A Global ILC Approach in Pixel Space using CMB Covariance Matrix at Large Angular Scales

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The Usual ILC Method - In Pixel Space

- A model independent method.
- The total number of available frequency bands be n
- Observed CMB map x_i at a given frequency can be modelled as:

$$\mathbf{X}_i(p) = \mathbf{Y}(p) + \mathbf{F}_i(p) + \mathbf{N}_i(p) \tag{1}$$

- A cleaned map is defined in as linear superposition of input maps,

$$\mathbf{Y}(p) = \sum_i w_i \mathbf{X}_i(p) \tag{2}$$

  with weights constrained by $\sum_{i=1}^{n} w_l^i = 1$
- Choice of Weights which minimizes the variance of cleaned map is given by,

$$\mathbf{W} = \frac{\mathbf{e}\mathbf{A}^{\dagger}}{\mathbf{e}\mathbf{A}^{\dagger}\mathbf{e}^{T}} \tag{3}$$

  where, **A** is a n×n matrix with its elements $A_{ij} = \mathbf{x}_i^T \mathbf{x}_j$ and $[e] = (1, 1, ..., 1)_{1\times n}$
- We use Planck three LFI maps, four HFI maps and WMAP 5 maps at N_{side} 16 with beam smoothing 9 degrees.

Cleaned Map - ILC Method

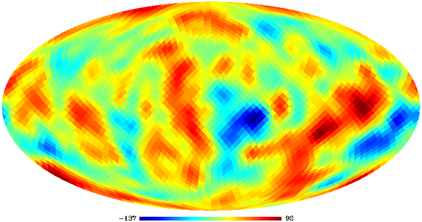

Figure 1: Cleaned CMB map obtained using ILC method.

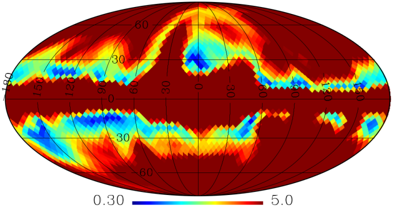

Figure 2: Reconstruction error in the Cleaned CMB map obtained using ILC method estimated through Monte Carlo simulations.

CMB-Foregrounds Chance Correlation

- We consider two cases:
  - Case 1

$$\hat{\mathbf{A}} = \hat{\mathbf{C}}_{fc} + \hat{\mathbf{C}}_f$$

The weights are obtained as follows,

$$\mathbf{W} = \frac{\mathbf{e}\hat{\mathbf{A}}^{\dagger}}{\mathbf{e}\hat{\mathbf{A}}^{\dagger}\mathbf{e}^T} \tag{4}$$

  - Case 2

$$\hat{\mathbf{A}} = \hat{\mathbf{C}}_f$$

The weights are obtained as follows,

$$\mathbf{W} = \frac{\mathbf{e}\hat{\mathbf{A}}^{\dagger}}{\mathbf{e}\hat{\mathbf{A}}^{\dagger}\mathbf{e}^T} \tag{5}$$

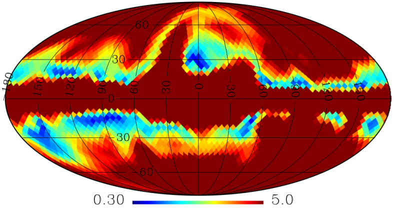

Figure 6: Standard Deviation map obtained in Case 1.

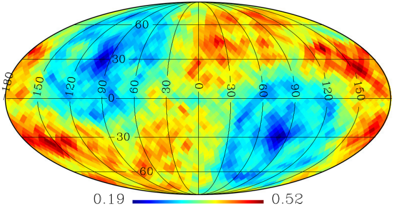

Figure 3: Standard Deviation map obtained in Case 2.

A Global ILC Method

- We incorporate the CMB Covariance structure information in the usual ILC method in pixel space.
- Instead of minimizing the cleaned map variance, we propose a new measure,

$$\sigma^2 = \mathbf{WAW}^T \tag{6}$$

where **W** = $(w_1, w_2, .., w_n)$ with a constraint $\sum_i w_i \ = \ 1.$

- Here **A** is a n×n matrix with its elements A_{ij} given by,

$$A_{ij} = \mathbf{X}_i^T \mathbf{C}^{\dagger} \mathbf{X}_j \tag{7}$$

$$C_{ij} = \sum_{l=2}^{l_{max}} \frac{2l+1}{4\pi} C_l B_l^2 \mathcal{P}_l(cos(\gamma_{ij})) P_l^2 \tag{8}$$

where

$$cos(\gamma_{ij}) = cos(\theta_i)cos(\theta_j) + sin(\theta_i)sin(\theta_j)cos(\phi_i - \phi j) \tag{9}$$

