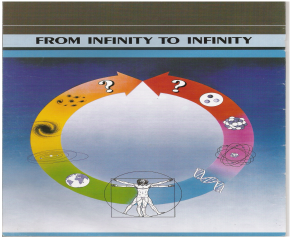
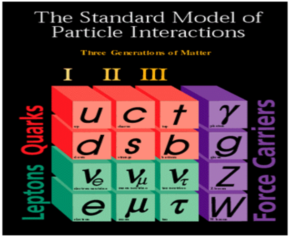
Motivation and brief background

Two major aspect of universe

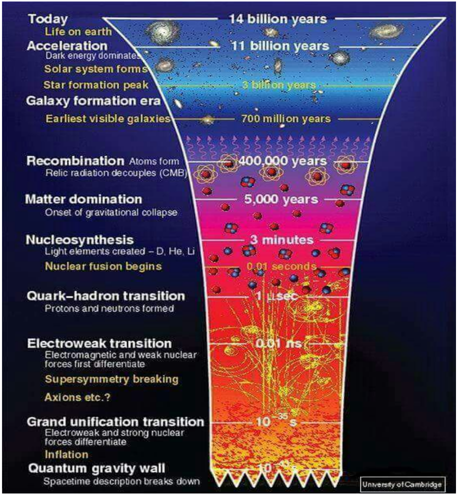
- Macro (Cosmic) universe: Galaxies, planets and other celestial bodies.
- Micro (Elementary) universe: Molecules, atoms, quarks, leptons, gauge bosons and other quantum particles.

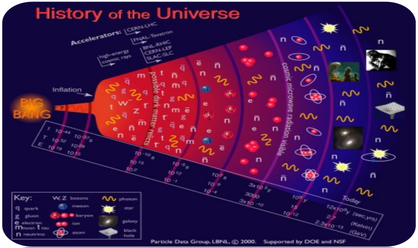




- HEP: Probing structure of matter inwards for exploring unity and simplicity of physical world.
  Result: Fundamental particles and Fundamental interactions of nature.
- Probing at 10^{-18} m- 100-200 GeV needed (age of universe 10^{-10} Sec.) LEP collider at CERN.
- Probing at 10^{-19} m- 1 TeV (Age of Universe 10^{-12} Sec.)
- Breaking of Electroweak Symmetry during this time (Massive $W^{\pm}$, Z^{0}).
- At 10^{12} TeV (10^{15} GeV): GUT operative.
- Chronology of Big-Bang Cosmology: chart.

Evolution of the Universe





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An estimate/indicator of energy achievement with High Energy Precession Machines

Lab energy for Colliders

$$E = \frac{E_{cm}^2}{2m_p}$$

Energy-time relationship(Livingstone)

- First phase

$$ln\left(\frac{E}{100KeV}\right) = \frac{1}{6}(t - 1930)$$

(Energy of accelerator increases by a factor of 10 in every 6 years) which for,

$$E_{lab} = \frac{(10^{19}GeV)^2}{2m_p} \approx 10^{38}GeV$$

$\Rightarrow$ t-1930 = 252 years, Reach of Planck Scale Energy - 2182 A.D.

- Second phase: Most optimistic Collider era:

$$ln\left(\frac{E}{1000GeV}\right) = \frac{1}{3}(t - 1990)$$

$\Rightarrow$ t-1990 = 105 years, Reach of Planck Scale Energy - 2095 A.D.

- Understanding of the (elementary) particle behaviour and their interactions may be extremely useful in interpreting history and future of our Universe.
- A leading role is played by symmetries and the resulting Gauge Theoretical formulations: Unifying tools of modern physics.
- A convincing fundamental theory of strong interactions among Hadrons:
- QCD: A non-Abelian color gauge theory- $SU(3)_c$.
- QCD behaviour in high energy sector: Perturbative, Property of Asymptotic freedom.
- In low Energy Sector: Non-perturbative (less explored). Increase in α_s at large hadronic distances (small momentum transfer) $\Rightarrow$ need for the analytic study of the important Non-perturbative features of QCD like:
  (a) Quark Confinement
  (b) Breakdown of Chiral Symmetry
  (c) Hadron mass Spectrum

