Neutron star properties from the extended linear sigma model

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Outline

1. Introduction
2. Methods
3. Results
4. Conclusion

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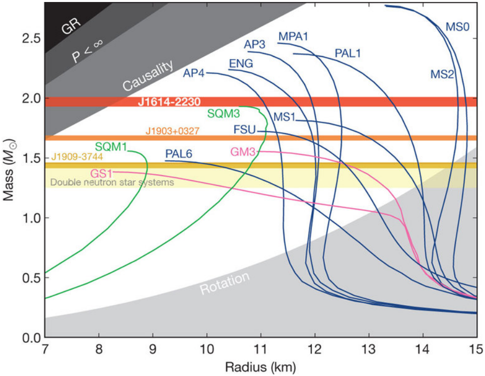
Motivation

- QCD is unsolvable on low energies and finite densities
- Effective models are used to describe strongly interacting matter in this region
- Observations of neutron stars put constraints on dense strongly interacting matter [1,2]



Mass-radius relation from different models [1]

[1] Demorest P., et al. (2010), Nature, 467, 1081

[2] Antoniadis J., et al. (2013), Science, 340, 6131

The extended linear sigma model [3]

The Lagrangian is constructed based on linearly realized global $U(3)_L \times U(3)_R$ chiral symmetry and its explicit breaking:

- The model includes three flavours of constituent quarks
- Vector and axialvector meson nonets are included in addition to scalar and pseudoscalar mesons
- Explicit symmetry breaking terms and spontaneous breaking by scalar condensates

Contribution from Polyakov loop potential is included:

- Polyakov loop variables constructed from gluon fields
- Modifies the Fermi-Dirac distribution function → mimics quark confinement

[3] Kovács P., Szép Zs., Wolf Gy. (2016), Phys. Rev., D93, 114014

Applied approximations and parameter determination

Calculating the grand potential:

- One-loop fermionic contributions are considered
- Mesonic contributions are considered at tree-level
- Thermal contributions of lowest mass mesons (π, K, f_0^L) are included in the pressure

Parametrization:

- 14 unknown parameters + renormalization scale
- Determined with χ^2 fit to particle masses and decay widths
- 16 experimental masses → fitted with curvature masses with vacuum contributions (assignment is not trivial)
- 12 experimental decay widths
- Good agreement with experimental values

The Tolman-Oppenheimer-Volkoff equation

Solving the Einstein-equations with spherical symmetry inside matter we get the TOV equation:

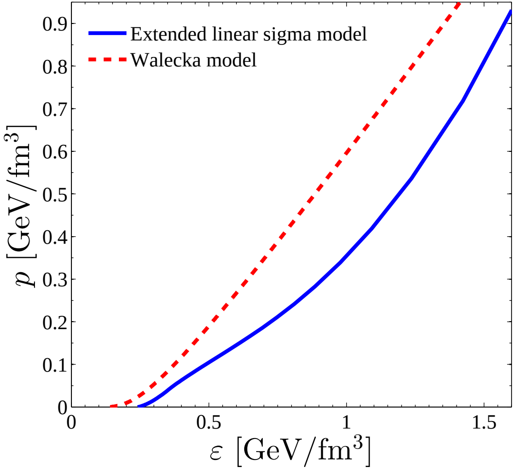
$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]} \tag{1}$$

We can integrate this equation numerically using a specific equation of state (i.e. $p(\varepsilon)$ function):

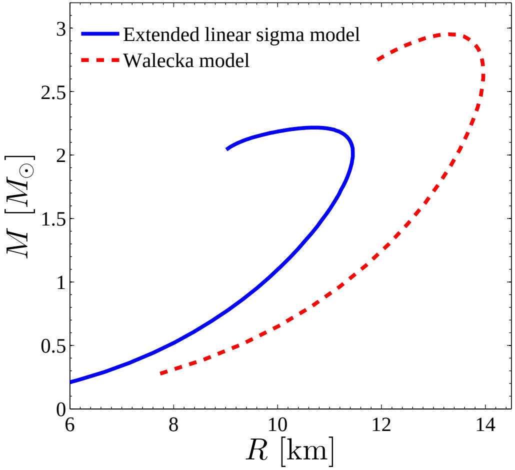
- For a fixed central energy density we solve (1) until $p = 0$
- By changing the central energy density we get a sequence of neutron stars parametrized by ε_c
- Once the maximum mass is reached the sequence of stable neutron stars end

Equation of state

- Compared the results with results from the Walecka model
- Pressure becomes zero at $\varepsilon \approx 240$ MeV/fm^3
- The equation of state is less "stiff" than the equation of state in the Walecka model



Mass-radius relation



- Lower radii due to higher energy density at saturation
- Lower value of maximal mass due to less "stiff" equation of state
- Managed to produce neutron stars with 2 $M_\odot$ while including three flavours

Conclusion

Conclusion

- The model is able to predict neutron stars with 2 $M_{\odot}$
- Unlike the Walecka model this model manages to do this while including three quark flavours

Future work

- Making the calculations consistent by including one-loop mesonic terms
- Including interactions of fermions through exchange of vector-mesons (e.g. the ρ-meson, which describes short-range repulsion)

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Thank you for your attention!

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Lagrangian I.

