Hubble-type solutions of hydrodynamics with bulk viscosity

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Introduction

- Ideal fluid (=perfect fluid) hydrodynamics:
  - Well-proven theory, clear basic equations
  - Good at describing bulk development in heavy-ion collisions
    $\Leftrightarrow$ QGP viscosity (experimentally) small
  - Analytic solutions (vs. numerical ones)
  - Non-relativistic: many well known simple fireball solutions
  - Relativistic: historical + (more or less) recent developments
- Viscous hydrodynamics:
  - Non-relativistic: basic equations clear (?)
  - Fireball NR solutions $\rightarrow$ generalization works
  - Relativistic: basic equations NOT well settled
    - Landau theory vs. Eckart's theory (1940s):
      1st order (parabolic PDEs, acausality, instability...)
    - Israel-Stewart theory: hyperbolic PDEs (complicated; more coefficients)
  - Exact relativistic viscous solutions **lacking**
- Goal: study relativistic viscous effect in simplest possible way

Basic equations

- Ideal fluid (=perfect fluid) case (a reminder):
  - $T_{\mu\nu}$ stress-energy-momentum tensor, $\partial_\nu T^{\mu\nu}=0$.
  - $T_{\mu\nu}=(\varepsilon+p)u_\mu u_\nu - p g_{\mu\nu}$, =definition of p, ε.
    $\Rightarrow$ energy equation: $(\varepsilon+p)\partial_\nu u^\nu + u^\nu \partial_\nu \varepsilon=0$
    $\Rightarrow$ Euler equation: $(\varepsilon+p)u^\nu \partial_\nu u^\mu=(g^{\mu\rho}-u^\mu u^\rho)\partial_\rho p$.
  - Need Equation of State (EoS): eg. $\varepsilon=\kappa(T)p$.
  - Simplest choice: κ=const.
  - Two cases in what follows:
    $p=nT$ (with conserved charge n, $\partial_\nu(nu^\nu)=0$) vs.
    $p=p_0(T/T_0)^{\kappa+1}$ (no conserved charge, only σ entropy density)
- Viscous hydrodynamics, first order theory:
  - $T_{\mu\nu}=(\varepsilon+p)u_\mu u_\nu - p g_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu + \pi_{\mu\nu}$
    q_μ: thermal conduction ($q_\mu u^\mu=0$), $\pi_{\mu\nu}$: viscous tensor ($\pi_{\mu\nu}u^\nu=0$).
  - Conservation of particle number (charge): $\partial_\mu N^\mu=0$
    $N^\mu=nu^\mu+j^\mu$ with $u_\mu j^\mu=0$.
    j^μ: ambiguity in definition of u^μ.

Eckart vs. Landau frame

- Eckart frame: $N^\mu = nu^\mu$ (ie: $j^\mu = 0$): choice for definition of u^μ
  - $q_\mu = \lambda(g_{\mu\nu} - u_\mu u_\nu)\cdot\left(\partial^\nu T - Tu^\rho\partial_\rho u^\nu\right)$
  - $\pi_{\mu\nu} = \eta\left[(g_{\mu\rho} - u_\mu u_\rho)\partial^\rho u_\nu + (g_{\nu\rho} - u_\nu u_\rho)\partial^\rho u_\mu - \frac{2}{d}(g_{\mu\nu} - u_\mu u_\nu)\partial^\rho u_\rho\right] +$
    $+\zeta(g_{\mu\nu} - u_\mu u_\nu)\partial^\rho u_\rho$
  - Expressions: from increase of entropy
- λ: thermal conductivity, η: shear viscosity, ζ: bulk viscosity, $d=3$.
- Landau frame: $q_\mu = 0$, choice for definition of u^μ
  - $j_\mu = \lambda\left(\frac{nT}{\varepsilon+p}\right)^2(g_{\mu\nu} - u_\mu u_\nu)\cdot\partial^\nu\frac{\mu}{T}$
  - $\pi_{\mu\nu}$ similar form as for Eckart (definition of u^μ not the same...)
- In what follows: take $\lambda=0$, investigate role of ζ
  - η: famous lower limit $\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$ from AdS/CFT
    P. Kovtun, D. T. Son, A. O. Starinets, PRL 94, 111601 (2005)
  - ζ: not straightforward; general consensus $\zeta \ll \eta$.
    Eg. in monatomic gases, $\zeta = 0$
    I. M. Khalatnikov, Sov. Phys. JETP **2**, 169 (1956)

Hubble-type solution & generalization

- Well-known (in fact, simplest) 3D solution in perfect fluid case:

$$u^\mu = \frac{x^\mu}{\tau} \qquad n = n_0 \left(\frac{\tau_0}{\tau}\right)^d, \qquad T = T_0 \left(\frac{\tau_0}{\tau}\right)^{d/\kappa}.$$

