

# $T$ -dependence of the axion mass when the $U_A(1)$ and chiral symmetry breaking are tied

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## Introduction

- QCD has the Strong CP problem: **no experimental evidence of any CP-symmetry violation in strong interactions, in spite of its  $\theta$ -term:**

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{CPsymmetric}}^{\text{QCD}} + \theta \frac{g^2}{32\pi^2} F_{\mu\nu}^b \tilde{F}^{b\mu\nu} \quad (\tilde{F}^{b\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^b)$$

$\theta$ -term cannot be discarded (unlike in QED), although it is a total divergence, due to **nontrivial topological structures in QCD** – e.g., **instantons** (probably yielding anomalously large  $M_{\eta'}$ ) **important for solving the  $U_A(1)$  problem.**

- In spite of this, the experimental bound is only  $|\theta| < 10^{-10}$
- Thus, a mystery: **why is  $\theta$  so small ?**
- Nowadays preferred solution: a new particle beyond SM: **axion** a
- **Axions are very interesting also for cosmology as candidates for dark matter.**

## Axions as quasi-Goldstone bosons

- Peccei & Quinn introduced a new axial global symmetry  $U(1)_{PQ}$  which is **broken spontaneously at some scale  $f_a$**  (= the free parameter of QCD axion theories. It ultimately determines the absolute value of the axion mass  $m_a$ , but cancels from combinations such as  $m_a(T)/m_a(0)$ .)
- **the pseudoscalar axion field  $a(x)$  is the (would-be massless) Goldstone boson of this spontaneous breaking.** Then,

$$\mathcal{L}_{\text{axion}}^{\text{QCD}+} = \mathcal{L}_{\text{CPsymmetric}}^{\text{QCD}} + \left( \theta + \frac{a}{f_a} \right) \frac{g^2}{32\pi^2} F_{\mu\nu}^b \tilde{F}^{b\mu\nu} + \frac{1}{2} \partial_\mu a \partial^\mu a + \mathcal{L}_{\text{int}}^{a\psi}$$

- But, the  $U(1)_{PQ}$  symmetry is **also broken explicitly** by the gluon axial anomaly through axion's coupling with gluons  $\Rightarrow m_a \neq 0$ . Gluons generate an effective axion potential, which leads to the axion expectation value  $\langle a \rangle$  such that  $(\theta + \langle a \rangle / f_a) = 0$ , minimizing the potential  $\Rightarrow$  **strong CP problem solved, irrespective of the initial  $\theta$ .**

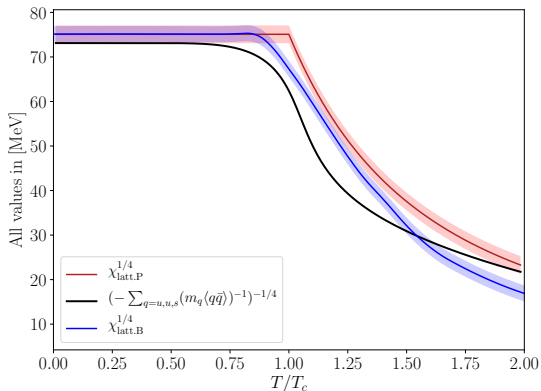
("Misalignment production" is relaxation from any value in the early Universe towards the minimum at  $(\dots) = 0$ , and the resulting axion oscillation energy is a "cold dark matter" candidate.)

## Axion mass

- For all temperatures:  $m_a^2(T) f_a^2 = \chi(T) =$  QCD topological susceptibility
- At  $T = 0$ ,  $m_a^2 f_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} f_\pi^2 M_\pi^2 \rightarrow$  **isospin limit**  $\rightarrow (78.9 \text{ MeV})^4$
- This agrees well with results, including  $\chi(T)$ , from “our” DS-BSE chirally well-behaved model (separable: simplified, but phenomenologically successful)

- $\chi(T)$  from lattice:  
**Petreczky & al. PLB (2016)** and  
**Borsany & al. Nature (2016)**

- $\chi(T)$  from our usual DS-BSE model: successful at  $T = 0$ ,  
**no additional fitting for  $T > 0$** :  
 condensates  $\langle \bar{q}q \rangle(T)$  of **massive  $q = u, d, s$**  essential to yield **good  $T$ -dependence** of  $\chi(T)$  for good  $T$ -dependence of  $\eta$  and  $\eta'$  masses.



