

Do we need viscosity to suppress v_2 ?

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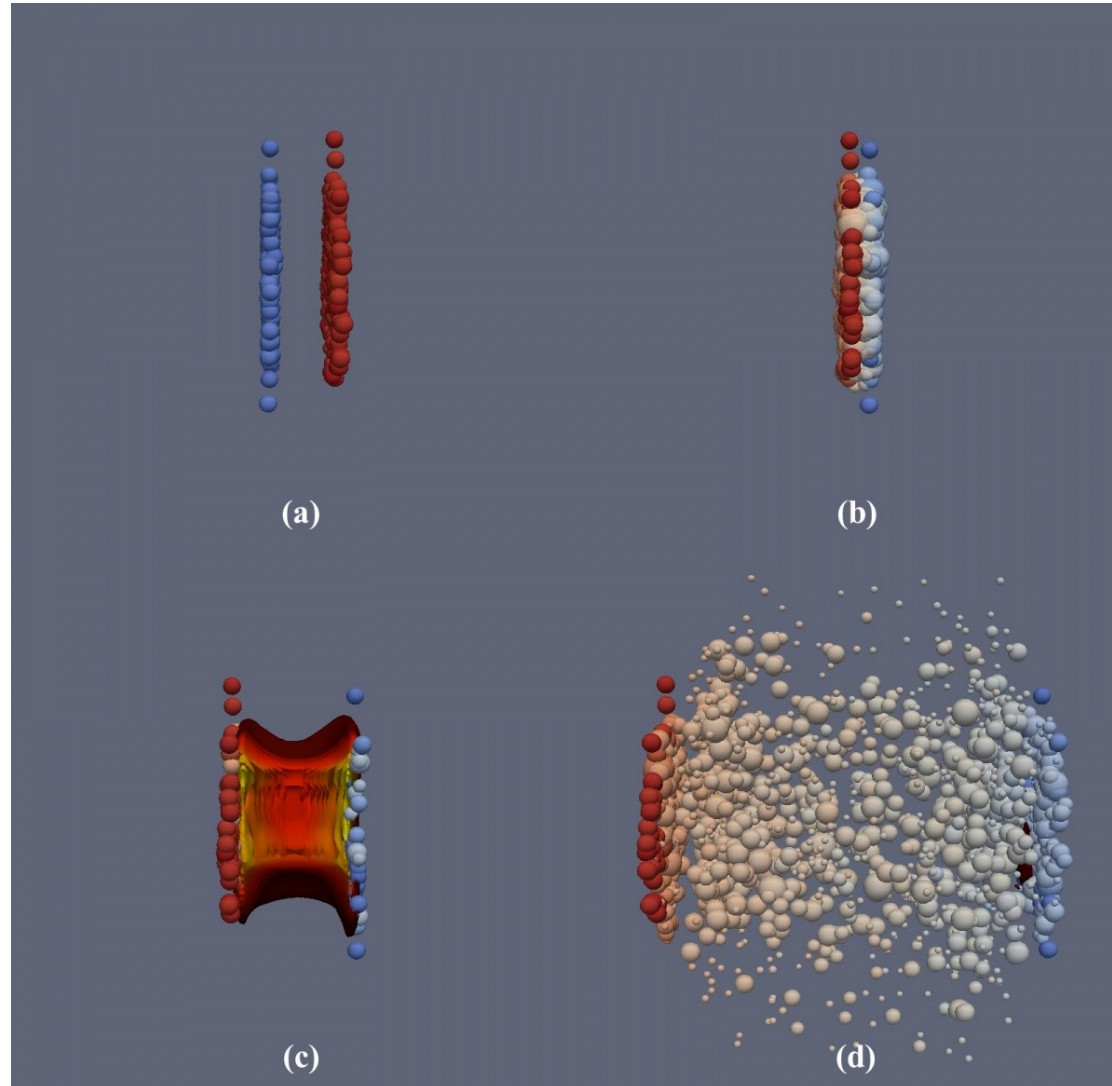
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Wigner GPU Laboratory



Outline

1. Motivation:
relativistic hydrodynamics to understand heavy ion collisions
2. Particlization:
basics, shortcomings, influence on observables
3. How much does f matter?
4. Results
5. Conclusion and outlook

Motivation



For students: see Gabriel Denicol's talk from Hot Quarks 2018
<https://indico.cern.ch/event/703015/contributions/3095199/>

Particlization

How is it done?

Particlization: particles from hydro

P. Hoven et al Phys. Lett. B 503, 58 (2001)

K.Dusling, G.D.Moore, D.Teaney, Phys. Rev. C81, 034907 (2010)

D.Molnar & Z.Wolf, Phys. Rev. C95, 024903 (2017)

Relativistic hydrodynamics [$u^\mu(x^\mu)$, $\varepsilon(x^\mu)$, $n(x^\mu)$]

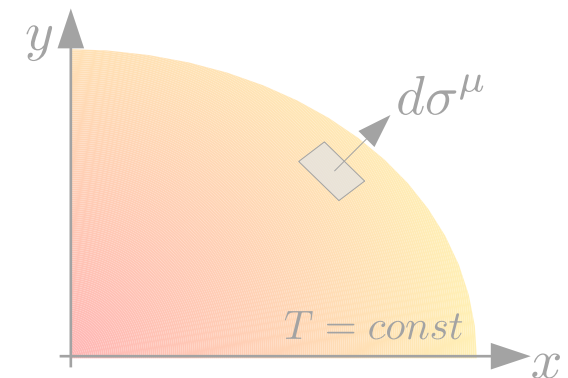
conversion fluid \rightarrow particles (*particlization*)

1. phase space density (particle specie i):

$$\left. \begin{aligned} T^{\mu\nu}(x^\lambda) &= \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(x^\lambda, \mathbf{p}) \\ N_c^\mu(x^\lambda) &= \sum_i q_{c,i} \int \frac{d^3p}{E} p^\mu f_i(x^\lambda, \mathbf{p}) \end{aligned} \right\} f_i(x^\mu, \mathbf{p}) = \frac{dN_i(t, \mathbf{r}, \mathbf{p})}{d^3r d^3p}$$

