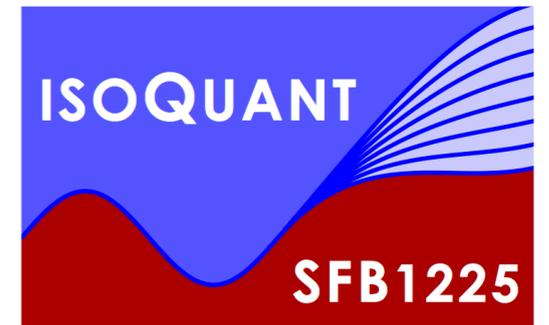




UNIVERSITÄT
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Quarkonium phenomenology from a generalised Gauss-law

David Lafferty

Institut für Theoretische Physik, Universität Heidelberg

with Alexander Rothkopf, University of Stavanger

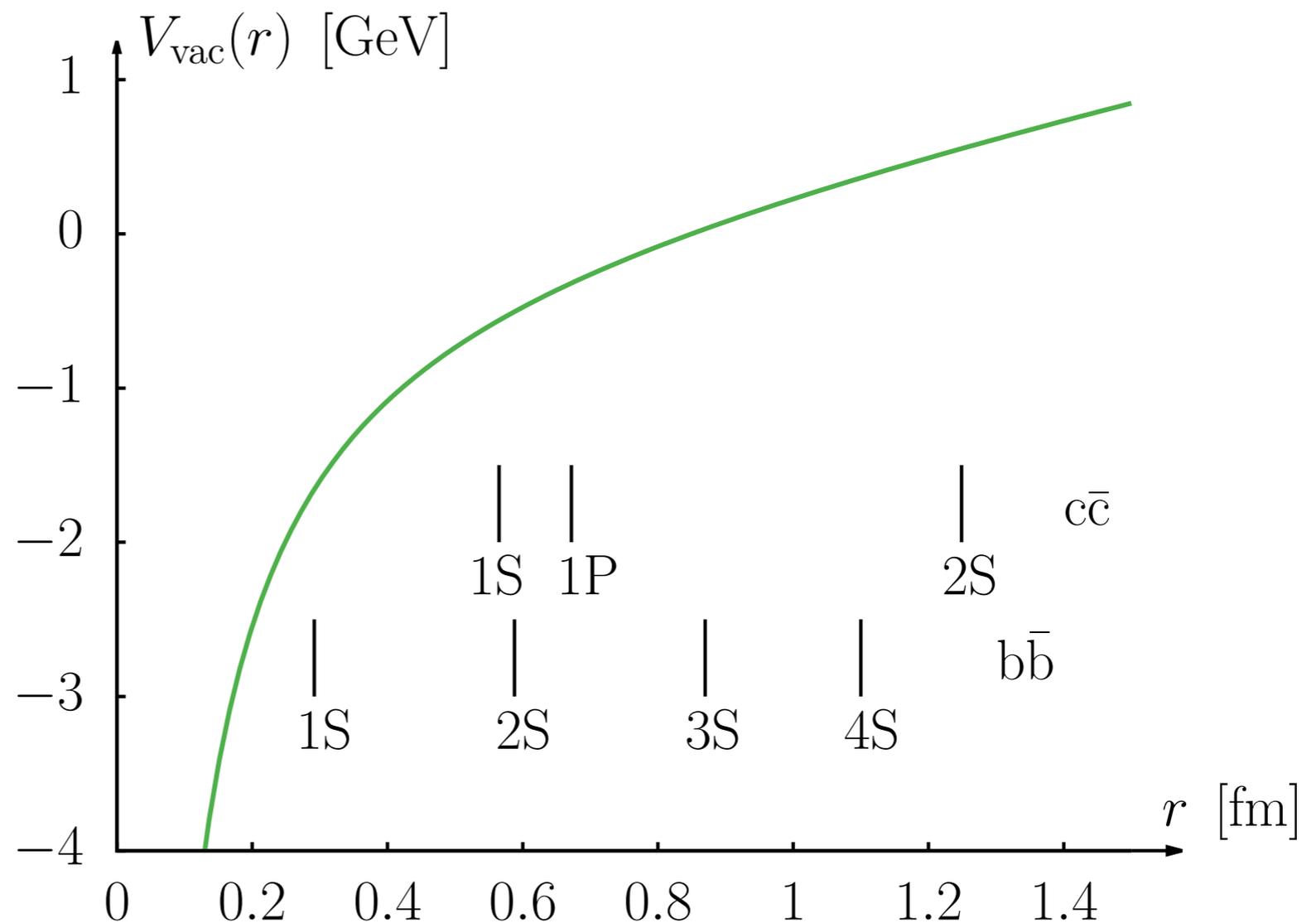
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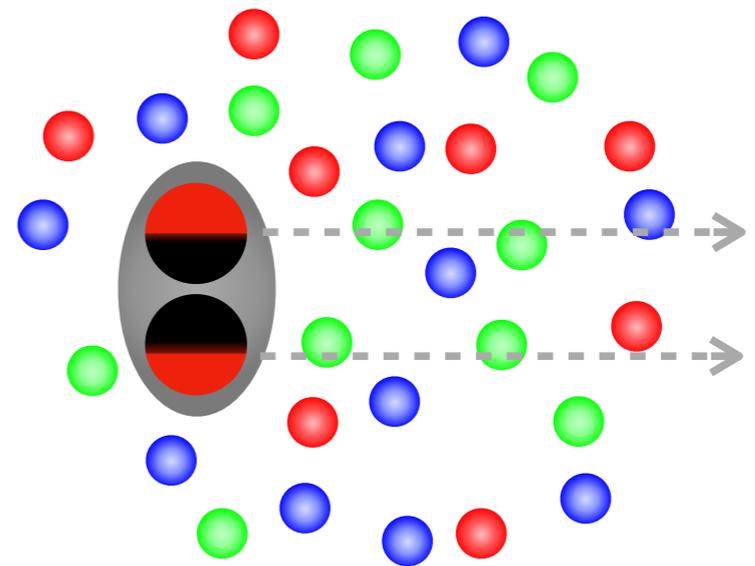