## Our various $\chi(T)$ – from our various calculations of anomalous $\eta'$ - $\eta$ masses

**QCD chiral behavior** (reproduced by, e.g., DS approach) **of the non-anomalous parts** of masses of light  $q\bar{q}'$  pseudoscalars:  $M_{q\bar{q}'}^2 = \text{const} (m_q + m_{q'})$ .

$\Rightarrow$  non-anomalous parts of the masses cancel in Witten-Veneziano rel. (WVR):

$$M_{\eta'}^2 + M_{\eta}^2 - 2 M_K^2 = \chi_{\text{YM}} \frac{2N_f}{f_\pi^2} = \text{anomalous mass}^2 \equiv M_{U_A(1)}^2 \approx \Delta M_{\eta_0}^2,$$

$$\chi = \int d^4x \langle 0 | Q(x) Q(0) | 0 \rangle, \quad Q(x) = \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^a(x) F_{\rho\sigma}^a(x)$$

QCD topological susceptibility  $\chi$  = a direct measure of  $U_A(1)$  breaking, so  $U_A(1)$  restoration is indicated by vanishing or asymptotic reduction of  $\chi$  and quantities related to it, like  $M_{U_A(1)} \approx \Delta M_{\eta_0} \approx \Delta M_{\eta'}$ .

- $Q(x)$  = topological charge density operator
- In WVR,  $\chi$  is pure-gauge, YM one,  $\chi_{\text{YM}} \leftrightarrow \chi_{\text{quench}}$ , obtained long ago by lattice - harder for  $\chi$  of light-flavor QCD, but can use DiVecchia-Veneziano

relation: 
$$\chi = - \frac{\langle \bar{q}q \rangle_0}{\sum_{q=u,d,s} \frac{1}{m_q}} + \mathcal{C}_m(\text{unknown corrections, higher } \mathcal{O} \text{ in small } m_q)$$

## The 1<sup>st</sup> example of tied breaking of $U_A(1)$ and chiral symmetries:

**Leutwyler-Smilga relation (LS)**, also connecting YM and full QCD quantities (like WVR), "making"  $\chi_{\text{YM}}$  out of much smaller  $\chi$ :

$$\chi_{\text{YM}} = \frac{\chi}{1 + \frac{\chi}{\langle \bar{q}q \rangle_0} \sum_{q=u,d,s} \frac{1}{m_q}} \equiv \tilde{\chi} \rightarrow \tilde{\chi}(T)$$

where for the light quarks

$$\chi = - \frac{1}{\sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0}} + C_m$$

- $C_m$  = small corrections of higher orders in small  $m_q$ , ... but  $C_m$  should not be neglected, since  $C_m = 0$  would imply that  $\chi_{\text{YM}} = \infty$ . However, note that for axions, the leading term of  $\chi$  would suffice.
- LS relation fixes the value of the correction at  $T = 0$ :

$$\frac{1}{C_m} = \sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0} - \chi_{\text{YM}}(0) \left( \sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0} \right)^2.$$

## Shore's generalization of WV = 2<sup>nd</sup> example of this tying

$$(f_{\eta'}^0)^2 M_{\eta'}^2 + (f_{\eta}^0)^2 M_{\eta}^2 = \frac{1}{3} (f_{\pi}^2 M_{\pi}^2 + 2f_K^2 M_K^2) + 6A \quad (1)$$

$$f_{\eta'}^0 f_{\eta}^8 M_{\eta'}^2 + f_{\eta}^0 f_{\eta}^8 M_{\eta}^2 = \frac{2\sqrt{2}}{3} (f_{\pi}^2 M_{\pi}^2 - f_K^2 M_K^2) \quad (2)$$

$$(f_{\eta'}^8)^2 M_{\eta'}^2 + (f_{\eta}^8)^2 M_{\eta}^2 = -\frac{1}{3} (f_{\pi}^2 M_{\pi}^2 - 4f_K^2 M_K^2) \quad (3)$$

(Large  $N_c$  limit,  $f_{\eta'}^0, f_{\eta}^8, f_K \rightarrow f_{\pi}$  & 'off-diagonal'  $f_{\eta'}^0, f_{\eta}^8 \rightarrow 0$  recovers standard WV.)