See e.g. T. Csörgő, *et al.*, PLB **565**, 107 (2003)

- Ansatz for viscous solution (with $\lambda = 0$, continuity works)

$$u^\mu = \frac{x^\mu}{\tau} \qquad n = n_0 \left(\frac{\tau_0}{\tau}\right)^d, \qquad T \equiv T(\tau), \quad \Rightarrow \quad p \equiv p(\tau)$$

- Choice for ζ bulk viscosity & Eos??? Cases investigated are:
  1. Case A: $\zeta = \zeta_0$ constant, no conserved n, $p = p_0 (T/T_0)^{\kappa+1}$
  2. Case B: $\zeta = \zeta_0$ constant, conserved n, $p = nT$
  3. Case C: $\zeta = \zeta_0 (T/T_0)^\kappa$ (ie. $\zeta \propto s$), no conserved n, $p = p_0 (T/T_0)^{\kappa+1}$
  4. Case D: $\zeta = \zeta_0 (n/n_0)$ (proxy for $\zeta \propto s$), conserved n, $p = nT$
  5. Case E: $\zeta = \zeta_0 (T/T_0)^\kappa$ (other proxy for $\zeta \propto s$), conserved n, $p = nT$

Solving the equations...

- Some intermediate steps:
  - All terms containing η **shear viscosity cancel**
    $\Rightarrow$ Hubble profile: ideal to study effect of ζ
  - Also λ (heat conductive) terms cancel (if present at all)
  - Euler equation is automatically **satisfied** (because assumed only τ dependence)
  - In simplest case (case A,B) possibility for ellipsoidal generalization with arbitrary $\mathcal{V}(S)$, $S=\frac{r_x^2}{X_0^2t^2}+\frac{r_y^2}{Y_0^2t^2}+\frac{r_z^2}{Z_0^2t^2}$.
- Only energy equation to solve: reduces to 1 ODE for $p(\tau)$

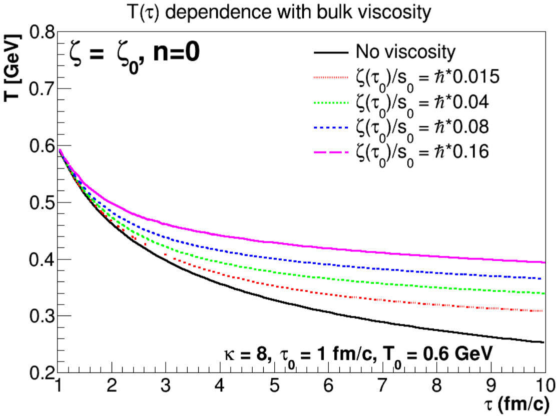
$$\kappa\frac{dp}{d\tau}+\frac{d(\kappa+1)}{\tau}p-\frac{d^2}{\tau^2}\zeta(p,\tau)=0.$$

Solution: straightforward, in each cases.

Case A

$\zeta = \zeta_0$ constant, no conserved n, $p = p_0 (T/T_0)^{\kappa+1}$

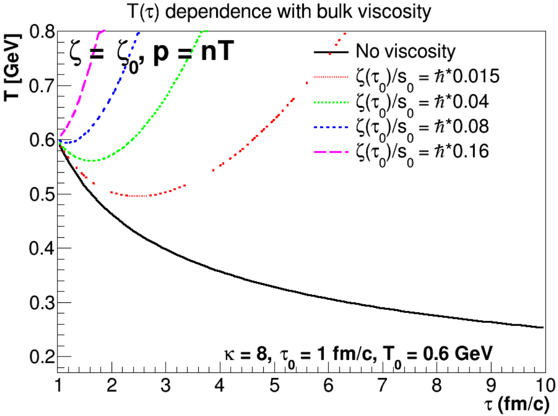
$$p(\tau) = \left[p_0 - \frac{d^2}{(\kappa+1)d - \kappa} \frac{\zeta_0}{\tau_0} \right] \left(\frac{\tau_0}{\tau} \right)^{d \frac{\kappa+1}{\kappa}} + \frac{d^2}{(\kappa+1)d - \kappa} \frac{\zeta_0}{\tau}.$$



