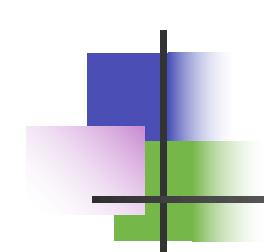


Spectra and flow of light nuclei in relativistic heavy ion collisions at RHIC and LHC

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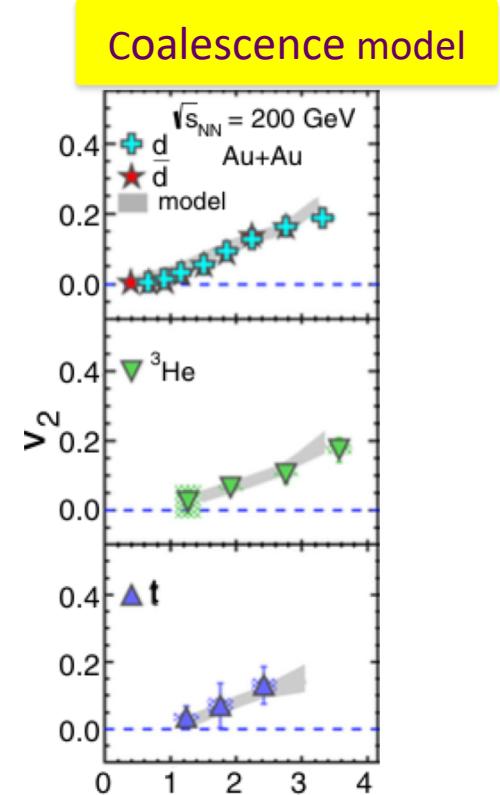
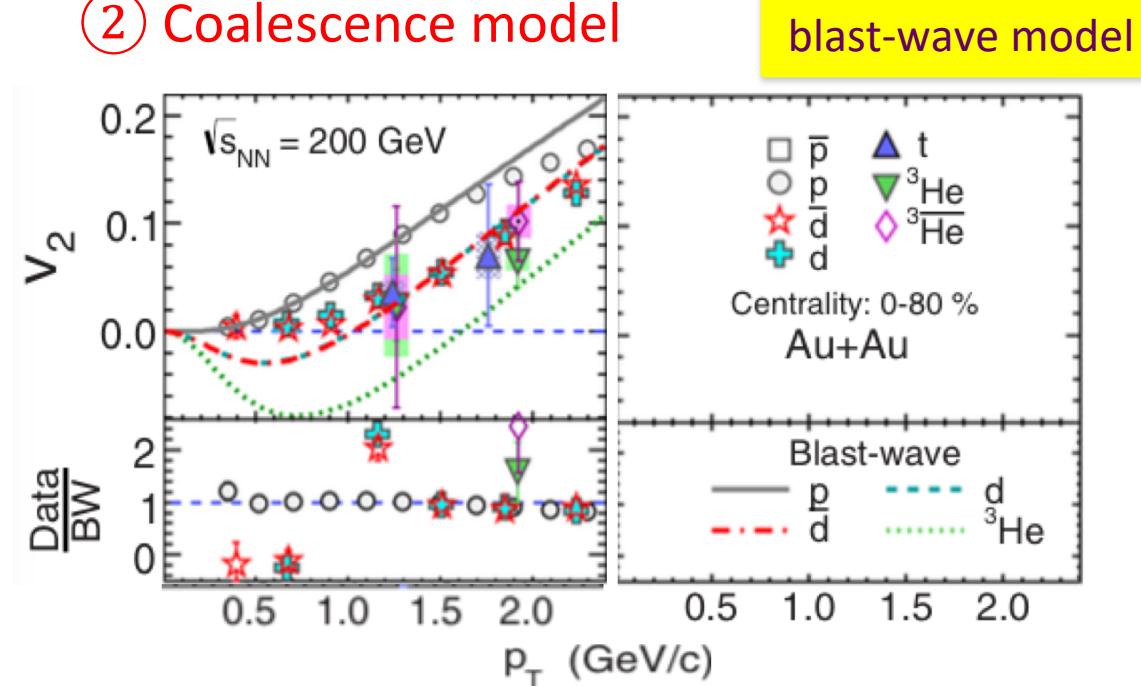
Outline

- Motivation
- Coalescence model and Set up
- Results and Discussions
- Summary

Motivation

Experimentally, the production of light nuclei is typically discussed within two models:

- ① Blast-wave model
- ② Coalescence model

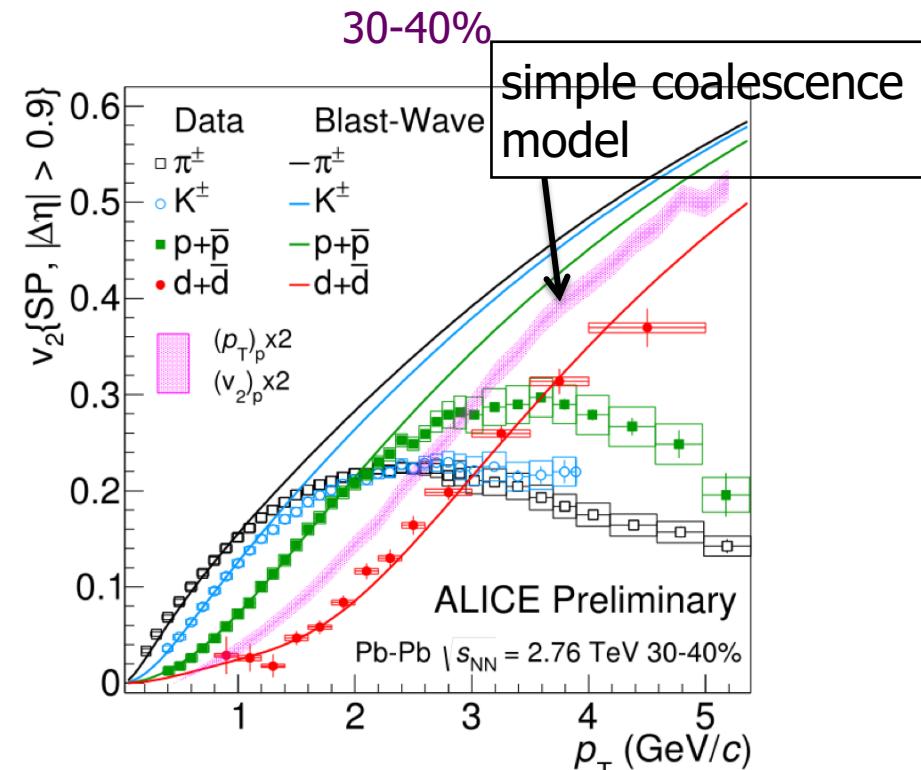
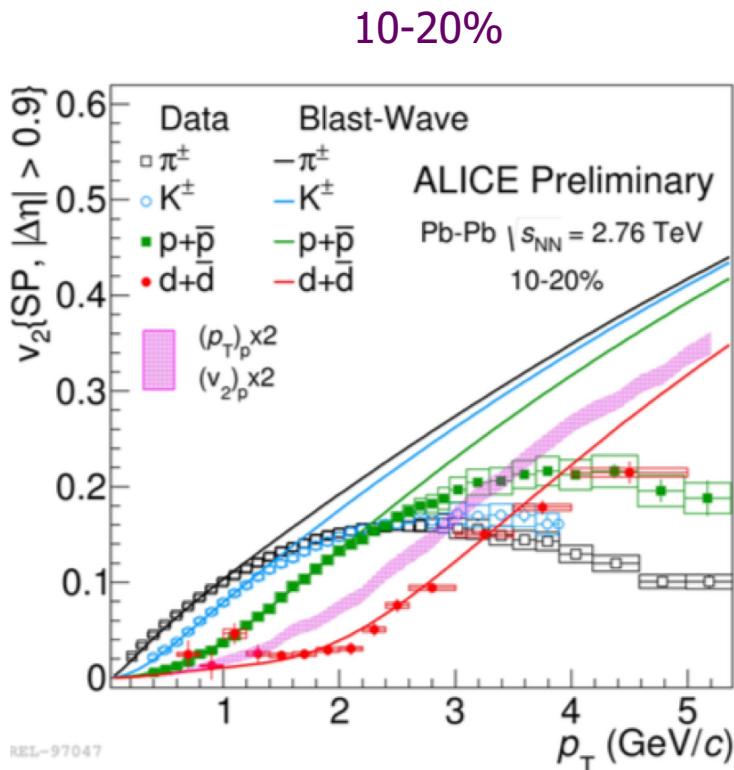