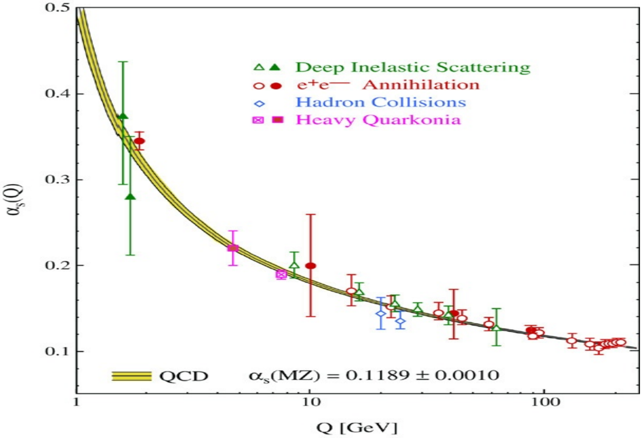
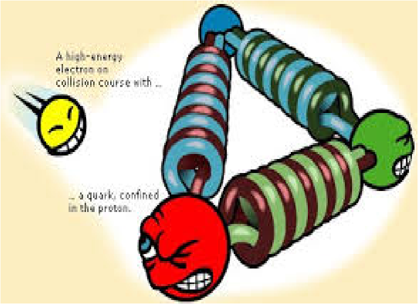


Figure: The running of the QCD coupling constant as a function of momentum transfer, experimental data vs. theoretical prediction

Quark confinement: The infrared slavery in QCD

Confinement

1. One of the fundamental and most challenging issue.
2. Effective models are needed to explore the underlying mechanism.
3. Dual superconducting model: Electric flux confinement via condensation of magnetic monopoles.
4. Introduction of monopoles: t' Hooft Abelian projection, D.G.L.theory and Field decomposition formulation.



Dual superconducting model of QCD: flux tube formulation

- First proposed by Nambu (1974),and Mandelstam (1976) based on the analogy between the superconductor and QCD vacuum.
- Color confinement: the dual version of superconductivity.
- Dual Meissner effect brought by monopole condensation in QCD vacuum.
- The squeezing of color electric flux into tube like structure leads to the confinement of color isocharges.
- In QCD: non-abelian gluons are present but no color monopoles.

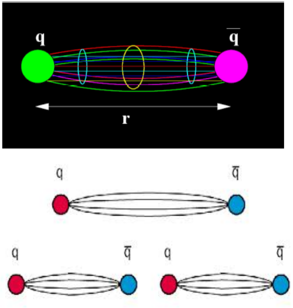


Figure: The flux tube fragmentation process into mesons

't Hooft abelian projection

- Abelian gauge: a special type of gauge where color monopoles naturally appear as the most essential degrees of freedom.
- Validity of Abelian dominance: The Abelian part of QCD appears to be responsible for the confinement.
- This defined as the diagonalization of an arbitrary gauge dependent variable $X(x) = T^a X^a(x)$
- In this gauge X(x) is diagonalized by the gauge transformation, in the fundamental representation of $SU(N_c)$,

$$X(x) \rightarrow X'(x) = \Omega(x) X(x) \Omega^{-1}(x) = diag(\lambda_1(x), \lambda_2(x), ...\lambda_{N_c}(x))$$

where $\Omega(x) = exp(iT^a \chi^a(x)) \in SU(N_c)$ is a gauge function in the Minkowski space.

- The residual symmetry as an abelian symmetry of $[U(1)]^{N_c - 1}]$.
- off-diagonal elements of gauge degrees of freedom are frozen, but the diagonal elements of the gluon field remain as the gauge degrees of freedom.
- In other words, non-abelian gauge theories are reduced to the abelian gauge theories by imposing the abelian gauge fixing conditions, and the off diagonal gluons are regarded as charged matter field in terms of the residual abelian gauge symmetry.
- The color monopole appears in the abelian space due to the abelian gauge fixing and it depends on the choice of the variable X to diagonalize.

- The non-abelian nature of the gauge theory plays an essential role on the appearence of the hedgehog configuration in the full gauge space corresponding to the non-trivial homotopy group $\prod_2(SU(N_c)/[U(1)]^{N_c-1}) = Z_\infty^{N_c-1}$ and this configuration behaves as the singularity or the Dirac monopole in terms of the residual abelian gauge field in the abelian gauge.
- Condensation of colored monopoles $\rightarrow$ the dual Meissner effect $\rightarrow$ the color confinement.
- The color electric flux between the static quarks then squeezed into an one dimensional string or a tube like vortex in the superconductivity.
- This flux tube picture of hadrons leads to a linearly rising potential between static quarks, characterizing the quark confinement, and is consistent with Regge trajectories of hadrons and lattice gauge theories.
- Thus color monopole condensation seems to give a physical explanation of color confinement and related phenomena in the non-perturbative region of QCD.

Limitations and drawbacks

- First of all, we must separate the Abelian part to prove this conjecture, but it does not tell us how to do that gauge independently.
- The popular way to obtain the Abelian part is to choose the so-called "maximal Abelian gauge". But strictly speaking,this is a gauge fixing and does not tell us exactly what constitutes the Abelian part.
- Obviously the Abelian part must contain the trivial Maxwell-type Abelian potential, but this U(1)potential is not supposed to produce the confinement. So the Abelian part must contain something else. But the maximal Abelian gauge does not specify what that is. This means that, even if we prove the Abelian dominance, we cannot tell what is really responsible for the confinement.

