A statistical model to estimate low energy hadronic cross sections

Gábor Balassa

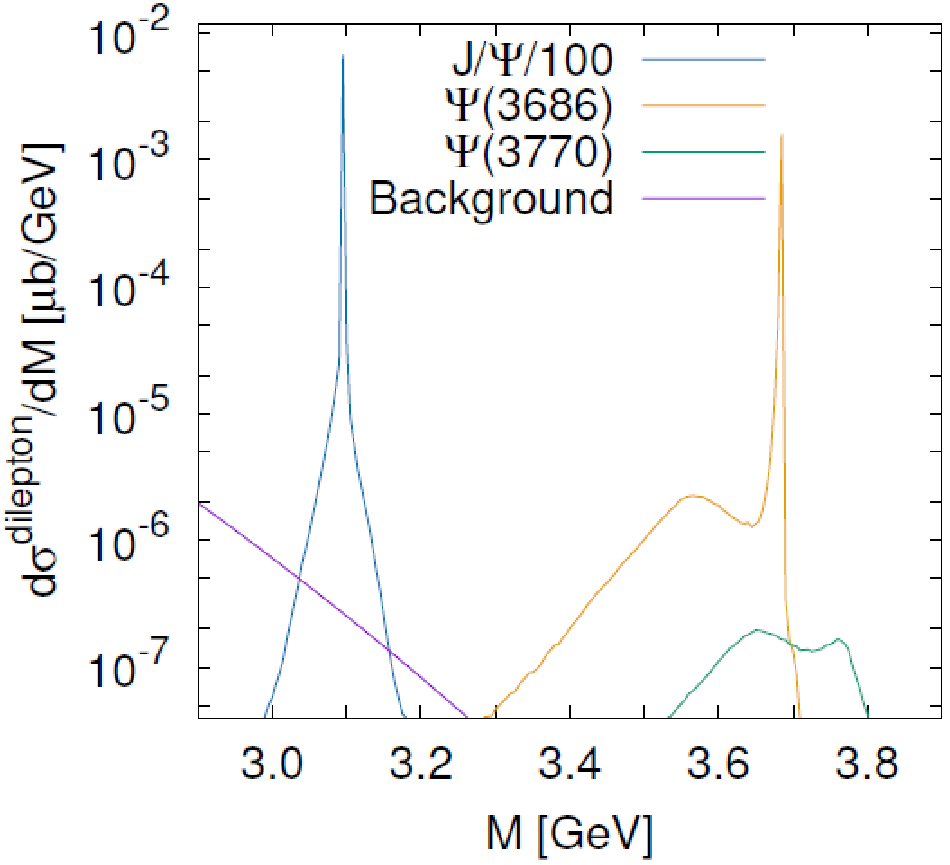
Wigner Research Centre for Physics

Outline

- Motivation
- The method
- Results
- Open problems

Motivation

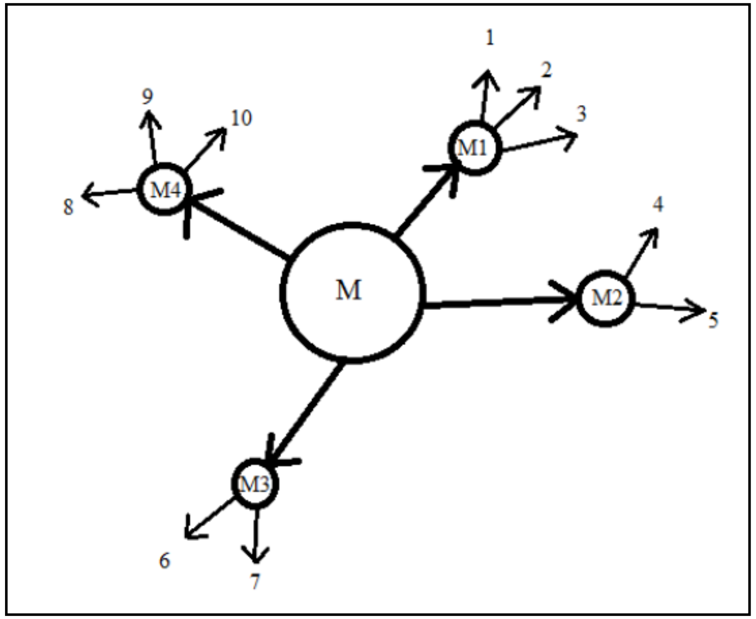
- Heavy ion collisions
  - Charmonium in dense matter (hivatkozás)
  - Off-shell transport (BUU)
  - Elementary cross sections are needed e.g. $p + \bar{p} \rightarrow charm + X$