L. Adamczyk, PRC94, 034908, (2016)

- ✓ At **RHIC**, the blast-wave model underestimates the elliptic flows of light nuclei of all species at low p_T .
- ✓ Coalescence model based on freeze-out nucleons from AMPT reproduces data.

Motivation

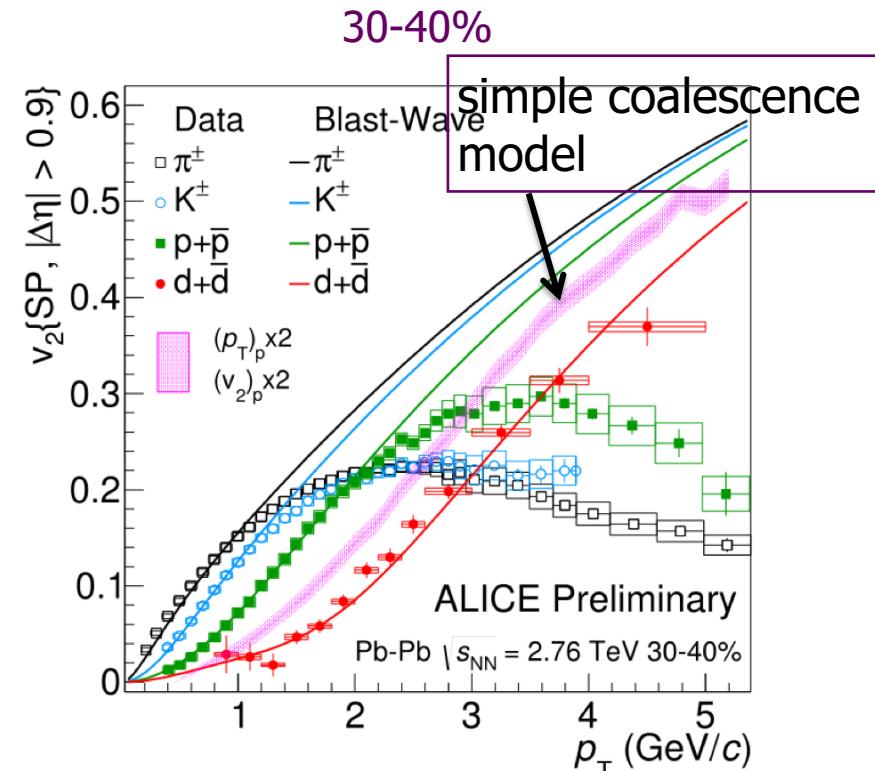
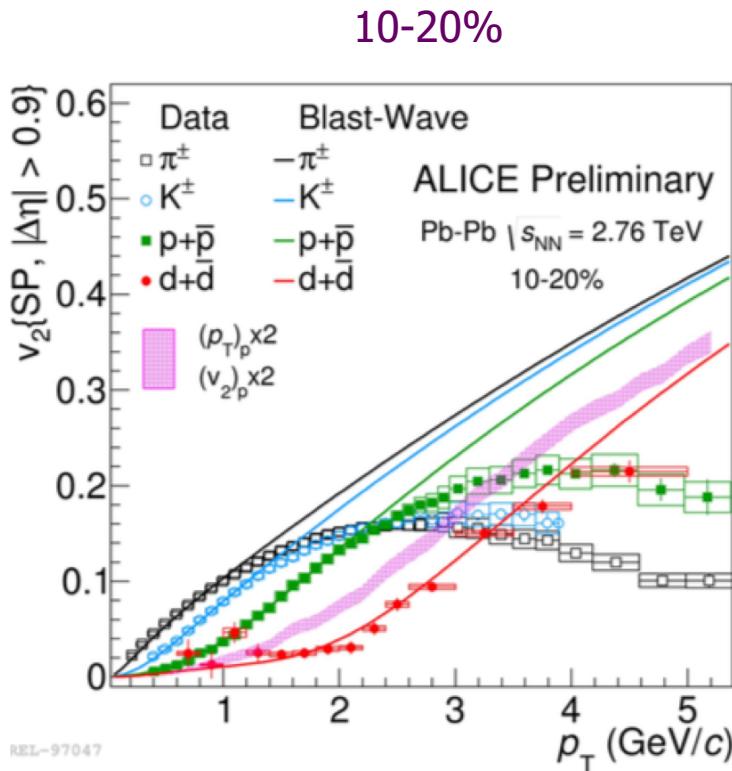
At LHC, the simple coalescence model does not describe deuteron v_2 , while the blast-wave model gives a good description.



R. Lea, NPA956, 264(2016)

Motivation

At LHC, the simple coalescence model does not describe deuteron v_2 , while the blast-wave model gives a good description.



R. Lea, NPA956, 264(2016)

Does there exist a more realistic coalescence model that could also describe the experimental data?

Coalescence model

Nuclei are produced by recombination of nucleons at freeze-out using the sudden approximation in the coalescence model.

