Unintegrated dipole gluon distribution (TMD PDF) at small transverse momentum

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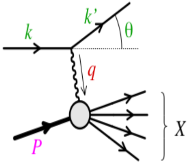
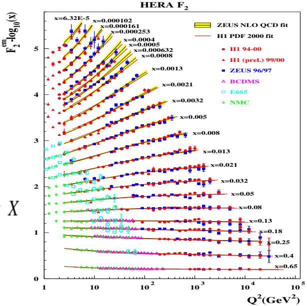
Outline

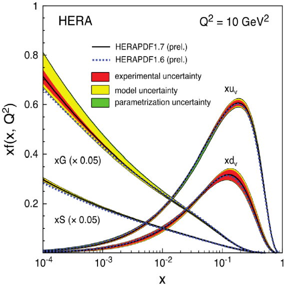
- Deep inelastic scattering
- QCD evolution equations
- Balistky-Kovchegov equation
- Parton distribution functions
- Unintegrated gluon distribution functions
- Conclusion

Deep Inelastic scattering (DIS) and QCD evolutions

- Deep inelastic scattering is the process used to probe the insides of hadrons (particularly the baryons, such as protons and neutrons), using highly energetic leptons.



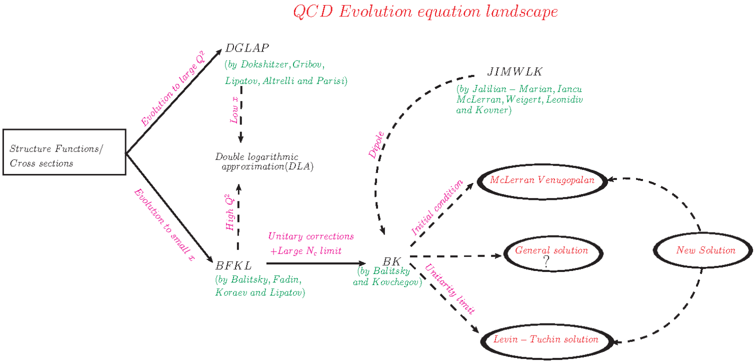




- Scattering cross section:

$$\frac{d^2\sigma^{ep}}{dE'\,d\Omega} = \frac{4\alpha^2 E'^2 \cos^2(\theta/2)}{q^4 M_P \nu}\left[2\nu \tan^2(\theta/2)\;\; F_1(x,Q^2) + M_P\;\; F_2(x,Q^2)\right]$$

QCD evolution landscape



Balistky-Kovchegove (BK) equation

- High energy evolution of S-matrix in the large N_c:

$$\frac{\partial S\left(x_{10}, Y\right)}{\partial Y} = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2} \left[S\left(x_{02}, Y\right) S\left(x_{12}, Y\right) - S\left(x_{10}, Y\right)\right]$$

Solutions and initial conditions of BK equation:

- McLerran Venugopalan

Outside Saturation region $(r_\perp Q_s \ll 1)$,

$$S(r_\perp, Y) = \exp\left(-\kappa\ r_\perp^2 Q_s^2(Y)\right)$$

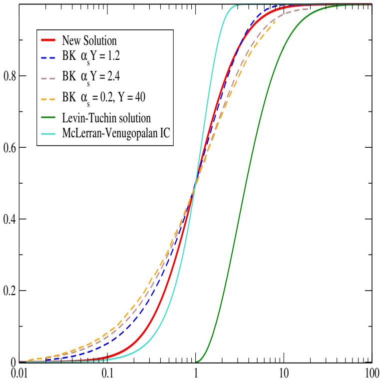
- Levin-Tuchin Solution

Deep inside the saturation region $(r_\perp Q_s \gg 1)$

$$S(r_\perp, Y) = \exp\left(-\tau\ \ln^2\left[r_\perp^2 Q_s^2(Y)\right]\right)$$

- New Solution

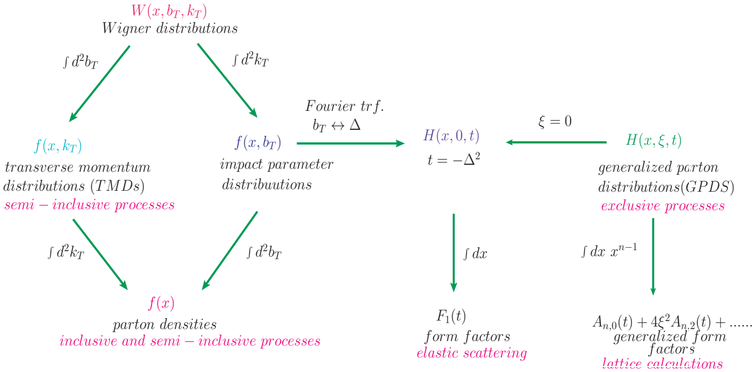
$$\boxed{S(r_\perp, Y) = \exp\left(\tau Li_2\left[-\lambda\ r_\perp^2 Q_s^2(Y)\right]\right)}$$



