



# Plan

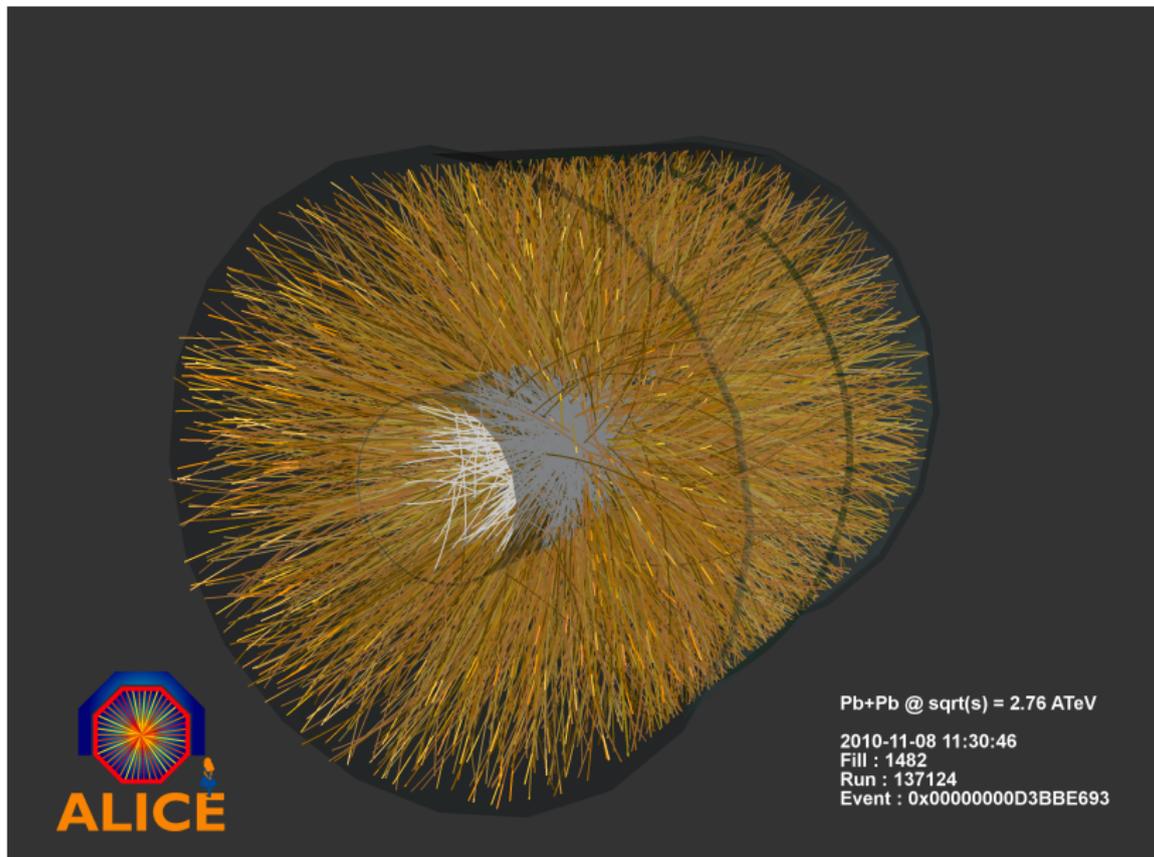
Introduction and motivation

Jet quenching and jet broadening

Fluctuations

Conclusions

# Difficulties in heavy ion collisions



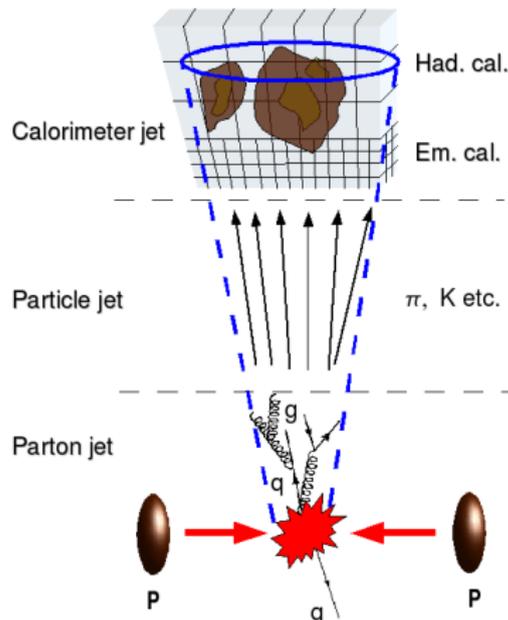
# Difficulties in heavy ion collisions

- ▶ We want to understand QGP.
- ▶ This is just **one of the many stages** that takes place in heavy ion collisions.
- ▶ At the end what we see in the detectors is the same type of particles as in proton-proton collisions, but **a lot of them**.