The momentum distribution

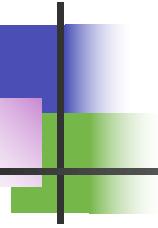
the phase-space distribution functions for protons and neutrons at freeze-out

$$\frac{d^3N_A}{dp_A^3} = g_A \int \prod_{i=1}^Z p_i^\mu d^3\sigma_{i\mu} \frac{d^3\bar{p}_i}{E_i} f_p(\bar{x}_i, \bar{p}_i, t_i) \times \int \prod_{j=1}^N p_j^\mu d^3\sigma_{j\mu} \frac{d^3\bar{p}_j}{E_j} f_n(\bar{x}_j, \bar{p}_j, t_j)$$
$$\times f_A(x_1', \dots, x_Z'; x_1', \dots, x_N'; p_1', \dots, p_Z'; p_1', \dots, p_N'; t')$$
$$\times \delta^{(3)}\left(\bar{P}_A - \sum_{i=1}^Z \bar{p}_i - \sum_{j=1}^N \bar{p}_j\right)$$

Wigner function of nuclei

In the following study, the phase-space distribution of nucleons are from two models:

- **Blast-wave model**
- **IEBE-VISHNU hybrid model**



Blast-wave model + Coalescence model

Blast-wave model

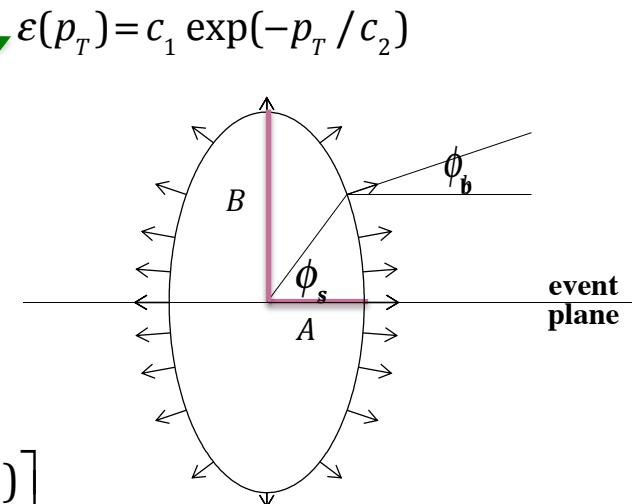
The invariant nucleon momentum distribution

$$E \frac{d^3N}{d^3\vec{p}} = \frac{2\xi\tau_0}{(2\pi)^3} \int_{\Sigma^\mu} d\eta r dr d\phi m_T \cosh(\eta - y) \exp \left[-\frac{\cosh\rho(m_T \cosh y \cosh\eta - \bar{p}_T \cdot \bar{\beta} - m_T \sinh y \sinh\eta)}{T_K} \right]$$

Parametrization:

The transverse flow velocity: $\beta = \beta(r) [1 + \varepsilon(p_T) \cos(2\phi_b)]$

The radial flow velocity: $\beta(r) = \beta_0 \sqrt{\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2}$

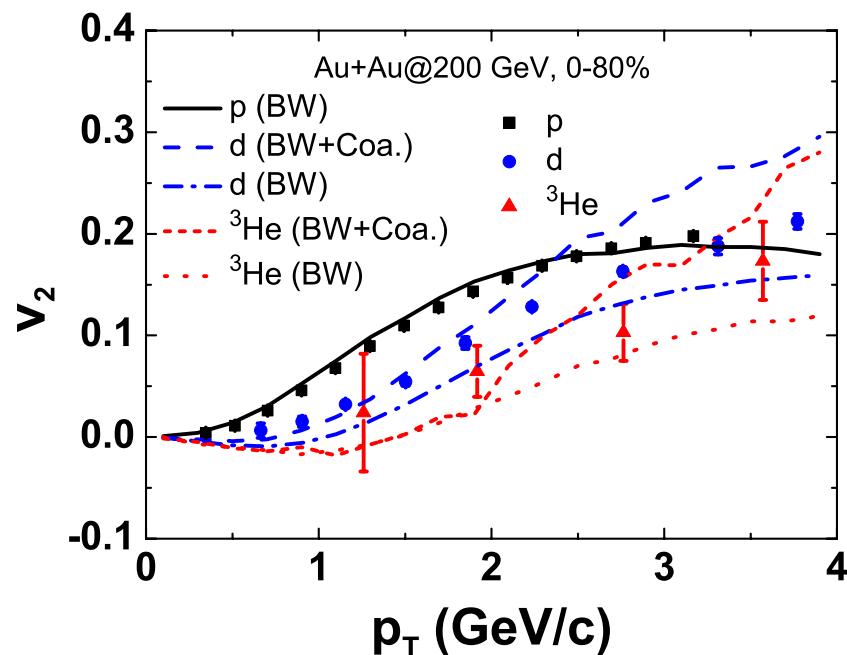
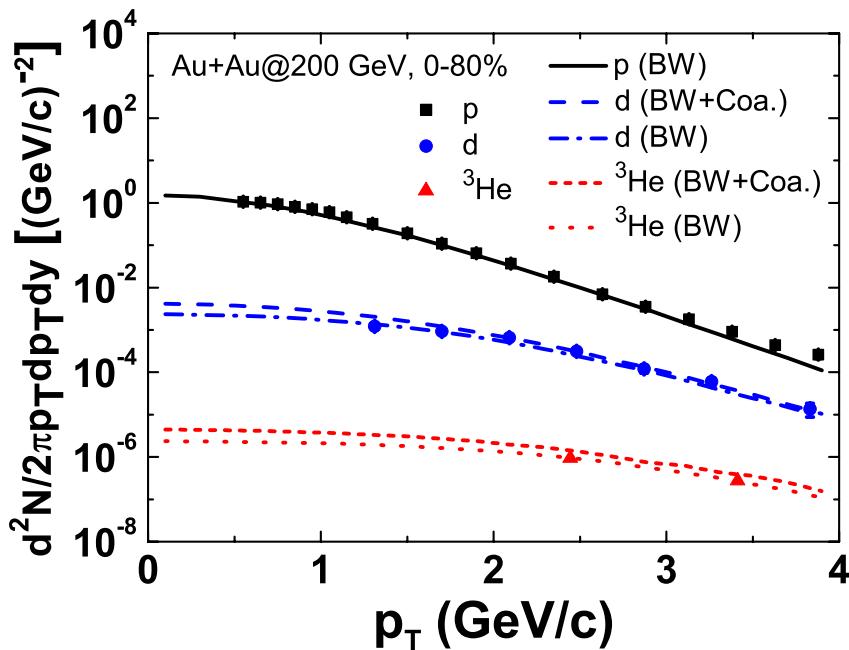


The spatial distribution of nucleon: $r \leq R_0 [1 + s_2 \cos(2\phi_s)]$

- All parameters are determined by fitting the proton transverse spectrum and elliptic flow.
- A large number of test nucleons are sampled by the nucleon phase-space distribution as the input of coalescence model.