Dual QCD formulation and its non-perturbative features

Dual QCD using field decomposition

- A restricted gauge theory as a self consistent subset of a non Abelian gauge theory by imposing an extra magnetic symmetry to the gauge symmetry leading to the monopoles as topological charges.
- The theory describes the dual dynamics between the color isocharges and the topological charges of the non Abelian symmetry im which color confinement can be made manifest, and contains two potentials the electric and the magnetic potentials in a dual symmetric way.
- The topological charge is identified as the dual of the Noether charge of the magnetic symmetry of the theory.

Dual QCD formulation and its non-perturbative features

Magnetic symmetry and Dual QCD

- Understanding of the full content of the color gauge theory and specification of global topology of gauge symmetry.
- QCD as a local gauge theory, involving non-abelian monopoles.
- Color confinement by dynamical breaking of magnetic symmetry.
- Geometrical structure- Higher dimensional metric formulation of gauge theory.
- Unified space: P $\rightarrow$ (4+n) dimensional metric manifold (g_{AB}).

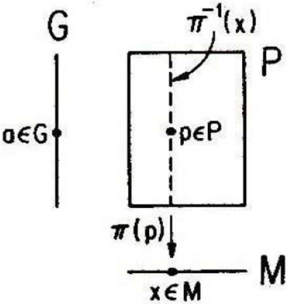


Figure: The Principal fiber bundle P with M as the base manifold and G as the structure group.

- Killing vector fields (ξ_i):$[\xi_i,\xi_j]=f_{ij}^k\xi_k$ and $\mathcal{L}_{\xi_i}g_{AB}=0$.
- Identification:P$\Rightarrow$ Principle fiberbundle P(M,G). P/G $\Rightarrow$ Quotient space M$\Rightarrow$base manifold, G $\Rightarrow$ Structure group.
- Canonical projection map: $\prod : P \longrightarrow M$

$$g_{AB}=\begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & g_{ij} \end{pmatrix} \; Internal \;\; metric \;\; g_{ij}=g_{AB}\xi_i^A\xi_j^B$$

- The topological charge in non-Abelian gauge theory can best be described by introducing the magnetic symmetry.
- Magnetic symmetry is defined as an additional isometry (H) of the internal space which may be described by a Killing vector m_a which has the Cartans subgroup of the gauge symmetry as its little group.

$$m_a=m_a^i\xi_i, \quad [m_a,\xi_i]=0, \quad \mathcal{L}_{m_a}g_{AB}=0$$

- A constraint on the internal metric and the gauge potential. H $\Rightarrow$ determined by m_a.
- Let there exists one such vector field, $\hat{m}$: $[\hat{m},\xi_i]$=0

$$\hat{m}=[m^1,m^2,....m^n]^T$$

- The magnetic symmetry may be imposed by insisting that the gauge potential W_μ must satisfy the gauge covariant constraint

$$D_\mu \hat{m} = \partial_\mu + g\mathbf{W}_\mu \times \hat{m} = 0, \Rightarrow \hat{m} : \; \textit{symmetry of the potential}.$$

- Magnetic symmetry: Topological structure of gauge symmetry.
- Monopole a topological object associated with $\prod_2(G/H)$ homotopy.
- For the simplest choice of the gauge group $G \equiv SU(2)$ with the little group $H \equiv U(1)$, the exact solution of equation(1) is written as

$$\mathbf{W}_\mu = \underbrace{A_\mu \hat{m}}_{\text{Abelian part}} - \underbrace{g^{-1}(\hat{m} \times \partial_\mu \hat{m})}_{\text{topological part}}$$

- The associated field strength:
$\mathbf{G}_{\mu\nu} = \mathbf{W}_{\nu,\mu} - \mathbf{W}_{\mu,\nu} + g\mathbf{W}_\mu \times \mathbf{W}_\nu \equiv (F_{\mu\nu} + B^{(d)}_{\mu\nu})\hat{m}$

$$\textit{where} \quad F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}, \quad B^{(d)}_{\mu\nu} = -g^{-1}\hat{m}.(\partial_\mu \hat{m} \times \partial_\nu \hat{m})$$

- The topological structure may therefore be explicitly brought into dynamics in a dual symmetric way when such separation of gauge fields is perform in the magnetic gauge obtained by rotating $\hat{m}$ to a prefixed direction in isospace using a gauge transformation (U) as

$$\hat{m} \to \xi_3 = (0, 0, 1)^T.$$

- For $\hat{m} = (sin\alpha cos\beta, sin\alpha sin\beta, cos\alpha)^T$, and $U = exp(-\alpha t_2 - \beta t_3)$.
- The gauge potential and the gauge field strength, in magnetic gauge are obtained in their simple form as

$$\mathbf{W}_\mu \to (A_\mu + B_\mu)\hat{\xi}_3 \; and \; \mathbf{G}_{\mu\nu} \to (F_{\mu\nu} + B^{(d)}_{\mu\nu})\hat{\xi}_3$$

where $B_\mu = g^{-1} cos\alpha \partial_\mu \beta$ and $B^{(d)}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

- The part B_μ, is fixed completely by $\hat{m}$, is thus identified as the magnetic potential associated with the topological monopole and therefore the topological properties of $\hat{m}$ are brought down to a dynamical variable (B_μ).
- The fields, thus, appear in a completely dual symmetric way and induce a dual dynamics between color isocharges and the topological charges.

