Search for the QCD critical point at RHIC

Adam Bzdak

AGH University of Science and Technology, Kraków

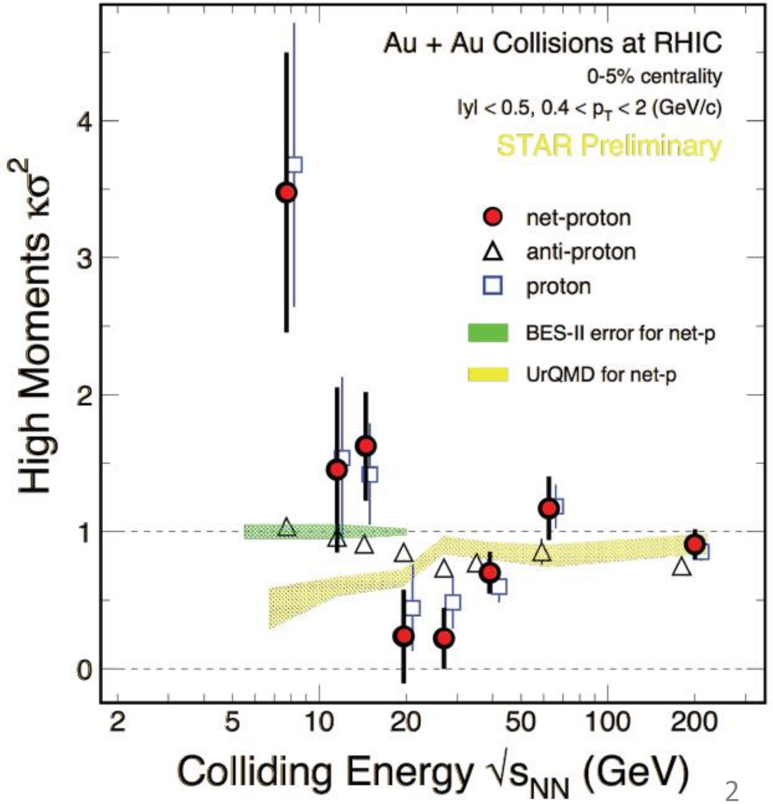
AGH

AB, V.Koch, N.Strodthoff, PRC 95 (2017) 054906
AB, V.Koch, V.Skokov, EPJC 77 (2017) 288
AB, V.Koch, D.Oliinychenko, J.Steinheimer, PRC 98 (2018) 054901
AB, V.Koch, 1811.04456

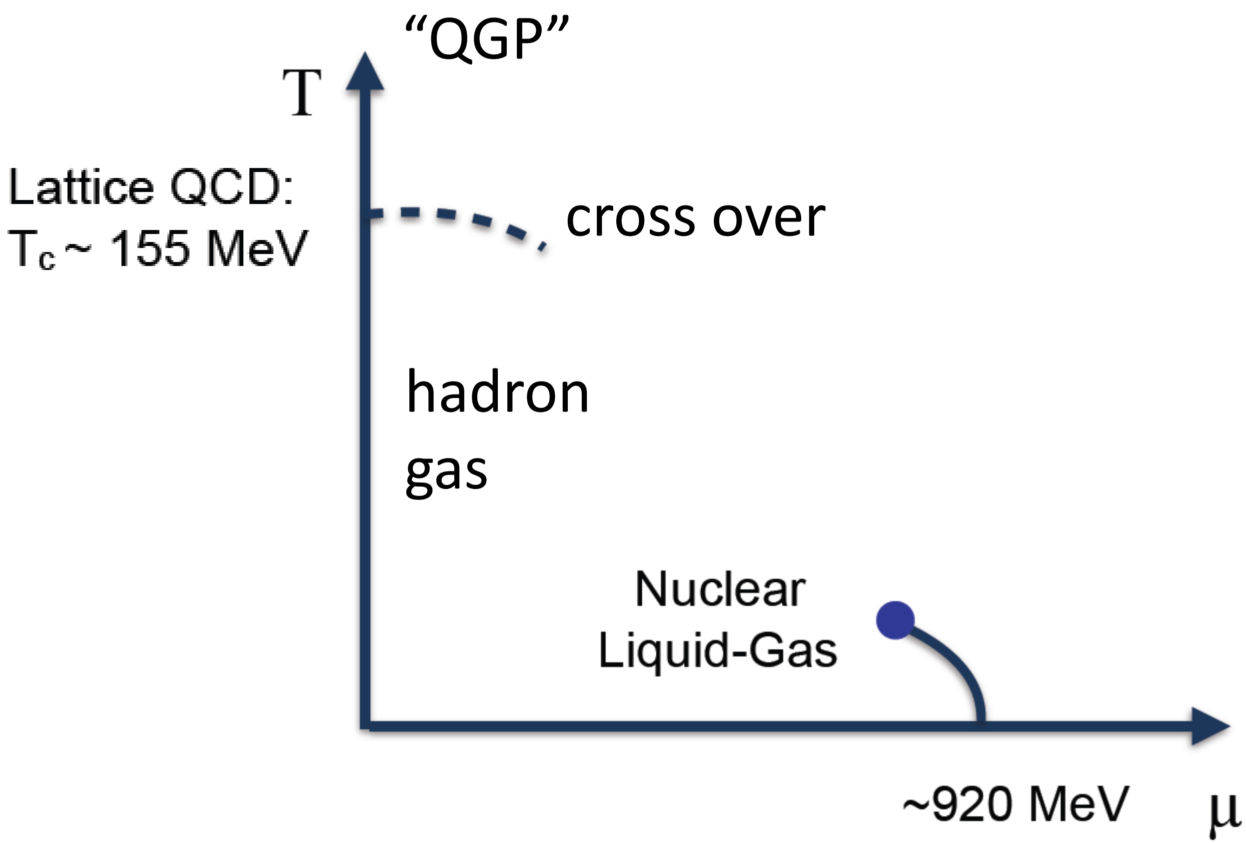
Outline:

- introduction
- factorial cumulants, cumulants
- STAR data
- bimodal distribution
- statistics friendly distributions
- summary

STAR Preliminary

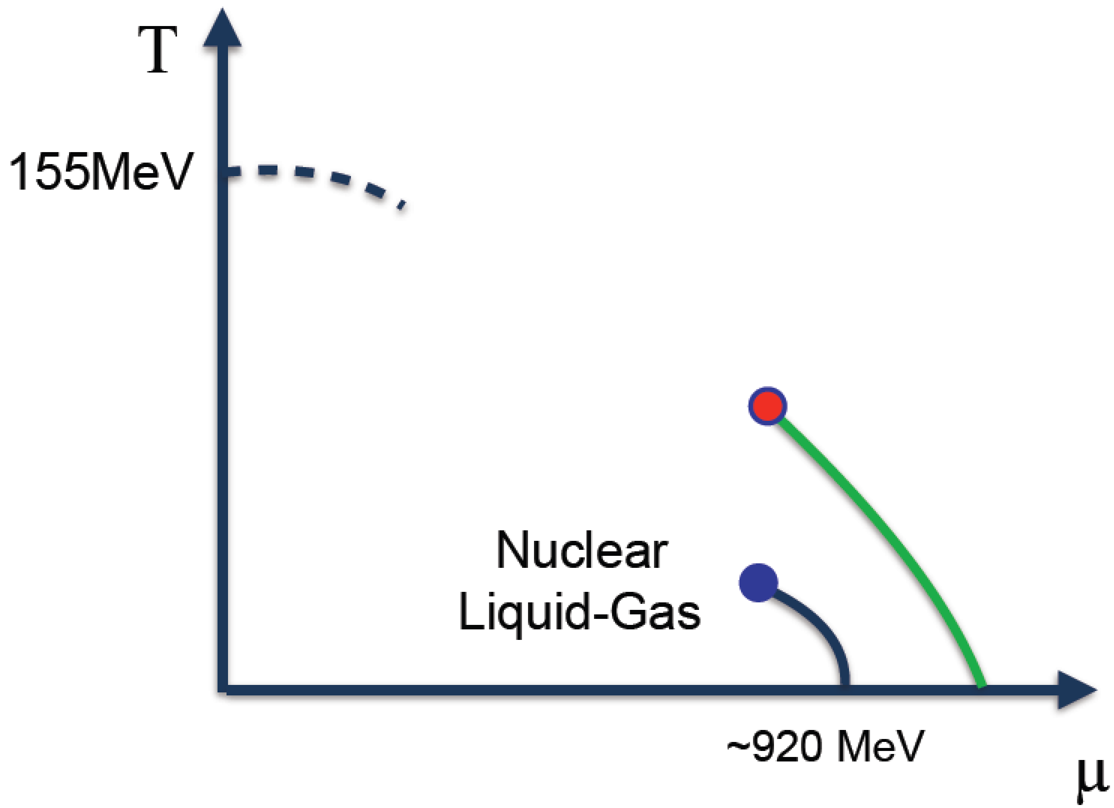


What we know about the QCD phase diagram



The rest is everybody's guess.