Methodology

- We use Planck three LFI maps, four HFI maps and WMAP 5 maps at N_{side} 16 with beam smoothing 9 degrees.
- Method 1
  We use entire sky in the foreground removal.
- Method 2
  We divide the sky into two regions:
  - Galactic Region
  - Outer Region

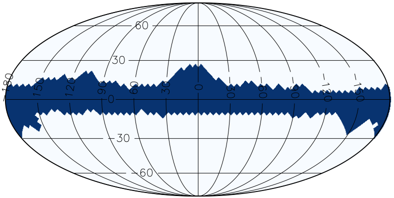

Figure 4: ThDust Mask 5000

- We obtain the CMB Covariance matrix for the sky region defined by ThDust 5000 mask.
- We obtain the Cleaned map corresponding to this sky region using our method.
- We replace the ThDust sky region of all input maps by the cleaned region obtained above.
- Using this as the new set of input maps, we find the full-sky cleaned map.

Results - Cleaned Map

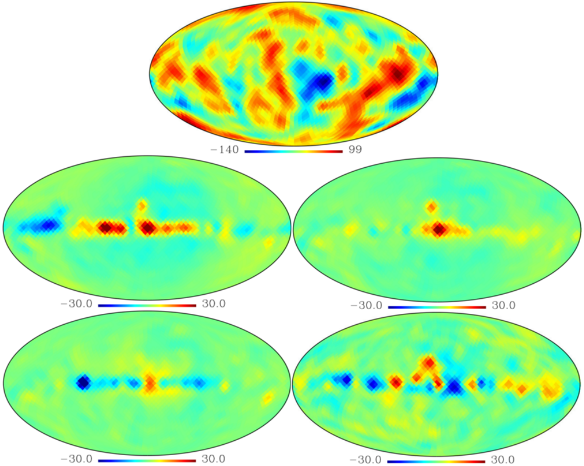

Figure 5: Foreground cleaned CMB map (CMap1) obtained following Method-1 is shown in the top panel. The middle left and middle right figures show the difference maps COMMANDER - CMap1 and NILC - CMap1 respectively. The bottom panel from left to right show SMICA - CMap1 and WMAP ILC - CMap1.

Results - Difference Map

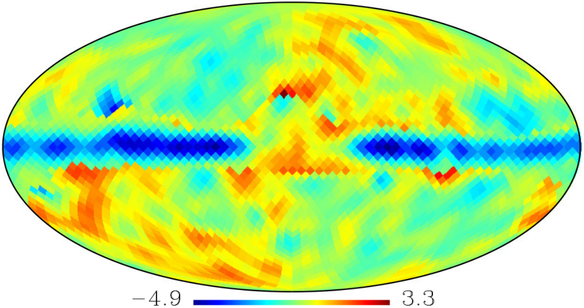

Figure 6: Difference between CMAP2 and CMAP1. CMAP2 appears to have lesser foreground contamination along the both sides of galactic plane.

Results - CMB Angular Power Spectrum

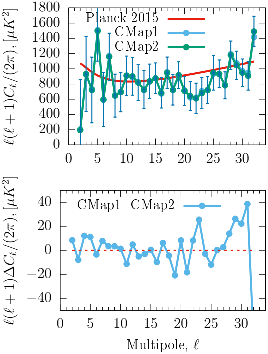

Figure 7: PS obtained from both the methods

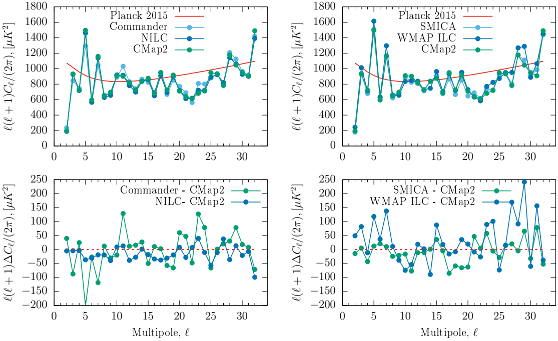

Figure 8: PS obtained from our method compared with PS obtained by Commnander, NILC, SMICA and WMAP