# Jets as a probe of the medium

We need to define observables that:

- ▶ Are easy to measure in heavy-ion collisions?
- ▶ Contain information about the medium.



Can we learn something about the medium by observing jets in heavy-ion collisions?

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Introduction and motivation

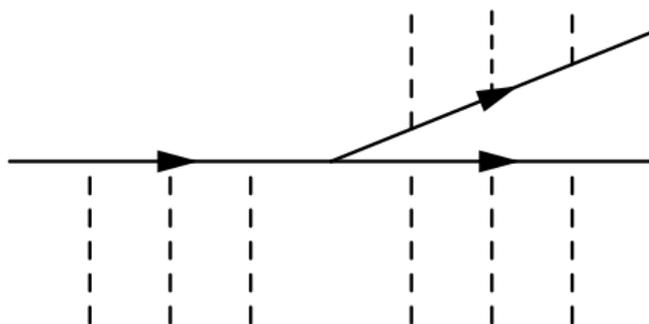
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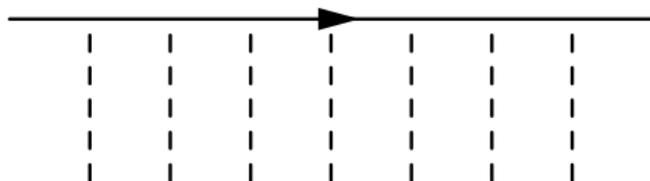
# Jet quenching

With jet quenching we refer to the energy that a jet loses when traversing a medium.



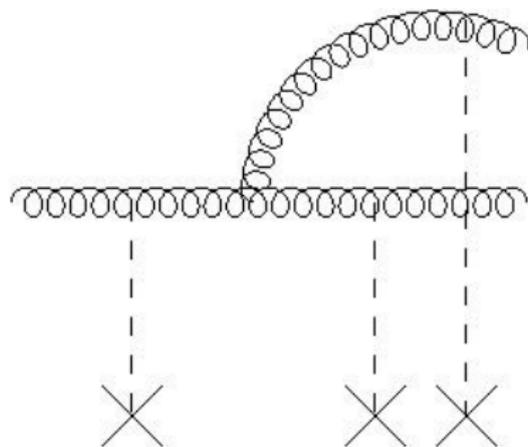
- ▶ Plane lines are partons with a high energy.
- ▶ Dashed lines are interaction with the medium with energy of order  $T$  (the temperature).

# Jet broadening



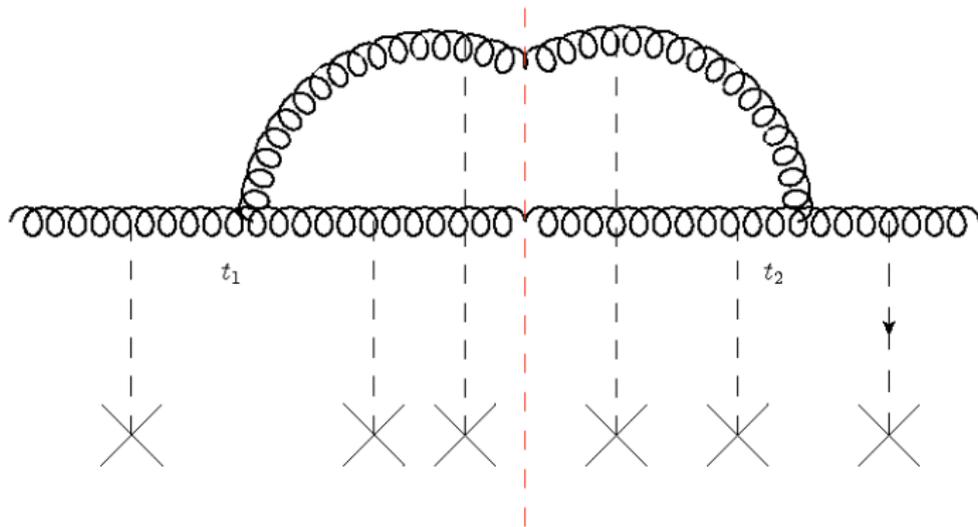
- ▶ In light-cone coordinates  $(p^+, p^-, p_\perp)$  the initial parton has  $(0, E, 0)$ , after the interaction with the medium it changes to  $(\frac{k_\perp^2}{2E}, E, k_\perp)$ . This is called jet broadening.
- ▶ It is different from jet quenching but in order to compute jet quenching one needs information on jet broadening.
- ▶ Can be seen as a Brownian motion in the transverse direction.  $\langle k_\perp^2 \rangle = \hat{q}L$ ,  $\hat{q}$  is a property of the medium.

