Phase Transition in Interacting Boson System at Finite Temperatures

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Outline

- Introduction: Thermodynamic mean-field model
- Parametrization of the mean field
- The Bose-Einstein condensation in the ideal gas
- Selfconsistent solution for interacting gas of “pions”
- Density of condensed particles
- Pressure of the condensate-gas phase competes with pressure of the liquid-gas phase
- Energy density
- Concluding remarks

Thermodynamically consistent mean-field model [1]

Density of the free energy

$$\phi(n,T) = \varepsilon(n,T) - T\,s(n,T)\,, \qquad \phi(n,T) = n\,\mu(n,T) - p(n,T)$$

$$\phi(n,T) = \frac{F(N,V,T)}{V}\,, \qquad \mu = \frac{\partial\phi}{\partial n}\,, \qquad s = -\frac{\partial\phi}{\partial T}$$

The system of interacting particles $\longrightarrow$ a sum of free and interacting contributions:

$$\phi(n,T) = \phi_0(n,T) + \phi_{\rm int}(n,T)$$

where $\varepsilon(n,T)$ is the energy density, $p(n,T)$ is the pressure.
We adopt the system of units $k_B = c = \hbar = 1$.

[1] D. Anchishkin, V. Vovchenko, J. Phys. G: Nucl. Part. Phys. **42**, 105102 (2015); arXiv:1411.1444 [nucl-th].

Thermodynamic mean-field

Definitions

$$U(n,T) \equiv \frac{\partial \phi_{\rm int}(n,T)}{\partial n}\,, \qquad P^{\rm ex}(n,T) \equiv n\,\frac{\partial \phi_{\rm int}(n,T)}{\partial n} - \phi_{\rm int}(n,T)$$

The condition of thermodynamic consistency

$$n\,\frac{\partial U(n,T)}{\partial n} = \frac{\partial P^{\rm ex}(n,T)}{\partial n}$$

Transition to the Grand Canonical Ensemble

$$p = p_0(T,\mu_0) + P^{\rm ex}(n,T) \quad \to$$

$$\to \quad p(T,\mu) = p_0\big[T,\mu - U(n,T)\big] + P^{\rm ex}(n,T)$$

$$p_0(T,\mu_0) = \frac{g}{3}\int \frac{d^3k}{(2\pi)^3}\,\frac{\boldsymbol{k}^2}{\sqrt{m^2+\boldsymbol{k}^2}}\, f(\boldsymbol{k};T,\mu_0)$$

Thermodynamic mean-field. The Grand Canonical Ensemble

Pressure

$$p(T,\mu) = \frac{g}{3}\int \frac{d^3k}{(2\pi)^3}\frac{\mathbf{k}^2}{\sqrt{m^2+\mathbf{k}^2}} f(\mathbf{k};T,\mu) + P^{\mathrm{ex}}(n,T)$$

$$f(\mathbf{k};T,\mu) = \left\{\exp\left[\frac{\sqrt{m^2+\mathbf{k}^2}+U(n,T)-\mu}{T}\right]+a\right\}^{-1}$$

with $a=+1$ for fermions, $a=-1$ for bosons and $a=0$ for the Boltzmann statistics.

Particle density. We use: $n\,\frac{\partial U(n,T)}{\partial n} = \frac{\partial P^{\mathrm{ex}}(n,T)}{\partial n}$

$$n(T,\mu) = \frac{\partial p(T,\mu)}{\partial \mu} \quad\rightarrow\quad n = g\int\frac{d^3k}{(2\pi)^3} f(\mathbf{k};T,\mu)$$

Entropy density and energy density

$$s = \frac{\partial p(T,\mu)}{\partial T}, \qquad \varepsilon + p = T\,s + \mu\,n$$

Interacting boson systems with zero chemical potential: Toy model [2]

Skyrme-like parametrization of the mean field

$$U(n) = -A\,n + B\,n^2\,,$$

where n is the particle density, A and B are the positive model parameters, which should be specified for each particle species.
To be specific we take bosons with mass $m_\pi = 139$ MeV and degeneracy factor $g = 3$, and we call conventionally these particles as "pions".

Parameter B

$$B = 10 m_\pi b^2 \quad \text{with} \quad b = 4\frac{4\pi}{3}r_0^3, \quad r_0 \approx 0.3 \text{ fm}\ .$$

The attractive coefficient A is considered as a model parameter.

[2] D. Anchishkin, I. Mishustin and H. Stoecker, arXiv:1806.10857 [nucl-th].