- With these considerations, for analyzing the color confinement mechanism, we use the SU(2) Lagrangian with a quark doublet source $\psi(x)$ given by:

$$\mathcal{L} = -\frac{1}{4}\mathbf{G}^2_{\mu,\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi - m_0 \bar{\psi}\psi,$$

- which, in the magnetic gauge, yields the dual symmetric field equations is given by

$$G_{\mu\nu},^{\nu} = F_{\mu\nu},^{\nu} = j_\mu \quad and \quad G^{(d)}_{\mu\nu},^{\nu} = B_{\mu\nu},^{\nu} = k_\mu$$

- In view to avoid the undesirable features of pointlikeness and the singular behaviour of potential, we introduce the dual magnetic potential $B^{(d)}_\mu$ coupled to a complex scalar field $\phi(x)$. The dual QCD Lagrangian becomes

$$\mathcal{L}' = \mathcal{L} + \left| \left[\partial_\mu + \frac{4\pi}{g}(A^{(d)}_\mu + B^{(d)}_\mu)\right] \phi\right|^2 - V(\phi^*\phi)$$

The Lagrangian in the quenched approximation takes the following form

$$\boxed{\mathcal{L}' = -\frac{1}{4}B_{\mu\nu}^2 + \left| \left[\partial_\mu + i\frac{4\pi}{g}B_\mu^{(d)}\right]\phi \right|^2 - V_{eff}(\phi^*\phi)} \quad (1)$$

where $V_{eff}(\phi^*\phi) = \Omega(\phi^*\phi - \phi_0^2)^2$
The field equations associated with the Lagrangian(5) are

$$\left(\partial_\mu - i\frac{4\pi}{g}B_\mu^{(d)}\right)\left(\partial^\mu + i\frac{4\pi}{g}B^{\mu(d)}\right)\phi + 2\Omega\phi(\phi\phi^* - \phi_0^2) = 0 \quad (2)$$

$$\partial^\nu B_{\mu\nu} + i\frac{4\pi}{g}(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi) - \frac{32\pi^2}{g^2}B_\mu^{(d)}\phi\phi^* = 0 \quad (3)$$

Considering the single flux tube solution using the cylinderical symmetry with the co-ordinates (ρ, φ, z), and the flux tube orientation inside the hadronic sphere along the direction of z-axis. For such system, the dual gauge field and the monopole field can be expressed as,

$$\mathbf{B}^{(\mathbf{d})}_{\mu} = \mathbf{g}^{-1}\mathbf{cos}\alpha(\partial_\mu\beta)\hat{\mathbf{m}} \tag{4}$$

$$\text{and} \quad \phi(x) = exp(in\varphi)\chi(\rho) \quad (n = 0, \pm 1, \cdots). \tag{5}$$

It leads to the following cylinderical components for the dual gauge field,

$$B^{(d)}_{\rho} = \frac{1}{g} cos\,\alpha\,\partial_\rho\,\beta\,, \;\; B^{(d)}_{\varphi} = \frac{1}{g} cos\,\alpha\,\frac{1}{\rho}\partial_\varphi\,\beta\,, \;\; B^{(d)}_{z} = \frac{1}{g} cos\,\alpha\,\partial_z\,\beta\,. \tag{6}$$

With the uniqueness of the function $\phi(x)$, the Nelson-Olesen ansatz for the dual gauge field and monopole field in static limit may be expressed as,

$$\mathbf{B}(\rho) = -\hat{\phi}B(\rho), \;\; as \;\; B_t = B_\rho = B_z = 0, \tag{7}$$

The color electric field along the z- direction then takes the following form,

$$E_m(\rho) = -\frac{1}{\rho}\frac{d}{d\,\rho}\Big(\rho\,B(\rho)\Big), \tag{8}$$

- Using the cylindrical symmetric form of the potential the field equations (6) and (7) may reduced to the following form

$$\chi^{''} + \frac{\chi^{'}}{\rho} - \left(\frac{n}{\rho} + \frac{4\pi}{g}B\right)^2 \chi + 2\Omega\chi(\chi^2 - \phi_0^2)^2 = 0 \tag{9}$$

$$B^{''} + \frac{B^{'}}{\rho} - \frac{B}{\rho^2} - \frac{8\pi}{g}\left(\frac{n}{\rho} + \frac{4\pi}{g}B\right) = 0. \tag{10}$$