## General picture



- ▶ All the needed information from the medium is encoded in  $\hat{q}$  and the length  $L$ .
- ▶ Emission probability given by the BDMPS-Z theory.  
Baier, Dokshitzer, Mueller, Peigné, Schiff, Nucl. Phys. B483, 291 and Zakharov, JETP Lett. 63 952.

## Medium-induced gluon emission: formation time



- ▶  $\tau_f = t_1 - t_2$ . By uncertainty relation  $\frac{1}{\tau_f} \sim \frac{k_{\perp}^2}{2\omega}$ .
- ▶ In a medium the acquired transverse momentum is  $k_{\perp}^2 \sim \hat{q}\tau_f$ .
- ▶  $\tau_f \sim \sqrt{\frac{2\omega}{\hat{q}}}$

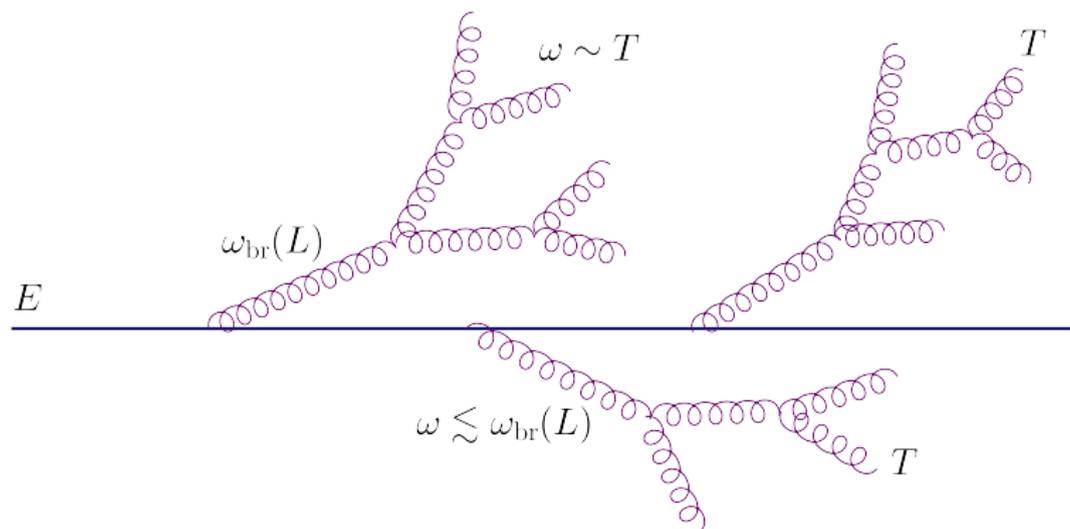
## Branching time

The probability of emitting a gluon  $P(\omega, \Delta t) \sim \mathcal{O}(1)$  when  $\Delta t =$  branching time  $\tau_{br}(\omega)$

$$\tau_{br}(\omega) \sim \frac{1}{\alpha_s} \tau_f(\omega) \sim \frac{1}{\alpha_s} \sqrt{\frac{\omega}{\hat{q}}}$$

- ▶  $\tau_{br} = L$  when  $\omega = \omega_{br} \equiv \frac{\alpha_s^2 N_c^2 \hat{q} L^2}{\pi^2}$ : "branching energy".
- ▶ Gluons with  $\omega \sim \omega_{br}(L)$  dominate the energy loss of the jet.
- ▶ Via their subsequent branching they transfer the energy from the leading particle to the medium.
- ▶ Soft gluons with  $\omega \ll \omega_{br}(L)$  are abundantly emitted but carry only little energy.