Usual expectation based on various effective models



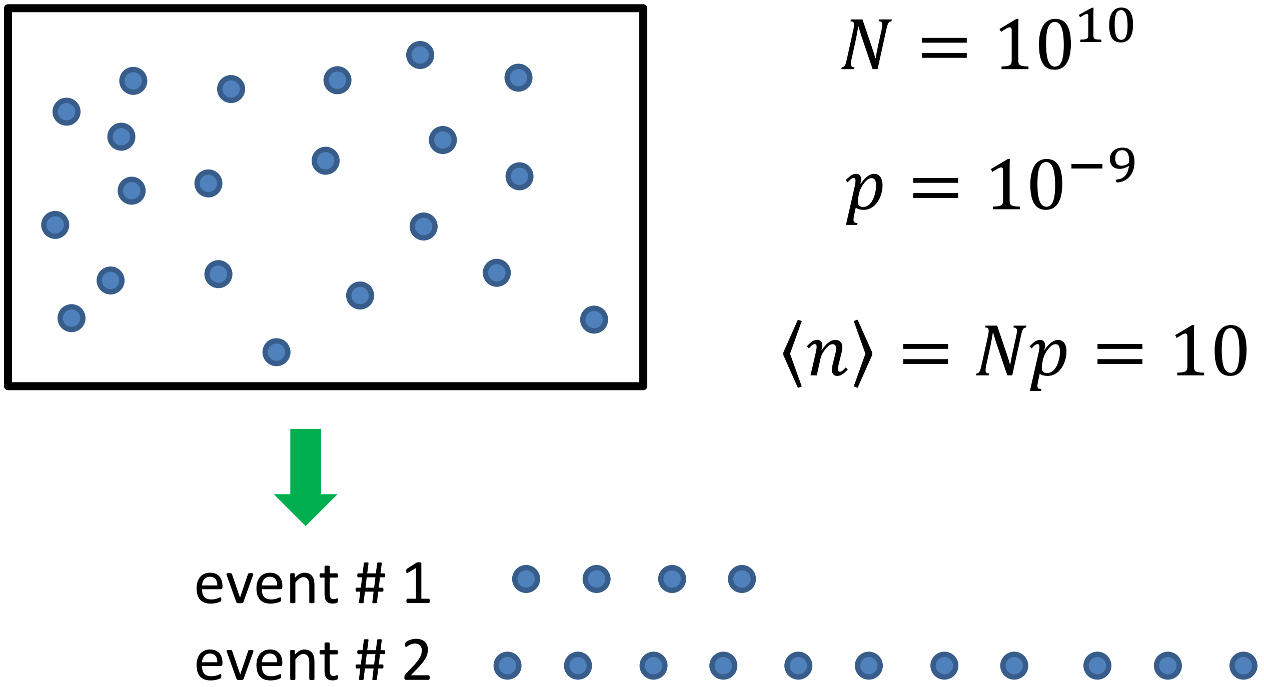
On the experimental side all we can do is to measure various fluctuation observables and hope to see some nontrivial energy or/and system-size dependence

see, e.g.,
Stephanov, Rajagopal, Shuryak, PRL (1998)
Stephanov, PRL (2009)
Skokov, Friman, Redlich, PRC (2011)

There are some intriguing results:

STAR, HADES	NA 49
Higher order cumulants	Intermittency in the transverse momentum phase space
Proton v_1 (STAR)	Strongly intensive variables
HBT radii (STAR)	

Poisson distribution



$$N = 10^{10}$$

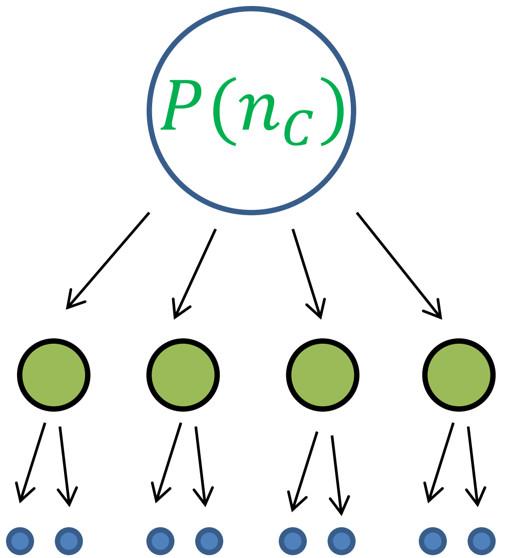
$$p = 10^{-9}$$

$$\langle n \rangle = Np = 10$$

$P(n)$ = Poisson if $N \to \infty,\ p \to 0,\ \ Np = \langle n \rangle$

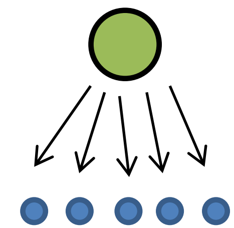
Such source (multiplicity distribution) is characterized by
All **factorial cumulants** $\boldsymbol{C_n = 0}$, $n = 2,3, \ldots$ ("no correlations")

Multiparticle correlations - **factorial cumulants**



Poisson

m particle cluster



$$\boldsymbol{C_2} \neq 0$$

$$\boldsymbol{C_k} = 0, k > 2$$

$$\boldsymbol{C_{2,3,\ldots,m}} \neq 0$$

$$\boldsymbol{C_k} = 0, k > m$$

factorial cumulants

$$\boldsymbol{C_k} = \frac{d^k}{dz^k} \ln \left(\sum_n P(n) z^n \right) \Big|_{z=1}$$

Two-particle correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

Integrating both sides over some bin in rapidity

$$\langle n(n-1)\rangle = \langle n\rangle^2 + \boldsymbol{C_2}$$

$$\langle n(n-1)\rangle = \int \rho_2(y_1, y_2)\,dy_1 dy_2$$

$$\langle n\rangle = \int \rho(y)\,dy$$

factorial cumulant (integrated correlation function)

$$\boxed{\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2)\,dy_1 dy_2}$$

Factorial cumulants vs cumulants

factorial cumulant

$$\boldsymbol{C_i} = \frac{d^i}{dz^i} \ln\left(\sum\nolimits_n P(n) z^n\right)\Big|_{z=1}$$

cumulant

$$K_{\boldsymbol{i}} = \frac{d^i}{dt^i} \ln\left(\sum\nolimits_n P(n) e^{tn}\right)\Big|_{t=0}$$

cumulants naturally appear in statistical physics

$$\ln(Z) = \ln\left(\sum_i e^{-\beta(E_i - \mu N_i)}\right)$$

Poisson:

$C_i = 0$

$K_i = \langle n \rangle$

We have

$$K_2 = \langle N \rangle + \boldsymbol{C_2}$$

$$K_3 = \langle N \rangle + 3\boldsymbol{C_2} + \boldsymbol{C_3}$$

$$K_4 = \langle N \rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

cumulants mix integ. correlation functions of different orders

$$K_5 = \langle N \rangle + 15C_2 + 25C_3 + 10C_4 + C_5$$

$$K_6 = \langle N \rangle + 31C_2 + 90C_3 + 65C_4 + 15C_5 + C_6$$

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

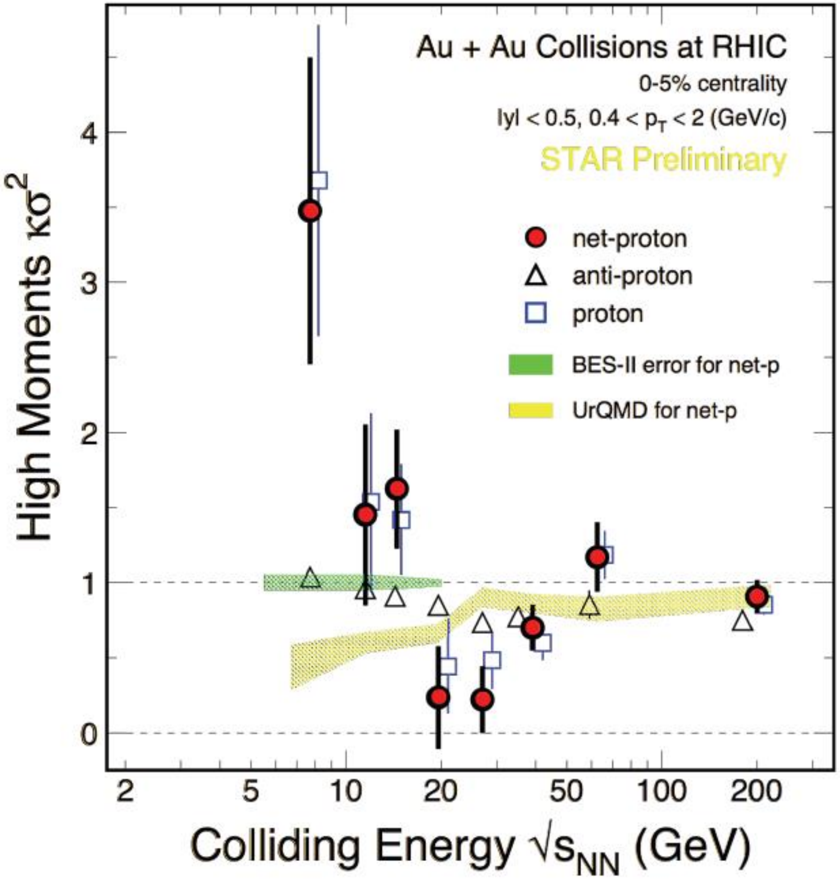
$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2) dy_1 dy_2$$