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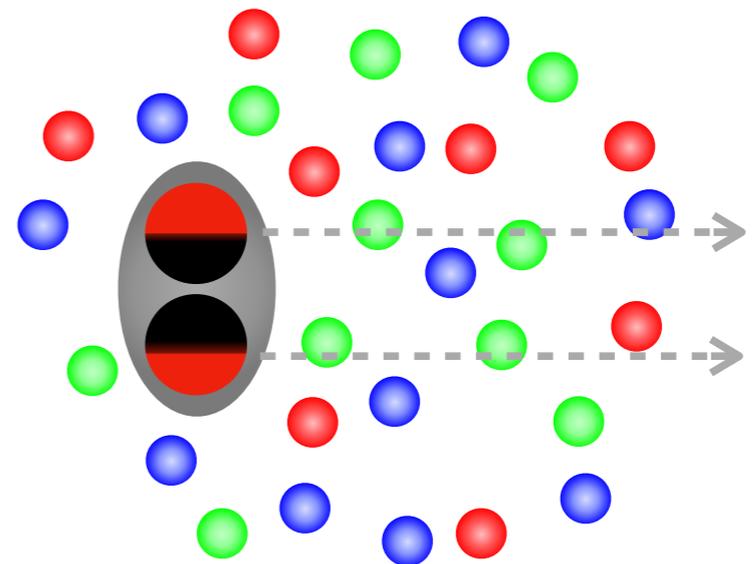


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$m_Q \rightarrow \infty$:

$$W_{\square}(r, t) =$$

[arXiv:hep-ph/0410047]

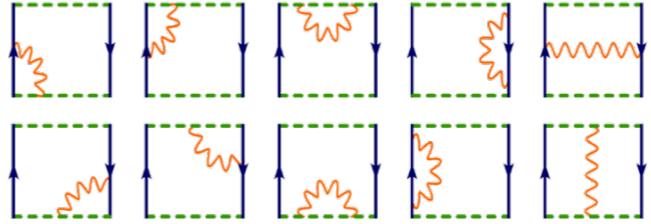
$$i\partial_t W_{\square}(r, t) = \Phi(r, t) W_{\square}(r, t)$$

[arXiv:0712.4394]

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$m_Q \rightarrow \infty$: $W_{\square}(r, t) =$  [arXiv:hep-ph/0410047]

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Perturbative (HTL): [arXiv:hep-ph/0611300]

$$V_{HTL}(r) = -\tilde{\alpha}_s \left[m_D + \frac{e^{-m_D r}}{r} + iT\phi(m_D r) \right] + \mathcal{O}(\tilde{\alpha}_s^2)$$

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Recovers HTL at large T

Our improved model employs the generalised
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Generalised Gauss law

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$$\varepsilon^{-1}(p, m_D) = \frac{p^2}{p^2 + m_D^2} - i\pi T \frac{pm_D^2}{(p^2 + m_D^2)^2} \quad [\text{arXiv:1401.0172}]$$

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$$\varepsilon^{-1}(r, m_D) = -\frac{m_D^2 e^{-m_D r}}{4\pi r} - i \frac{m_D T}{4r\sqrt{\pi}} G_{1,3}^{2,1} \left(\begin{matrix} -\frac{1}{2} \\ -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{4} m_D^2 r^2 \right)$$

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**HTL
result!**

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$$\text{Re}V_S(r) = \frac{2\sigma}{m_D} - \frac{e^{-m_D r} (2 + m_D r) \sigma}{m_D}$$

$$\text{Im}V_S(r) = \frac{\sqrt{\pi}}{4} m_D T \sigma r^3 G_{2,4}^{2,2} \left(\begin{matrix} -\frac{1}{2}, -\frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, -\frac{3}{2}, -1 \end{matrix} \middle| \frac{1}{4} m_D^2 r^2 \right)$$

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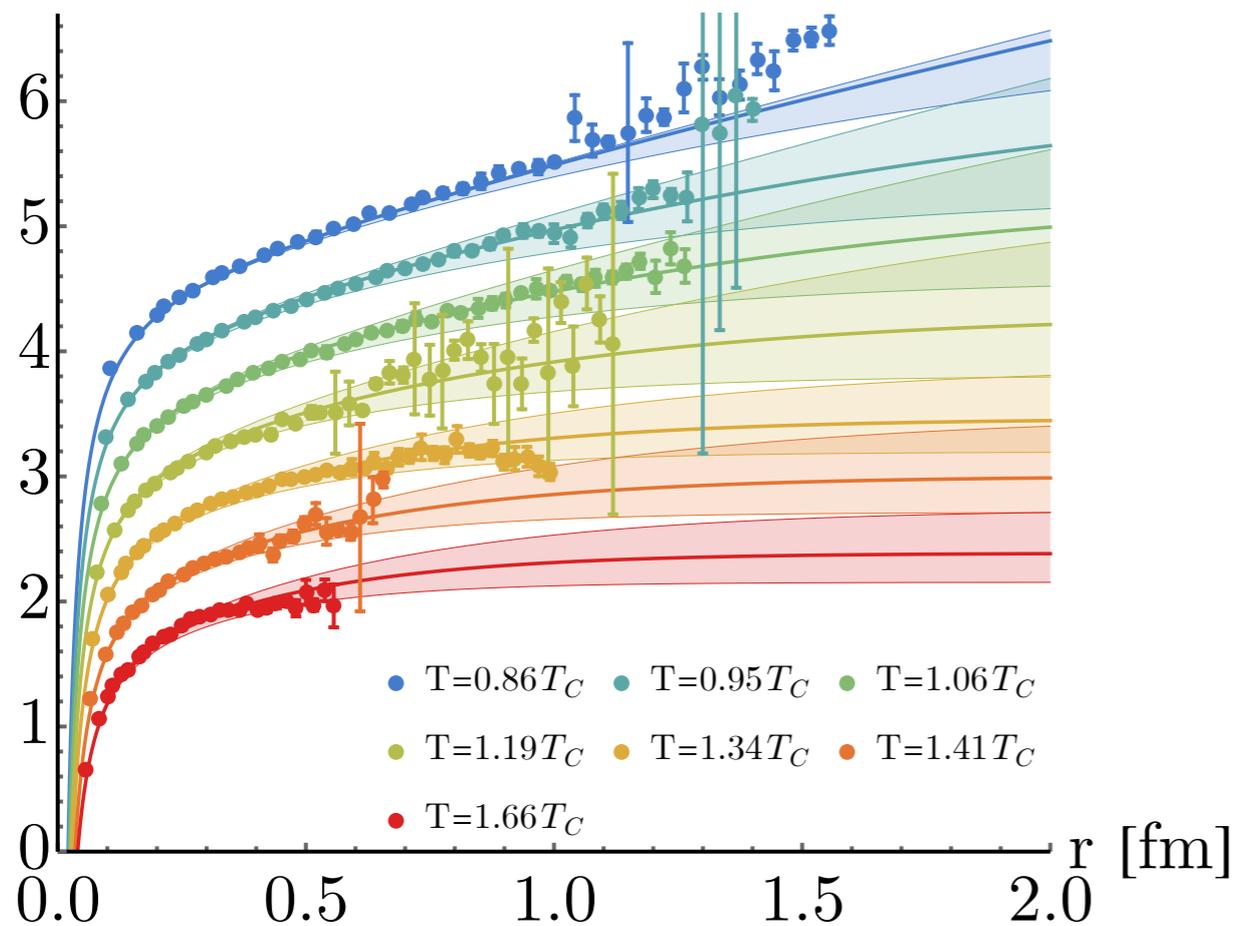
$$\text{Im}V_S(r) = \frac{\sigma T}{m_D^2} \chi(m_D r)$$