## Multiple branching



Successive branching are quasi-independent  $\rightarrow$  a Markovian stochastic process. Blaizot, Dominguez, Iancu and Mehtar-Tani JHEP06(2014)075.

# The gluon spectrum

Energy density per unit  $x \equiv \frac{\omega}{E}$

$$D(x, t) = x \left\langle \sum_i \delta(x_i - x) \right\rangle$$

$$\frac{\partial}{\partial \tau} D(x, \tau) = \int dz \mathcal{K}(z) \left[ \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \tau\right) - \frac{z}{\sqrt{x}} D(x, \tau) \right]$$

where  $\tau = \frac{\alpha_s N_c}{\pi} \sqrt{\hat{q}} t = \frac{t}{\tau_{br}(E)}$ .

Two cases:

- ▶  $\tau_{br}(E) \gg L$ . Interesting for LHC physics.
- ▶  $\tau_{br}(E) \sim L$ . Interesting to study the case in which the jet is completely absorbed by the medium.

This equation also has an important role in bottom-up thermalization. Baier, Mueller Schiff and Son  
*Phys.Lett.B502(2001) 51-58.*

# The gluon spectrum

- ▶ Due to the assumptions the formulas are only valid up to an infrared cut-off  $x_0 \sim \frac{T}{E}$ .
- ▶  $E \left( 1 - \int_{x_0}^1 dx D(x, \tau) \right)$  is the energy that is lost to the medium.
- ▶ Setting  $x_0 = 0$  in the formulas is a good approximation.

## The gluon spectrum

With the initial condition  $D(x, 0) = \delta(x - 1)$  there is an analytic solution with an approximate kernel  $\mathcal{K} \rightarrow \mathcal{K}_0$ .

$$D(x, \tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}} \exp\left\{-\frac{\pi\tau^2}{1-x}\right\}$$

Average energy inside the jet

$$E\langle X(\tau) \rangle = E \int_0^1 dx D(x, \tau) = E e^{-\pi\tau^2},$$

Then the average energy loss for  $\tau \ll 1$  is

$$E(1 - \langle X(\tau) \rangle) = E\pi\tau^2 = \frac{\alpha_s^2 N_c^2 \hat{q} t^2}{\pi}.$$

Blaizot, Iancu and Mehtar-Tani *Phys.Rev.Lett* 111, 052001

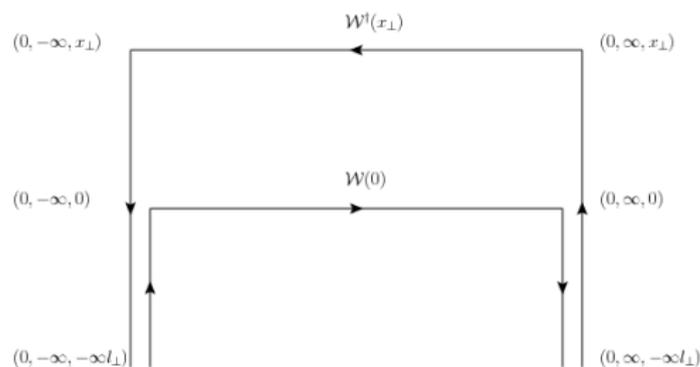
# The broadening probability

- ▶ We have seen that the energy loss is proportional to  $\hat{q}$ . Is it possible to compute it?

$P(k_{\perp})$  is the probability that an initial parton with momentum  $(0, E, 0)$  transforms into a parton with momentum  $(\frac{k_{\perp}^2}{2E}, E, \mathbf{k}_{\perp})$ .  $\hat{q}$  can be defined as

$$\hat{q} = \frac{1}{L} \int \frac{d^2 k_{\perp}}{(2\pi)^2} k_{\perp}^2 P(k_{\perp})$$