The model

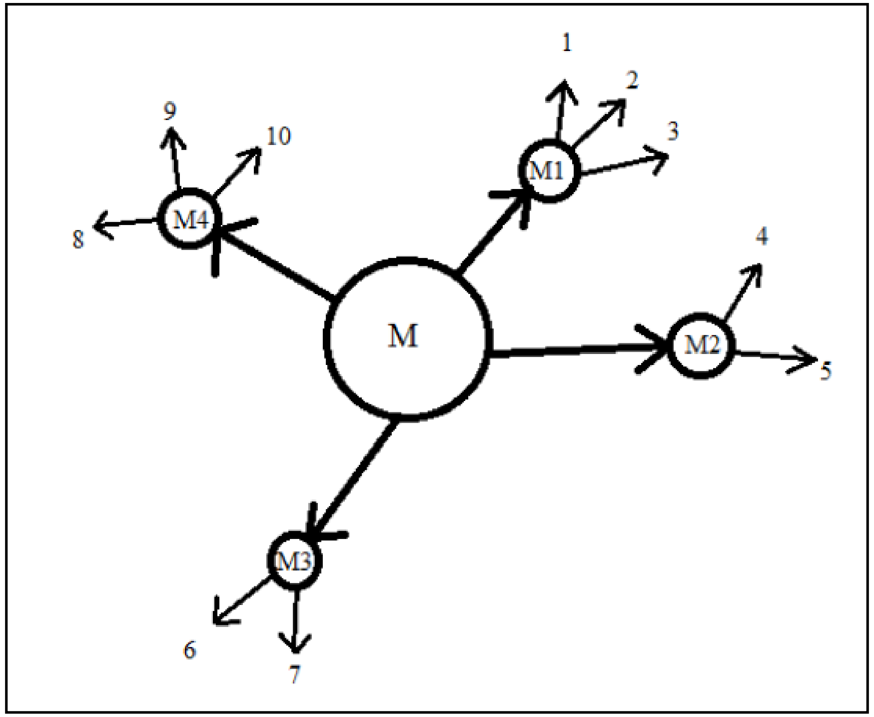
- Phenomolegical model from the Statistical Bootstrap (Hagedorn 1965) *R. Hagedorn, Nuovo Cim. Suppl. 3, 147 (1965)*
- Factorize the probability (generalized cross section for n-bodies):

$$\sigma^{n\to k}(M) = \left(\int \prod_{i=1}^{n} d^3p_i R(M, p_1, \ldots, p_n)\right) \times \left(\int \prod_{j=1}^{k} d^3q_j w(M, q_1, \ldots, q_k)\right)$$



- Assumptions:
  - Initial state is described by the total cross section for two bodies.
  - Final state probabilities are described by mixed statistical/dynamical factors

- Collision scheme:
  - Fireball formation with invariant mass M
  - The initial fireball decays to n smaller fireballs so that $\sum m_i = M$
  - Each small fireball hadronize to m-hadrons
  - We need models for the:
    - Fireball decay scheme
    - Hadronization



$$W_k^{n_1,\ldots,n_k}(M) = N_k(M) P_k^{\text{fb}}(M) C_Q(M) \int_{x_{1,min}}^{x_{1,max}} \cdots \int_{x_{k,min}}^{x_{k,max}} \prod_{i=1}^{k} dx_i P_{n_1}^{H,1}(x_1) P_{n_2}^{H,2}(x_2) \cdots P_{n_k}^{H,k}(x_k) \delta\left(\sum_{i=1}^{k} x_i - M\right)$$