The role of  $\chi_{\text{YM}}$  taken over by the full QCD topological charge parameter  $A$ ,

$$A = \frac{\chi}{1 + \chi \left( \frac{1}{\langle \bar{u}u \rangle m_u} + \frac{1}{\langle \bar{d}d \rangle m_d} + \frac{1}{\langle \bar{s}s \rangle m_s} \right)} \quad (4)$$

$A$  behaves with  $T$  as a full QCD quantity, **but**, at  $T = 0$ ,  $A = \chi_{\text{YM}} + \mathcal{O}(\frac{1}{N_c})$

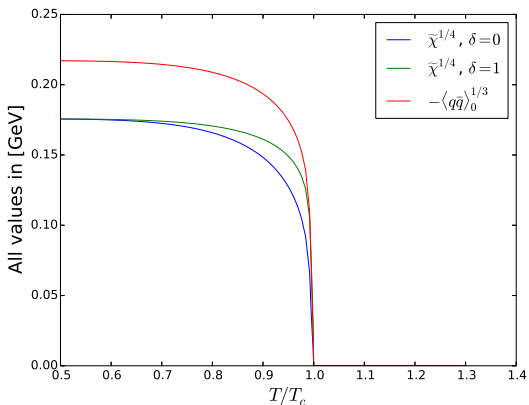
Gives the now established crossover behavior around  $T \sim T_{\text{Ch}}$  when also

$$\chi = \frac{-1}{\frac{1}{m_u \langle \bar{u}u \rangle} + \frac{1}{m_d \langle \bar{d}d \rangle} + \frac{1}{m_s \langle \bar{s}s \rangle}} + C_m \quad (5)$$

Large  $N_c$  limit & approximating 3 condensates by  $\langle \bar{q}q \rangle_0$ , returns the LS relation.

## Chiral condensate $\langle q\bar{q} \rangle_0(T)$ and resulting $\tilde{\chi}(T)$

Would something go wrong in axion theory because of  $\chi(T) \rightarrow 0$  as  $T \rightarrow T_c$ ?



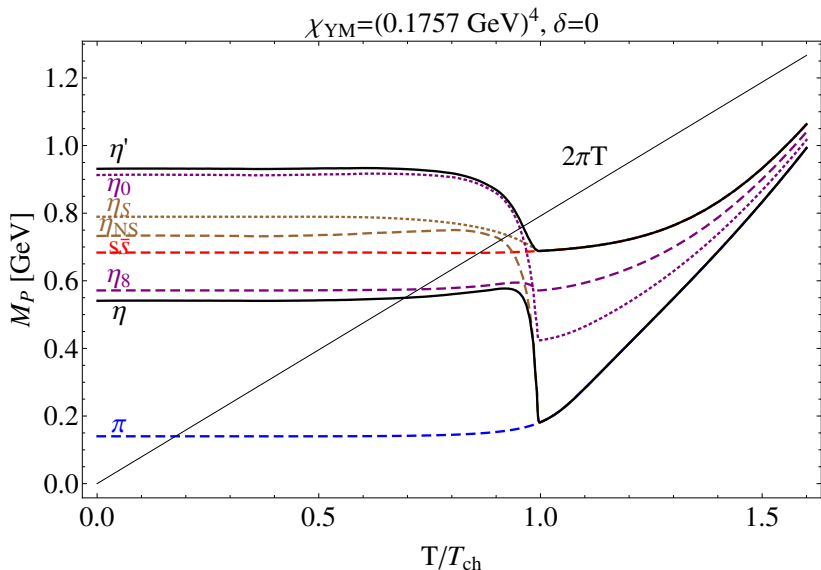
Anyhow, this sharp chiral transition enforces at  $T = T_c \equiv T_{\text{Ch}}$  the abrupt transition to the NS-S asymptotic regime of vanishing  $U_A(1)$  anomaly influence:  $M_{\eta'}(T) \rightarrow M_{s\bar{s}}(T)$ , and  $M_{\eta}(T) \rightarrow M_{\text{NS}}(T) \rightarrow M_{\pi}(T)$ , and  $\phi(T) \rightarrow 0$ .