- Dimensionless parameters:

$$r = 2\sqrt{\Omega}\phi_0\rho, \quad F(r) = \frac{4\pi\rho}{g}B(\rho), \quad H(r) = \frac{\chi(\rho)}{\phi_0}$$

Then the field equations become

$$H^{''} + \frac{H^{'}}{r} + \frac{(n+F)^2}{r^2}H + \frac{1}{2}H(H^2 - 1) = 0 \tag{11}$$

$$F^{''} - \frac{F^{'}}{r} - \gamma(n+F)H^2 = 0 \tag{12}$$

- Boundary conditions: $F \to -n, \quad H \to 1, \quad J \to 0 \;\; as \;\; r \to \infty$

- The equation for $F(\rho)$: $F(\rho) = -n + C\rho^{\frac{1}{2}} exp(-m_B^0(\rho))$ where $C = 2B\pi(3\sqrt{2}\lambda g^{-3}\phi^0)^{\frac{1}{2}}$. Since, this function associated with the color electric field through the gauge potential $B(\rho)$ it shows the emergence of DME and the confinement of color isocharges in magnetically condensed dual QCD vacuum.
- Energy per unit length:

$$k = 2\pi\phi_0^2 \int_0^\infty rdr \left[\frac{6\lambda}{g^2}\frac{F'}{r^2} + \frac{(n+F)^2}{r^2}H^2 + (H')^2 + \frac{1}{4}(H^2-1)^2\right] = \gamma\phi_0^2 \quad (13)$$

- Regge slope parameter: $\alpha' = (2\pi k)^{-1} = (2\pi\gamma\phi_0^2)^{-1}$

Mass scales and other parameters in dual QCD

- Onset of charaecteristic mass scales:

$$Vector\ mode:\ m_B = (\lambda_{QCD}^{(D)})^{-1},\quad Scalar\ mode:\ m_\phi = (\xi_{QCD}^{D})^{-1}$$

- Ratio: $\frac{m_\phi}{m_B} = \sqrt{3\lambda}(2\pi\alpha_s)^{-1/2}$, $G.L\ parameter:\ \kappa_{QCD}^{D} = \frac{\lambda_{QCD}^{D}}{\xi_{QCD}^{D}}$

α_s	$\phi_0(GeV)$	$m_B(GeV)$	$m_\phi(GeV)$	k_{QCD}
0.12	0.143	2.11	4.20	1.99
0.22	0.149	1.51	2.22	1.47
0.47	0.167	1.21	1.22	1
0.96	0.181	0.929	0.655	0.71

QCD thermofield dynamics and QGP

- Useful for exploring the true phase structure of QCD : Study of non-perturbative features under unusual conditions (High temperature/High density).
- Thermal behaviour of QCD: At low temperatures (T-Small): QCD system as a Gas of colourless Hadronic states (mainly Pions).
- T $\rightarrow$ increasing : Pion density grows, other Hadrons also appear in medium (Theoretical analysis - ?)
- T $\rightarrow$ high: Proper basis for theoretical analysis turns to the elementary fields in QCD Lagrangian (Quarks and Gluons) and not the Hadronic states.
- Hadrons get ionised to Quarks and Gluons. A Heat Bath System of quasi-free coloured particles with small effective couplings.
- Useful in understanding dynamics of QGP-phase of matter with its immediate implications for Early Universe in Big-Bang cosmology.

Basic thermodynamic relations

- Using the grand canonical ensamble formalism partition function for a thermodynamical system in thermal and chemical equilibrium is expressed as

$$Z(T, V, \mu) = Trexp[-(\hat{H} - \mu\hat{N})/T] = exp[-\Omega(T, V, \mu)/T]$$

- The thermodynamical potential is related to the grand canonical partition functions as

$$\Omega(T, V, \mu) = -TlnZ(T, V, \mu)$$

- The thermodynamical variables are related with the grand canonical partition function and expressed as,

$$P = \frac{\partial}{\partial V}(TlnZ), \varepsilon = \frac{T^2}{V}\frac{\partial}{\partial T}lnZ + \mu n, s = \frac{1}{V}\frac{\partial}{\partial T}(TlnZ)$$

$$n = \frac{T}{V}\frac{\partial}{\partial \mu}lnZ, C_v = \left(\frac{\partial \varepsilon}{\partial T}\right)_V, C_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{s(T)}{C_V(T)}$$