Interacting boson systems with zero chemical potential: Toy model

Selfconsistent equation with respect to particle density n

$$n = \frac{g}{2\pi^2} \int_0^\infty dk\, k^2 \left\{ \exp\left[\frac{\sqrt{m^2 + k^2} + U(n)}{T} \right] - 1 \right\}^{-1} ,$$

$$\text{where} \quad \mu = 0$$

Critical values of the particle density n: n_1 and n_2

$$\text{when} \quad |\boldsymbol{k}| \to 0 \quad \Rightarrow \quad m + U(n) \geq 0$$

$$m - A\,n + B\,n^2 = 0 \quad \Rightarrow \quad n_{1,2} = \sqrt{\frac{m}{B}} \left(\kappa \mp \sqrt{\kappa^2 - 1} \right) ,$$

$$\kappa \equiv \frac{A}{2\sqrt{mB}} , \quad \kappa_c = 1 , \quad A_c = 2\sqrt{mB} \quad \to \quad A = \kappa A_c$$

Mean field: Variations of $A = \kappa A_c$

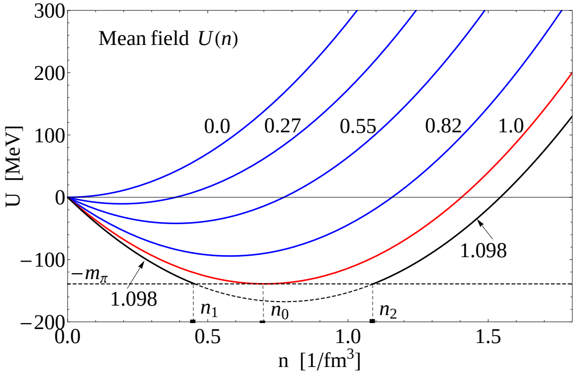

Figure: The mean field potential U versus particle density n, $\mu = 0$. The blue curves are labeled by value of the parameter κ: $\kappa = 0$, $\kappa = 0.27$, $\kappa = 0.55$, $\kappa = 0.82$. In the case $\kappa = \kappa_c = 1.0$ (red solid curve), the corresponding curve touches the negative energy level $-m_\pi$. For the case $\kappa = 1.098$ (black solid curve) the curve contains a segment below $-m_\pi$ (black-dashed curve), which corresponds to a super-critical strength.

The Bose-Einstein condensation in the ideal gas

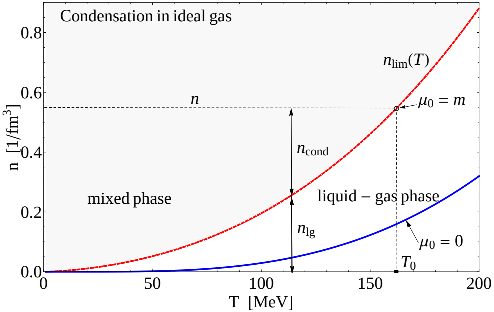

Figure: Particle density versus temperature for the noninteracting pion gas with $\mu_0 = m$ (dashed red curve $n_{\rm lim}$). The same dependence corresponds to the interacting pion system with mean field $U(n) = -m$ and $\mu = 0$.

$$n_{\rm lim} = \frac{g}{2\pi^2}\int_0^\infty dk\, k^2 \left\{\exp\left[\frac{\sqrt{m^2+k^2} - m}{T}\right] - 1\right\}^{-1}$$

Selfconsistent solution for interacting boson gas with different values of $A = \kappa A_c$

$$A_c = 396\ \text{MeV}\cdot\text{fm}^3, \qquad n = \frac{g}{2\pi^2}\int_0^\infty dk\, k^2 \left\{\exp\left[\frac{\sqrt{m^2+k^2}+U(n)}{T}\right]-1\right\}^{-1}$$

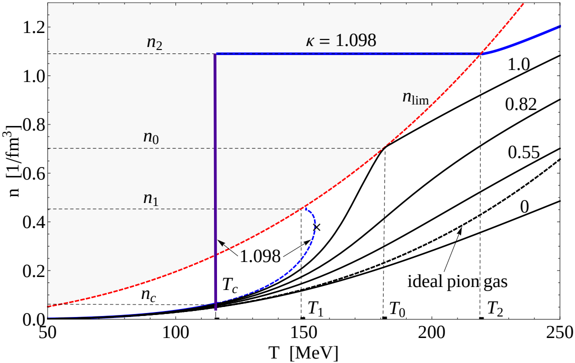

Figure: Particle density versus temperature, $\mu = 0$. The curves are labeled by the values $\kappa = A/A_c$. For $\kappa = 1.098$ the lower piece of the blue dashed curve corresponds to the meta-stable states, while the upper piece is unstable. The phase transition occurs at $T_c = 116$ MeV and $n_c = 0.065\ \text{fm}^{-3}$.