2. Cooper-Frye formula at $T(or \varepsilon)=const$

$$E \frac{d^3 N_i}{d^3 \mathbf{p}} = \frac{g_i}{(2\pi)^3} \int d\sigma^\mu p_\mu f_i(x^\nu, p)$$



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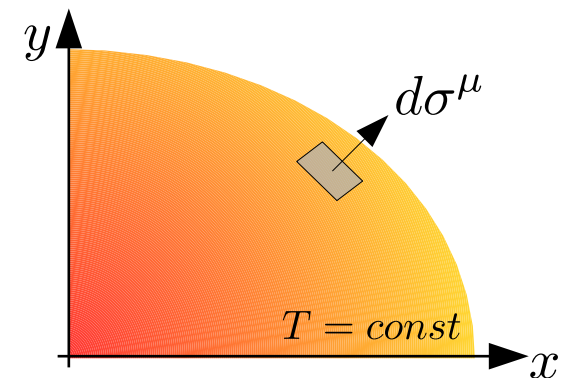
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Assumption 1.: local equilibrium \rightarrow thermal and chemical equilibrium

$$f_i^{eq}(x, \mathbf{p}) = \frac{g_i}{(2\pi)^3} e^{\frac{\mu_i(x) - p_\alpha u^\alpha(x)}{T(x)}} \quad \mu_i = \sum_c q_{c,i} \mu_c(x)$$

Local rest frame [$u^\mu_{LR} = (1, \mathbf{0})$] + Equation of state

$$\left. \begin{array}{l} f_i^{eq}(x, \mathbf{p}) \\ T_{id}^{\mu\nu}(x^\lambda) = \text{diag}(\varepsilon(x^\lambda), -\mathbf{1}P(x^\lambda)) \\ N_{c,id}^\mu(x^\lambda) = n_c(x^\lambda) u^\mu(x^\lambda) \end{array} \right\} T(x^\nu), \mu_i(x^\nu)$$

Assumption 2.: near to the local equilibrium

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Problem: finite set of conditions, can be satisfied with infinitely many different $\delta f_i!$

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Assumption 2.: close to the local equilibrium

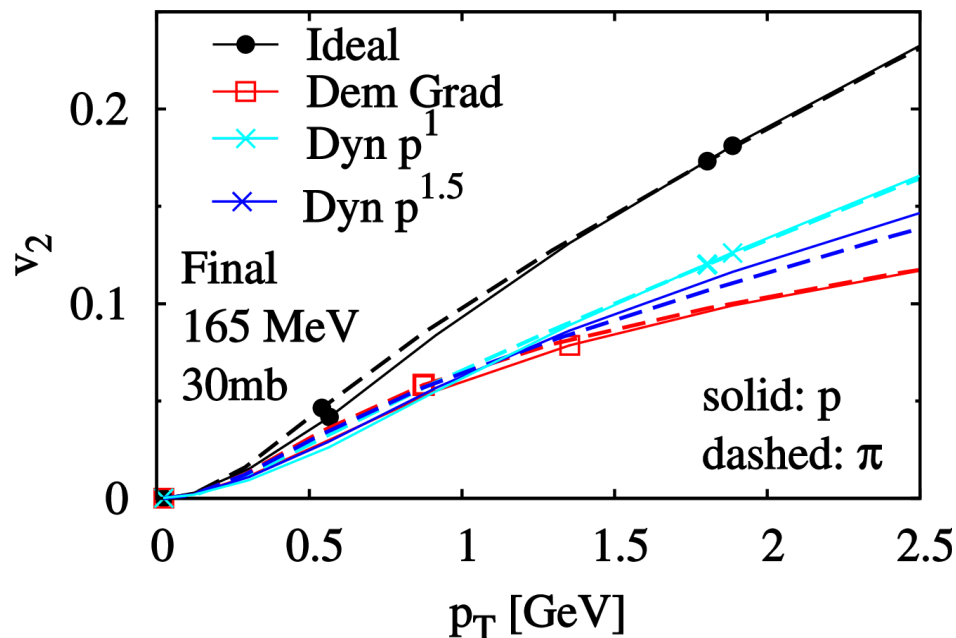
$$f_i(x^\mu, \mathbf{p}) = f_i^{eq}(x^\mu, \mathbf{p}) + \delta f_i(x^\mu, \mathbf{p}) \rightarrow \begin{aligned} T^{\mu\nu} &= T_{id}^{\mu\nu} + \delta T^{\mu\nu} \\ N_c^\mu &= N_{c,id}^\mu + N_c^\mu \end{aligned} \rightarrow \begin{aligned} T(x^\nu), \mu_i(x^\nu) \\ \delta f_i(x^\mu, \mathbf{p}) \end{aligned}$$

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Particlization: particles from hydro

Assumptions for δf_i (for dissipative fluids):

- Democratic Grad (scalar theory and momentum diffusion): $\delta f_i \sim p^2$ all i
M.Luzum & P.Romatschke, Phys.Rev. C78, 034915 (2008)
- Power law dependence: $\delta f_i \sim p^\alpha$ all i or multicomponent gas
K.Dusling, G.D.Moore, D.Teaney, Phys.Rev. C81, 034907 (2010)
- Dynamic (linearized kinetic transport)



D.Molnar & Z.Wolff, Phys.Rev. C95, 024903 (2017)
 Z.Wolff & D.Molnar, Phys.Rev. C96, 044909 (2017)
 Reprint from: Phys.Rev. C95, 024903 (2017)

**Problem: theoretical uncertainty
 in viscosity!**

Particlization

Do we know f_{eq} ?
(What if it is NOT Boltzmann?)