$$\chi(x) = 2 \int_0^\infty dz \frac{2 - 2 \cos(xz) - xz \sin(xz)}{\sqrt{z^2 + \Delta_D^2} (z^2 + 1)^2}$$

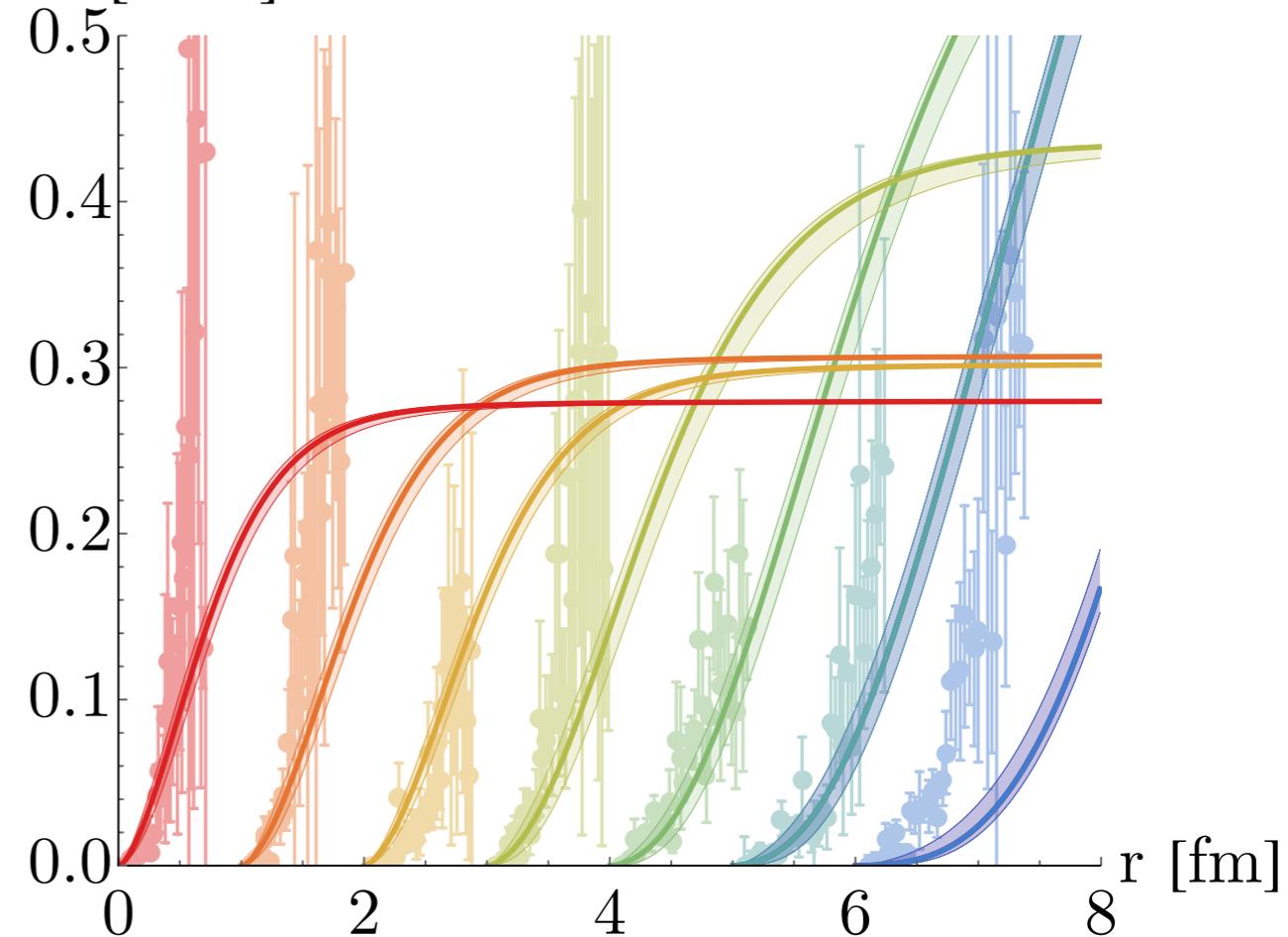
The resulting functional form is fitted to the real lattice data and the imaginary part is postdicted.