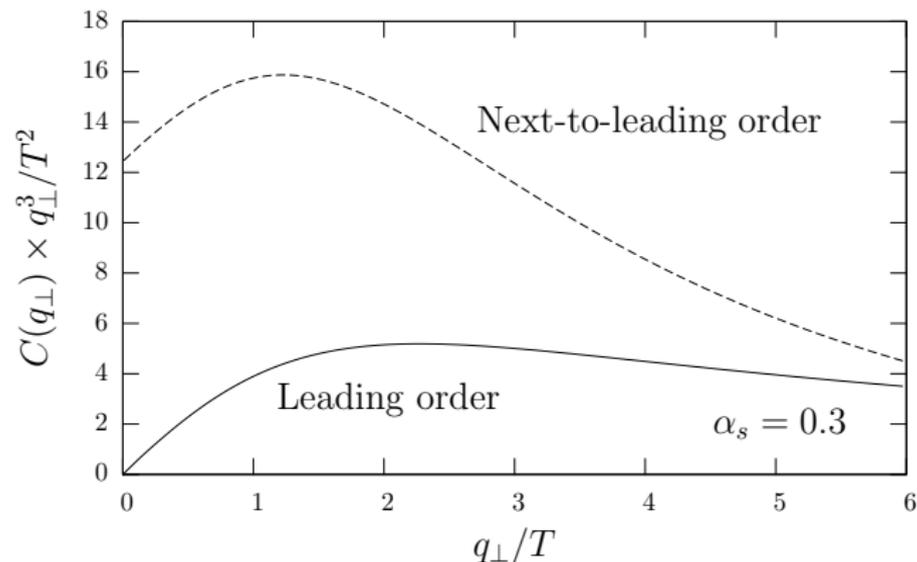
# Gauge-invariant $P(x_{\perp})$



- ▶ Lines in the diagram represent Wilson lines.
- ▶ Result in normal gauges: Baier et al. (1997), Zakharov (1996), Casalderrey-Solana and Salgado (2007). Gauge-invariant generalization (Brambilla, Benzke, M.A.E. and Vairo (2013)) (also valid for  $A^+ = 0$  gauge).

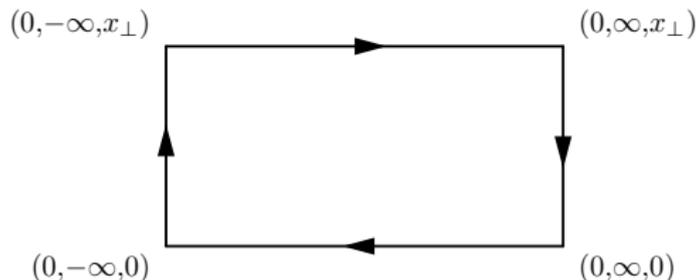
## $\hat{q}$ in perturbation theory

Computed in Arnold and Xiao (2008) and Caron-huôt (2009) (plot taken from here)



Caveats:  $g$  is not so small. The scale  $g^2 T$  is non-perturbative even if  $g \rightarrow 0$  and  $\hat{q}$  is sensitive to it.

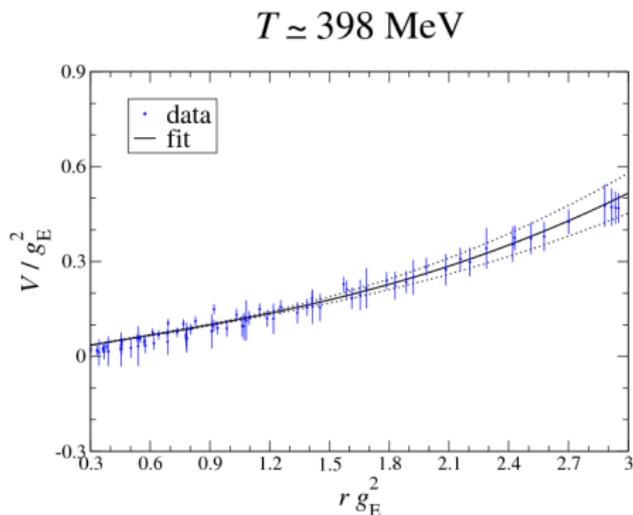
## $P(x_{\perp})$ in Lattice QCD



- ▶ In Caron-Huôt (2009) it was shown that the contribution from the scales below  $\pi T$  to  $P(k_{\perp})$  can be computed in Euclidean space without need of analytic continuation.
- ▶ It is possible to use lattice computations to learn about jet broadening.

# $P(x_{\perp})$ in Electrostatic QCD and in Lattice QCD

Results from Panero, Rummukainen and Schäfer (2013)



$$V = - \lim_{L \rightarrow \infty} \frac{1}{L} \log P(x_{\perp}, L)$$

$$g_E^2 = g^2 T$$

A similar study has been made in Classical Lattice QCD (Laine and Rothkopf (2013)). A computation with a slightly different definition of  $\hat{q}$  can be found in Majumder (2012).