Origin of $f_{eqi}(x,p)$

- 1) Microscopic theory (kinetic transport) with *simple* interaction \rightarrow interaction thermalizes
- 2) Microscopic theory with long-range interaction or fluctuation + relaxation \rightarrow non-Boltzmann f_{eq} (e.g., Tsallis)
- 3) Conformal hydro ($\varepsilon=3P$) doesn't require equilibrium. Any isotropic $f_{eq}(p)$ would do

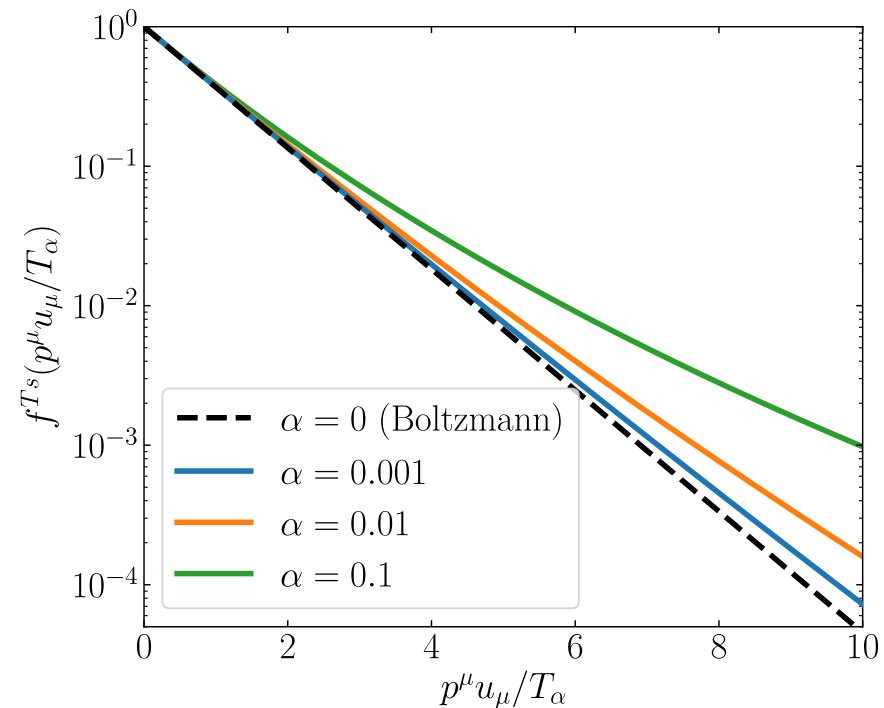
P.Arnold, J.Lenaghan, G.D.Moore, L.G. Yaffe, Phys. Rev. Lett. 94, 072302 (2005)

Isortopic Tsallis phase space density

Ideal hydro with Tsallis distribution:

$$f^{Ts}(x, \mathbf{p}) = N \left(1 + \frac{\alpha}{T_\alpha} p^\mu u_\mu(x) \right)^{-1/\alpha}$$

$$\lim_{\alpha \rightarrow 0} T_\alpha = T \quad (\text{Boltzmann limit})$$



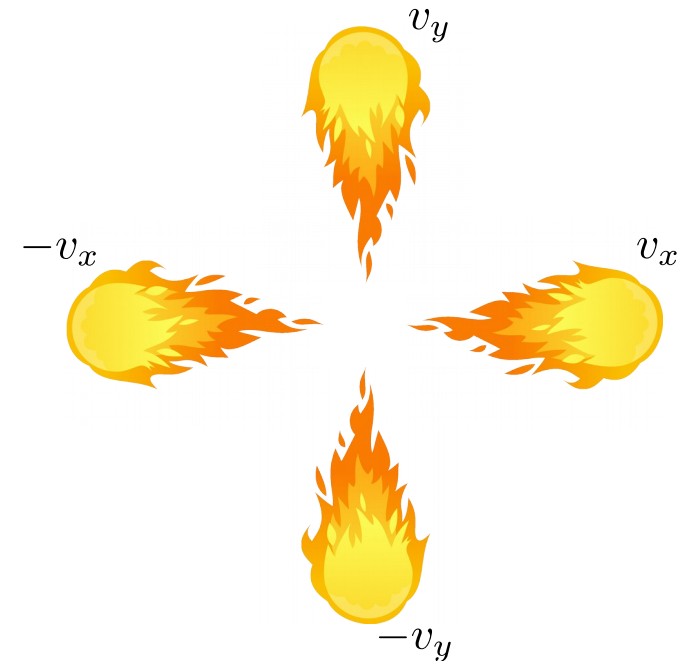
To set (N, α, T_α) :

Fix to partial ε and P (T_α changes): simplest assumption.

Results: 4-Source Fireball Model

P. Hoven et al Phys. Lett. B 503, 58 (2001)

- Simple way to model $v_2(p_T)$
- 4-uniform fireballs (same V and T), no longitud. Expansion boosted symmetrically ($\pm v_x, \pm v_y$) and $t=const$ freezes-out



$$f_4(p_T, \phi) = f_{(+v_x)} + f_{(-v_x)} + f_{(+v_y)} + f_{(-v_y)}$$

$$p_{v_x}^\mu = (\gamma_x(m_T x - v_x p_T \cos \phi), \gamma_x(p_T \cos \phi - v_x m_T \cosh y), p_T \sin \phi, m_T \sinh y)$$