[arXiv:1111.1710]

ReV [GeV]



ImV [GeV]



Spectral functions are computed by solving an appropriate Schrödinger equation.

[arXiv:0711.1743]

$$\left[\hat{H} \mp i|\text{Im}V(r)| \right] D^>(t; \mathbf{r}, \mathbf{r}') = i\partial_t D^>(t; \mathbf{r}, \mathbf{r}'), \quad t \geq 0$$

$$\hat{H} \equiv 2m_Q - \frac{\nabla_{\mathbf{r}}^2}{m_Q} + \frac{l(l+1)}{m_Q r^2} + \text{Re}V(r)$$

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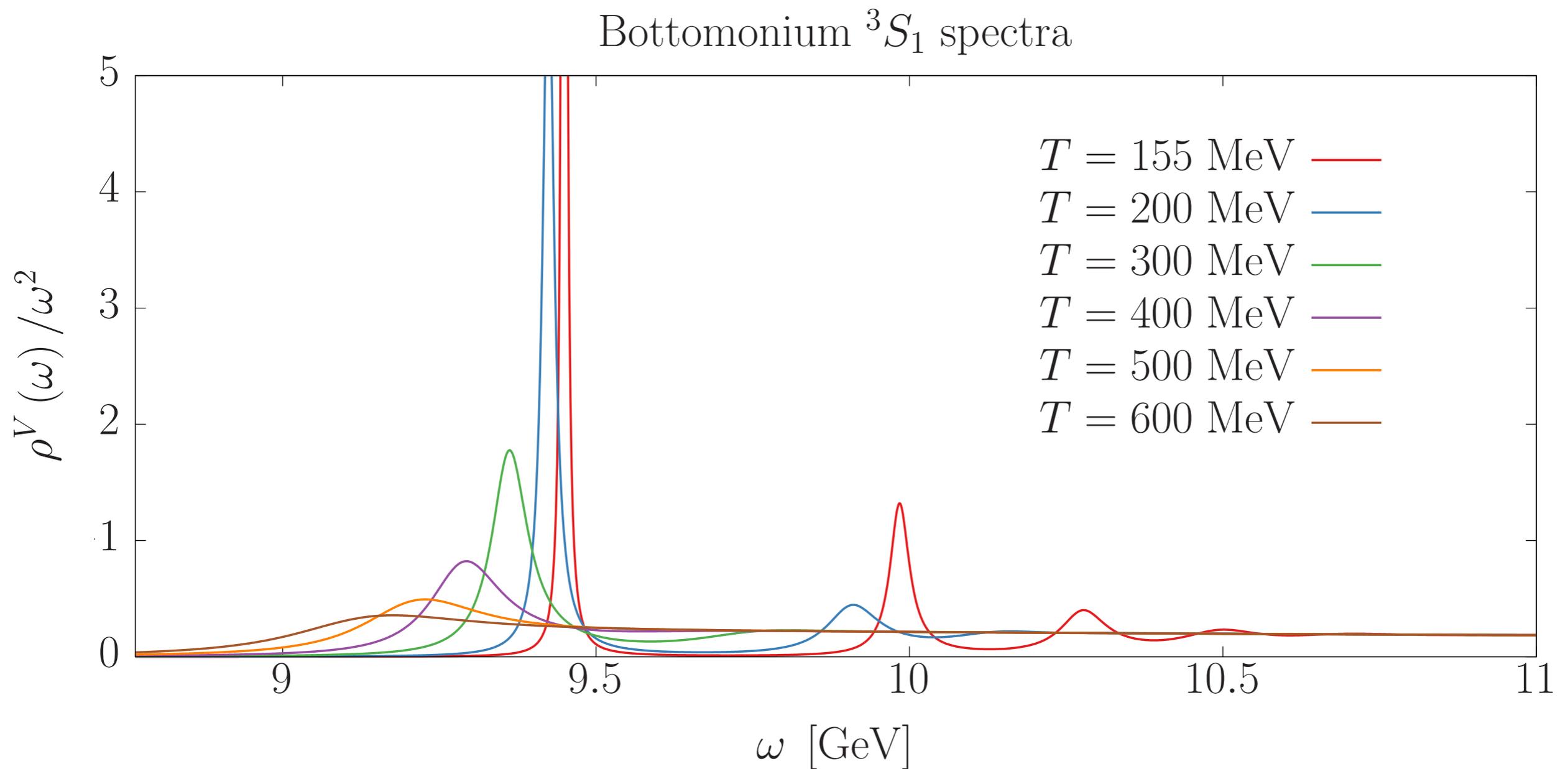
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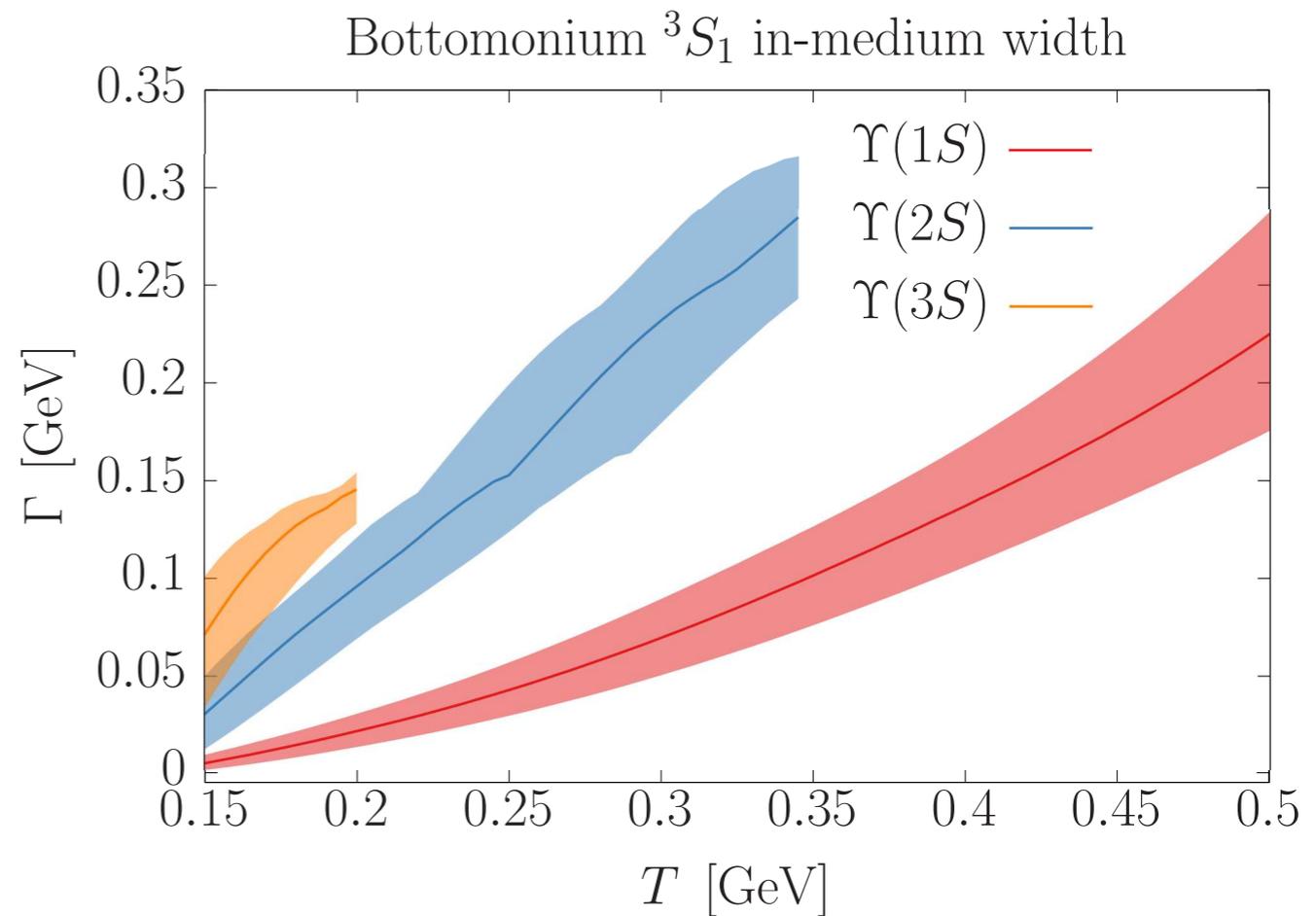
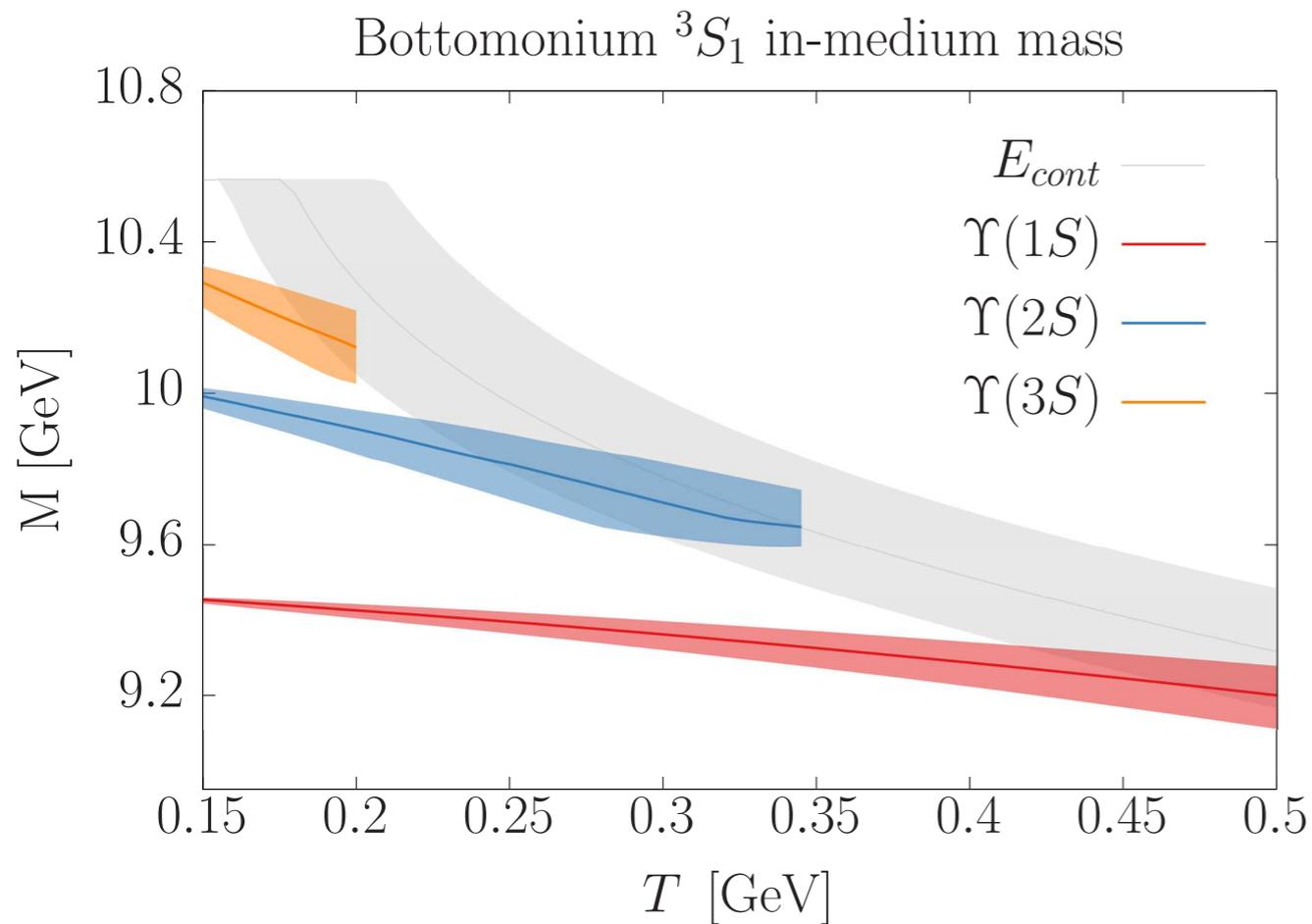
$$\tilde{D}(\omega, \mathbf{r}, \mathbf{r}') = \int_{-\infty}^{\infty} dt e^{i\omega t} D^>(t; \mathbf{r}, \mathbf{r}')$$