Phase transition parameters

$$(T\ln Z)_b = \frac{g_b V}{90}\pi^2 T^4, \; (T\ln Z)_f = \frac{g_f V}{24}\left(\frac{7}{30}\pi^2 T^4 + \mu^2 T^2 + \frac{1}{2\pi^2}\mu^4\right)$$

$$\Rightarrow (T\ln Z)_p = \frac{2V}{3}\left(\frac{\pi^2}{3}T^4 + \mu_q^2 T^2 + \frac{1}{2\pi^2}\mu_q^4\right) - BV$$

Using Grand Canonical Ensamble formalism and Bag model of hadrons, the bag parameter in the infrared sector of QCD as derived from magnetic symmetry based flux tube formulation of dual QCD, using confining part of the energy of a multiflux tube structure on a S^2-sphere with periodically distributed flux tube

$$B^{1/4} = \left(\frac{12}{\pi^2}\right)^{1/4}\frac{m_B}{8}$$

Energy density, pressure and entropy density

QGP phase

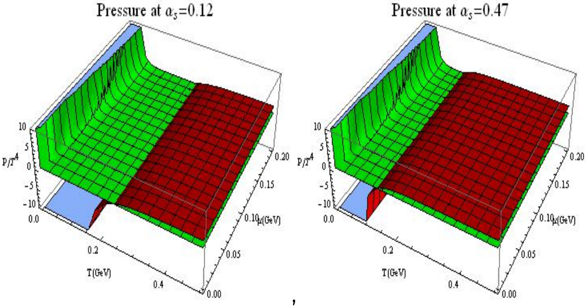
$$\epsilon_p = \frac{2}{3}\pi^2 T^4 + 2\mu_q^2 T^2 + \frac{\mu_q^4}{\pi^2} + B, \;\; P_p = \frac{2}{9}\pi^2 T^4 + \frac{2}{3} T^2 \mu_q^2 + \frac{\mu_q^4}{3\pi^2} - B$$

$$s_p = \frac{8}{9}\pi^2 T^3 + \frac{4}{3} T \mu_q^2$$

Hadronic phase

$$\epsilon_h = \frac{7}{60}\pi^2 T^4 + \frac{1}{2}\mu_q^2 T^2 + \frac{1}{4\pi^2}\mu_q^4, \;\; P_h = \frac{7}{180}\pi^2 T^4 + \frac{1}{6}\mu_q^2 T^2 + \frac{1}{12\pi^2}\mu_q^4$$

$$s_h = \frac{7}{45}\pi^2 T^3 + \frac{1}{3}\mu^2 T$$



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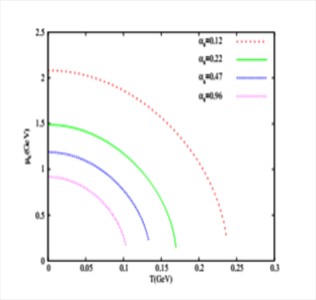
Gibbs criteria

$$P_h = P_p = P_c, \quad T_h = T_p = T_c, \quad \mu = 3\mu_q = \mu_c$$

$$\Rightarrow \frac{11}{60}\pi^2 T_c^4 + \frac{1}{18} T_c^2 \mu_c^2 + \frac{\mu_c^4}{324\pi^2} = B$$


2.5
2
1.5
1
0.5
0
0
0.05
0.1
0.15
0.2
0.25
0.3
T(GeV)
α_s=0.12
α_s=0.22
α_s=0.47
α_s=0.96

Critical parameters of QGP-phase transition

Table: Normalized pressure in (T-μ)*plane*

α_s	$T_c^0(GeV)$	$(\mu_c, T_c^\mu)(GeV)$	$(\mu_E, T_E^\mu)(GeV)$	$T_d(GeV)$
0.12	0.239	(0.66,0.071)	(0.29,0.236)	0.312
0.47	0.136	(0.35,0.065)	(0.22,0.133)	0.221
0.96	0.105	(0.25,0.060)	(0.16,0.103)	0.174

Normalised pressure difference, quark number density and susceptibility

$$\frac{\triangle P_p}{T^4} = \frac{P_p(T,\mu)}{T^4} - \frac{P_p(T,0)}{T^4} = \frac{2\mu_q^2}{3T^2} + \frac{\mu_q^4}{3\pi^2 T^4}$$

$$\frac{n_q}{T^3} = \frac{\partial P}{\partial \mu_q} = \frac{4}{3}\frac{\mu_q}{T} + \frac{4\mu_q^3}{3\pi^2 T^3}$$

$$\frac{\chi_q}{T^2} = \frac{\partial^2 P}{\partial \mu_q^2} = \frac{4}{3} + \frac{4\mu_q^2}{\pi^2 T^2}$$

α_s	$\Delta P_p/T^4$	n_q/T^3	χ_q/T^2
0.12	309.6	120.8	24.4
0.47	47.7	28.2	5.44
0.96	21.7	15.3	3.09

