Averaging and the shape of the correlation function

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The goal(s) of this talk

- An introduction into formalism of femtoscopy, correlation function, and its parametrisation

- Comments on the Levy-stable shape of the correlation function

Correlation function from the source

Notation:

$$q = p_1 - p_2 \, , \qquad K = \frac{1}{2}(p_1 + p_2)$$

Emission function: the Wigner function of the source $S(x, K)$

$$C(q, K) \approx 1 + \frac{\left| \int d^4x \, S(x, K) e^{iqx} \right|^2}{\left(\int d^4x \, S(x, K) \right)^2}$$

Direct Fourier transform

$$C(q, K) \approx 1 + \frac{\int d^4r \, D(r, K) e^{iqr}}{\left(\int d^4x \, S(x, K) \right)^2} \, ,$$

where

$$D(r, K) = \int d^4X \, S\left(X + \frac{r}{2}, K \right) S\left(X - \frac{r}{2}, K \right)$$

Correlation function measures the Fourier transform of the distribution of emission point differences. This is often "bell shaped".

Gaussian parametrisation

Coordinates: out (direction of K_T), side (normal to K_T), long (beam axis)
On-shell constraint

$$q^0 = \frac{\vec{q}\vec{K}}{K^0} = \vec{q}\vec{\beta}$$

$$\begin{aligned} &C(q,K) - 1 \\ &\quad = \exp\left[-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2 - 2q_o q_s R_{os}^2 - 2q_o q_l R_{ol}^2 - 2q_s q_l R_{sl}^2\right] \end{aligned}$$

Equate with correlation function up to 2nd order in q: get correlation radii ($\tilde{x} = x - \langle x \rangle$)

$$\begin{aligned} R_o^2 &= \left\langle (\tilde{x} - \beta_T \tilde{t})^2 \right\rangle & R_{os}^2 &= \langle (\tilde{x} - \beta_T \tilde{t})\tilde{y} \rangle \\ R_s^2 &= \left\langle \tilde{y}^2 \right\rangle & R_{ol}^2 &= \langle (\tilde{x} - \beta_T \tilde{t})(\tilde{x} - \beta_l \tilde{t}) \rangle \\ R_l^2 &= \left\langle (\tilde{z} - \beta_l \tilde{t})^2 \right\rangle & R_{sl}^2 &= \langle \tilde{y}(\tilde{z} - \beta_l \tilde{t}) \rangle \end{aligned}$$

1-D correlation function

Due to lack of statistics, sometimes correlation function is parametrised as function of

$$q_{inv}^2 = q_o^2 + q_s^2 + q_l^2 - q_0^2 = |\vec{q}|^2 - \vec{q}\vec{\beta}$$

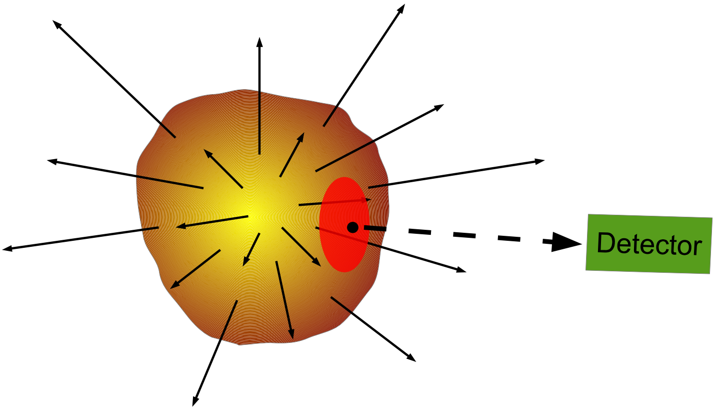
as $C(q_{inv}, K)$.