factorial cumulant

See, e.g.,
B. Ling, M. Stephanov, PRC 93 (2016) 034915
AB, V. Koch, N. Strodthoff , PRC 95 (2017) 054906
AB, V.Koch, D.Oliinychenko, J.Steinheimer, PRC 98 (2018) 054901

Preliminary STAR data

X.Luo, N.Xu, 1701.02105



my notation

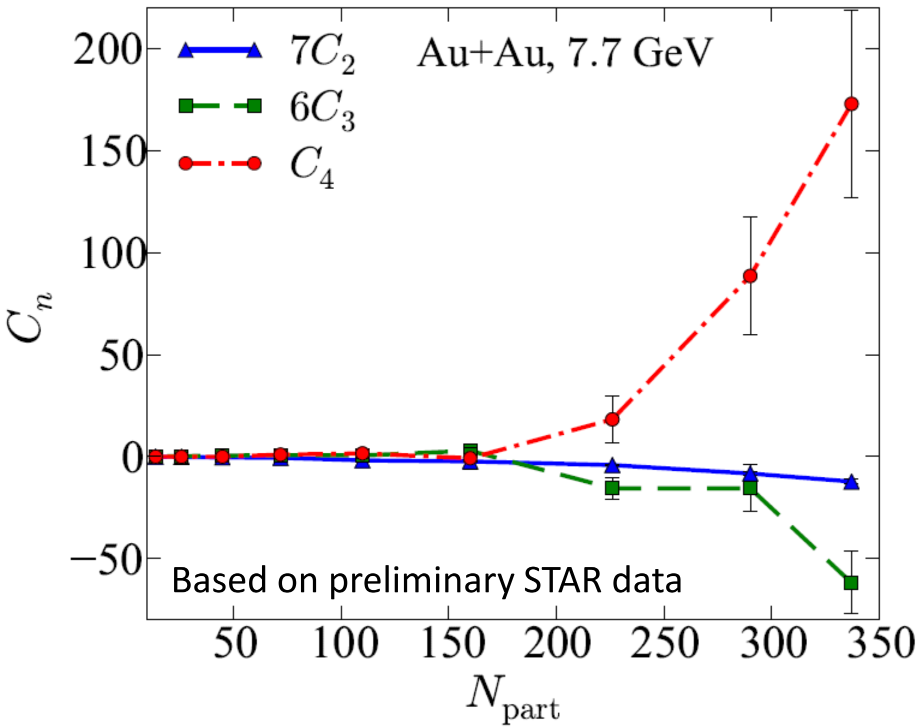
K_4/K_2

Is proton signal at 7.7 GeV large?

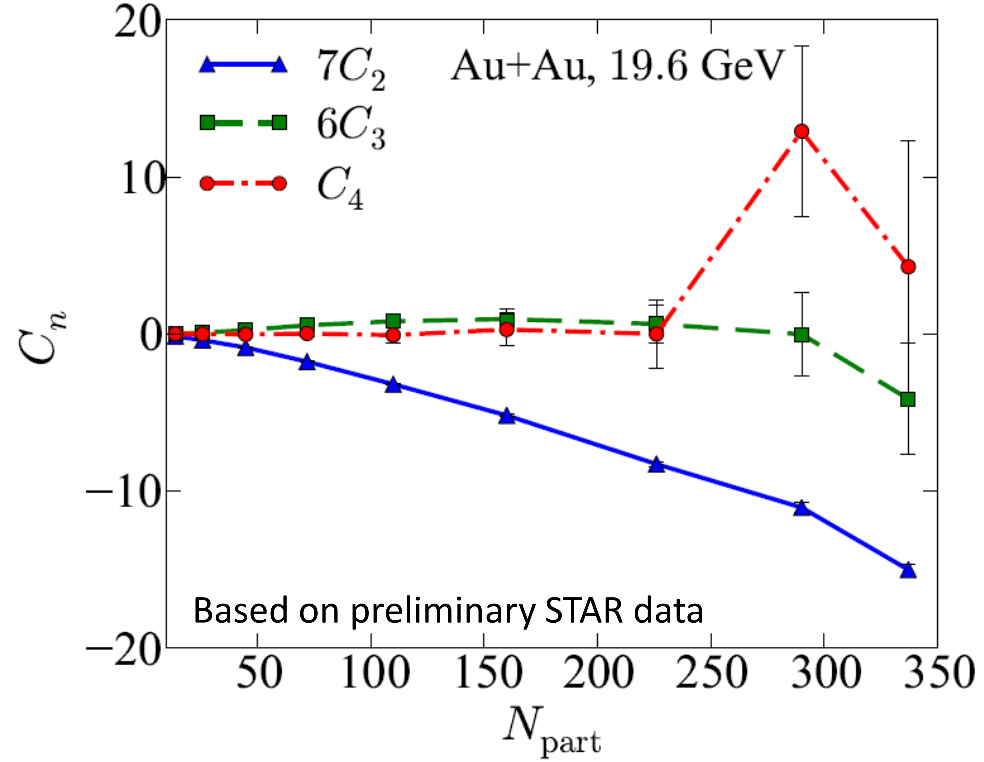
Is physics changing between 7 and 19 GeV?

Using preliminary STAR data we obtain C_n

central signal at $\mathbf{7.7}$ GeV is driven by large 4-particle correlations



central signal at $\mathbf{19.6}$ GeV is driven by 2-particle correlations



$$\boldsymbol{C_4}(7.7) \sim \mathbf{170}$$

$\boldsymbol{C_4}$ and $\boldsymbol{6C_3}$ cancelation in most central coll.

AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906

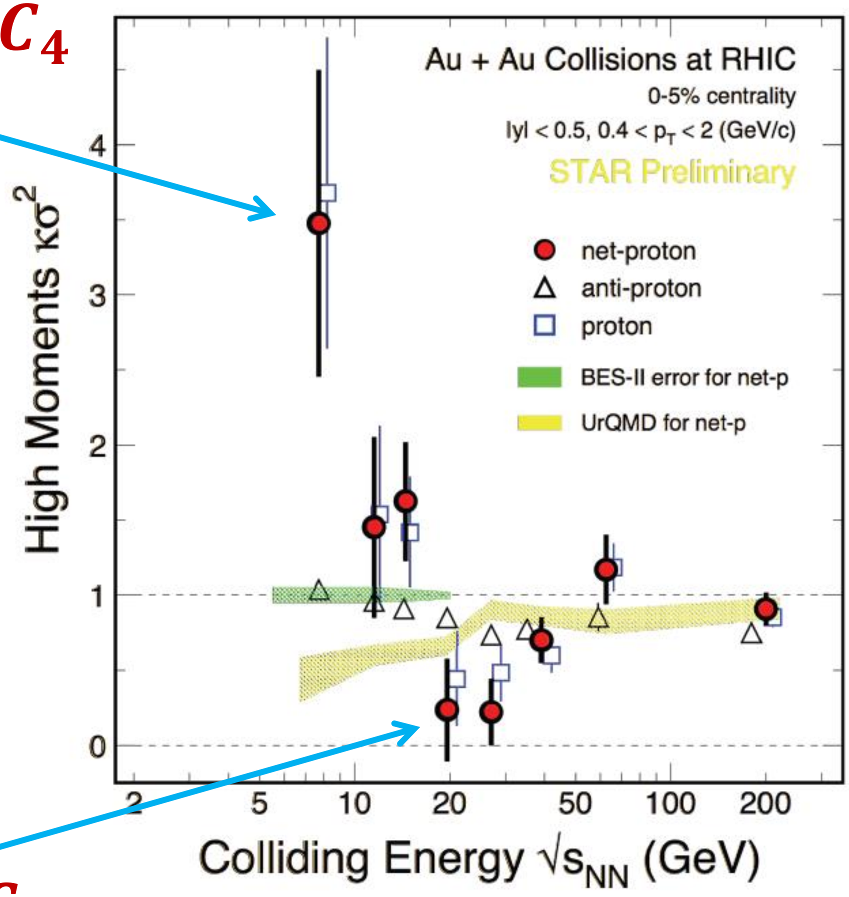
STAR preliminary
X.Luo, N.Xu, 1701.02105