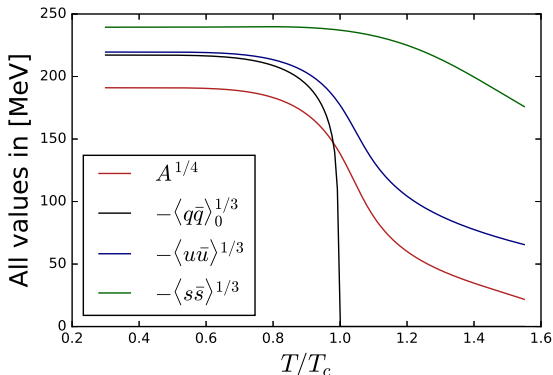
## Prediction good for $\eta'$ , but for $\eta$ not supported by any experiment

[Benić, Horvatić, Kekez and Klabučar, Phys. Rev. D **84** (2011) 016006.]:

Anomalous contribution obtained via LS with chiral-limit condensate  $\langle \bar{q}q \rangle_0$ :



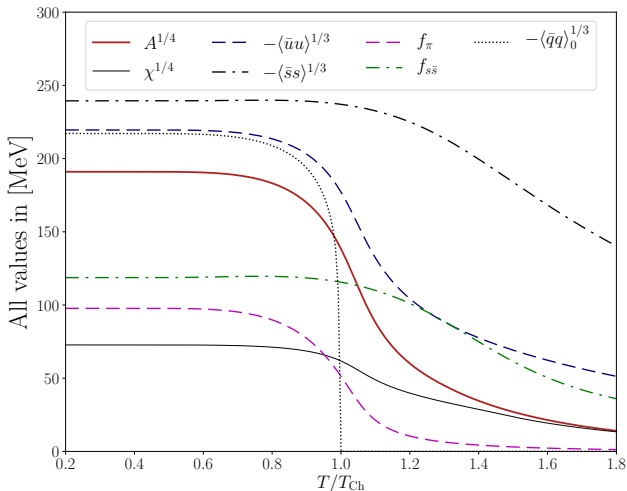
## A solution: $U_A(1)$ breaking from realistic condensates



Instead of the fast-falling **chiral-limit** condensate  $\langle \bar{q}q \rangle_0$ , try  $\langle \bar{q}q \rangle$  condensates with realistic explicit chiral symmetry breaking: replace  $m_q \langle \bar{q}q \rangle_0 \rightarrow m_q \langle \bar{q}q \rangle$ , ( $q = u, d, s$ ) in  $\chi$ , like in the original  $A$ .

- **Introduces crossover behavior, also in  $\chi(T)$ , and thus in  $m_a(T)$**

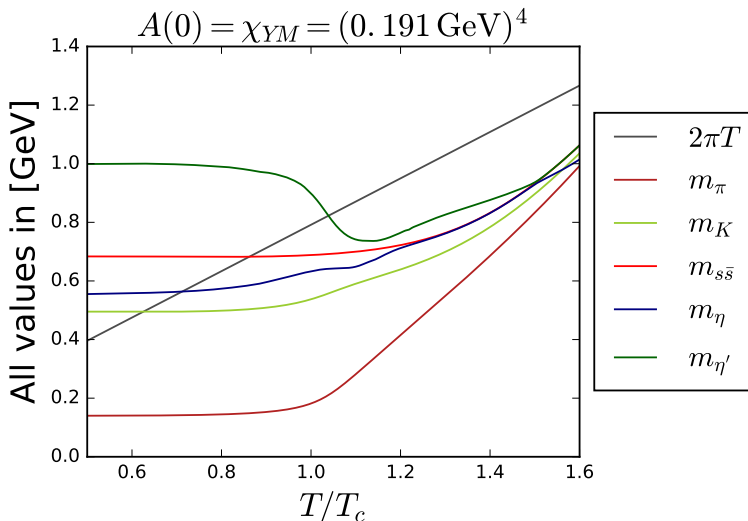
## Compare $T$ -dependence of $\langle \bar{q}q \rangle$ & decay const's $f_P$ with $\chi$ & $A$



**FKS scheme on Shore**  $\Rightarrow$  how  $f_P$  influence elements of the  $\eta$ - $\eta'$  mass matrix:

$$X = \frac{f_\pi}{f_{s\bar{s}}}, \quad M_{NS}^2 = M_\pi^2 + \frac{4A}{f_\pi^2}, \quad M_{NS}^2 = \frac{2\sqrt{2}A}{f_\pi f_{s\bar{s}}}, \quad M_S^2 = M_{s\bar{s}}^2 + \frac{2A}{f_{s\bar{s}}^2}$$

⇒  $T$  dependence of light pseudoscalars including  $\eta$  and  $\eta'$



Simplest Ansatz  $\mathcal{C}(T) = \mathcal{C}(0)$  fails above  $T > 1.6T_c$ , as  $\chi(T)$  and  $A(T) < 0$ .

⇒ Try different  $\mathcal{C}(T)$  ...