This measures some effective size of the source in the frame co-moving with β.

$$C(q_{inv}, K) \approx 1 + \frac{\int dq_o dq_s dq_l \delta(|\vec{q}|^2 - \vec{q}\vec{\beta} - q_{inv}^2) \left| \int d^4x\, S(x, K) e^{iqx} \right|^2}{\left(\int d^4x\, S(x, K) \right)^2}$$

A homogeneity region

- fixed momentum is only produced from a part of the fireball: homogeneity region
- homogeneity region moves roughly with the same velocity as the produced particles



Averaging

Momentum

Hadrons with different momenta (p_T and/or ϕ) come from different homogeneity regions.
Schematically:

$$C(q,K) \approx 1 + \frac{\int_{bin} dK \left| \int d^4x\, S(x,K) e^{iqx} \right|^2}{\left(\int_{bin} dK \int d^4x\, S(x,K) \right)^2}$$

Events

The correlation function is a superposition of many correlation functions.
Schematically, $\rho(R)$ is distribution of a size parameter:

$$C(q,K) \approx \int dR\, \rho(R) \left(1 + \frac{\left| \int d^4x\, S(x,K;R) e^{iqx} \right|^2}{\left(\int d^4x\, S(x,K;R) \right)^2} \right)$$

also: [C. Plumberg, U. Heinz, Phys. Rev. C **92** 044906 (2015), **92** 049901 (2015)]

The Blast-Wave model

The emission function (no viscous corrections)

$$S(x,K)\,d^4x = \frac{m_t \cosh(\eta - y)}{(2\pi)^3}\,\tau\,d\tau\,d\eta\,r\,dr\,d\theta\,\left(\exp\left(\frac{u^\mu(x)p_\mu}{T}\right) \pm 1\right)^{-1} \times \Theta(r - R(\theta))\,\delta(\tau - \tau_{fo})$$

space coordinates: r, θ, $\tau = \sqrt{t^2 - z^2}$, $\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$

- phase-space cell volume: $(2\pi)^3$
- flux of particles across the freeze-out hypersurface : $m_t \cosh(\eta - y)$
- thermal distribution, energy in the rest-frame of the fluid: $E_{fl} = u^\mu p_\mu$
- sharp cutoff in transverse direction: $\Theta(r - R(\theta))$
- infinite in space-time rapidity
- freeze-out along $\sqrt{t^2 - z^2} = \tau_{fo}$: $\delta(\tau - \tau_{fo})$

The Blast-Wave model, cont'd

The emission function (no viscous corrections)

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collective expansion velocity field

$$u^\mu(x) = (\cosh\eta\,\cosh\eta_t(r,\theta_b),\,\cos\theta_b\,\sinh\eta_f(r,\theta_b),\, \sin\theta_b\,\sinh\eta_f(r,\theta_b),\,\sinh\eta\,\cosh\eta_t(r,\theta_b))$$

transverse expansion:

$$\eta_t(r,\theta_b) = \rho_0 \frac{r}{R(\theta)}\left[1 + 2\rho_2\cos(2(\theta_b - \theta_2))\right]$$

Transverse shape:

$$R(\theta) = R_0\left[1 + 2a_2\cos(2(\theta_b - \theta_2))\right]$$

Resonance production

- resonances are produced according to the same emission function
- resonances decay exponentially in time: $\rho(\tau_d) \propto e^{-\Gamma_R \tau_d}$
- 2-body and 3-body decays included
- branching ratios for decays via different channels
- chain decays accounted for in the model

Does averaging over events matter?

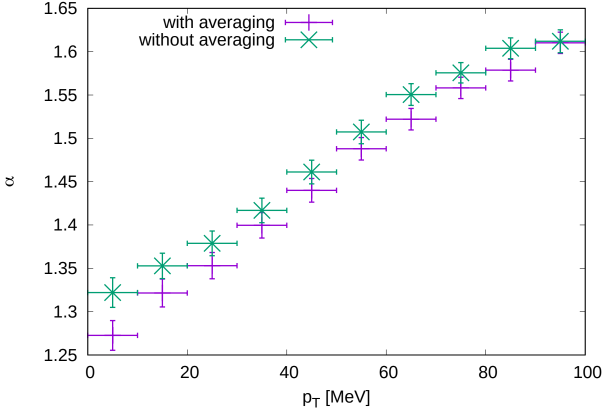
Simulation with: $T = 0.12$ GeV, $R_0 = 7$ fm, $\tau_{fo} = 10$ fm$/c$, $\rho_0 = 0.8$, no resonances

a_2 fluctuates within $(-0.1, 0.1)$, or it is fixed to 0.05

Fit function:

$$1 + \lambda e^{-(q_{inv} R)^\alpha}$$

shown values of α



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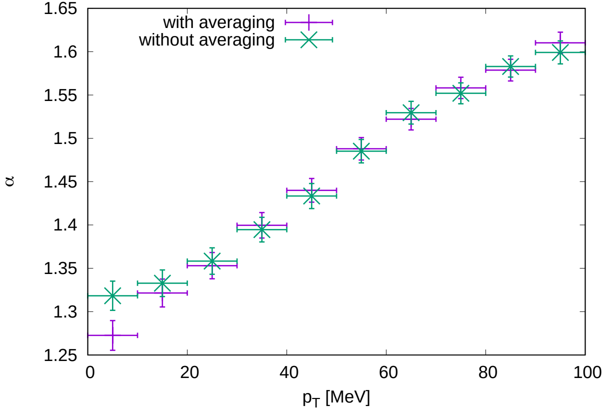

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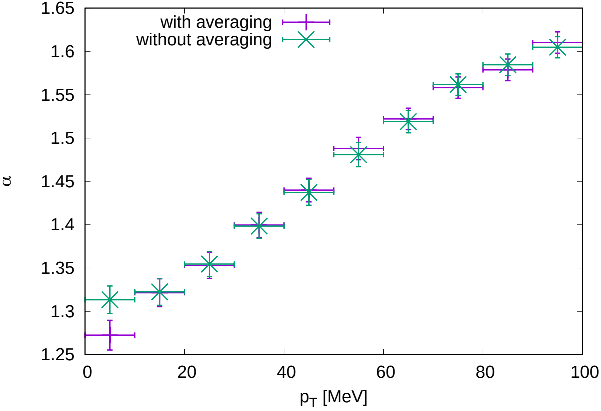
shown values of α



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But: correlation function in q_{inv} seems pretty non-Gaussian itself!
And averaging over p_T could matter!

The influence of resonances

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Fit function:

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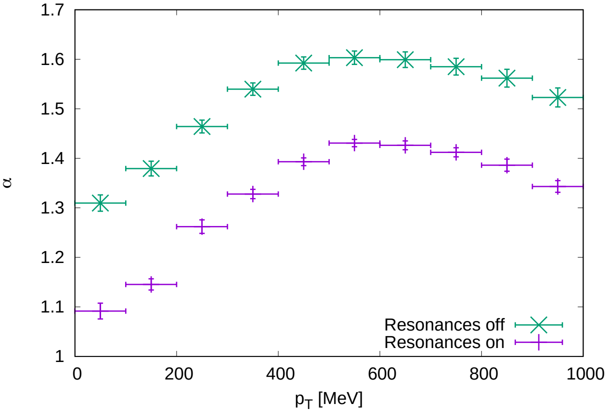
shown values of α



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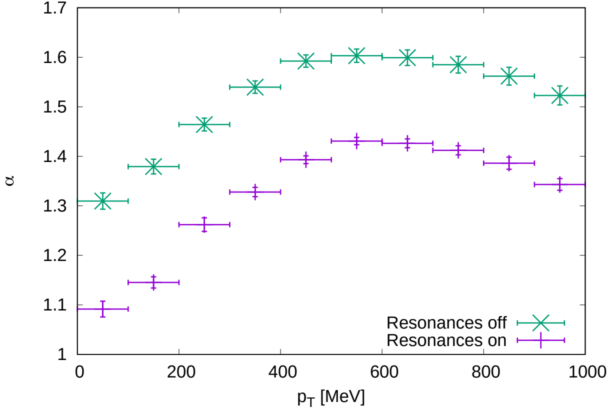
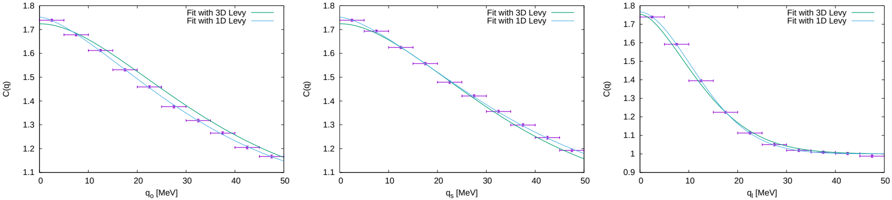


The resonances have a big influence!