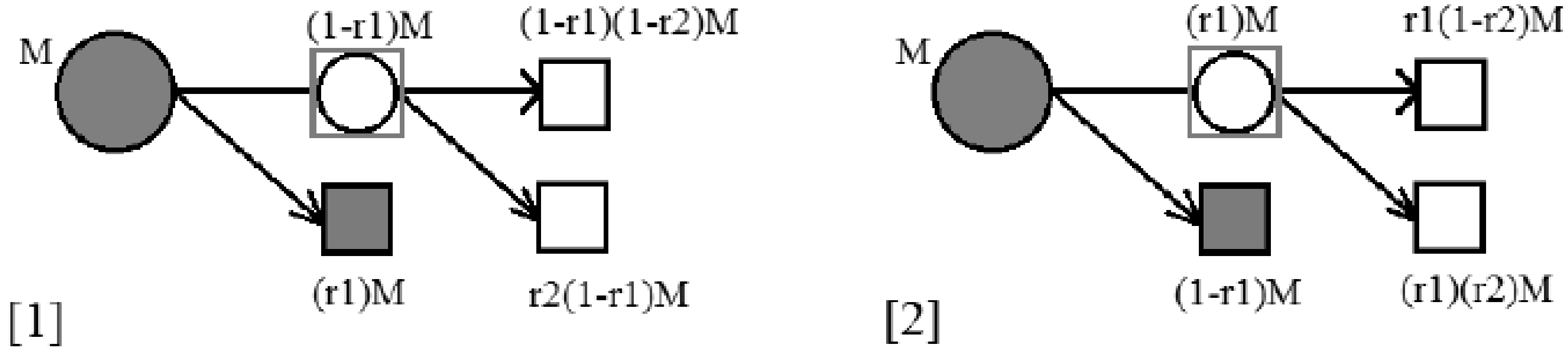
Normalization integral

Fireball decay probabilities

Quark combinatorial probabilities

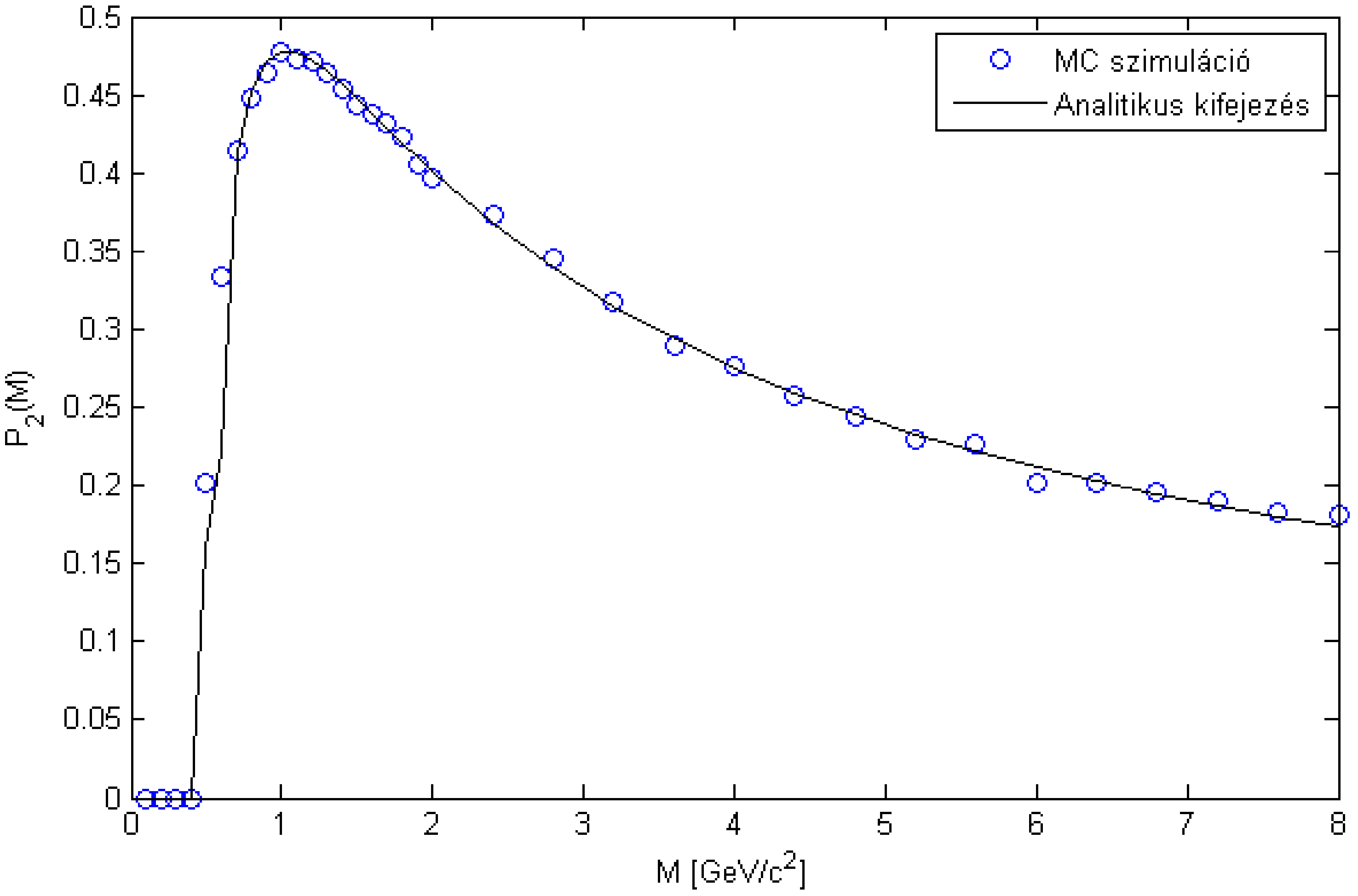
Hadronization probabilities

- Model for fireball decay and the total probability:



- $P_1(E), P_2(E)$ can be calculated analytically
- $P_3(E), \dots.$ Can only be integrated out numerically
- For 2 final fireballs the maximum number of particles are 6 (2x3 decay)

$$P_2(M) = \begin{cases} \frac{2C}{M}\left[\ln\left(\frac{M}{2C} - \frac{1}{2}\right) + \frac{1}{2}\right], & 0 < \frac{C}{M} < \frac{1}{3} \\ 1 - \frac{2C}{M}, & \frac{1}{3} < \frac{C}{M} < \frac{1}{2} \end{cases}$$



Hadronization probabilities

- DOS from statistical Bootstrap:

$$\rho(M) = \frac{a\sqrt{M}}{(M_0+M)^{3.5}} e^{\frac{M}{T_0}}$$

Phys. Rev. D 3, 2821 (1971).

$$P_2^{H,i}(x_i) = \prod_{l=1}^{2}(2s_l+1)P_2^d \frac{\Phi_2(x_i,m_1,m_2)}{\rho(x_i)(2\pi)^3 N_I!},$$

$$P_3^{H,i}(x_i) = \prod_{l=1}^{3}(2s_l+1)P_3^d \frac{\Phi_3(x_i,m_1,m_2,m_3)}{\rho(x_i)(2\pi)^6 N_I!}$$