here we see $\boldsymbol{C_4}$

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$



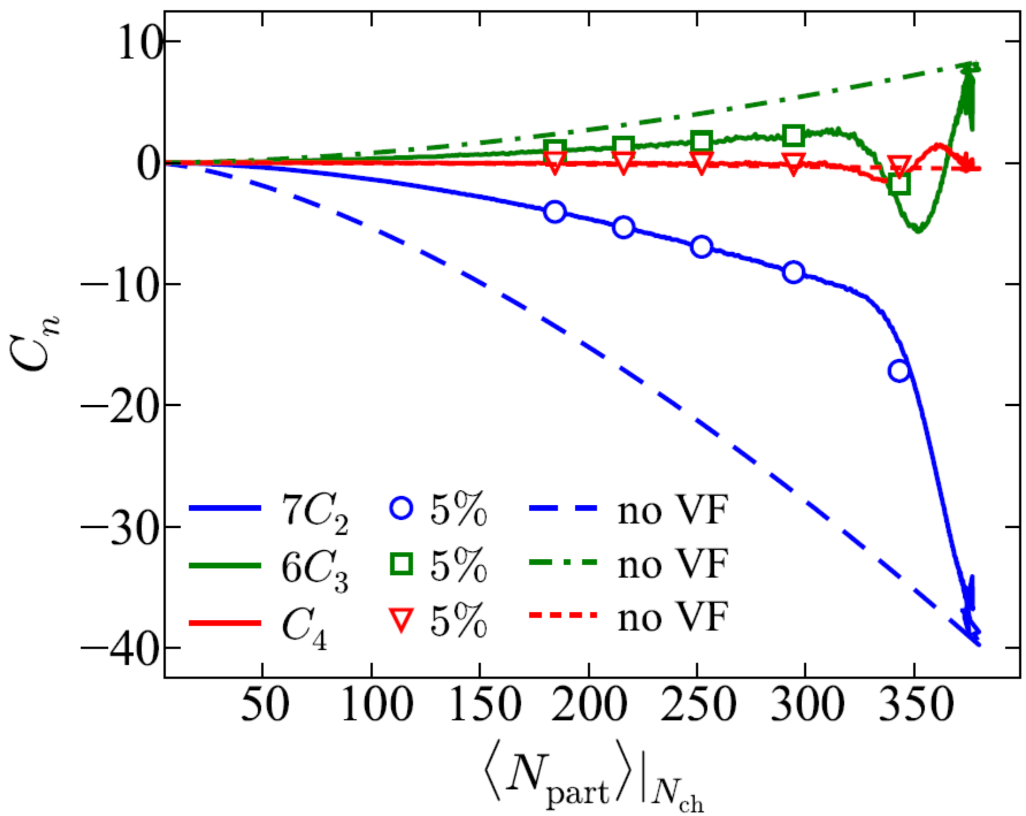
and here $\boldsymbol{C_2}$

AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906

Baryon conservation + volume fluctuation (minimal model)

- independent baryon stopping (baryon conservation by construction)
- N_{part} fluctuations (volume fluctuation - VF)

Au+Au, $\sqrt{s} = 7.7$ GeV



STAR

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$

we follow the STAR way (centrality etc.) as closely as possible

AB, V. Koch, V. Skokov, EPJC 77 (2017) 288

See also, P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114

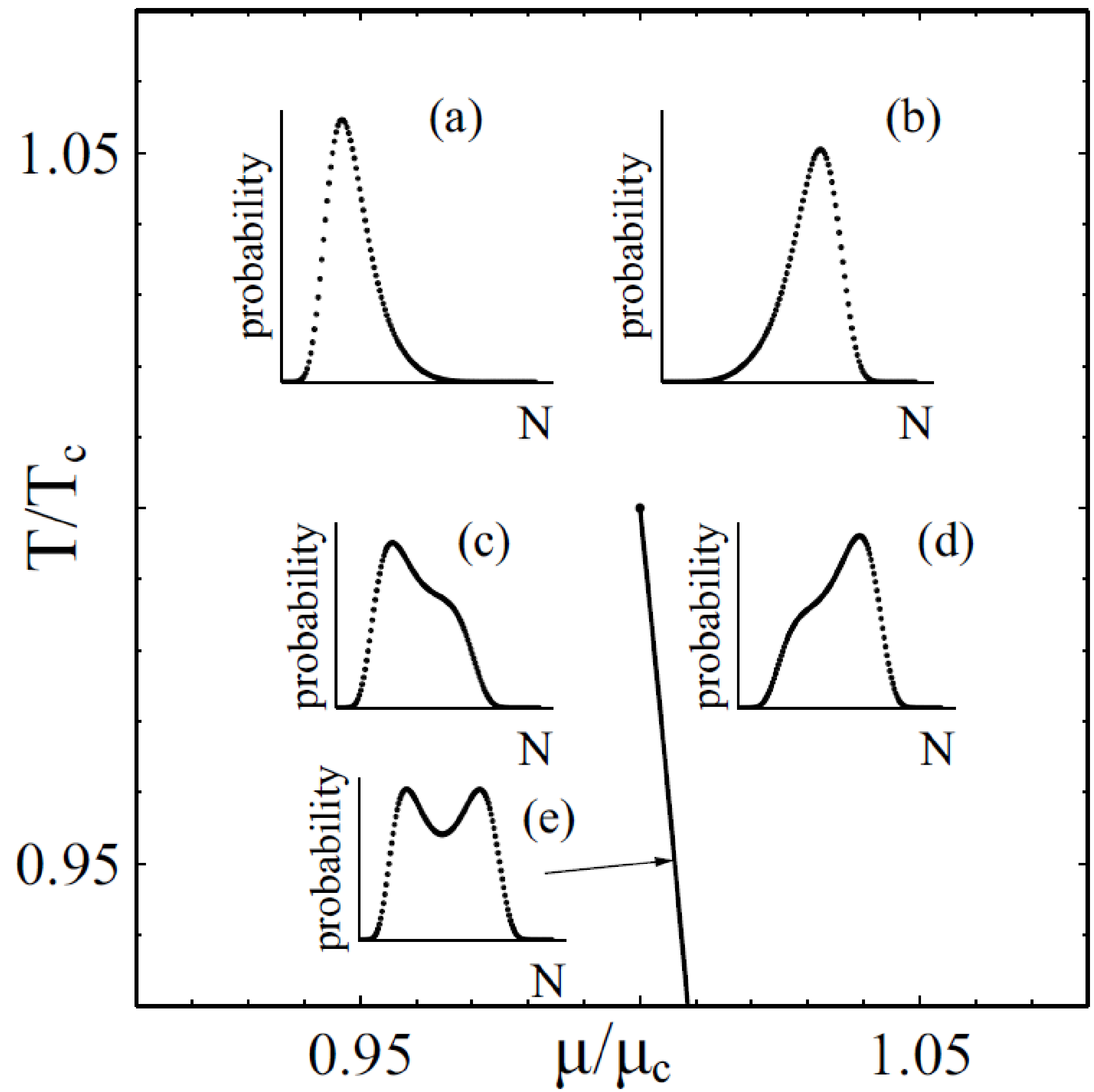
Can we describe the STAR data at 7.7 GeV with simple multiplicity distributions?