Case B

$\zeta = \zeta_0$ constant, conserved n, $p = nT$

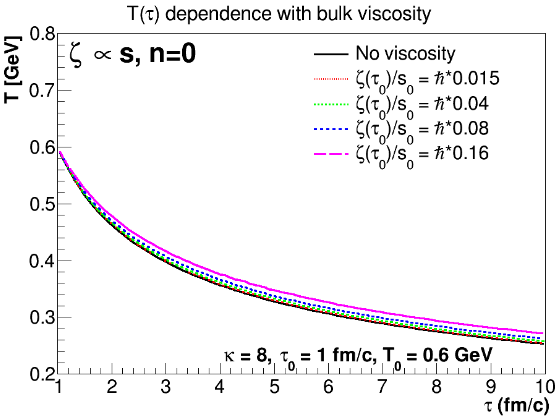
$$p(\tau) = \left[p_0 - \frac{d^2}{(\kappa+1)d - \kappa} \frac{\zeta_0}{\tau_0} \right] \left(\frac{\tau_0}{\tau} \right)^{d\,\frac{\kappa+1}{\kappa}} + \frac{d^2}{(\kappa+1)d - \kappa} \frac{\zeta_0}{\tau}.$$



Case C

$\zeta = \zeta_0 (T/T_0)^\kappa$ (ie. $\zeta \propto s$), no conserved n, $p = p_0 (T/T_0)^{\kappa+1}$

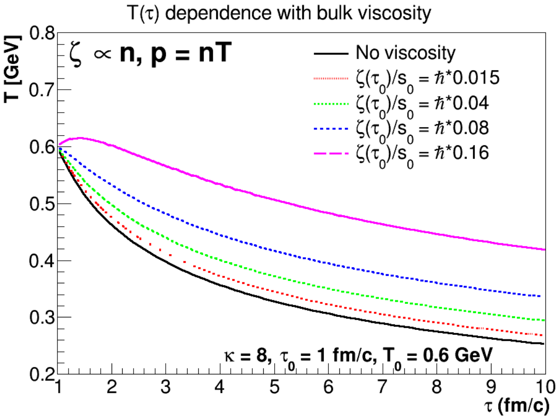
$$p(\tau) = p_0 \left\{ \left(1 + \frac{d^2}{(\kappa+1)(\kappa-d)} \frac{\zeta_0}{p_0 \tau_0} \right) \left(\frac{\tau_0}{\tau} \right)^{\frac{d}{\kappa}} - \frac{d^2}{(\kappa+1)(\kappa-d)} \frac{\zeta_0}{p_0} \frac{1}{\tau} \right\}^{\kappa+1}$$



Case D

$\zeta = \zeta_0 (n/n_0)$ (proxy for $\zeta \propto s$), conserved n, $p = nT$

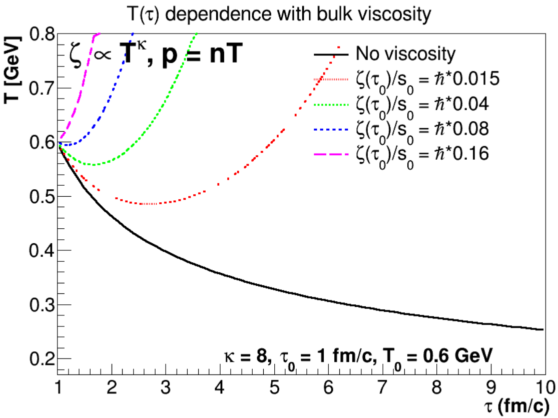
$$p(\tau) = \left[p_0 + \frac{d^2}{\kappa - d} \frac{\zeta_0}{\tau_0} \right] \left(\frac{\tau_0}{\tau} \right)^{\frac{\kappa+1}{\kappa} d} - \frac{d^2}{\kappa - d} \frac{\zeta_0}{\tau_0} \frac{\tau_0^{d+1}}{\tau^{d+1}}$$



Case E

$\zeta = \zeta_0 (T/T_0)^\kappa$ (other proxy for $\zeta \propto s$), conserved n, $p = nT$

$$\frac{p(\tau)}{p_0} = \left\{ \left(1 - \frac{d^2 \zeta_0/(p_0\tau_0)}{(d-1)(\kappa^2+\kappa)+d}\right)\left(\frac{\tau_0}{\tau}\right)^{\frac{d}{\kappa}} + \frac{d^2\zeta_0/(p_0\tau_0)}{(d-1)(\kappa^2+\kappa)+d}\left(\frac{\tau}{\tau_0}\right)^{d\frac{\kappa}{\kappa+1}-1}\right\}^{\kappa+1}.$$



Summary and outlook

- Hydrodynamical models: perfect fluid vs. viscous fluids
- A (simple) viscous solution presented: Hubble flow, energy development distorted by bulk viscosity
- Illustrations plotted for (somewhat) equivalent values of ζ in different scenarios
- Different assumptions, different quantitative effects
- Further steps
  - Of course many possibilities (beyond simplest Hubble velocity field. . .)

Thank you for your attention!

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