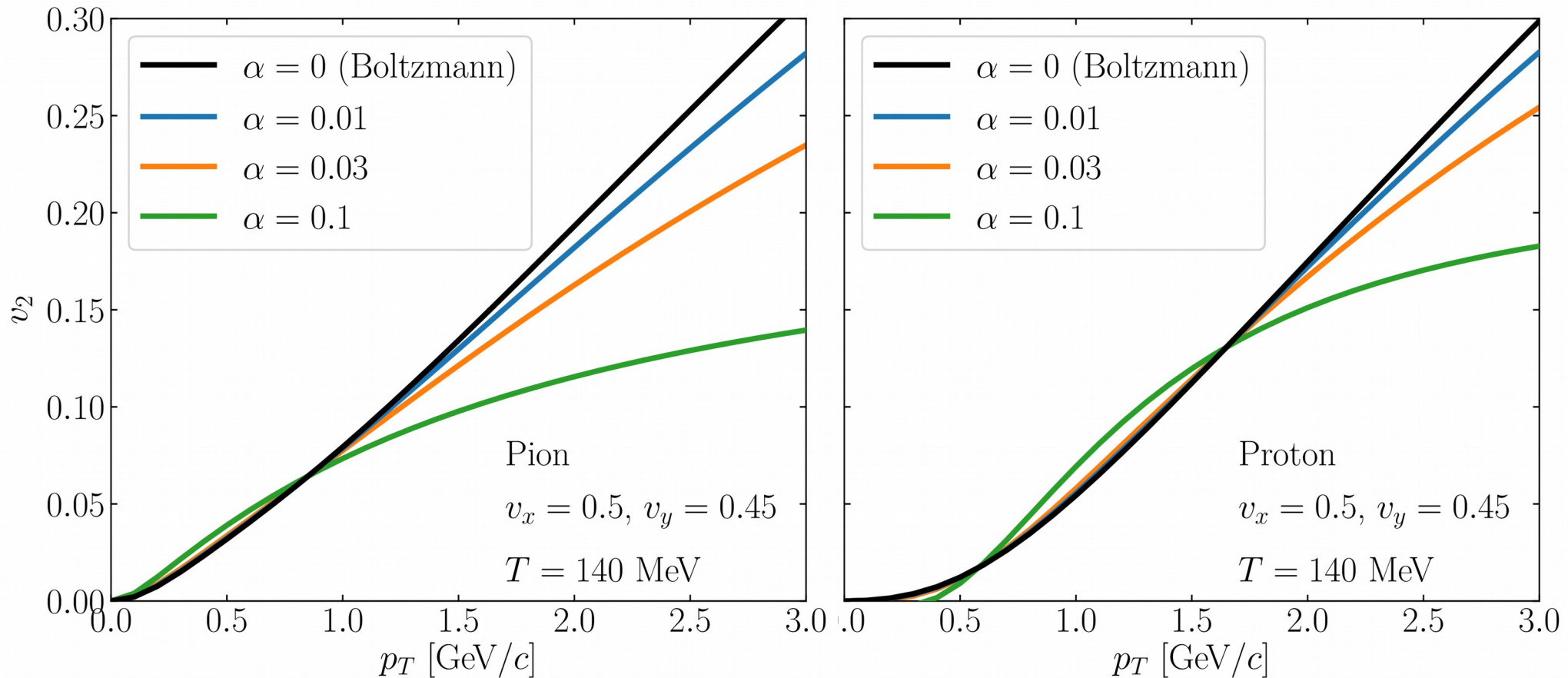
$$v_n(p_T) = \frac{\int_0^{2\pi} d\phi f_4(p_T, \phi) \cos(n\phi)}{\int_0^{2\pi} d\phi f_4(p_T, \phi)}$$

Similar setup to:

D.Molnar & Z.Wolff, Phys.Rev. C95, 024903 (2017)

Z.Wolff & D.Molnar, Phys.Rev. C96, 044909 (2017)

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Hydrodynamic Simulation

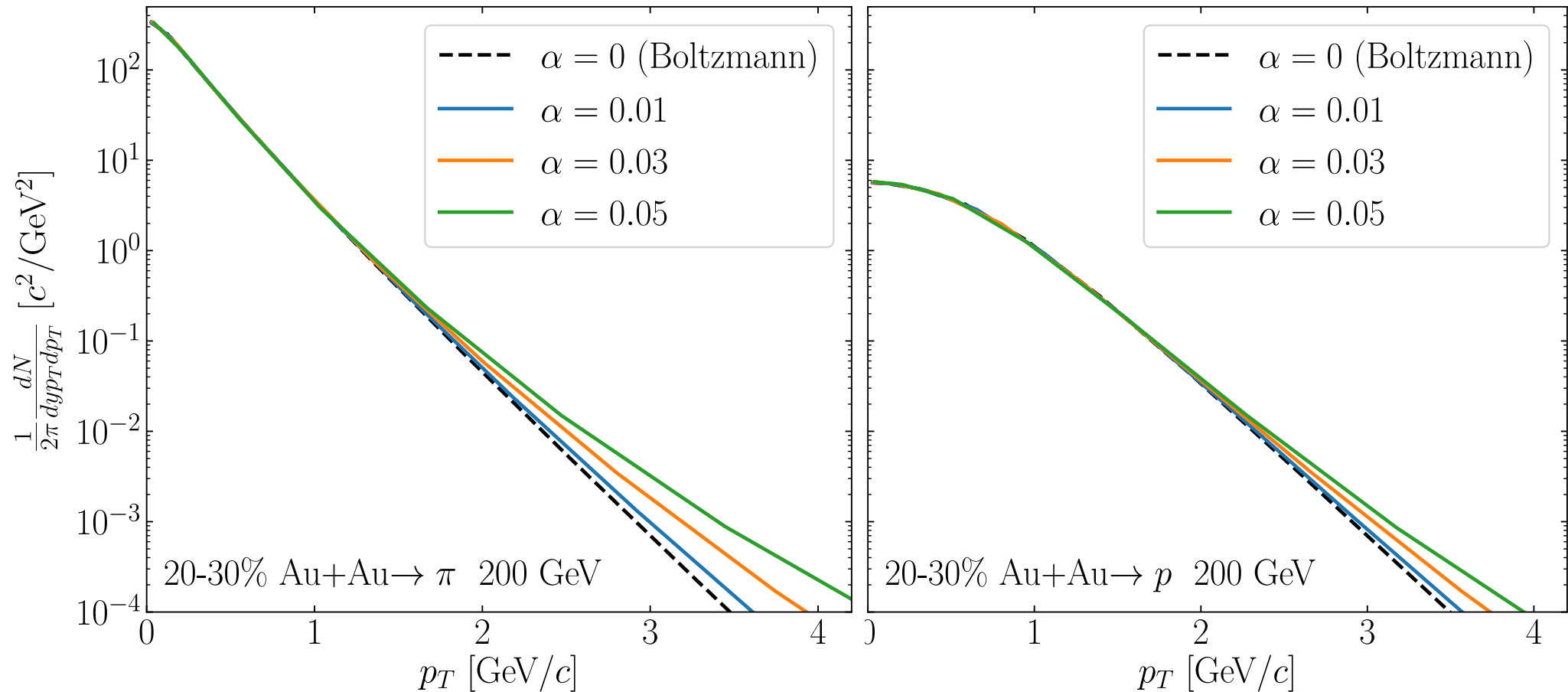
1. Initial condition: 20-30% Au+Au @ 200 GeV analytic Glauber Model
 $S_0=110 \text{ fm}^{-3}$, $b=7 \text{ fm}$ ~ 20-30% centrality
2. Ideal hydro 2+1D with AZHYDRO package $\mu=0$
3. Cooper–Frye freeze-out with Tsallis phase space density
 $\varepsilon(x)$ and $P(x)$ are fixed, $T_f=165 \text{ MeV}$
4. Resonance decay is included with RESO