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Introduction and motivation

Jet quenching and jet broadening

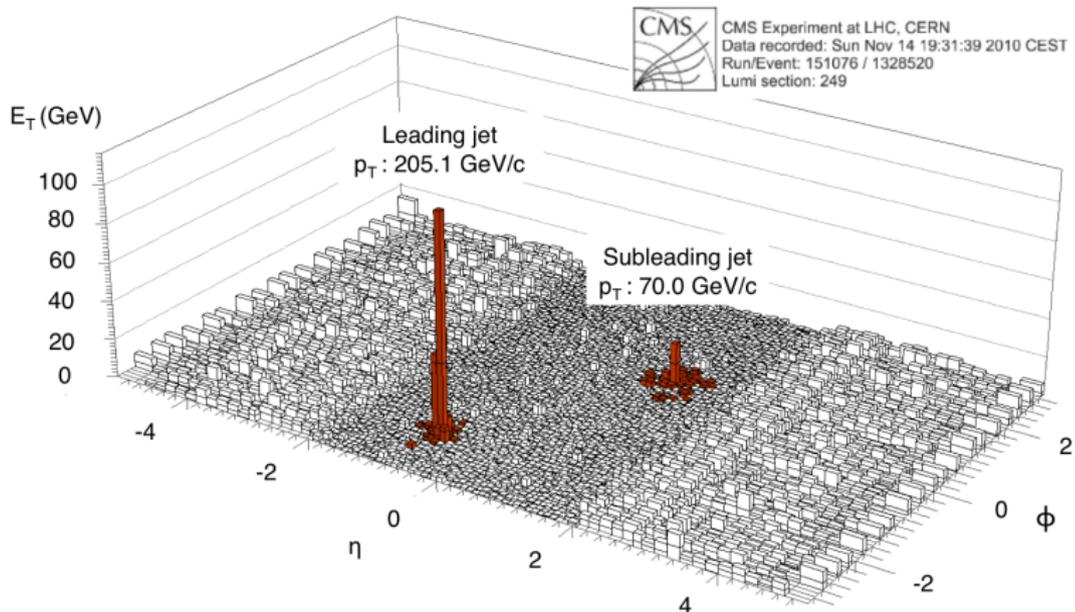
**Fluctuations**

Conclusions

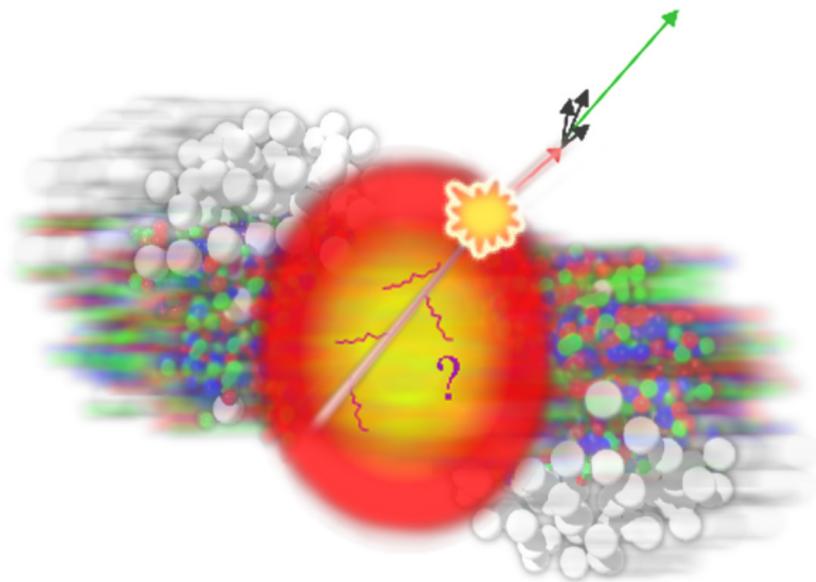
# The average energy loss

- ▶ The average energy loss is proportional to  $\hat{q}$ .
- ▶  $\hat{q}$  is well-defined in terms of QCD operators and it is even possible to extract information non-perturbatively.
- ▶ Are all experimental observations related to the average energy loss?