Bimodal distribution

$$P(N) = (1-\alpha)\underbrace{P_{(a)}(N)}_{\text{Poisson, binomial, etc..}} + \alpha \underbrace{P_{(b)}(N)}_{\text{Poisson, binomial, etc.}}$$

AB, V. Koch, D. Oliinychenko, J. Steinheimer, PRC 98 (2018) 054901

A finite volume van der Walls model



AB, V. Koch, D. Oliinychenko, J. Steinheimer, PRC 98 (2018) 054901

$$C_2 = \alpha(1-\alpha)\overline{N}^2 \approx \alpha\overline{N}^2,$$
$$C_3 = -\alpha(1-\alpha)(1-2\alpha)\overline{N}^3 \approx -\alpha\overline{N}^3,$$
$$C_4 = \alpha(1-\alpha)(1-6\alpha+6\alpha^2)\overline{N}^4 \approx \alpha\overline{N}^4,$$

$$\overline{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle$$

$$\boxed{\frac{C_6}{C_5} \approx \frac{C_5}{C_4} \approx \frac{C_4}{C_3} = -17 \pm 6}$$

parameter-free
prediction at 7.7 GeV ($\alpha \ll 1$)

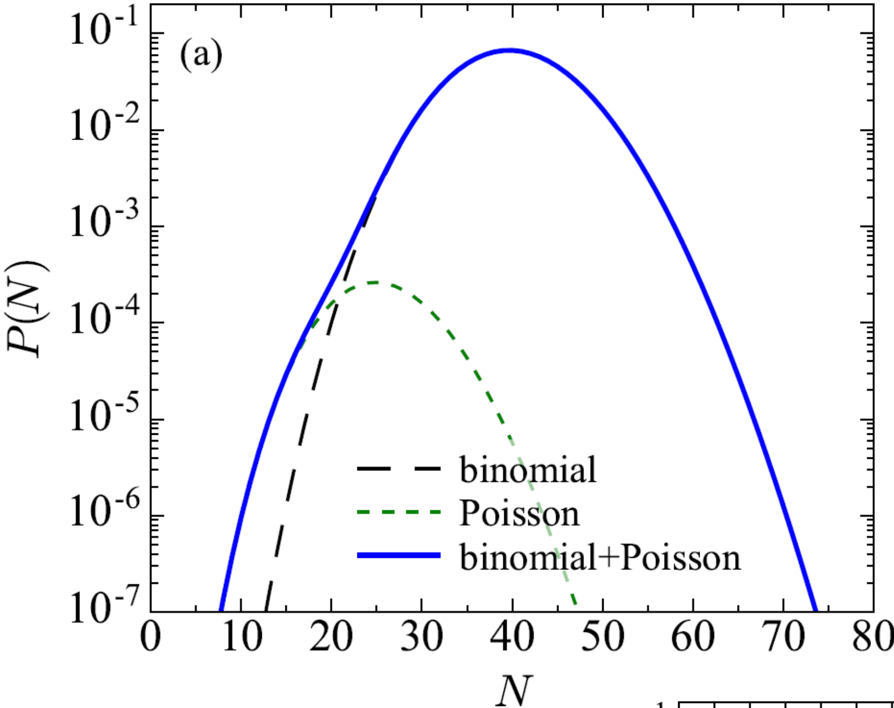
$$\boxed{\begin{aligned} C_5 &\approx -2645, \\ C_6 &\approx 40900, \end{aligned}}$$

assuming $C_4 = 170$

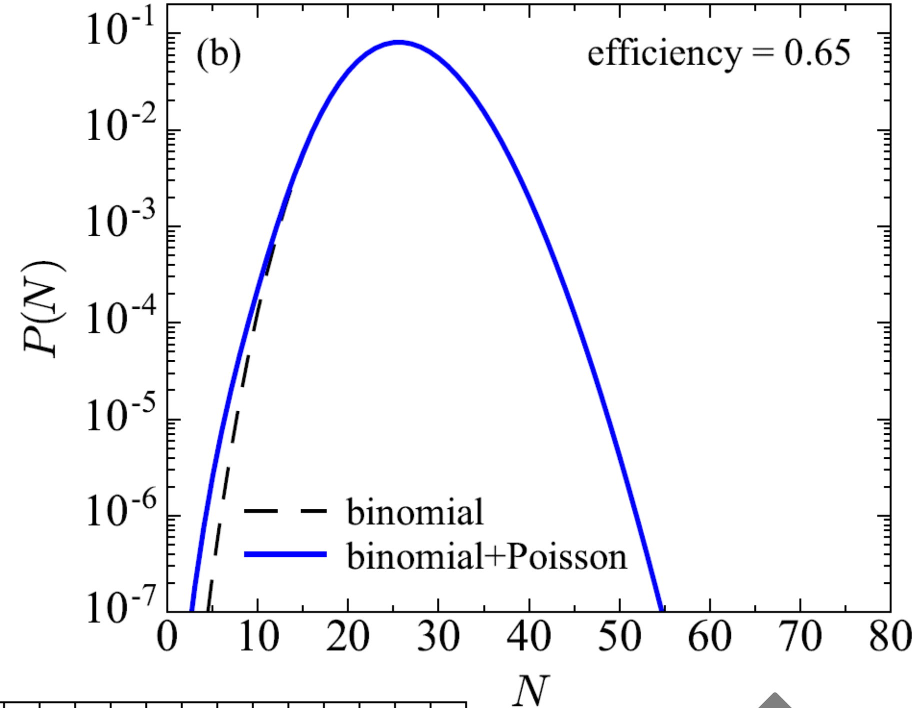
We can describe the data with $\alpha \approx 0.0033$

$$\langle N_{(a)} \rangle \approx 40,\ \langle N_{(b)} \rangle \approx 25$$

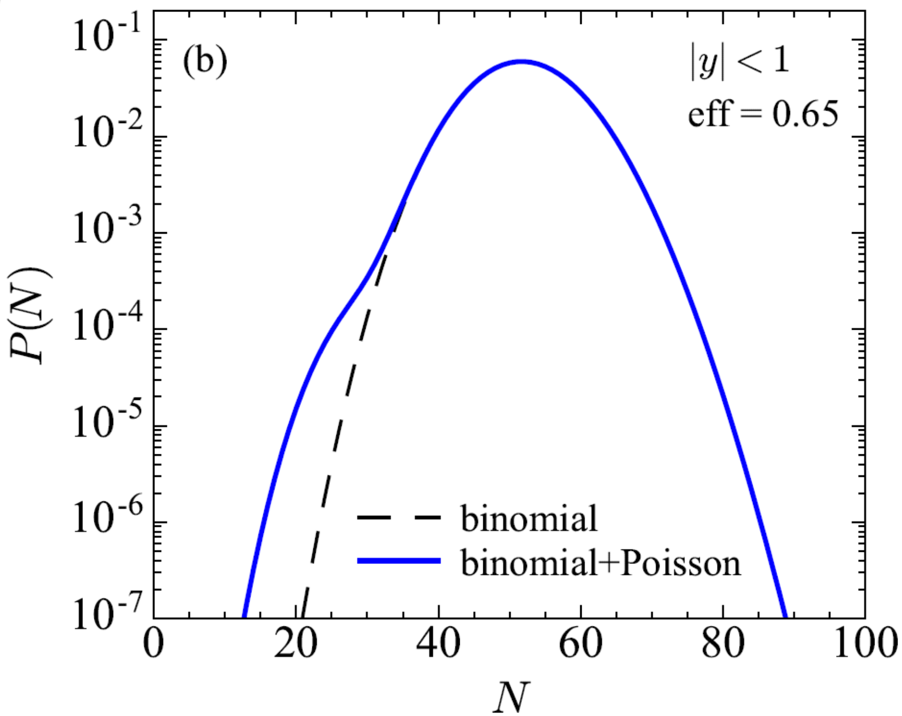
Now we can plot $P(N)$



(a)
binomial
Poisson
binomial+Poisson
P(N)
N



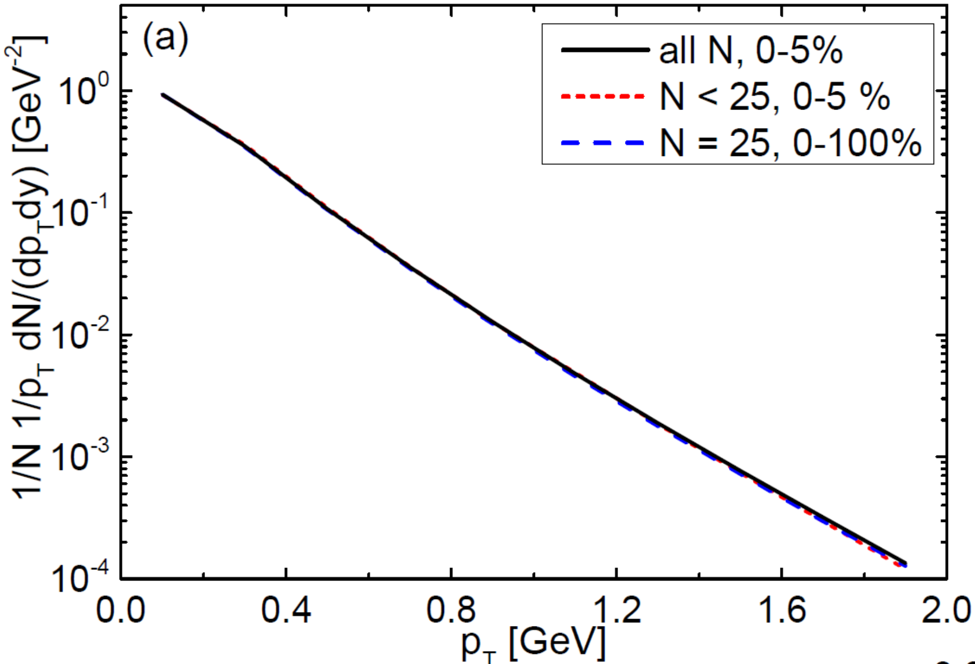
(b)
efficiency = 0.65
binomial
binomial+Poisson
P(N)
N