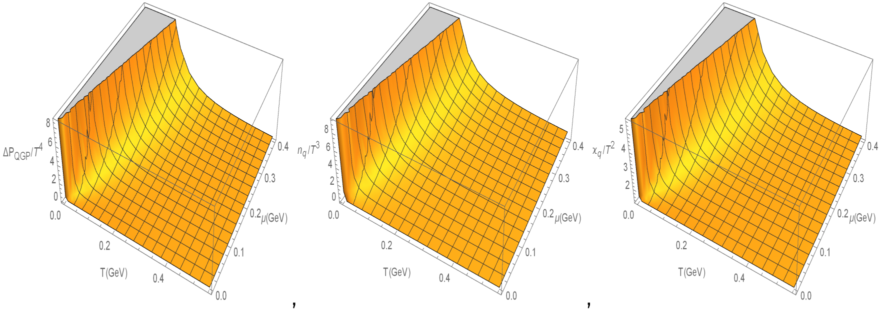


Figure: Normalised pressure difference, quark number density and susceptibility

Energy density, entropy density, specific heat and speed of sound

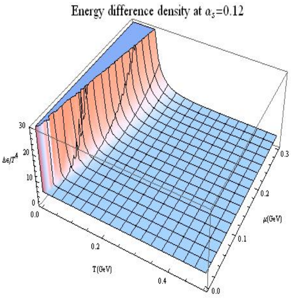
$$\Delta\epsilon = \epsilon_p(T_c^\mu) - \epsilon_h(T_c^\mu) = \frac{33}{60}\pi^2 T_c^{\mu^4} + \frac{3}{2}\mu_q^2 T_c^{\mu^2} + \frac{3}{4\pi^2}\mu_q^4 + B$$

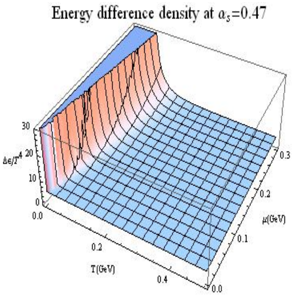
$$\Delta s = s_p T_c^\mu - s_h T_c^\mu = \frac{11}{15}\pi^2 T_c^3 + \frac{2}{3}\mu_q^2 T_c^\mu$$

$$\frac{C_V}{T^3} = (8\pi^2/9 + 4\mu_q^2/3T^2)\frac{(2\pi^2 T^4/3 + 2\mu_q^2 T^2 + \mu_q^4/\pi^2 + B)}{(2\pi^2 T^4/9 + 2T^2\mu_q^2/3 + \mu_q^4/3\pi^2 - B)}$$

$$C_s^2 = \frac{(2\pi^2 T^4/9 + 2T^2\mu_q^2/3 + \mu_q^4/3\pi^2 - B)}{(2\pi^2 T^4/3 + 2\mu_q^2 T^2 + \mu_q^4/\pi^2 + B)}$$

α_s	$\Delta\epsilon(GeV/fm^3)$	$\Delta s(GeV/fm^2)$	$C_s^2(\mu_c, T_c^\mu)$	$C_v/T^3(\mu_c, T_c^\mu)$
0.12	3.10	0.60	2659.4	0.069
0.47	0.35	0.19	1360.9	0.081
0.96	0.12	0.10	1316.4	0.077


Energy difference density at α_s=0.12
Δε/T⁴
T(GeV)
μ(GeV)


Energy difference density at α_s=0.47
Δε/T⁴
T(GeV)
μ(GeV)

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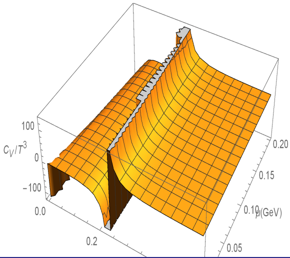
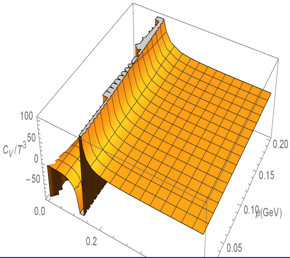
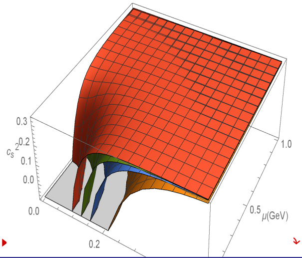
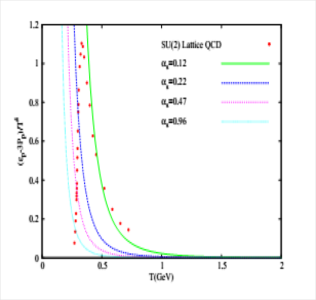

C_V/T^3
μ(GeV)


C_V/T^3
μ(GeV)


$c_s{}^2$
μ(GeV)