Parton distribution Functions (PDFs)

- Nucleon structure functions can be written in terms of the PDFs,

$$F_{2N}^{EM}(x) = 2xF_{1N}^{EM}(x) = \sum_i e_i^2 x\,[q_i(x) + \bar{q}_i(x)]$$



Unintegrated gluon distribution function

In small-x physics, two gluon distributions are widely used

1. Weizsacker Williams gluon distribution:

$$xG^{WW}(x,k_\perp) = \frac{S_\perp}{\pi^2\alpha_s}\frac{N_c^2-1}{N_c}\int\frac{d^2r_\perp}{(2\pi)^2}\frac{e^{-ik_\perp.r_\perp}}{r_\perp^2}N(x,r_\perp)$$

2. Color Dipole gluon distribution:

$$xG^{DP}(x,k_\perp) = \frac{S_\perp N_c}{2\pi^2\alpha_s}k_\perp^2\int\frac{d^2r_\perp}{(2\pi)^2}\quad e^{-ik_\perp.r_\perp}S(x,r_\perp)$$

Operator definition

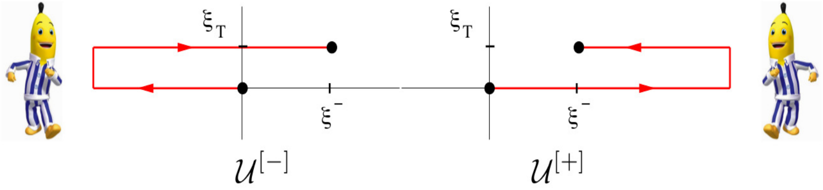
In terms of operators, two gauge invariant gluon definitions:

1. Weizsacker Williams gluon distribution:

$$xG^{WW}(x,k_\perp) = 2\int \frac{d\xi^- d^2\xi\perp}{(2\pi)^3 P^+} \quad e^{ixP^+\xi^- - ik_\perp.\xi_\perp}\langle P|Tr\Big[F^{+i}(\xi^-,\xi_\perp)U^{[+]\dagger}F^{+i}(0,0)U^{[+]}\Big]|P\rangle$$

2. Color Dipole gluon distribution:

$$xG^{DP}(x,k_\perp) = 2\int \frac{d\xi^- d^2\xi\perp}{(2\pi)^3 P^+} \quad e^{ixP^+\xi^- - ik_\perp.\xi_\perp}\langle P|Tr\Big[F^{+i}(\xi^-,\xi_\perp)U^{[-]\dagger}F^{+i}(0,0)U^{[+]}\Big]|P\rangle$$



TMD at small transverse momentum

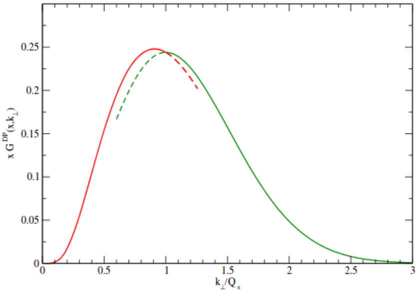
- Initiating with Color dipole gluon distribution, at small transverse momentum, we picked Levin-Tuchin (LT) solution of S-matrix :

$$xG^{DP}(x,k_\perp) = \frac{S_\perp N_c}{2\pi^2\alpha_s} k_\perp^2 \int \frac{d^2 r_\perp}{(2\pi)^2} \; e^{-ik_\perp . r_\perp} \exp\left(-\tau \ln^2 \left[r_\perp^2 Q_s^2(Y)\right]\right)$$

Under leading log approximation, the solution took the form:

$$\boxed{xG^{DP}(x,k_\perp)|_{Q_s > k_\perp \gg \lambda_{QCD}} \approx -\frac{S_\perp N_c \tau}{\pi^3 \alpha_s} \ln\left(\frac{k_\perp^2}{4Q_s^2(Y)}\right) \exp\left[-\tau \ln^2\left(\frac{k_\perp^2}{4Q_s^2(Y)}\right)\right]}$$

- Inside the saturation region, the dipole gluon distribution is expected to go to zero $k_\perp^2 \to 0$.
- When resumming the series in leading log accuracy, the results showing up striking similarity with the Sudakov form factor.



Conclusion

↬ We derived a general form of solution which reproduce both McLerran-Venugopalan initial conditions and Levin-Tuchin solution, in their appropriate limits.

↬ This new solution involving dilogarithm function connects both this limits smoothly and better approximates the numerical estimation of full leading order Balitsky-Kovchegov equation particularly inside saturation region.

↬ Inside the saturation region, the dipole gluon distribution is expected to go to zero as $k_\perp^2 \to 0$.

↬ When resumming the series in leading log accuracy, the results showing up striking similarity with the Sudakov form factor.


Thank You