Spectrum and elliptic flow for Au+Au@200 GeV

Yin, et al, PRC95, 054913(2017)

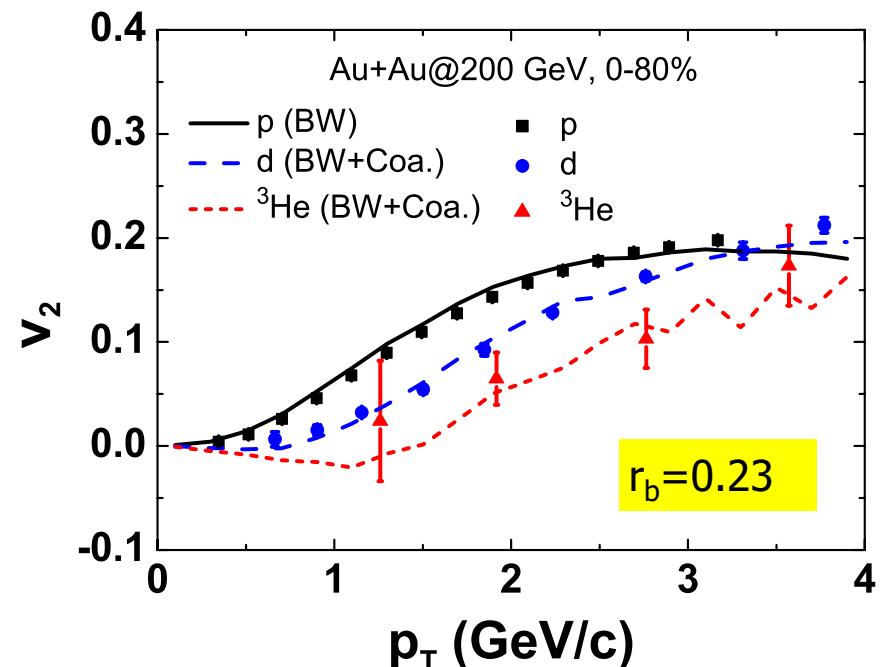
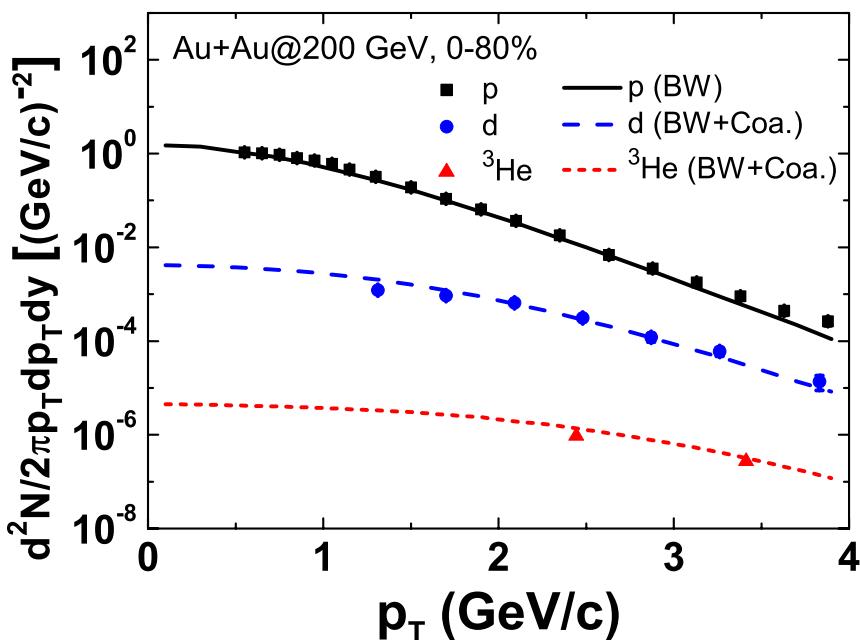


- ✓ Blast-wave model can't simultaneously describe the spectra and elliptic flow of light nuclei at RHIC.
- ✓ The coalescence model with the spatial distribution of nucleons independent of their momenta fails to describe experimentally measured elliptic flow of light nuclei.

Extended blast-wave model

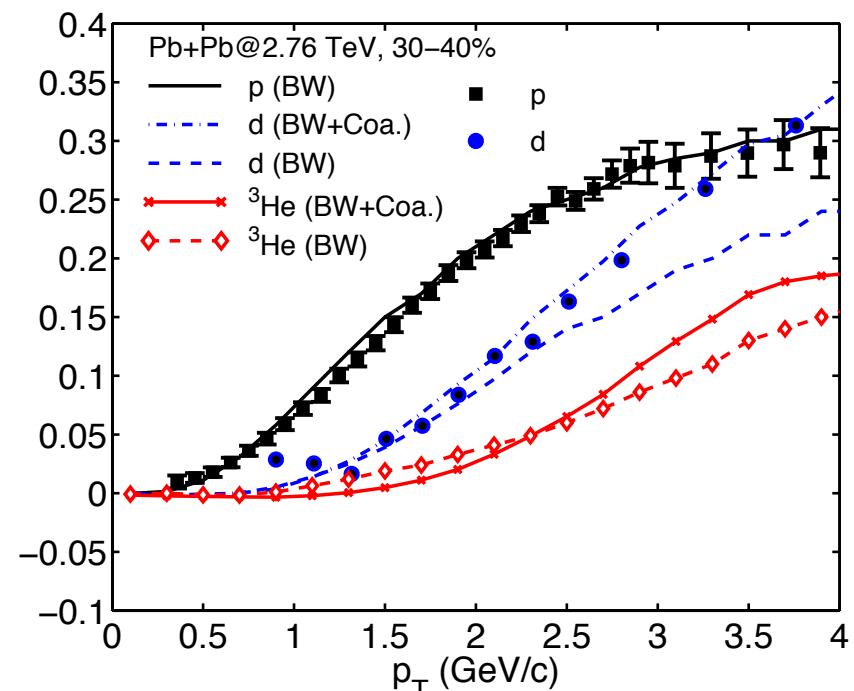
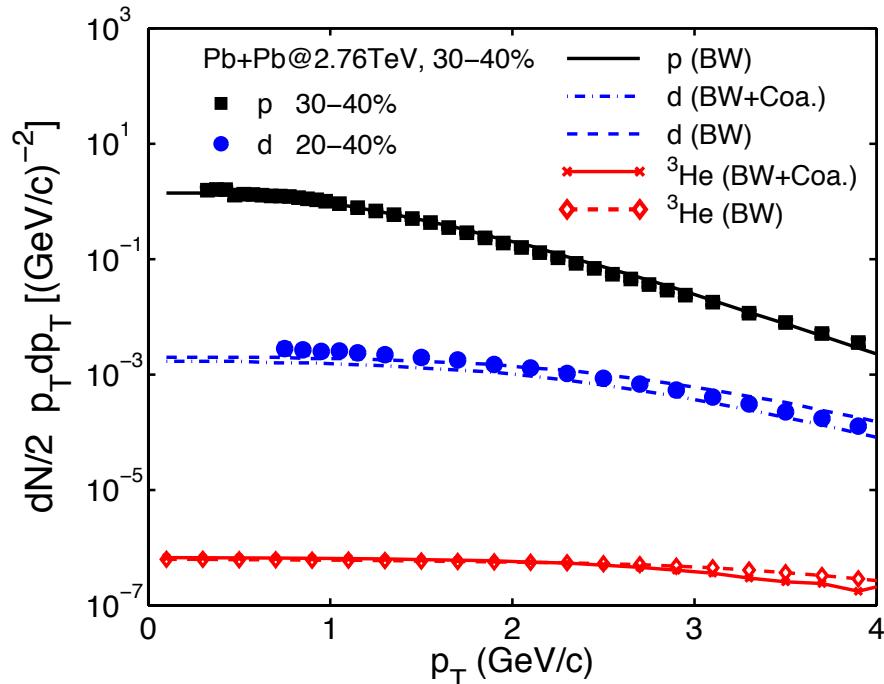
Introduce **a space-momentum correlation** to reduce the coalescence probability of nucleons moving along the reaction plane.