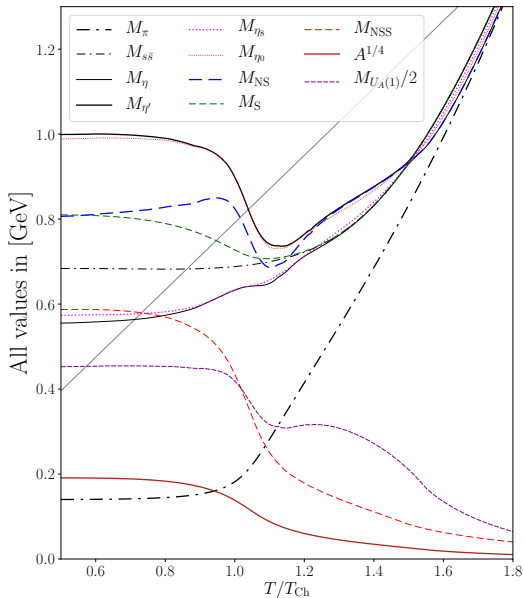
## $T$ -dependence of $M_P(T)$ up to $T = 1.8T_{\text{Ch}}$ [arXiv:1809.00379]

- $\mathcal{C}(T) \neq \text{const}$ , adjusted to enable reaching arbitrary high  $T$ 's, results otherwise very similar to previous case with  $\mathcal{C}(T) = \mathcal{C}(0)$ .
- Other limitations of rank-2 separable model make it hard to find solutions beyond  $\sim 1.8T_c$ .

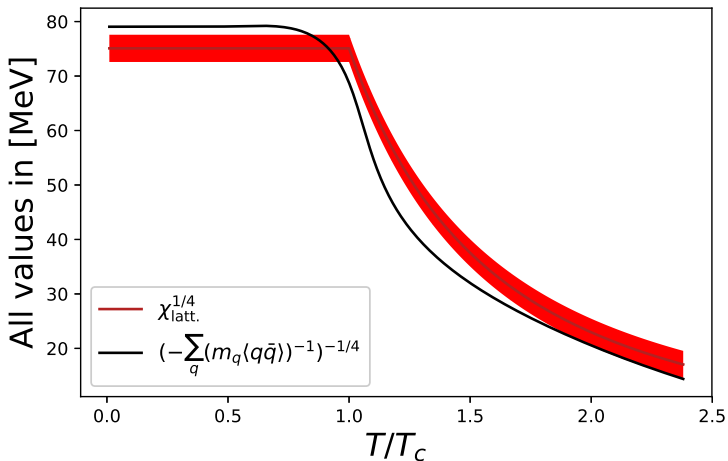
But it is enough to exhibit cleanly the asymptotic regime beyond anticrossing at  $1.5T_{\text{Ch}}$ .

Along with  $A$ , influence on anomalous masses is given by  $M_{\text{NSS}}$  and  $(\frac{1}{2})M_{U_A(1)}$ .

Utopistic in practice? - but in principle, accurate experimental knowledge of  $M_{\eta'}(T)$  would tell about  $A(T)$  and thus about  $\chi(T) \propto m(T)^2$ .



Pleasant surprise from even more simplified model ("rank-1 separable"):  
compare with  $\chi(T)$  extracted from Petreczky *et al.* PLB (2016).



Seems roughly OK, especially since Borsany *et al.* Nature (2016), lower at high  $T$ . In the future, we would like to employ condensates from the lattice.

## Summary

- The axion mass (up to the free scale parameter  $f_a$ ) in terms of the topological susceptibility can be calculated in an effective model of nonperturbative QCD, presently in the SD-BSE approach.
- Our approach to  $\eta'-\eta$  complex, tying the  $U_A(1)$  SB to the DChSB very closely, supports that also for axions  $\chi(T)$  should be in terms of the “massive” condensates exhibiting crossover around  $T \sim T_{\text{Ch}}$ , and there exhibits qualitative agreement with lattice results.
- Our  $\chi(T)$  for axions can be less modeled than for  $\eta'-\eta$ , since it can stop at the leading term; *i.e.*, the presently unknown correction term  $\mathcal{C}_m$  is not mandatory and can be omitted, thus avoiding assumptions about its  $T$ -dependence.
- Our calculations have so far reached  $T$ 's of only a couple of  $T_{\text{Ch}}$ , reaching the power-law regime  $1/T^b$  with smaller exponent ( $b \approx 5$ ) than the lattice ( $b \approx 6$  Petreczky et al. and  $b \approx 8$  Borsany et al.) Some models promise improvement in that sense – more work is needed.