$$\rho^V(\omega) = \lim_{\mathbf{r}, \mathbf{r}' \rightarrow 0} \frac{1}{2} \tilde{D}(\omega; \mathbf{r}, \mathbf{r}')$$

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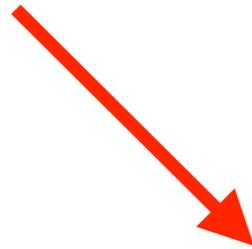


We explore the effects of finite baryo-chemical potential via linear extrapolation in the Debye mass.

$$m_D(T) = \underbrace{Tg(\Lambda) \sqrt{\frac{N_c}{3} + \frac{N_f}{6}}}_{\text{LO}} + \overbrace{\frac{N_c T g(\Lambda)^2}{4\pi} \log \left(\frac{\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}}{g(\Lambda)} \right)}^{\text{NLO}} + \underbrace{\kappa_1 T g(\Lambda)^2 + \kappa_2 T g(\Lambda)^3}_{\text{Non-perturbative}}$$

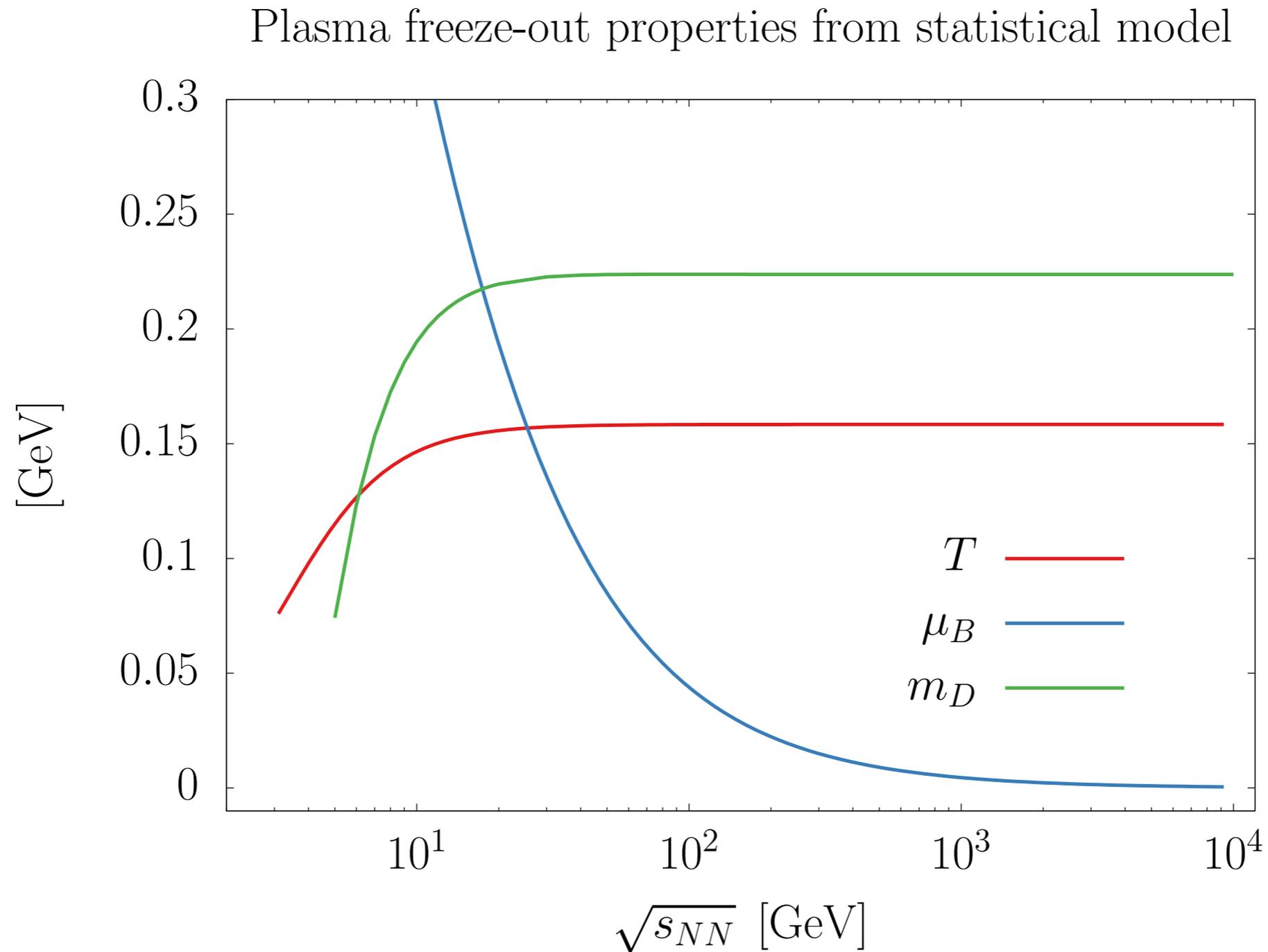
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$$m_D^2(T, \mu_B) = T^2 g^2(\Lambda) \left(\frac{N_c}{3} + \frac{N_f}{6} + \frac{N_f}{18\pi^2} \frac{\mu_B^2}{T^2} \right) + \dots$$

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[arXiv:1310.0164]