- From the Frautschi picture:
  - $P_2^d \approx 0.7$
  - $P_3^d \approx 0.28$
- Phase-space for 2&3 bodies:

$$\Phi_3(M,m_1,m_2,m_3) = V^2 \int d^3q_1 d^3q_2 d^3q_3 \times \delta(M-E_1-E_2-E_3)\delta^{(3)}(q_1+q_2+q_3)$$
$$= V^2 \left\{ \frac{M^5}{120} - \frac{M^3}{12}\sum_{i=1}^{3} m_i^2 + \frac{M^2}{6}\sum_{i=1}^{3} m_i^3 + \frac{M}{4}\left[\sum_{i=1}^{3}\sum_{\substack{j=2\\ i\neq j}}^{3} m_i^2 m_j^2 - \frac{1}{2}\sum_{i=1}^{3} m_i^4\right] + \frac{1}{30}\sum_{i=1}^{3} m_i^5 - \frac{1}{6}\sum_{i=1}^{3}\sum_{\substack{j=1\\ i\neq j}}^{3} m_i^3 m_j^2 \right\}$$

$$\Phi_2(M,m_1,m_2) = V\int d^3q_1 d^3q_2 \delta(M-E_1-E_2) \times \delta^{(3)}(q_1+q_2) = \frac{V\pi}{2M^4}\left(M^4-(m_1^2-m_2^2)^2\right) \times \sqrt{\lambda(M^2,m_1^2,m_2^2)},$$

Quark combinatorial factors

- How many ways a hadron can built up by nu,nd,ns,nc,... number of quarks
- Probability is described by the multinomial distribution:

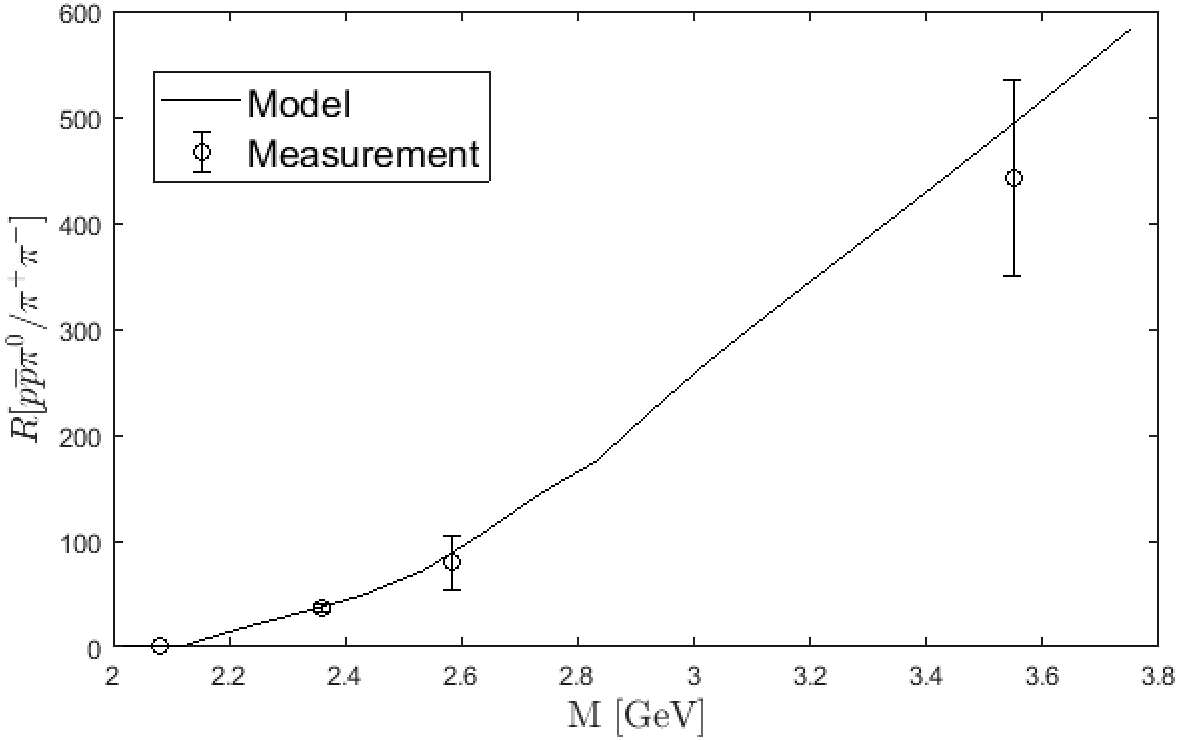
$$F = \frac{\left(\frac{N!}{n_u! n_d! n_s! n_c!}\right) P_u^{n_u} P_d^{n_d} P_s^{n_s} P_c^{n_c}}{P_{tot}(N)}$$