For in-plane ($|p_{Tx}| > |p_{Ty}|$) nucleons, the spatial distribution of high momentum nucleons ($p_T > 0.9$ GeV) has a larger radius parameter $R_0 = 10e^{r_b(p_T - 0.9)}$.



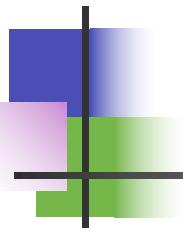
Results for Pb+Pb@2.76 TeV

Zhu, et al, EPJA54, 175(2018)



- ✓ The coalescence model can describe measured spectra and elliptic flow for deuteron data very well at LHC.
- ✓ Besides ten parameters, one more additional space-momentum correlation is also included.

A standard model is needed for the nucleon phase-space distribution.



IEBE-VISHNU hybrid model + coalescence model

IEBE-VISHNU hybrid model

- ✓ For hydrodynamics part, VISH2+1 describes the QGP expansion.
- ✓ Switch from hydrodynamics to hadron cascade (Cooper-Frye formula):

$$E \frac{d^3 N_i}{d^3 p}(x) = \frac{g_i}{(2\pi)^3} p \cdot d^3 \sigma(x) f_i(x, p)$$

- ✓ Hadron cascade simulated by UrQMD by:

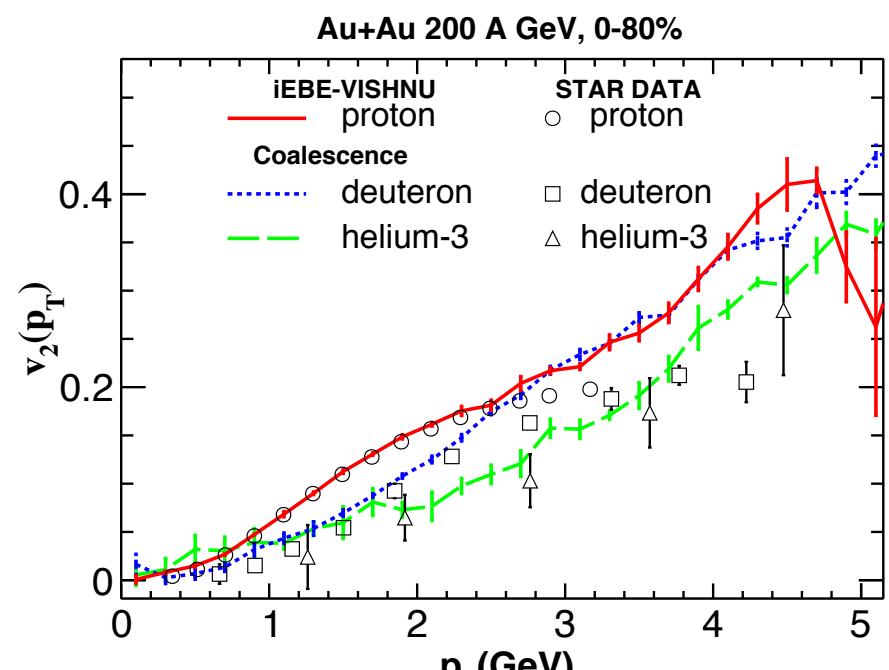
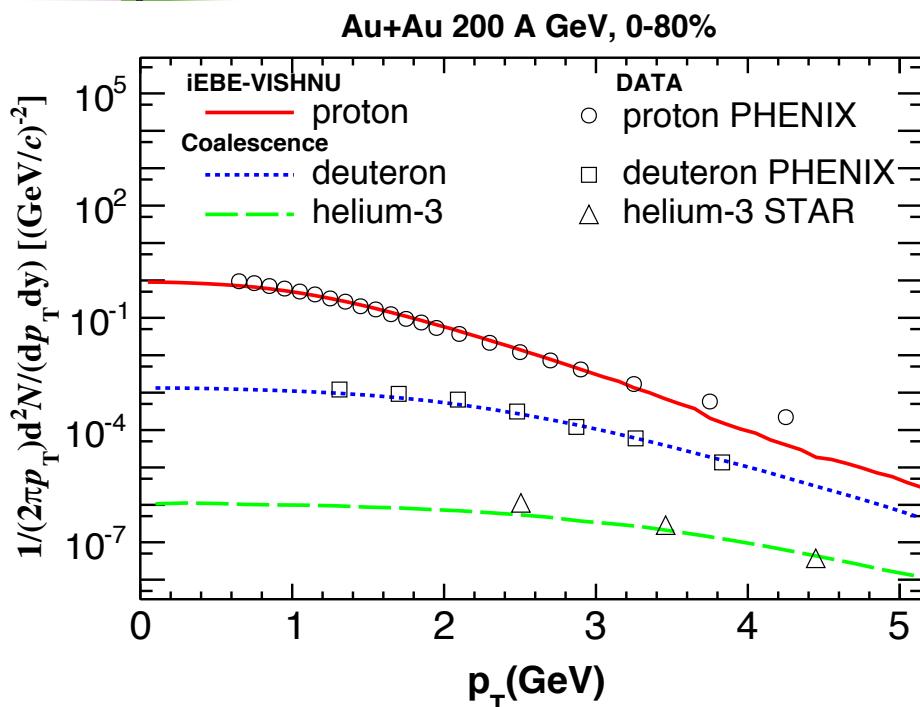
$$\frac{df_i(x, p)}{dt} = C_i(x, p)$$

- The parameters in IEBE-VISHNU simulations are fixed by the yields, spectra and flow harmonics of all charged hadrons.
- The space-momentum correlations of nucleons are naturally included through the dynamically generated nucleon distribution function without additional parameters.