# Dijet asymmetry



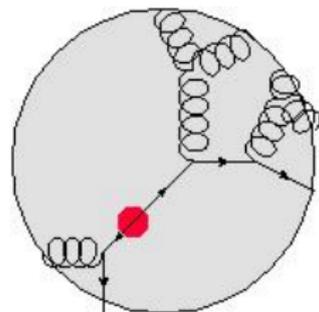
## Dijet asymmetry, the generally expected picture



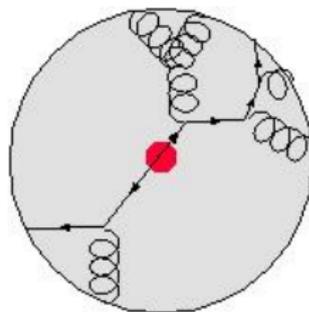
Mediums of different sizes are seen by the two jets → asymmetry, but that might not be the full story.

## Fluctuations in size vs fluctuations in energy loss

Asymmetry due to different path length.



Asymmetry due to fluctuations in the branching process.



To estimate the size of the fluctuations in energy is important to better understand the physics behind the dijet asymmetry.

## Fluctuations in the energy loss

$$\langle \mathcal{E}^2(t) \rangle - \langle \mathcal{E}(t) \rangle^2 = E^2(\langle X^2(t) \rangle - \langle X(t) \rangle^2),$$

We need a **new ingredient**, the average energy squared carried by a pair of gluons with energy fractions equal to  $x$  and  $x'$ .

$$D^{(2)}(x, x', t) = xx' \left\langle \sum_{i \neq j} \delta(x_i - x) \delta(x_j - x') \right\rangle,$$

with this we can compute  $\langle X^2(t) \rangle$

$$\langle X^2(t) \rangle = \int_0^1 dx x D(x, t) + \int_0^1 dx \int_0^1 dx' D^{(2)}(x, x', t).$$

Escobedo and Iancu, JHEP 1605 (2016) 008

## Analytic solution of $D^{(2)}$

$$D^{(2)}(x, x', \tau) = \int_0^\tau d\tau' (2\tau - \tau') \frac{e^{-\frac{\pi(2\tau - \tau')^2}{1-x-x'}}}{\sqrt{xx'}(1-x-x')^{3/2}},$$

The integrand is the contribution to  $D^{(2)}$  of all the branchings that happened at  $\tau'$ .

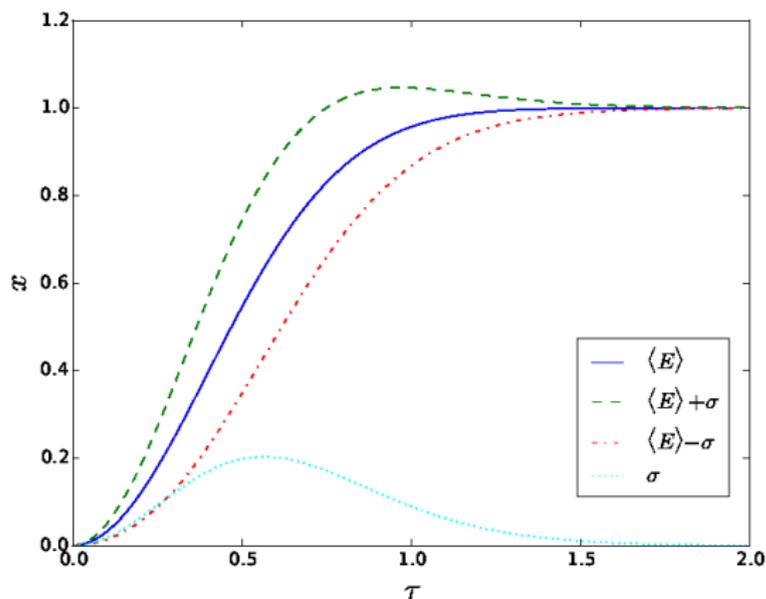
$$D^{(2)}(x, x', \tau) = \frac{1}{2\pi} \frac{1}{\sqrt{xx'}(1-x-x')} \left[ e^{-\frac{\pi\tau^2}{1-x-x'}} - e^{-\frac{4\pi\tau^2}{1-x-x'}} \right].$$

- ▶ Correlation due to common ancestors.
- ▶ The first term correspond to correlations created by late time branching.
- ▶ The second to early time branching.

## Fluctuations in energy

$$\sigma_{\mathcal{E}}^2 = \frac{\pi^2}{3} w_{br}^2(L) = \frac{\alpha_s^4 \hat{q}^2 N_c^4 L^4}{3\pi^2} \sim \langle \mathcal{E} \rangle^2$$

Dispersion in energy loss at large angles is comparable with its mean value.