(b)
|y| < 1
eff = 0.65
binomial
binomial+Poisson
P(N)
N

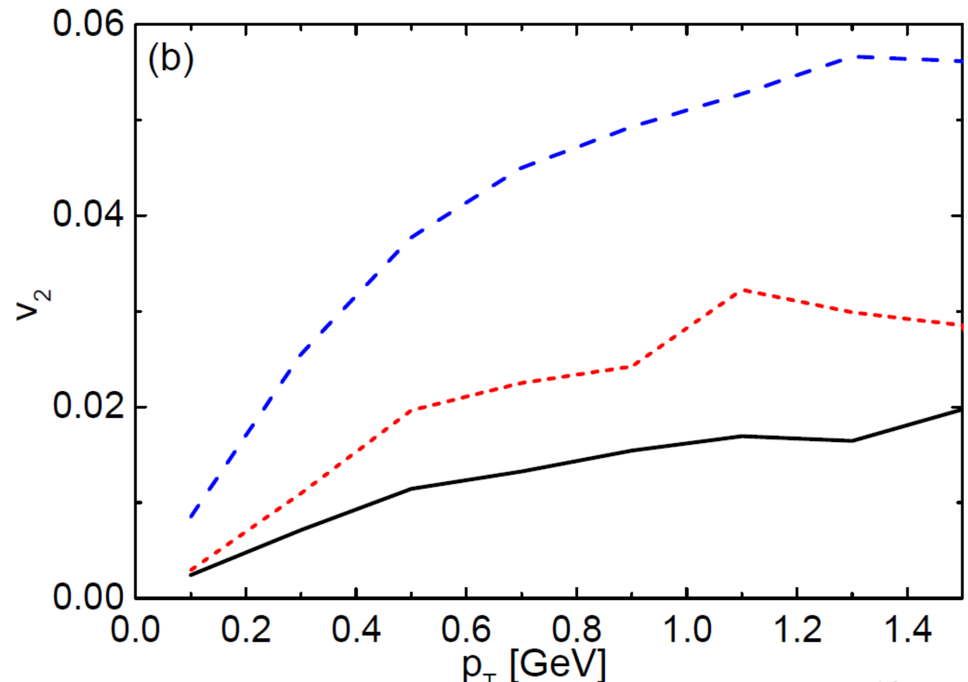
$|y| < 0.5$

$|y| < 1$



Cuts in the number of protons for central collisions

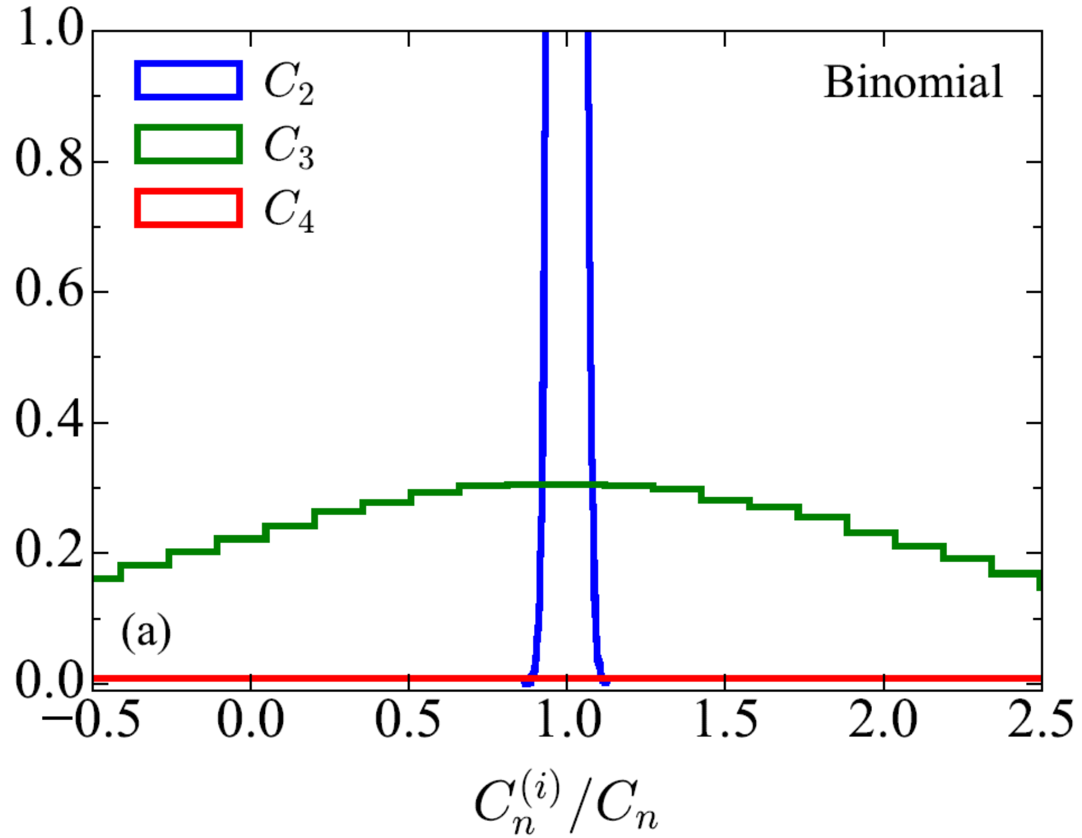
UrQMD



AB, V. Koch, D. Oliinychenko, J. Steinheimer, PRC 98 (2018) 054901

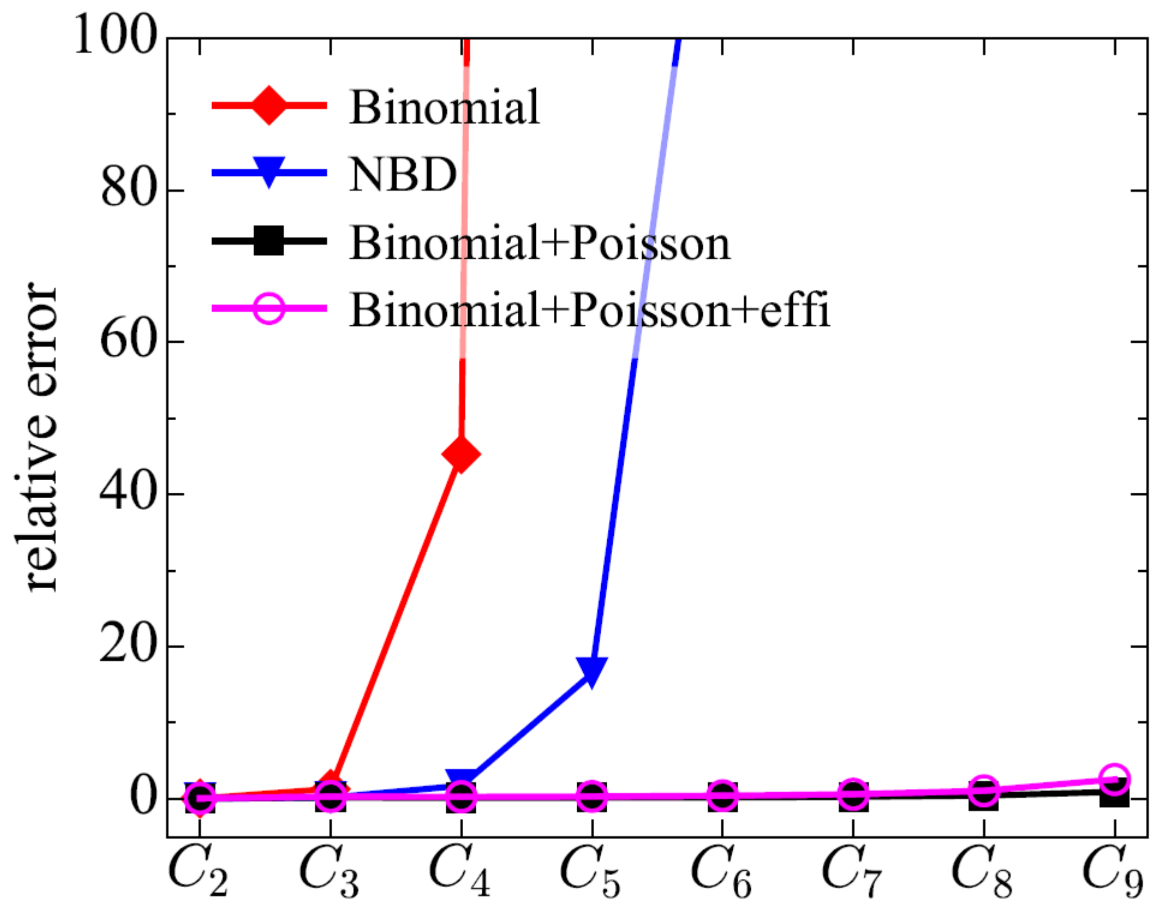
Can we verify the model (C_5, C_6, ...) with 144393 events available at STAR?