$$\mathcal{N}_{C,2}(M) = \left[\sum_{<ij>\in S} C_{Q,(ij)} \right]^{-1}$$

$$C_Q(M) = C_{Q,pn} \mathcal{N}_{C,2}(M)$$

- Probabilities are the only free parameters of the model.
  - Fit from experiments.
  - Use theoretical models.

$R(p\bar{p} \to p\bar{p}\pi^0 \mid p\bar{p} \to \pi^+\pi^-)$

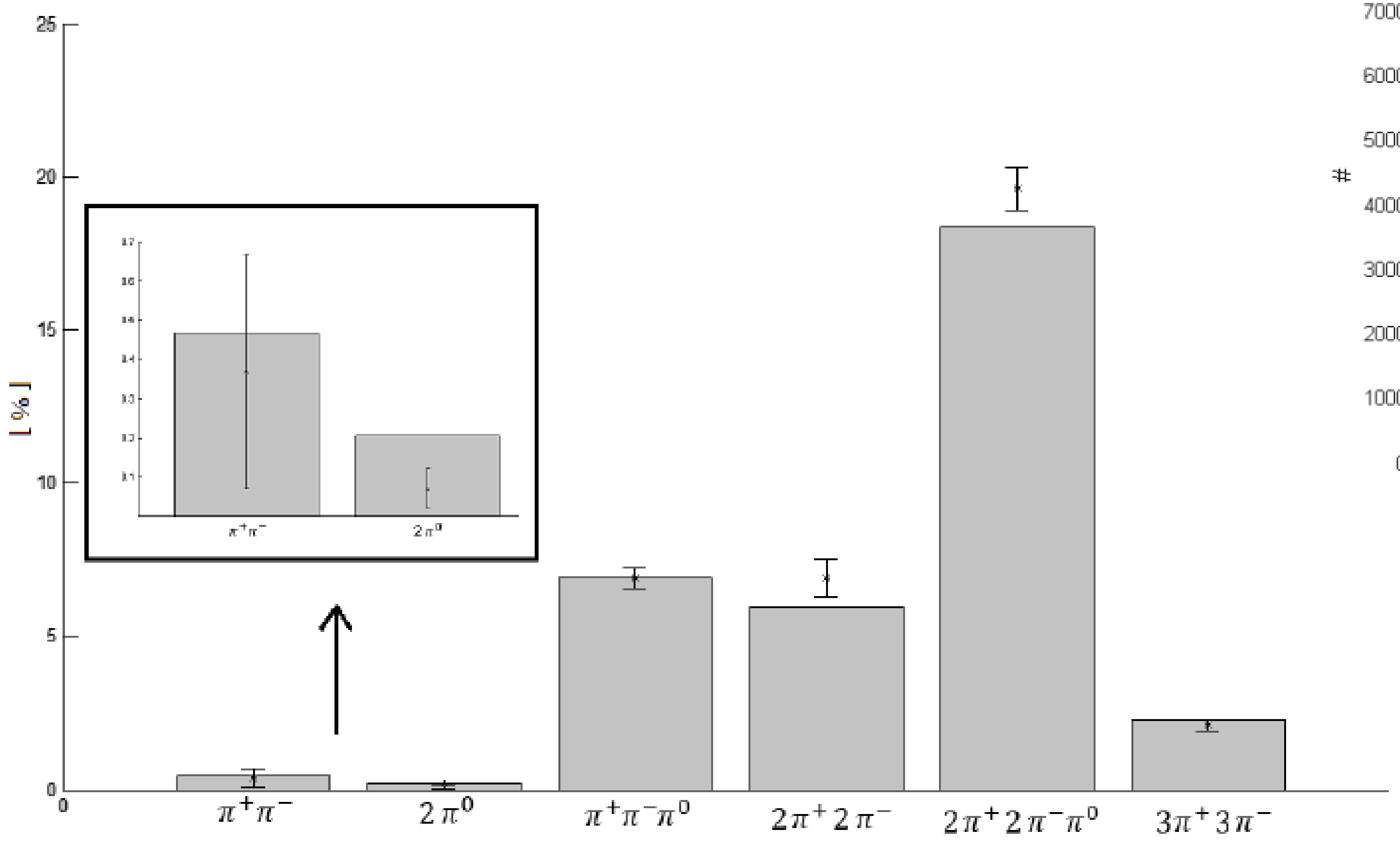


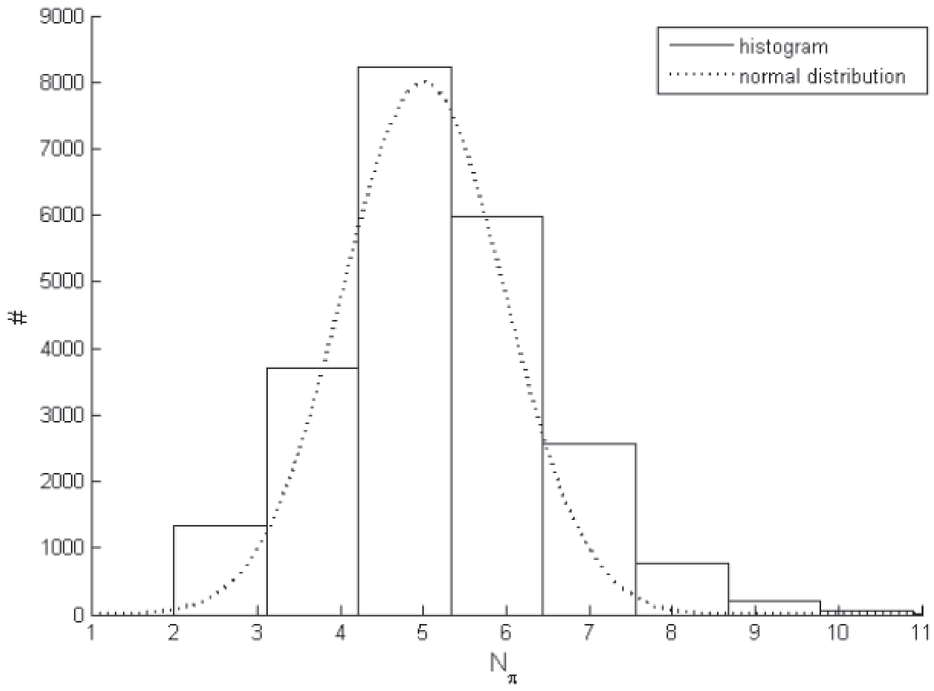
i	R_i	m_{R_i} [GeV]	s_{R_i}	$B_i^{p\pi^0}$
1	N_{1440}	1.430	1/2	0.22
2	N_{1520}	1.515	3/2	0.20
3	N_{1535}	1.535	1/2	0.15
4	N_{1650}	1.655	1/2	0.23
5	N_{1680}	1.685	5/2	0.23
6	Δ_{1232}	1.232	3/2	0.66
7	Δ_{1620}	1.630	1/2	0.17
8	Δ_{1910}	1.890	1/2	0.15
9	Δ_{1950}	1.930	7/2	0.27

$$R^{p\bar{p}\pi^0}_{\pi^+\pi^-}(M) = \frac{W_{p\bar{p}\pi^0}}{W_{\pi^+\pi^-}} = \frac{(n_u-1)^2}{\Phi_2(M,m_\pi,m_\pi)} \times \left[\frac{6P_3^d}{P_2^d(2\pi)^3}\frac{\mathcal{N}_{\mathcal{C},3}}{\mathcal{N}_{\mathcal{C},2}}\left[(n_u-2)^2+(n_d-1)^2\right] \times \Phi_3(M,m_p,m_p,m_{\pi^0}) + 4\sum_{i=1}^{9} B_i^{p\pi^0}(2s_{R_i}+1) \times \Phi_2(M,m_p,m_{R_i})\right],$$