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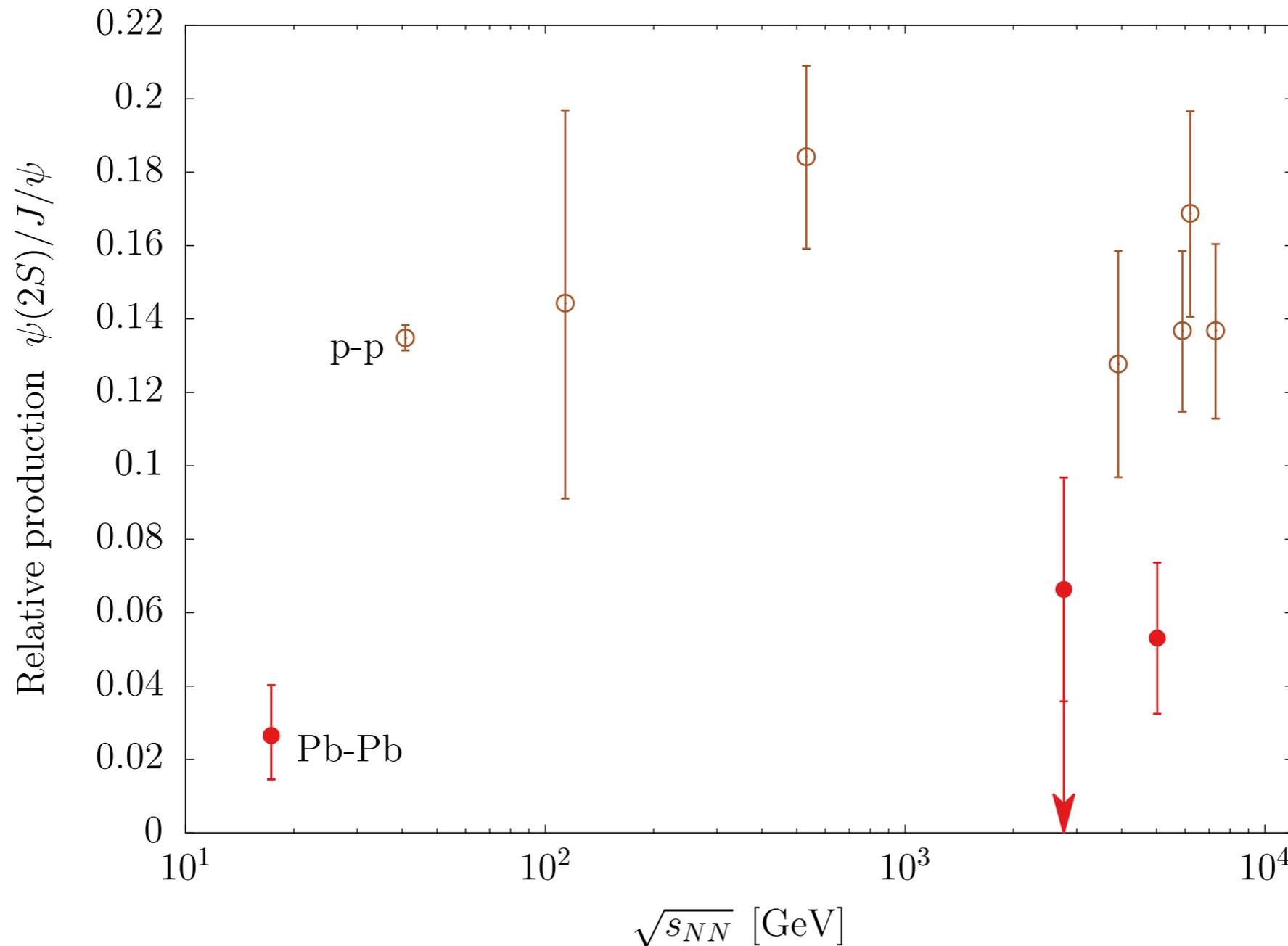
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Quarkonium production yields show good agreement with the latest LHC data.