Statistics hungry distributions (SHD): binomial, Poisson, NBD,...



Histogram of $C_n^{(i)}/C_n$ based on 144393 events.

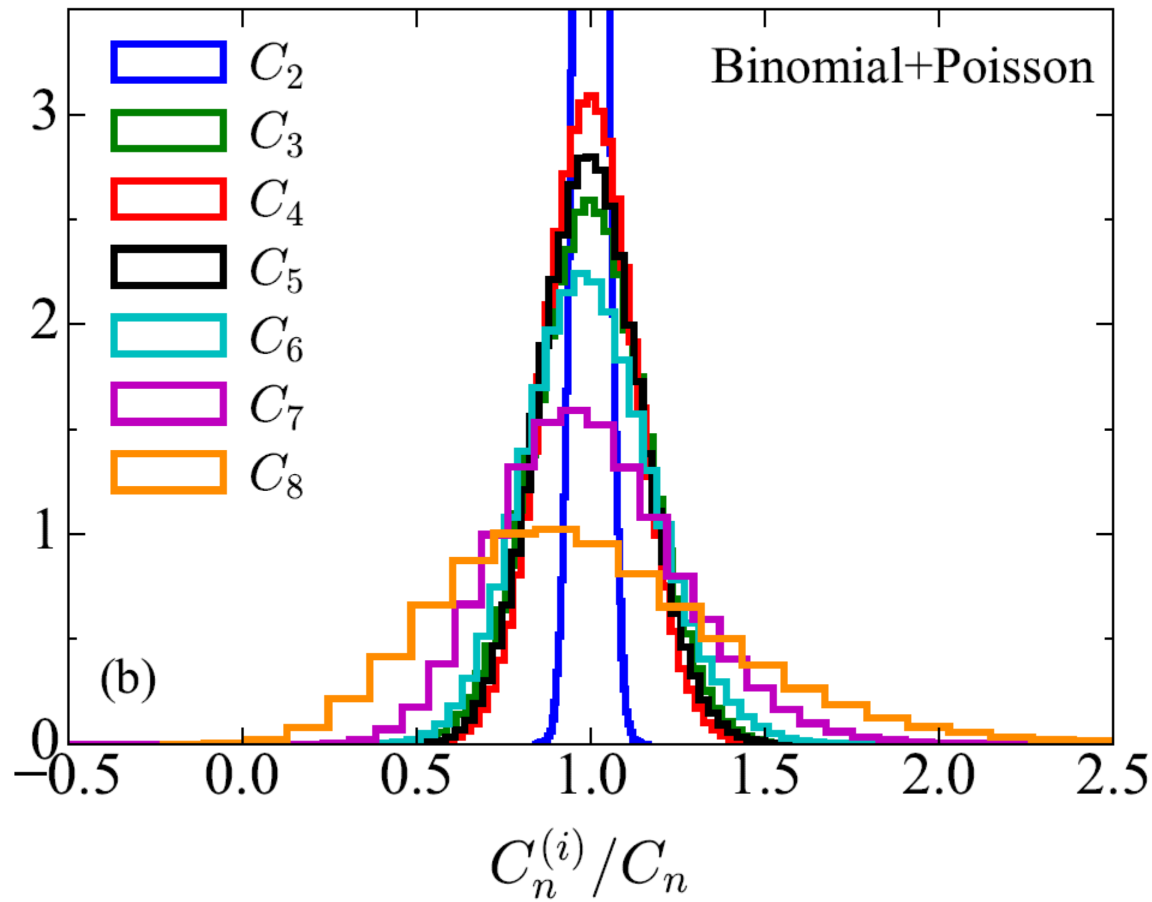
Are there **statistics friendly distributions** (SFD)? Yes.



based on 144393 events

AB, V.Koch, 1811.04456

Histogram of $C_n^{(i)}/C_n$ based on 144393 events.



It survives when hit with efficiency of 0.65

Our request to STAR.

Measure C_5, C_6, C_7, C_8 (**factorial cumulants**) and you should see:

$$C_5 \sim -2650$$

$$C_6 \sim +40900$$

$$C_7 \sim -615000$$

$$C_8 \sim +8520000$$

For Poisson $C_n = 0$

More details in 1811.04456

Conclusions:

Four-proton factorial cumulant (int. correlation function) at 7.7 GeV is surprisingly large. Three orders of magnitude larger than the minimal model.

Volume fluctuation and baryon conservation seem to be irrelevant for C_3 and C_4. C_2 (and K_2) is likely contaminated by background.

Proton clusters?

Bimodal $P(N)$. Parameter-free predictions.

Statistics friendly distributions close to the phase transition?

Backup

Critical point: everybody's guess

M. Stephanov, hep-lat/0701002

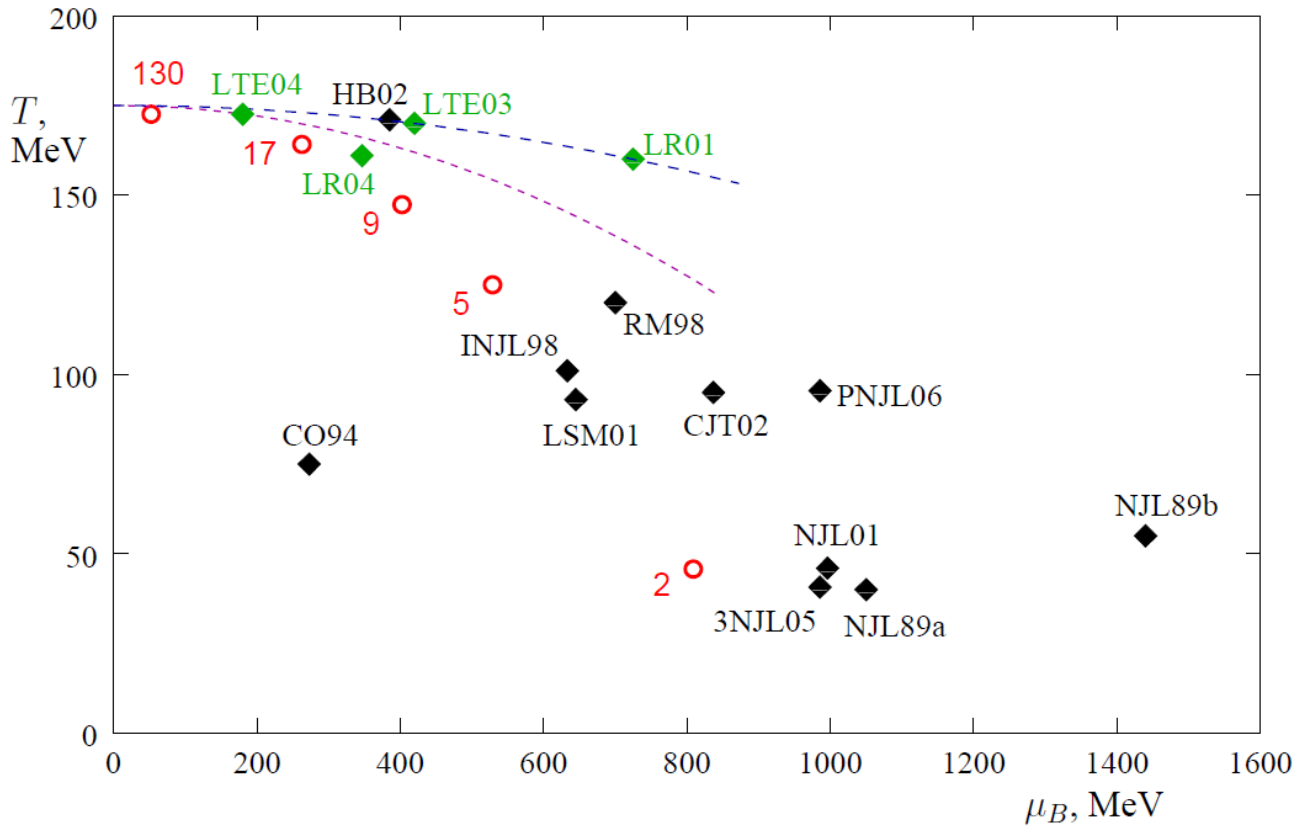
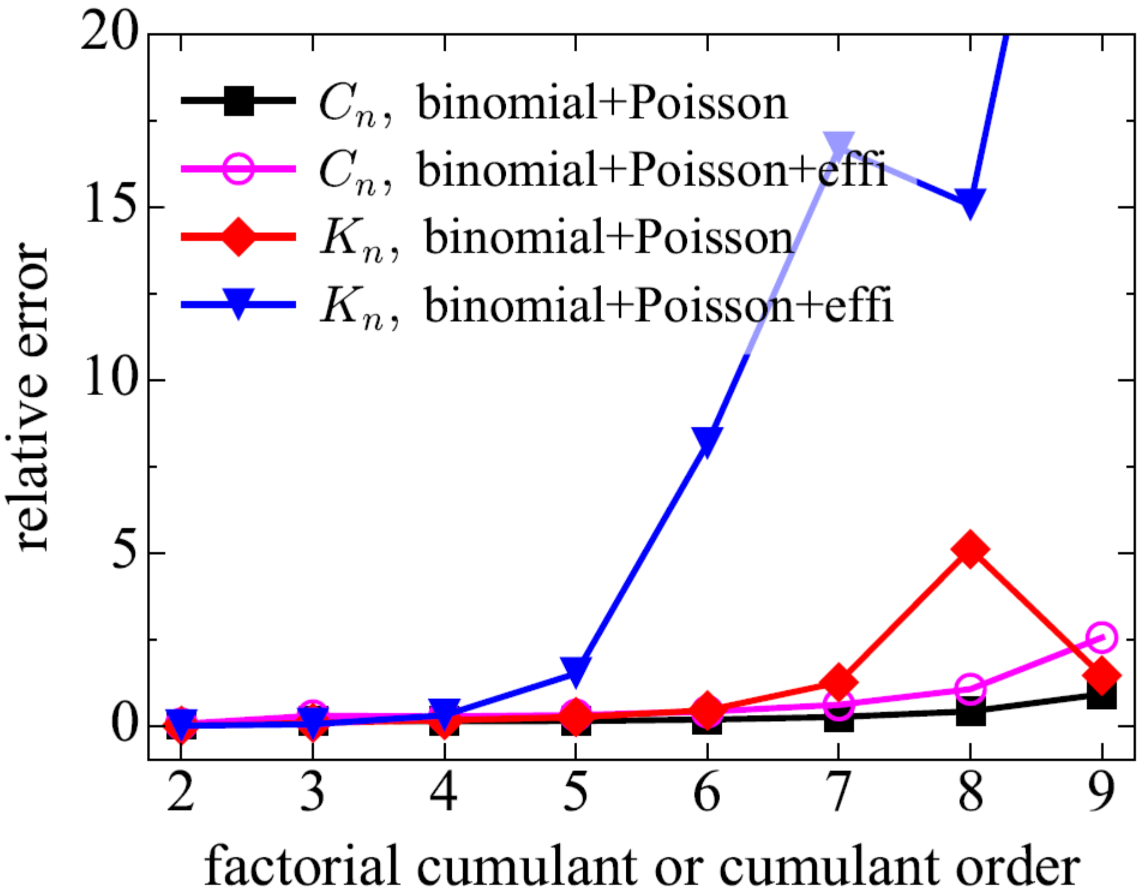