J. S. Moreland, J. E. Bernhard and S. A. Bass, PRC 92, 011901(2015)

H. J. Xu, Z. Li and H. Song, PRC 93, 064905 (2016)

Spectra and flow for Au+Au@200 GeV

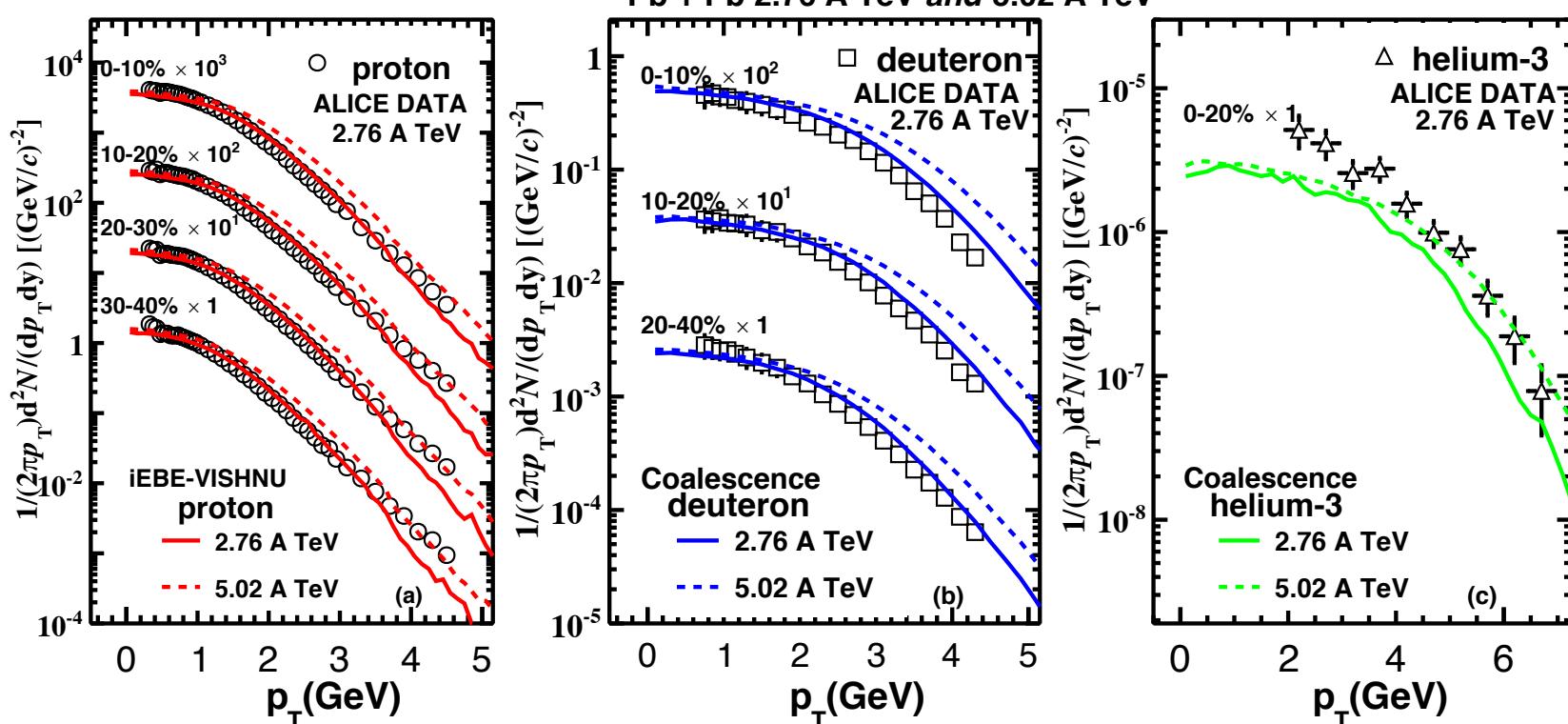


Zhao, et al, PRC98, 054905(2018)

- ✓ The spectra of deuteron and helium-3 are reproduced very well.
- ✓ The discrepancy between the model calculation and data from 2 to 4 GeV for elliptic flow of deuteron requires further study.
- ✓ The elliptic flow of helium-3 can roughly describe the measured data.

Spectra for Pb+Pb@2.76 and 5.02 TeV

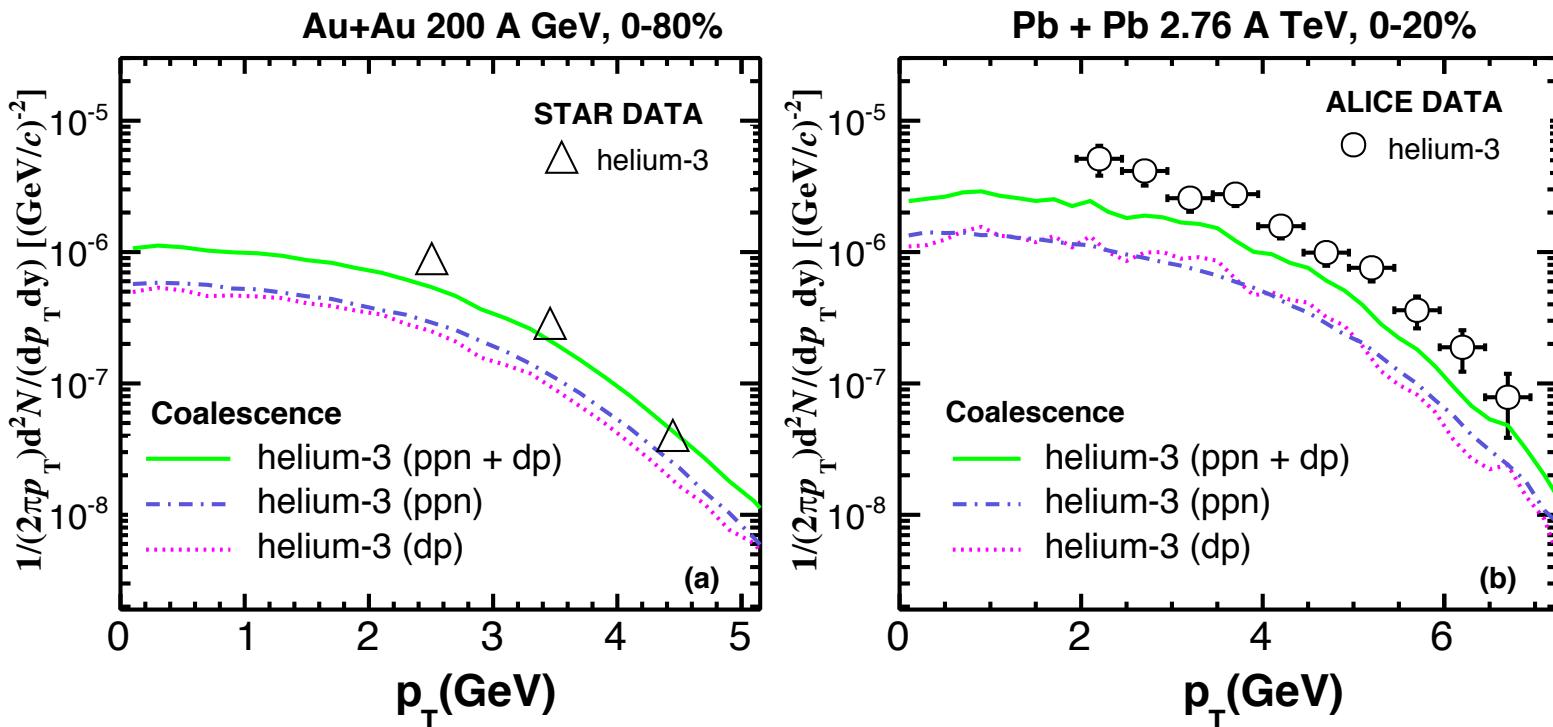
Zhao, et al, PRC98, 054905(2018)