Proton-antiproton annihilation at rest

(NORMALIZATION PROBLEM at high energies)





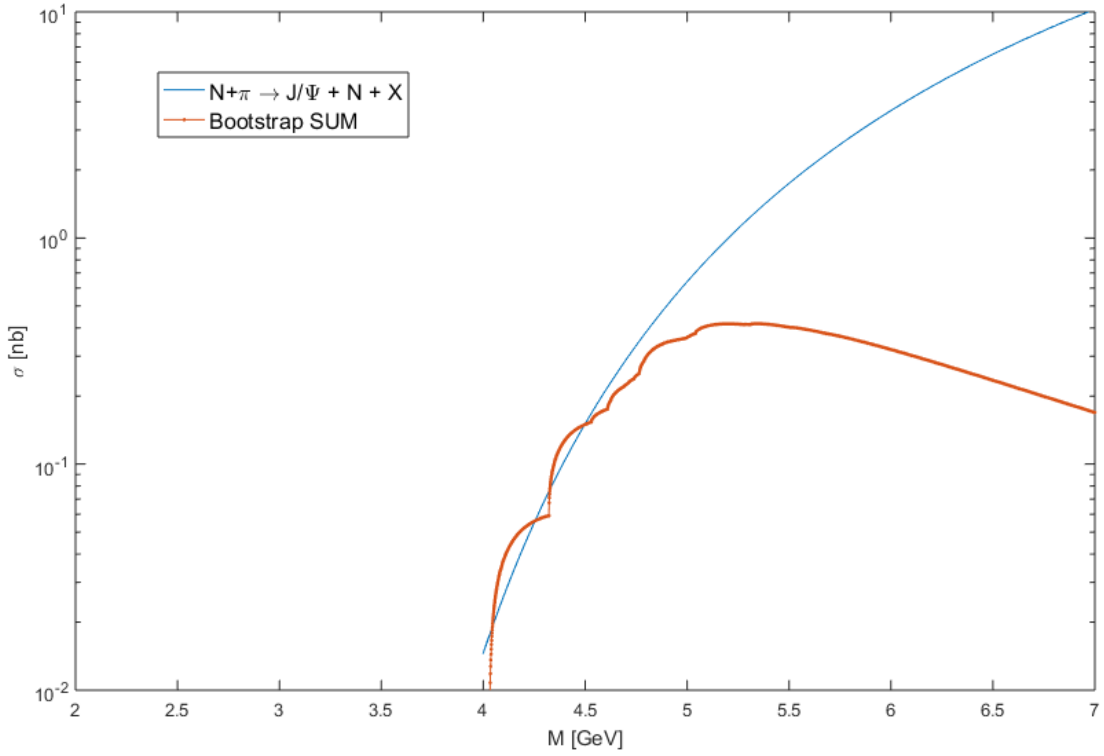
Pion distribution

Charmonium probabilities

- Pc – charm-anticharm creational probability must be fixed from experiment.
- We only have inclusive cross section -> impossible to calculate everything be hand -> simulations -> normalization problem.
- Preliminary results for:

$$\pi N \rightarrow charm + N + X$$

  - Sum the first 50 dominant channels.



Nuclear Physics A 786 (2007) 183–200

Summary

- Mixed statistical/dynamical model to calculate cross sections at a few GeV.
- Estimate unknown cross sections for transport simulations.
- Open problems:
  - Normalization problem
  - We need a good Pc fit.
  - Elastic cross sections are higher than what comes from the model.
  - Extend to many body collisions.