Similar setup to:

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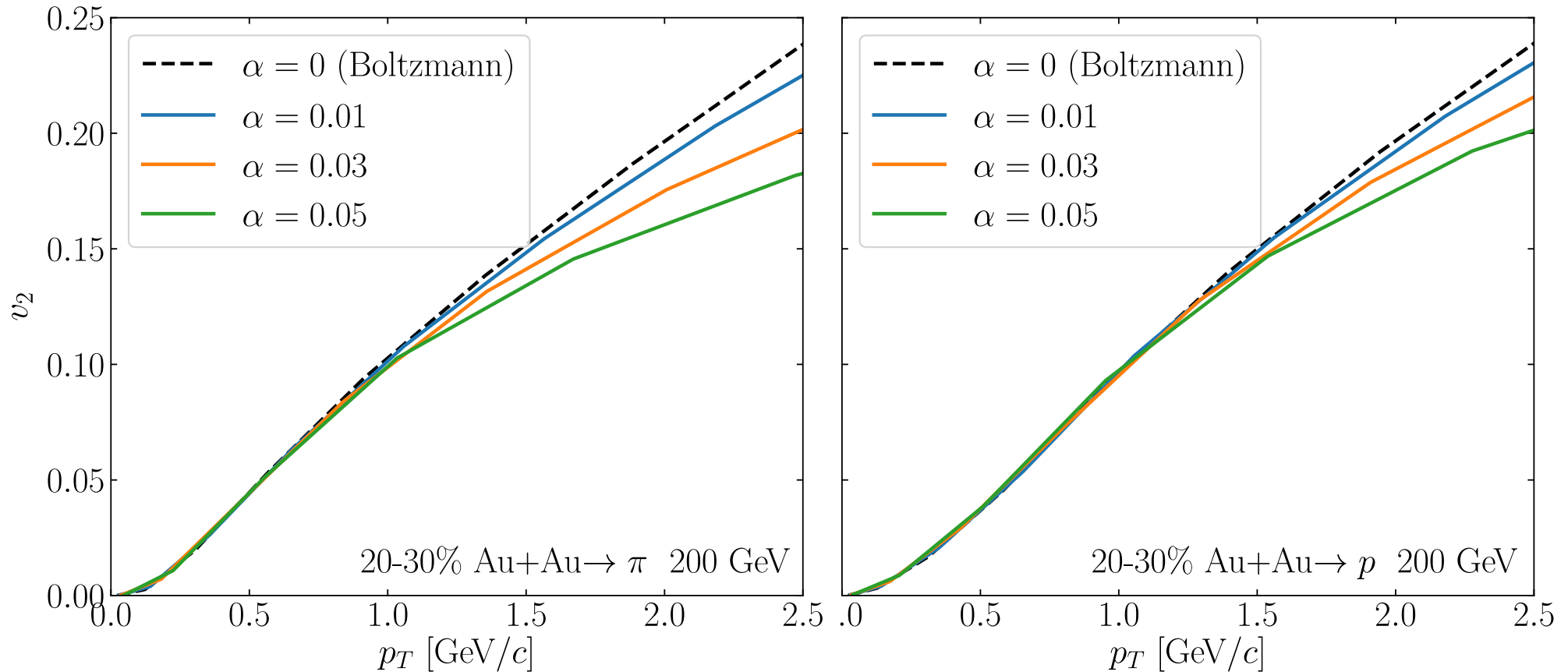
Results: hydrodynamic simulation



- The $\alpha \rightarrow 0$ limit gives back the Boltzmann case
- Beginning doesn't change, α describes the tail