Pressure of the condensate-gas phase competes with pressure of the liquid-gas phase

$$p_{\rm mix}(T) = \frac{g}{3}\int \frac{d^3k}{(2\pi)^3}\frac{\mathbf{k}^2}{\sqrt{m^2+\mathbf{k}^2}}\left\{\exp\left[\frac{\sqrt{m^2+\mathbf{k}^2}-m}{T}\right]-1\right\}^{-1} + P^{\rm ex}(n_2)$$

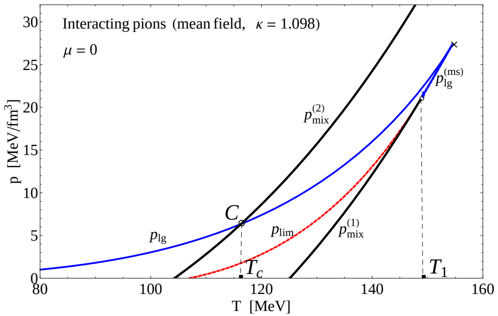

Figure: Pressure versus temperature, $\mu = 0$. Solid blue line $p_{\rm lg}$ corresponds to the pressure in the liquid-gas phase, the dark blue line $p_{\rm lg}^{\rm (ms)}$ corresponds to meta-stable states. Solid black lines $p_{\rm mix}^{(1)}$ and $p_{\rm mix}^{(2)}$ correspond to the pressures in the mixed phase for the particle densities $n = n_1$ and $n = n_2$, respectively. The branch $p_{\rm mix}^{(1)}$ is unstable. C is the critical point.

Excess pressure

$$P^{\rm ex}(n) = \int_0^n dn'\, n' \frac{\partial U(n')}{\partial n'} = -\frac{1}{2} A n^2 + \frac{2}{3} B n^3$$

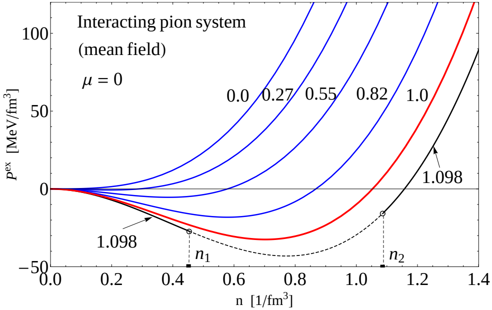

Figure: Excess pressure versus pion density for different values of parameter κ, $\mu = 0$. The lowest curve is calculated for $\kappa = 1.098$ contains a segment $n_1 < n < n_2$, where $U(n) + m_\pi < 0$ (black dashed line).

Energy density

$$\varepsilon^{(2)}_{\rm mix} = m n_{\rm cond}(T) + g \int \frac{d^3k}{(2\pi)^3} \sqrt{m^2 + \boldsymbol{k}^2} \left[\exp\left(\frac{\sqrt{m^2 + \boldsymbol{k}^2} - m}{T} \right) - 1 \right]^{-1} + \varepsilon^{\rm ex}(n_2)$$

$$\varepsilon^{\rm ex}(n) = n\,U(n) - P^{\rm ex}(n) = -\frac{1}{2} A n^2 + \frac{1}{3} B n^3$$

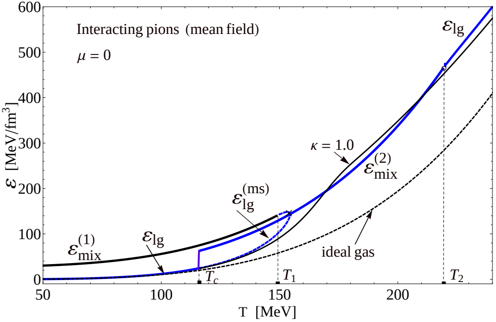

Figure: The energy density versus temperature, $\mu = 0$. The blue solid curve, which consists of several segments, labeled as $\varepsilon_{\rm lg}$ and $\varepsilon^{(2)}_{\rm mix}$, corresponds to $\kappa = 1.098$. The energy density of the mixed phase $\varepsilon^{(2)}_{\rm mix}(T)$ is calculated for $n = n_2$ in the temperature interval $T_{\rm c} \leq T \leq T_2$. The energy density in the mixed phase $\varepsilon^{(1)}_{\rm mix}(T)$ is calculated for $n = n_1$ (black solid segment). Blue dashed segment is the energy density of meta-stable states in the liquid-gas phase $\varepsilon^{(\rm ms)}_{\rm lg}$ for $\kappa = 1.098$.

Concluding remarks

- Thermodynamically consistent method to describe dense bosonic systems at high temperatures and zero chemical potential is presented.
- A central step of the approach is to solve self-consistent equation for the particle density at a given temperature with account that the distribution function is positive definite when the condition $U(n) \geq -m$ is fulfilled.
- If the attractive mean field is so strong that the latter condition is violated, the multi-boson system can develop a Bose condensate at finite temperature.

Thank you for attention!

Acknowledgements

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