Trace anomaly and conformal measure

$$\Delta(T) = \frac{\epsilon_p - 3P_p}{T^4}, \quad \zeta = 1 - \frac{3P_p}{\epsilon_p}$$



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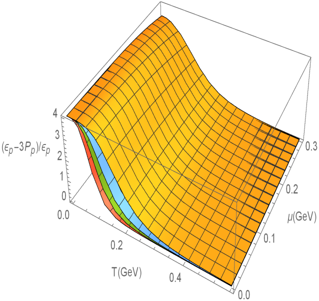


Conclusions

- Dual QCD based on topological structure of gauge theory+magnetic symmetry.
- Dynamical breaking of magnetic symmetry $\rightarrow$ Confinement (flux tube structure.)
- Thermal response of dual QCD and non perturbative vacuum.
- Grand canonical ensamble formalism for QGP phase transition with $\mu \neq 0$, Equation of state constructed with hadronic bag.
- Gibbs criteria $\rightarrow$ critical parameters for phase transition.
- T and $\mu \rightarrow$ complimentary behavior.
- Around critical values T_c and μ_c, pressure derivatives (ΔP_p, n_q, χ_q) show peaks with temperature decreasing monotonously.
- Growing fluctuations in mass quantities are supressed considerably as the CEP is reached so that the first order phase transition is expected to reconcile into an analytic crossover.

- Similar results from normalised energy density and entropy density as (jump around critical point providing finite latent heat during phase transition). Similar behavior of specific heat (temperature derivative of energy density) converging to $4\epsilon/T^4$ as high T.

$C_s^2 \rightarrow$ a sensitive indicator of the critical behavior or softening of equation of state at critical point.

At high T $\rightarrow C_s^2 \approx \frac{1}{3}$ (ideal real gas) $\Rightarrow$ Conformal symmetry.

At $\mu \neq 0$ and low T $\rightarrow C_s^2$ drops to minimum $\rightarrow 0$ as critical point is approximated.

$\Rightarrow$ Deviation or breaking from conformal symmetry estimated via trace anomaly and conformal measure.

References

G.'t Hooft: Nucl. Phys. **B190**, 455 (1981).

Y.M. Cho: Phys. Rev. **D21**, 1080 (1980); ibid **D23**, 2415 (1981).

H.C. Panndey and H.C. Chandola: Phys. Lett. **B476**, 193 (2000).

H.C. Chandola, D.Yadav, H.C.Pandey and H. Dehnen: Int. J. Mod. Phys. **A20**, 2743 (2005).

H.C. Chandola and D. Yadav: Nucl. Phys. **A829**, 151 (2009).

H.C. Chandola, G. Punetha, H. Dehnen: Nucl. Phys. **A945**, 226 (2016).

H. Neilsen and B. Olesen: Nucl. Phys. **B61**, 45 (1973).

Journey to be continued...

Thanks for your kind attention

Free energy change for quark-hadron phase transition

$$F_{q,g} = -\left[\frac{2}{9}\pi^2 T^4 + \frac{2}{3}\mu_q^2 T^2 + \frac{1}{3\pi^2}\mu_q^4\right] V$$

$$F_{interface} = \frac{1}{4} R^2 T^3 \gamma$$

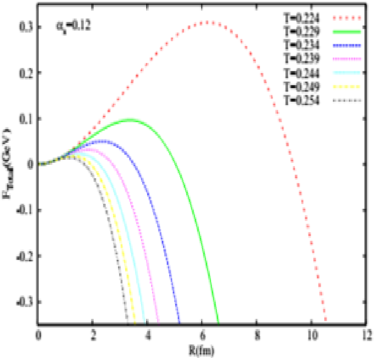
$$F_h = -\left[\frac{7}{180}\pi^2 T^4 + \frac{1}{6}\mu_q^2 T^2 + \frac{1}{12\pi^2}\mu_q^4\right] V$$

$$F_{Total} = F_{q,g} + F_h + F_{interface} + BV$$

$$\Delta F = -\frac{4\pi R^3}{3}[P_{had}(T,\mu_B) - P_{QGP}(T,\mu_B)] + 4\pi R^2 \sigma + \tau_{crit} T ln\left[1 + \frac{4\pi R^3}{3} s_{q,g}\right]$$

$$\sigma = \frac{3\Delta F_c}{4\pi R_c^2}$$

α_s	$T_c^0(GeV)$	$R_c(fm)$	$\Delta F_c(GeV)$	$\sigma^{1/3}(GeV)$
0.12	0.239	1.8307	0.03261	0.04483
0.47	0.136	3.2551	0.01899	0.02551
0.96	0.105	4.2162	0.01466	0.01969



α_s=0.12
T=0.224
T=0.229
T=0.234
T=0.239
T=0.244
T=0.249
T=0.254
R(fm)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
2
4
6
8
10
12