Figure 4: Comparison of predictions for the location of the QCD critical point on the phase diagram. Black points are model predictions: NJLa89, NJLb89 – [12], CO94 – [13, 14], INJL98 – [15], RM98 – [16], LSM01, NJL01 – [17], HB02 – [18], CJT02 – [19], 3NJL05 – [20], PNJL06 – [21]. Green points are lattice predictions: LR01, LR04 – [22], LTE03 – [23], LTE04 – [24]. The two dashed lines are parabolas with slopes corresponding to lattice predictions of the slope $dT/d\mu_B^2$ of the transition line at $\mu_B = 0$ [23, 25]. The red circles are locations of the freezeout points for heavy ion collisions at corresponding center of mass energies per nucleon (indicated by labels in GeV) – Section 5.

Regular cumulants are not as friendly



All of this can be understood

AB, V.Koch, 1811.04456

AB, V. Koch, V. Skokov, EPJC 77 (2017) 288

Let's put the STAR numbers in perspective.

Suppose that we have **clusters** (distributed according to Poisson) decaying always to 4 protons

$$\boldsymbol{C_k} = \langle N_{\rm cl} \rangle \cdot 4!/(4-k)!$$

$\uparrow$ mean number of clusters

$$\boldsymbol{C_4} = \langle N_{\rm cl} \rangle \cdot 24$$

for 5-proton clusters:

$$\boldsymbol{C_k} = \langle N_{\rm cl} \rangle \cdot 5!/(5-k)!$$

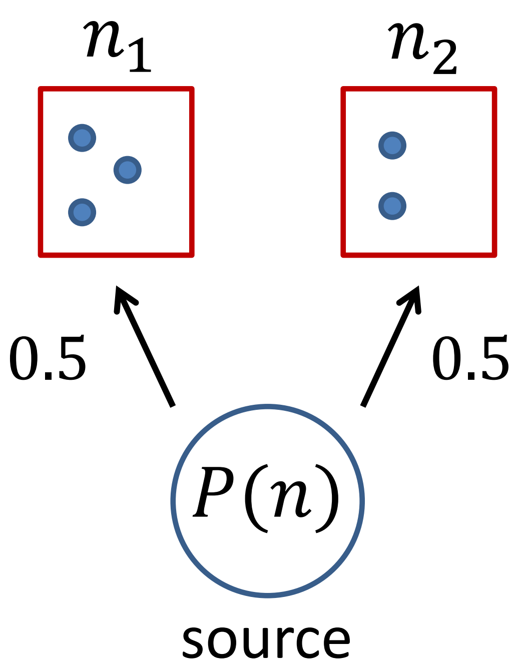
$$\boldsymbol{C_4} = \langle N_{\rm cl} \rangle \cdot 120$$

and $\langle N_{\rm cl} \rangle \sim 1$

To obtain $\boldsymbol{C_4} \approx 170$ we need $\langle N_{\rm cl} \rangle \sim 7,$ it means 28 protons.
STAR sees on average 40 protons in central collisions.

In this model $C_2 > 0$ and $C_3 > 0$ contrary to the STAR data

In what sense "no correlations"?



$$P(n_1, n_2) \stackrel{?}{=} P(n_1)P(n_2)$$

It is true for $P(n)$ = Poisson only

fixed N

finite N

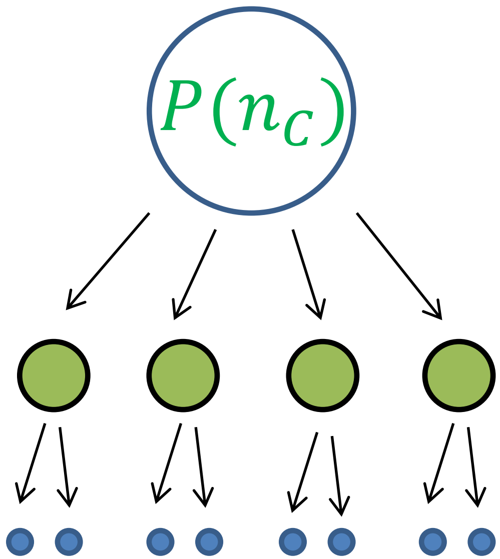
resonances

volume fluctuation

$$P(n_1, n_2) = P(n)\frac{n!}{n_1!\, n_2!}\left(\frac{1}{2}\right)^{n_1}\left(\frac{1}{2}\right)^{n_2}$$

$$n = n_1 + n_2$$

Suppose we have a system with two-particle clusters only



Poisson

In this case all information is contained in $\langle n \rangle$ and K_2.

No point to measure $K_{3,4,\dots}$

$$\boldsymbol{C_2} = 2\langle n_C \rangle \qquad \boldsymbol{C_{3,4,\dots}} = 0$$

$$\boxed{K_i = 2^i \langle n_C \rangle}$$

and for example: $\dfrac{K_4}{K_2} = 4$

looks nontrivial but no new information

Genuine three-particle correlation

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3) + \rho(y_1)\boldsymbol{C_2}(y_2, y_3) + \cdots$$

three possibilities

$$+ \boldsymbol{C_3}(y_1, y_2, y_3)$$

Integrating both sides

$$\langle n(n-1)(n-2)\rangle = \langle n\rangle^3 + 3\langle n\rangle \boldsymbol{C_2} + \boldsymbol{C_3}$$

factorial cumulant (integrated correlation function)

$$\boxed{\boldsymbol{C_3} = \int \boldsymbol{C_3}(y_1, y_2, y_3)\, dy_1\, dy_2\, dy_3}$$

and analogously for higher-order correlation functions

Comparison of 7.7, 11.5 and 19.6 GeV