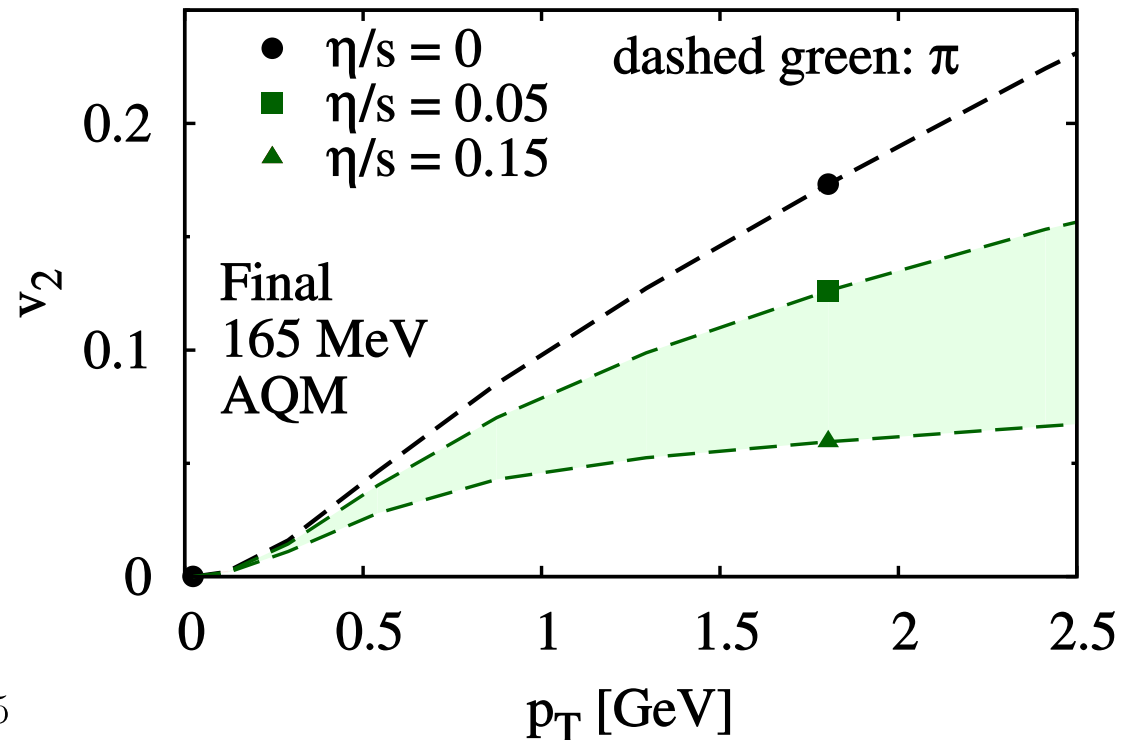
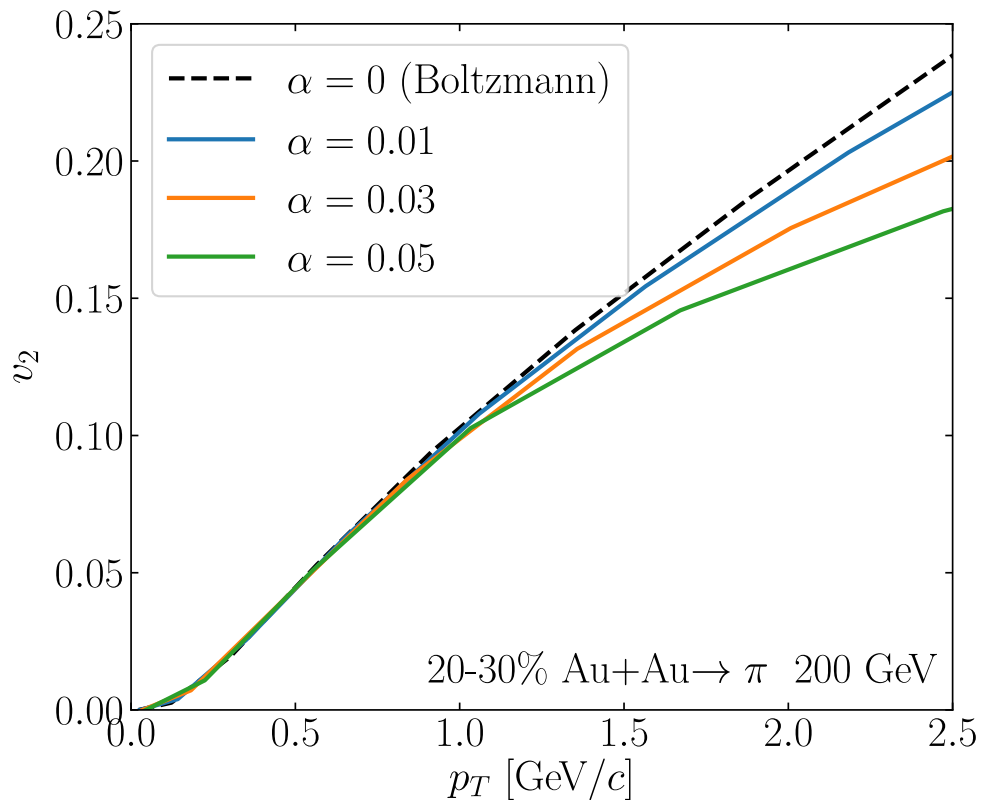
- More suitable shape for experimental data

Results: hydrodynamic simulation



- The $\alpha \rightarrow 0$ limit gives back the Boltzmann case
- Beginning doesn't change
- The α present a suppression in flow
- Less suppression for proton than pion
Just like shear viscosity

Results: hydrodynamic simulation



- The α present a suppression in flow
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Figure from:

D.Molnar & Z.Wolff, Phys.Rev. C95, 024903 (2017)

Summary

- The choice of δf_i is arbitrary \rightarrow uncertainty in observables (known)
- The choice of f_{eq} is arbitrary too \rightarrow additional uncertainty
- Keeping the full ideal hydro, but using Tsallis as a phase space density
 - Better shape for the spectra!
 - Suppression of flow without dissipation but similar to shear viscosity
 $\eta_s/s \sim 0.05$ for $\alpha \sim 0.05$. From previous spectrum studies: $\alpha \sim 0-0.07$
K.Urmosy, G.G.Barnafoldi, T.S.Biro, J. of Phys. Conf.Ser. 612 012048 (2015)
- Why does hydrodynamics work?
 - Can isotropization occur for heavy ions?
 - If it does, can the system be stuck in that state?

Conformal theory

Thank you for your attention!

Dissipative fluid

M.Luzum & P.Romatschke, Phys.Rev. C78, 034915 (2008)
D.Molnar & P.Houvinen, J.Phys. G35, 104125 (2008)
K.Dusling, G.D.Moore, D.Teaney, Phys.Rev. C81, 034907 (2010)
D.Molnar & Z.Wolff, Phys.Rev. C95, 024903 (2017)