## Computation of $D^{(n)}$

$\langle E^n \rangle$  and  $\langle N^n \rangle$  can be computed if you know  $D^{(n)}$ .

Recently we have found an exact expression for these quantities

$$D^{(n)}(x_1, \dots, x_n | \tau) = \frac{(n!)^2}{2^{n-1} n} \frac{(1 - \sum_{i=1}^n x_i)^{\frac{n-3}{2}}}{\sqrt{x_1 \cdots x_n}} h_n \left( \frac{\tau}{\sqrt{1 - \sum_{j=1}^n x_j}} \right),$$

where

$$h_n(l) = \int_0^l dl_{n-1} \cdots \int_0^{l_2} dl_1 (nl - \sum_{i=1}^{n-1} l_i) e^{-\pi(nl - \sum_{j=1}^{n-1} l_j)^2}.$$

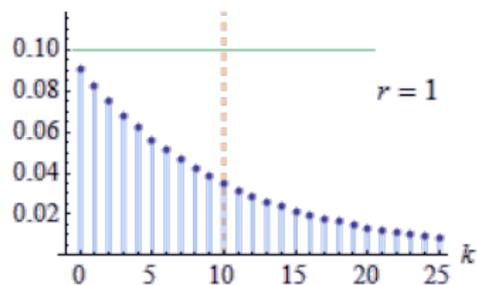
Time dependence enters only through the combination

$$l = \frac{\tau}{\sqrt{1 - \sum_{j=1}^n x_j}}.$$

## Things that can be learned from $D^{(n)}$

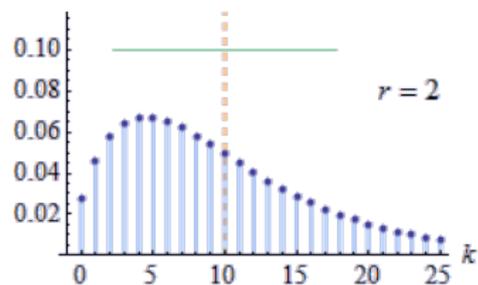
- ▶ The probability distribution that describes the emission of small energy gluons (still above the cut-off  $x_0$ ) is very similar to a negative binomial distribution with parameter  $r = 2$ .
- ▶ Probability of having  $n$  successful attempts in a Bernoulli trial before having  $r$  failures.
- ▶ A jet in the vacuum can be approximately described by a negative binomial with  $r = 3$ . (Dokshitzer, Khoze, Mueller and Troian *Basics of perturbative QCD*).

# Negative binomial distribution



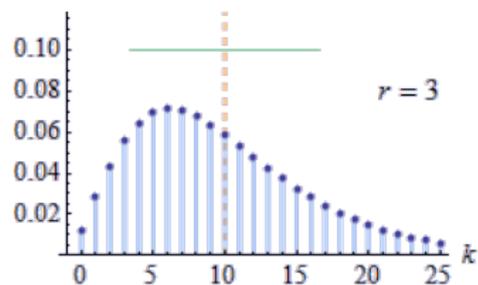
- ▶ NBD has more fluctuations and correlations than a Poisson distribution. The smaller  $r$  the more this is so.
- ▶ In the limit  $r \rightarrow \infty$  it goes to the Poisson distribution.
- ▶ Conclusion: Vacuum radiation has already more fluctuations than independent emissions (Poisson), medium radiation has even more.

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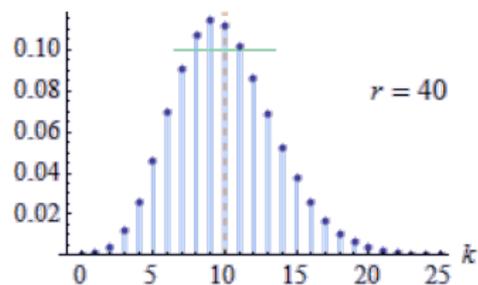
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- ▶ Jets and high energy particles allow to obtain information of the properties of the medium created in heavy-ion collisions.
- ▶ Studying energy loss we can obtain information about  $\hat{q}$ . On the other hand,  $\hat{q}$  can also be computed. Information on the non-perturbative sector can be obtained using lattice QCD.
- ▶ Not only the average energy loss is important. The whole probability distribution is interesting and related with observables.