Results - Error Maps

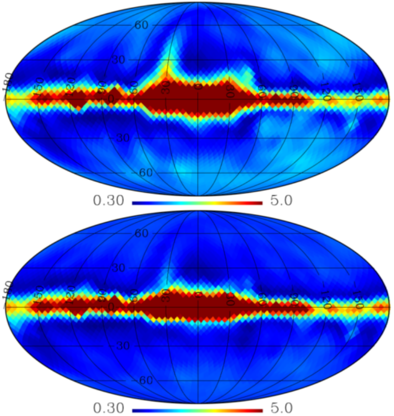

Figure 9: Top panel shows standard deviation map obtained from the difference of cleaned map and corresponding randomly generated input CMB map using 200 MC simulations of foreground minimization following Method-1 and bottom for Method-2.

Results - Average Angular Power Spectrum

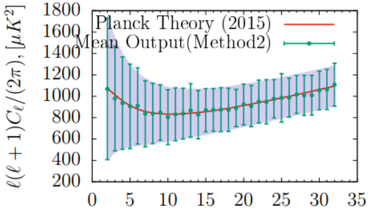

Figure 10: CMB power spectrum (in green) obtained from 200 Monte Carlo simulations of Method2 along with the theoretical CMB angular power spectrum (red line) consistent with Planck 2015 results

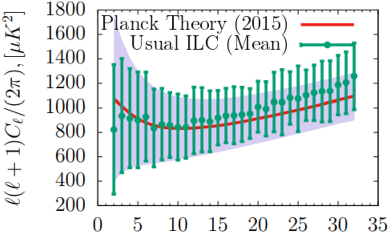

Figure 11: CMB power spectrum (in green) obtained from 200 Monte Carlo simulations of usual ILC approach along with Planck 2015 results

Weights

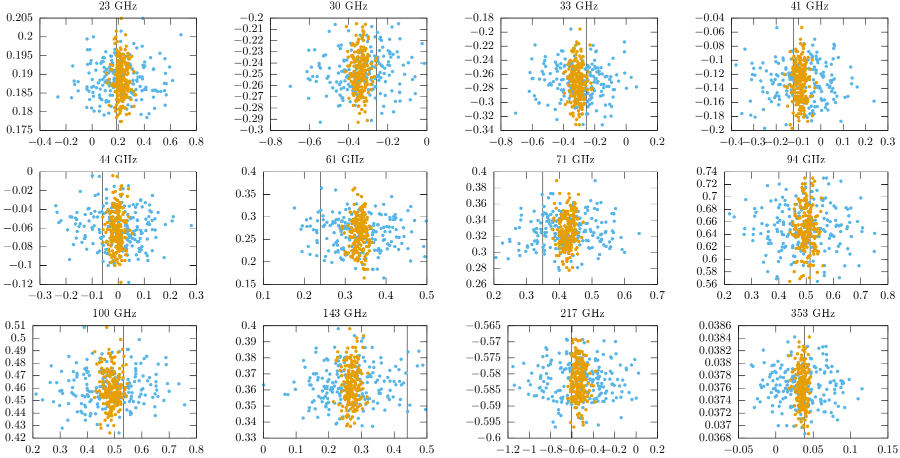

Figure 11: Scatter plot obtained from Monte Carlo simulations showing lower dispersion (along horizontal axis) of weights (yellow) when we follow global ILC method with prior information from theoretical CMB covariance matrix on large scales on the sky for different WMAP and Planck frequency bands. The y coordinates of blue or yellow points represent weights obtained from Monte Carlo simulations using Eqn. 5. The larger dispersion of blue points along the horizontal axes causes larger reconstruction error in cleaned maps for usual ILC methods at low resolution. The vertical lines represent values of weights obtained using Eqn. 5 but without any detector noise in the simulations.

Summary

- Cleaned CMB maps and it power spectrum has lower reconstruction error.
- Standard deviations of CMB angular power spectrum estimated from MC simulations agrees with those estimated by the cosmic variance.
- Angular power spectrum does not have any visible negative bias.
- We can use this procedure for removing foregrounds from the CMB Polarization maps.

References

- Sudevan, V. & Saha, R., 2018, ApJ867, 74
- Sudevan, V., et.al. 2017, ApJ, 842, 62
- Sudevan, V. & Saha, R., [Accepted in ApJ]
- Saha, R., et.al. 2010, ApJ, 714, 840

Thank You !!!!!