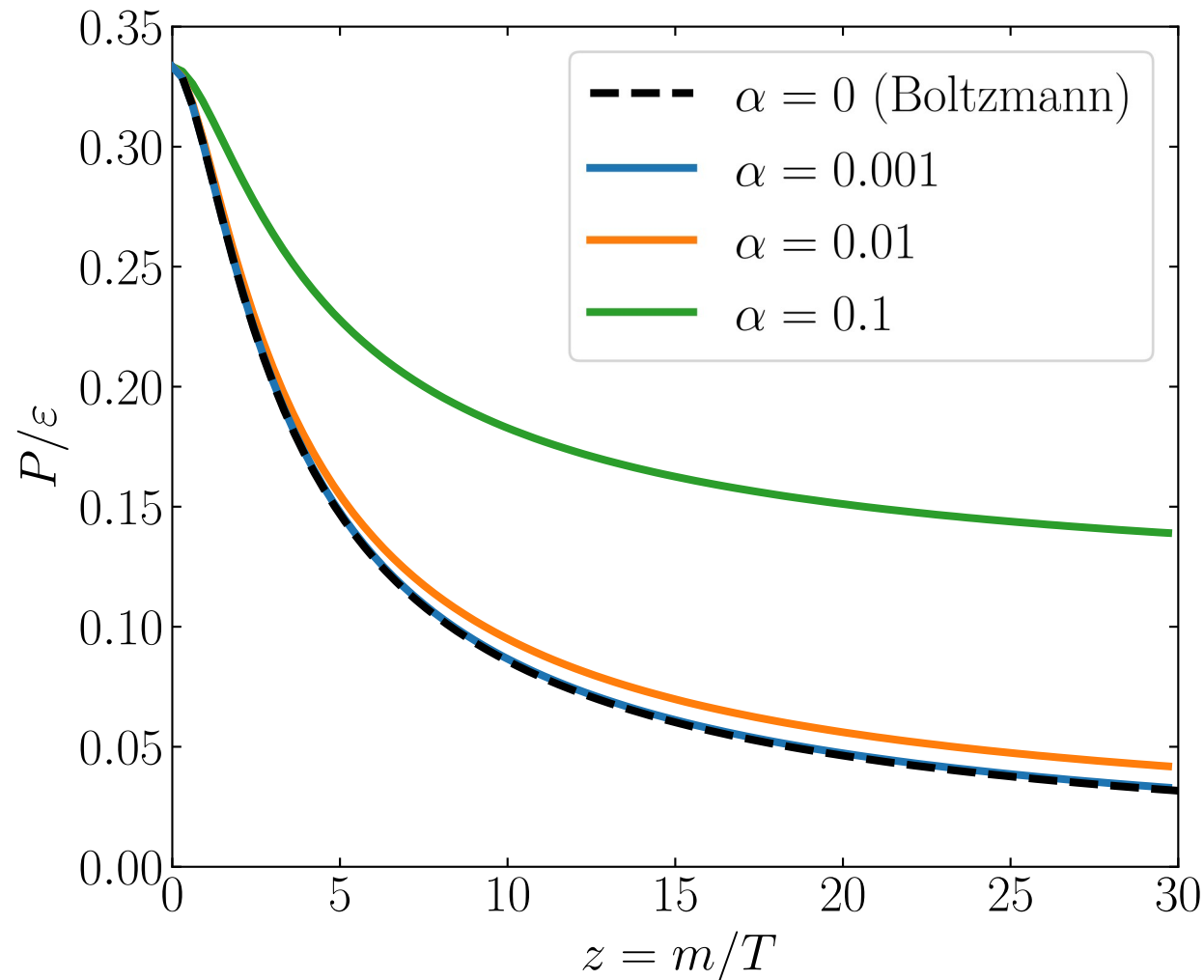
Close to local equilibrium: $f_i(x, \mathbf{p}) = f_i^{eq}(x, \mathbf{p}) + \delta f_i(x, \mathbf{p})$

$$T^{\mu\nu} = T_{id}^{\mu\nu} + \delta T^{\mu\nu} \qquad N_c^\mu = N_{c,id}^\mu + N_c^\mu$$

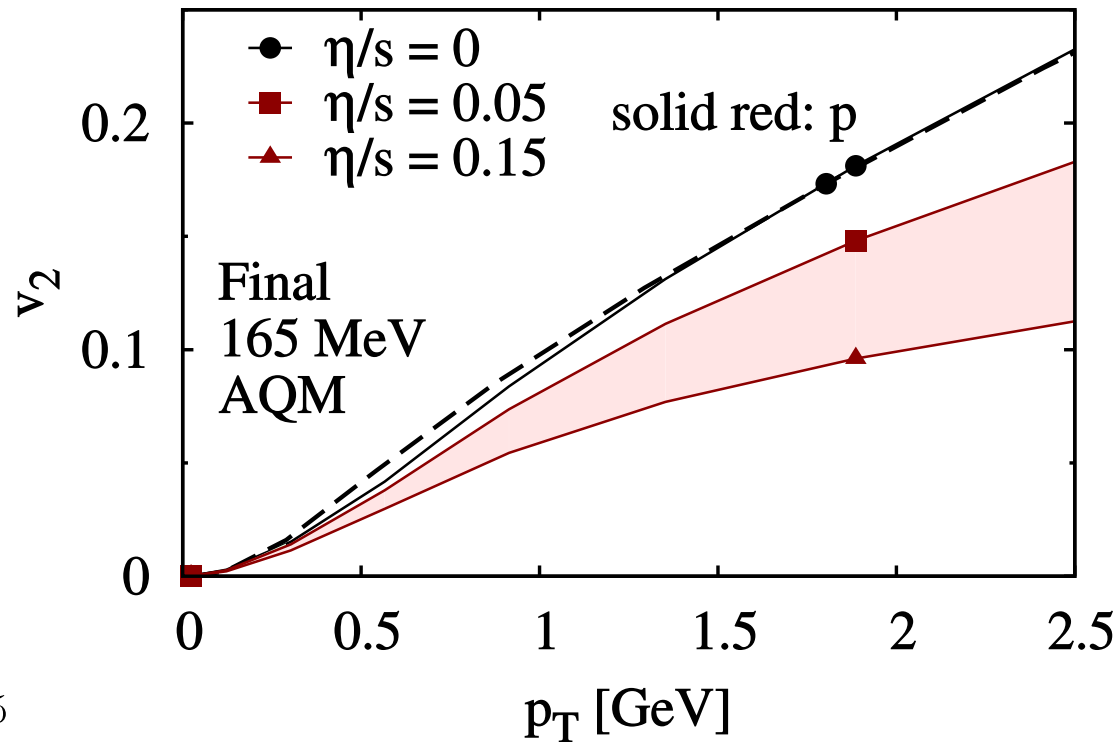
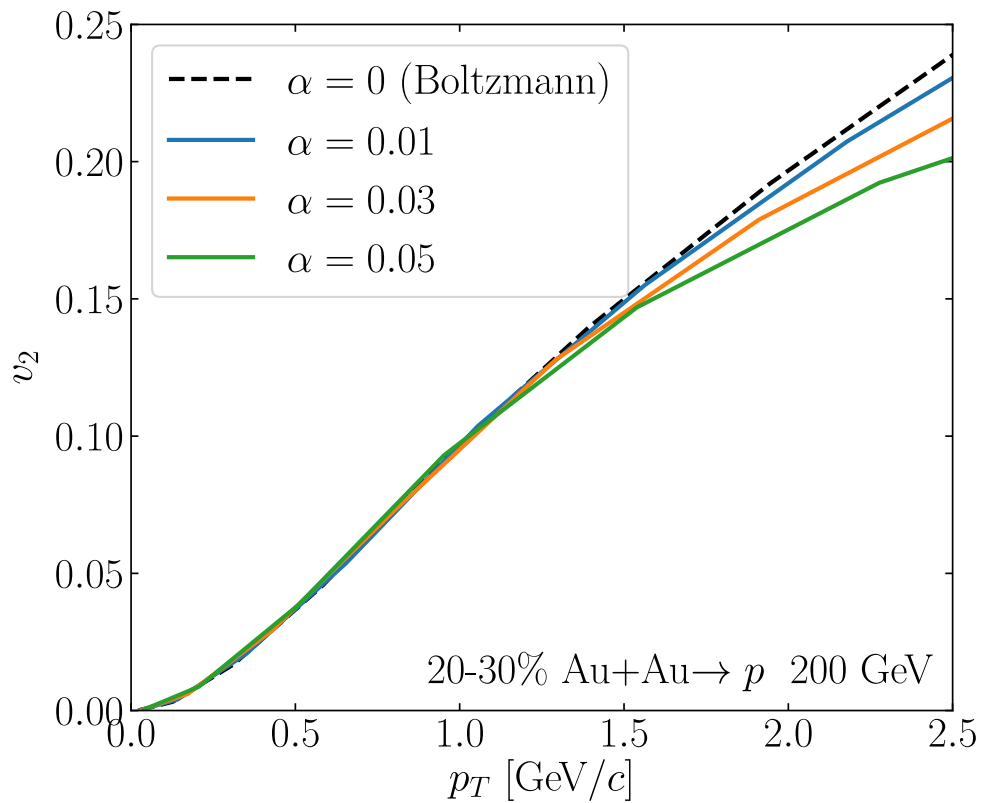
$$\left. \begin{aligned} \delta T^{\mu\nu} &= \sum_i \int \frac{d^3p}{E} p^\mu p^\nu \delta f_i(x, \mathbf{p}) \\ \delta N_c^\mu(x) &= \sum_i q_{c,i} \int \frac{d^3p}{E} p^\mu \delta f_i(x, \mathbf{p}) \end{aligned} \right\} \delta f_i(x, \mathbf{p})$$