- ✓ iEBE-VISHNU model can nicely describe deuteron data at 2.76 TeV.
- ✓ Stronger radial flow is developed at 5.02 TeV.
- ✓ The yield of helium-3 at 2.76 TeV is largely underestimated.

Helium-3 production

Zhao, et al, PRC98, 054905(2018)



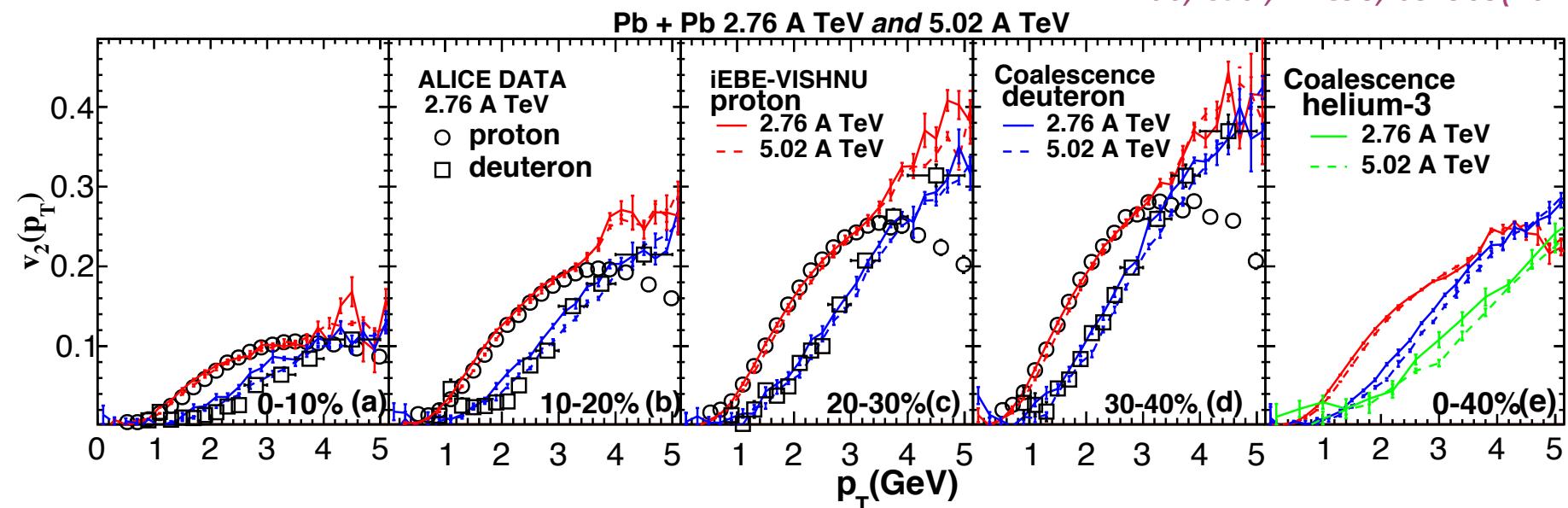
- ✓ Two processes are included in the coalescence calculation of helium-3:



- ✓ The spectrum of helium-3 in Pb+Pb collisions at 2.76 TeV is still largely underestimated.

Elliptic flow for Pb+Pb@2.76 and 5.02 TeV

Zhao, et al, PRC98, 054905(2018)



- ✓ The coalescence model can reproduce deuteron data up to 4 GeV/c at 2.76 TeV.
- ✓ The prediction for deuterons at 5.02 TeV are slightly lower than the ones at 2.76 TeV due to the slightly increased radial flow at large collision energy.

Summary

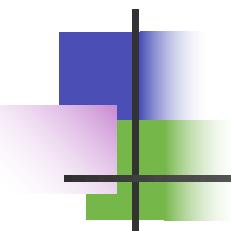
Two approaches are applied to study the p_T spectra and elliptic flow of deuterons and helium-3 in Au+Au and Pb+Pb collisions:

✓ **Blast-wave model + Coalescence model**

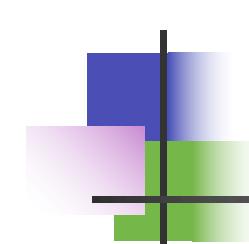
- It can nicely describe the spectra and elliptic flow of deuteron and helium-3 in Au+Au collisions at 200 GeV and Pb+Pb collisions at 2.76 TeV.
- **But besides the usual parameters, additional space-momentum correlations between nucleons are required.**

✓ **IEBE-VISHNU hybrid model + Coalescence model**

- It also works very well for deuteron production. **The space-momentum correlations of nucleons are naturally included without additional parameters.**
- Two production processes are considered for helium-3, but the spectra of helium-3 is still underestimated for Pb+Pb at 2.76 TeV.
- We also make the predictions of light nuclei production in Pb+Pb at 5.02 TeV, which are being studied at LHC.



Thanks



The naïve coalescence model

The spectrum of the composite nuclei is related to the one of the primordial nucleons via,

$$E_i \frac{d^3N_i}{dp_i^3} = B_A \left(E_p \frac{d^3N_p}{dp_p^3} \right)^A$$

B_A is the coalescence parameter for nuclei with mass number A and a momentum of $p_i = Ap_p$

$$B_A = \left(\frac{4\pi}{3} p_0^3 \right)^{A-1} \frac{M}{m^A}$$

Blast-wave model

In the blast-wave model, all produced particles are assumed in thermal equilibrium and undergo collective expansion.