$\mathcal{L}$ constructed based on linearly realized global $U(3)_L \times U(3)_R$ symmetry and its explicit breaking

$$
\begin{aligned}
\mathcal{L} = & \operatorname{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \operatorname{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\operatorname{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \operatorname{Tr}(\Phi^\dagger \Phi)^2 \\
& + c_1 (\det \Phi + \det \Phi^\dagger) + \operatorname{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \operatorname{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
& + \operatorname{Tr}\left[\left(\frac{m_1^2}{2}\mathbb{I} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\operatorname{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \operatorname{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2}\operatorname{Tr}(\Phi^\dagger \Phi)\operatorname{Tr}(L_\mu^2 + R_\mu^2) + h_2 \operatorname{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \operatorname{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
& + \bar{\Psi} i \gamma_\mu D^\mu \Psi - g_F \bar{\Psi}(\Phi_S + i\gamma_5 \Phi_{PS})\Psi,
\end{aligned}
$$

$$
\begin{aligned}
D^\mu \Phi &= \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi], \\
L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}, \\
R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}, \\
D^\mu \Psi &= \partial^\mu \Psi - iG^\mu \Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.
\end{aligned}
$$

+ Polyakov loop potential

Lagrangian II.

the matter and external fields are

$$\Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i, \quad \Delta = \sum_{i=0}^{8} \delta_i T_i$$

$$\Psi = (u, d, s)^{\mathrm{T}}$$

non strange – strange base:

$$\xi_N = \sqrt{2/3}\xi_0 + \sqrt{1/3}\xi_8,$$

$$\xi_S = \sqrt{1/3}\xi_0 - \sqrt{2/3}\xi_8, \qquad \xi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle \sigma_{N/S} \rangle \equiv \bar{\sigma}_{N/S}$

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Particle content

- Vector and Axial-vector meson nonets

$$V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star0} \\ K^{\star-} & \bar{K}^{\star0} & \omega_S \end{pmatrix}^\mu \quad A^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$\rho \to \rho(770), K^\star \to K^\star(894)$ $\quad a_1 \to a_1(1230), K_1 \to K_1(1270)$
$\omega_N \to \omega(782), \omega_S \to \phi(1020)$ $\quad f_{1N} \to f_1(1280), f_{1S} \to f_1(1426)$

- Scalar $(\sim \bar{q}_i q_j)$ and pseudoscalar $(\sim \bar{q}_i \gamma_5 q_j)$ meson nonets

$$\Phi_S = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star+} \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_0^{\star0} \\ K_0^{\star-} & \bar{K}_0^{\star0} & \sigma_S \end{pmatrix} \quad \Phi_{PS} = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

unknown assignment $\quad \pi \to \pi(138), K \to K(495)$
mixing in the $\sigma_N - \sigma_S$ sector $\quad$ mixing: $\eta_N, \eta_S \to \eta(548), \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$

fields shifted by their expectation value: $\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S}$

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau \, G_4(\vec{x}, \tau)\right]$$

↪ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement transition

low T: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$
high T: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

- Polyakov gauge: $G_4(\vec{x}, \tau) = G_4(\vec{x})$, plus gauge rotation to diagonal form in color space
- further simplification: $\vec{x}$-independence

$$\hookrightarrow \quad L = e^{i\beta G_4} = \mathrm{diag}(a, b, c) \left(\overset{!}{\in} SU(3)^{\mathrm{color}} \right); \quad a, b, c \in \mathbb{Z}$$

↪ use this to calculate partition function of free quarks

Form of the potential

I.) Simple polynomial potential invariant under $\mathbb{Z}_3$ and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}^{\rm YM}_{\rm poly}(\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2$$

with $b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2} + a_3\frac{T_0^3}{T^3}$

II.) Logarithmic potential coming from the $SU(3)$ Haar measure of group integration K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}^{\rm YM}_{\rm log}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T)\ln\left[1-6\Phi\bar{\Phi}+4\left(\Phi^3+\bar{\Phi}^3\right)-3\left(\Phi\bar{\Phi}\right)^2\right]$$

with $a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}, \qquad b(T) = b_3\frac{T_0^3}{T^3}$

$\mathcal{U}^{\rm YM}(\Phi,\bar{\Phi})$ models the free energy of a pure gauge theory

Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F) \longrightarrow$ determined by the min. of χ^2:

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \ldots, x_N) - Q_i^{\exp}}{\delta Q_i} \right]^2 ,$$

$(x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1, \ldots, x_N) \longrightarrow$ from the model, $Q_i^{\exp} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$
multiparametric minimalization $\longrightarrow$ MINUIT

- Curvature masses $\to$ 16 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\to$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$

Result of the parametrization

- 40 possible assignments of scalar mesons to the scalar nonet states
- 3 values of M_0 are used $\Longrightarrow$ 120 cases to investigate
  for each case $5 \cdot 10^4 - 10^5$ configurations are used for the χ^2 minimization
- lowest χ^2 obtained for $M_0 = 0.3$ GeV $\quad \chi^2 = 18.57$ and $\chi^2_{\mathrm{red}} \equiv \frac{\chi^2}{N_{\mathrm{dof}}} = 1.16$
  assignment: $a_0^{\bar{q}q} \to a_0(980)$, $K_0^{\star,\bar{q}q} \to K_0^\star(800)$, $f_0^{L,\bar{q}q} \to f_0(500)$, $f_0^{H,\bar{q}q} \to f_0(980)$

  problems: $m_{a_0} < m_{K_0^\star}$, $m_{f_0^{H/L}}$ too light
- by minimizing also for M_0 we obtain using $\mathcal{U}^{\mathrm{YM}}_{\log}(\Phi, \bar{\Phi})$ with $T_0 = 182$ MeV:

Parameter	Value	Parameter	Value
ϕ_N [GeV]	0.1411	g_1	5.6156
ϕ_S [GeV]	0.1416	g_2	3.0467
m_0^2 [GeV^2]	2.3925_{E-4}	h_1	27.4617
m_1^2 [GeV^2]	6.3298_{E-8}	h_2	4.2281
λ_1	-1.6738	h_3	5.9839
λ_2	23.5078	g_F	4.5708
c_1 [GeV]	1.3086	M_0 [GeV]	0.3511
δ_S [GeV^2]	0.1133		

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