Problem: finite set of conditions can be satisfied with infinitely many different δf_i !

Backup: 4-Fireball Source Model



Results: hydrodynamic simulation

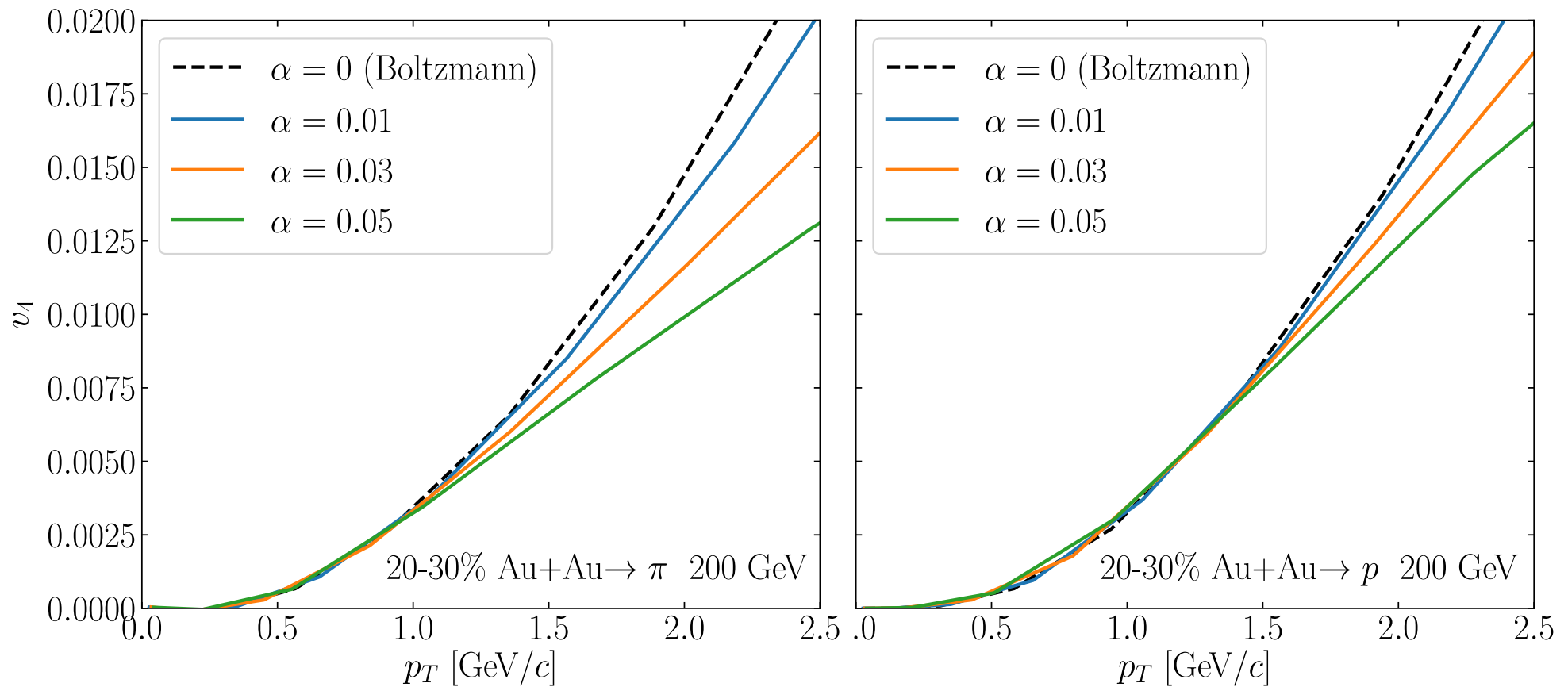


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