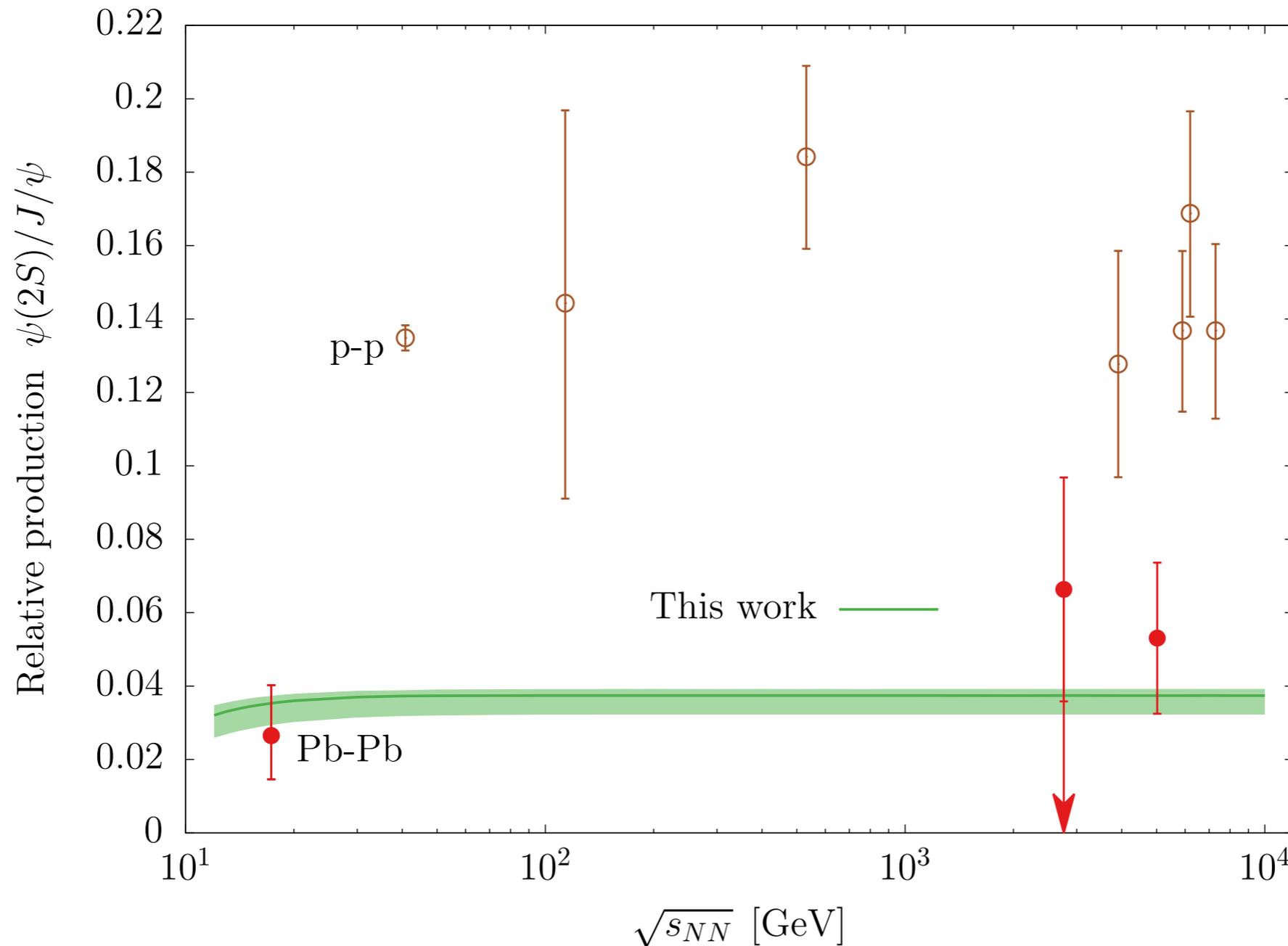


[arXiv:1506.08804]

[arXiv:1410.1804]

[arXiv:1611.01438]

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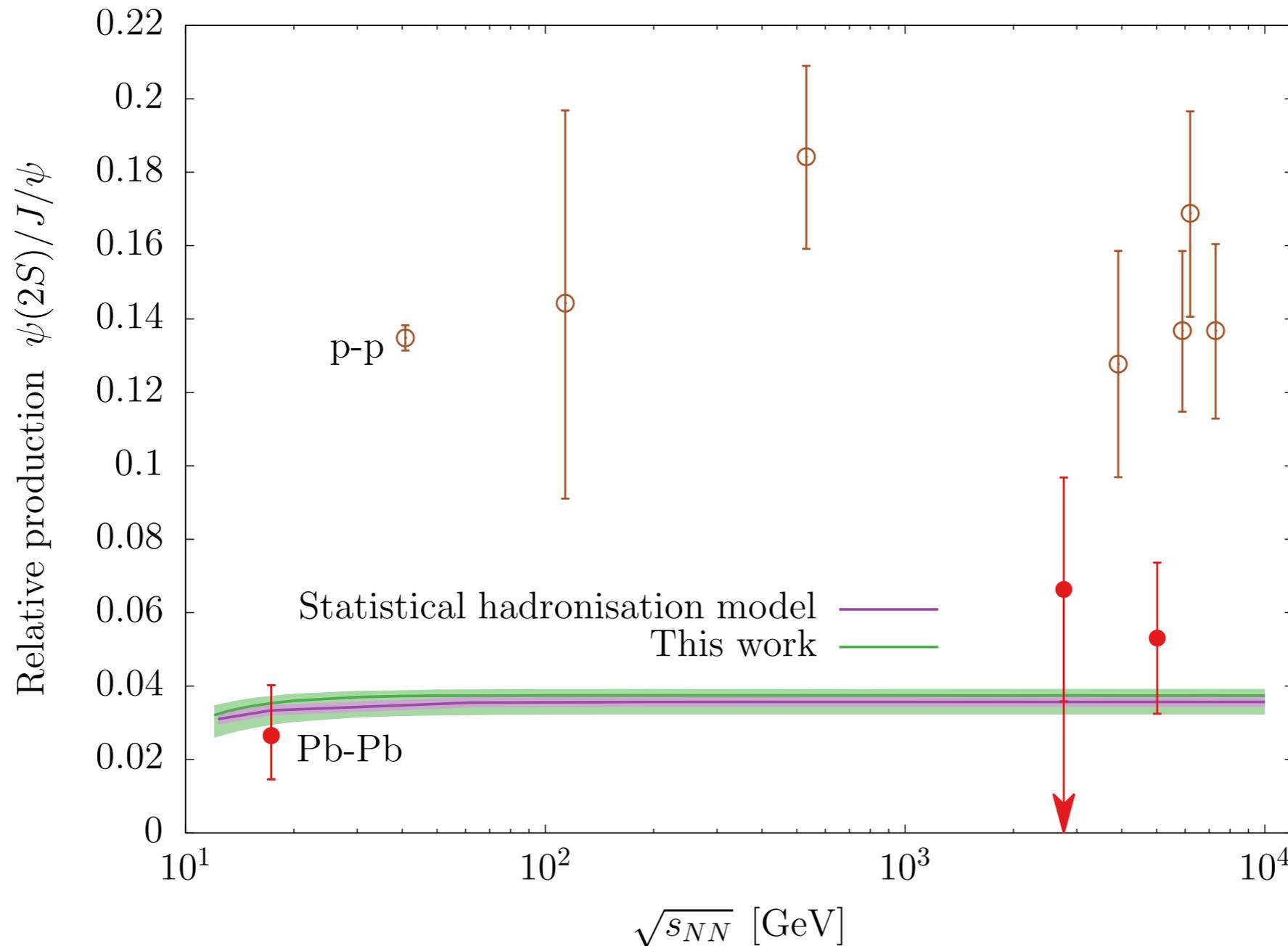


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Outlook: Extension to finite velocity spectra and investigation of transverse momentum dependence.