The invariant momentum distribution

$$E \frac{d^3N}{d^3\bar{p}} = \frac{d^3N}{p_T dp_T dy d\phi_p} = \int_{\Sigma^\mu} d^3\sigma_\mu p^\mu f(x, p)$$

$f(x, p)$: the Lorentz-invariant thermal distribution of nucleons

Neglect the effect of quantum statistics at high temperature

$$f(x, p) = \frac{2\xi}{(2\pi)^3} \exp(-p^\mu u_\mu / T_K)$$

four momentum $p^\mu = (p^0, \vec{p}) = (m_T \cosh y, p_T \cos \phi_p, p_T \sin \phi_p, m_T \sinh y)$

the azimuthal angle of the nucleon momentum with respect to x axis

flow four-velocity $u^\mu = \cosh \rho (\cosh \eta, \tanh \rho \cos \phi_b, \tanh \rho \sin \phi_b, \sinh \eta)$

the azimuthal angle of the transverse flow velocity with respect to x axis

$$\rho: \text{transverse flow rapidity } \rho = \frac{1}{2} \ln \frac{1+|\vec{\beta}|}{1-|\vec{\beta}|}$$

$$E \frac{d^3N}{d^3\bar{p}} = \frac{2\xi \tau_0}{(2\pi)^3} \int_{\Sigma^\mu} d\eta r dr d\phi m_T \cosh(\eta - y) \exp \left[-\frac{\cosh \rho (m_T \cosh y \cosh \eta - \vec{p}_T \cdot \vec{\beta} - m_T \sinh y \sinh \eta)}{T_K} \right]$$

Parametrization

The invariant momentum distribution

$$E \frac{d^3N}{d^3\vec{p}} = \frac{2\xi\tau_0}{(2\pi)^3} \int_{\Sigma^\mu} d\eta r dr d\phi m_T \cosh(\eta - y) \exp \left[-\frac{\cosh\rho(m_T \cosh y \cosh\eta - \vec{p}_T \cdot \vec{\beta} - m_T \sinh y \sinh\eta)}{T_K} \right]$$

$$\epsilon(p_T) = c_1 \exp(-p_T / c_2)$$

Parametrization:

The transverse flow velocity: $\beta = \beta(r) [1 + \epsilon(p_T) \cos(2\phi_b)]$

the azimuthal angle of the transverse flow velocity with respect to the x axis

The radial flow velocity: $\beta(r) = \beta_0 \sqrt{\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2}$

A and B are related to the transverse size R_0 and the elliptic eccentricity s_2

The spatial distribution of nucleon: $r \leq R_0 [1 + s_2 \cos(2\phi_s)]$

the azimuthal angle of nucleon position in the transverse plane

All parameters are determined by fitting the proton transverse spectrum and elliptic flow. Results for light nuclei are then obtained by replacing the mass and introducing the fugacity.

Light clusters production in coalescence model

The momentum distribution of triton (Helium-3)

$$\frac{d^3N_{t(^3He)}}{dp^3} = g_{t(^3He)} \int \left[\prod_{i=1}^3 d^3x_i d^3p_i \right] \frac{d^6N_p}{dx_1^3 dp_1^3} \frac{d^6N_{n(p)}}{dx_2^3 dp_2^3} \frac{d^6N_n}{dx_3^3 dp_3^3} \times f^W \delta^{(3)}(p_{t(^3He)} - p_1 - p_2 - p_3) - \frac{\rho^2 + \lambda^2}{2\sigma^2}$$

Gaussian wave function

$$\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) = (3\pi^2 \sigma^4)^{-3/4} e^{-\frac{\rho^2 + \lambda^2}{2\sigma^2}}$$

Jacobi coordinates

$$\vec{R} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3), \quad \vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \quad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

$$\vec{K} = \vec{k}_1 + \vec{k}_2 + \vec{k}_3, \quad \vec{k}_\rho = \frac{1}{\sqrt{2}}(\vec{k}_1 - \vec{k}_2), \quad \vec{k}_\lambda = \frac{1}{\sqrt{6}}(\vec{k}_1 + \vec{k}_2 - 2\vec{k}_3)$$

So $f_{t(^3He)}^W(\rho, \lambda, k_\rho, k_\lambda) = 8^2 \exp(-\frac{\rho^2 + \lambda^2}{\sigma^2}) \exp(-(k_\rho^2 + k_\lambda^2)\sigma^2)$

The above formula can be straightforwardly generalized to multi-particle coalescence.

Wigner functions of d and ^3He

In this study, we approximate the wave functions of deuteron and Helium-3 by those of the ground state of a harmonic oscillator.

Deuteron

C. M. Ko, et al, NPA928, 234(2014)

$$f_2(\bar{\rho}, \bar{p}_\rho) = \int d^3 \bar{y} \phi^*(\bar{\rho} - \frac{\bar{y}}{2}) \phi(\bar{\rho} + \frac{\bar{y}}{2}) e^{-i \bar{\rho} \cdot \bar{p}_\rho}$$

$$f_2(\bar{\rho}, \bar{p}_\rho) = 8g_2 \exp(-\frac{\bar{\rho}^2}{\sigma_\rho^2} - \bar{p}_\rho^2 \sigma_\rho^2) \quad \text{with} \quad \bar{\rho} = \frac{\bar{x}'_1 - \bar{x}'_2}{\sqrt{2}}, \bar{p}_\rho = \frac{\bar{p}'_1 - \bar{p}'_2}{\sqrt{2}}$$

the width parameter $\sigma_\rho = 1/\sqrt{\mu_1 \omega}$

ω is the oscillator frequency in
the harmonic wave function

$$\mu_1 = 2 / (1/m_1 + 1/m_2)^{-1}$$

Helium-3

$$f_3(\bar{\rho}, \bar{\lambda}, \bar{p}_\rho, \bar{p}_\lambda) = 8^2 g_3 \exp(-\frac{\bar{\rho}^2}{\sigma_\rho^2} - \frac{\bar{\lambda}^2}{\sigma_\lambda^2} - \bar{p}_\rho^2 \sigma_\rho^2) \quad \text{with}$$

$$\bar{\lambda} = \frac{\bar{x}'_1 + \bar{x}'_2 - 2\bar{x}'_3}{\sqrt{6}}$$

$$\bar{p}_\lambda = \frac{\bar{p}'_1 + \bar{p}'_2 - 2\bar{p}'_3}{\sqrt{6}}$$

10-20% in Pb+Pb@ 2.76 TeV

Zhu, et al, EPJA54, 175(2018)