$C(q, K)$ from BW model is non-Gaussian

Shapes of $C(q, K)$ fitted in each direction separately

Correlation functions with resonances



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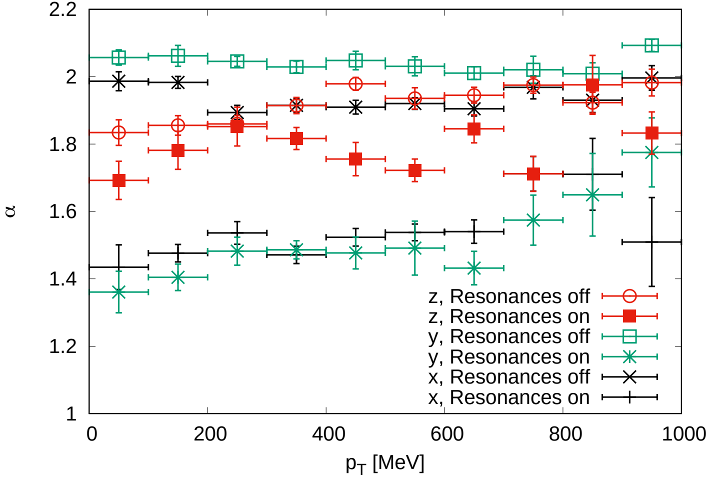
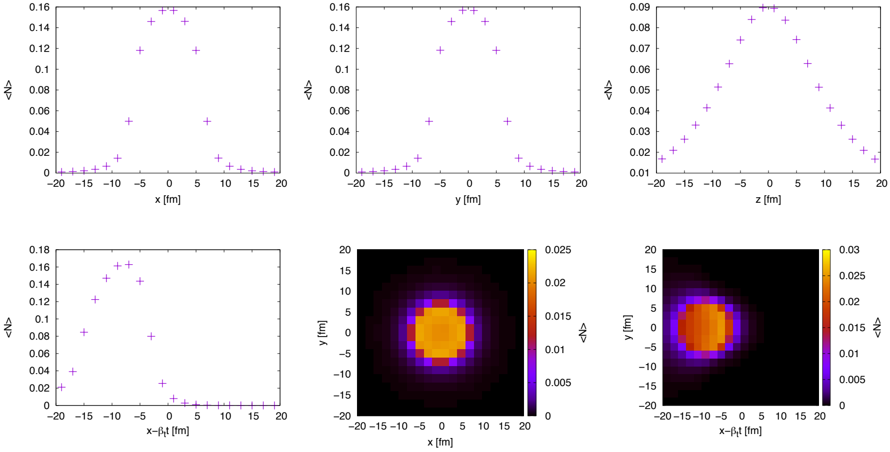
Shapes of $C(q, K)$ fitted in each direction separately

Results for the parameter α



The profile of the source

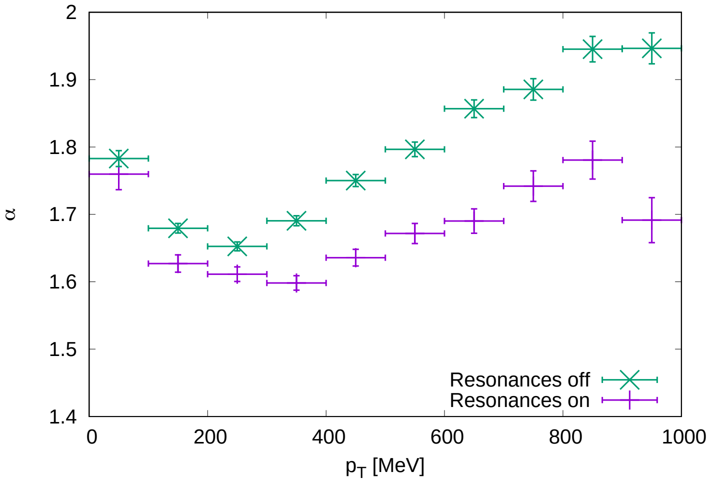
emssion points obtained from 50k events, resonances included



Fitting with Levy-inspired in 3D

fitting function:

$$C(q,K) = 1 + \lambda \exp\left[-\left(R_o'^2 q_o^2 + R_s'^2 q_s^2 + R_l'^2 q_l^2\right)^{\alpha/2}\right]$$



Summary

Reasons for small α

- non-Gaussian source in q_l
- resonances push down α by 0.2–0.6
- **the use of q_{inv} has a strong effect**