

Lifetime estimation from RHIC Au+Au data

arXiv:1811.09990

Gábor Kasza, Tamás Csörgő

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Outline

- New, exact solution of relativistic perfect fluid hydro [arXiv:1805.01427](https://arxiv.org/abs/1805.01427)
- Observables – $dN/d\eta$ [arXiv:1806.06794](https://arxiv.org/abs/1806.06794)
- Observables – R_{long} [arXiv:1810.00154](https://arxiv.org/abs/1810.00154)
- Inverse slope fits
- $dN/d\eta$ fits
- R_L fits
- Exact result of initial energy density [arXiv:1806.11309](https://arxiv.org/abs/1806.11309)
- Method is submitted for a publication in the MDPI Journal Universe (Zimányi School + analytic hydro special volume): [arXiv:1811.09990](https://arxiv.org/abs/1811.09990)

Perfect fluid hydrodynamics

- Energy and momentum conservation:

$$\partial_\nu T^{\mu\nu} = 0$$

- Energy-momentum tensor (perfect fluid):

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

- Continuity equation:

$$\partial_\mu (\sigma u^\mu) = 0$$

- Equation of state (EoS): closes the equation system

$$\varepsilon = \kappa p$$

$$\kappa = c_s^{-2}$$

- In this work κ is constant and the speed of sound (c_s) as well

New, exact solutions of perfect hydro

- Rindler coordinates, velocity field:

$$(\tau, \eta_x) = \left(\sqrt{t^2 - r_z^2}, \frac{1}{2} \ln \left[\frac{t + r_z}{t - r_z} \right] \right)$$

$$u^\mu = (\cosh(\Omega), \sinh(\Omega))$$

- 1+1 dimensional, parametric solution (in $\kappa \rightarrow 1$ limit: CNC):

$$\eta_x(H) = \Omega(H) - H,$$

$$\Omega(H) = \frac{\lambda}{\sqrt{\lambda - 1} \sqrt{\kappa - \lambda}} \arctan \left(\sqrt{\frac{\kappa - \lambda}{\lambda - 1}} \tanh(H) \right)$$

$$\sigma(\tau, H) = \sigma_0 \left(\frac{\tau_0}{\tau} \right)^\lambda \mathcal{V}_\sigma(s) \left[1 + \frac{\kappa - 1}{\lambda - 1} \sinh^2(H) \right]^{-\frac{\lambda}{2}},$$

$$T(\tau, H) = T_0 \left(\frac{\tau_0}{\tau} \right)^{\frac{\lambda}{\kappa}} \mathcal{T}(s) \left[1 + \frac{\kappa - 1}{\lambda - 1} \sinh^2(H) \right]^{-\frac{\lambda}{2\kappa}},$$

$$\mathcal{T}(s) = \frac{1}{\mathcal{V}_\sigma(s)},$$

$$s(\tau, H) = \left(\frac{\tau_0}{\tau} \right)^{\lambda - 1} \sinh(H) \left[1 + \frac{\kappa - 1}{\lambda - 1} \sinh^2(H) \right]^{-\lambda/2}$$

**λ : acceleration
parameter**



**accelerating
solution**

Csörgő, Nagy, Csanád
solution
[arXiv:0709.3677](https://arxiv.org/abs/0709.3677)

[arXiv:1805.01427](https://arxiv.org/abs/1805.01427)

[arXiv:1806.06794](https://arxiv.org/abs/1806.06794)

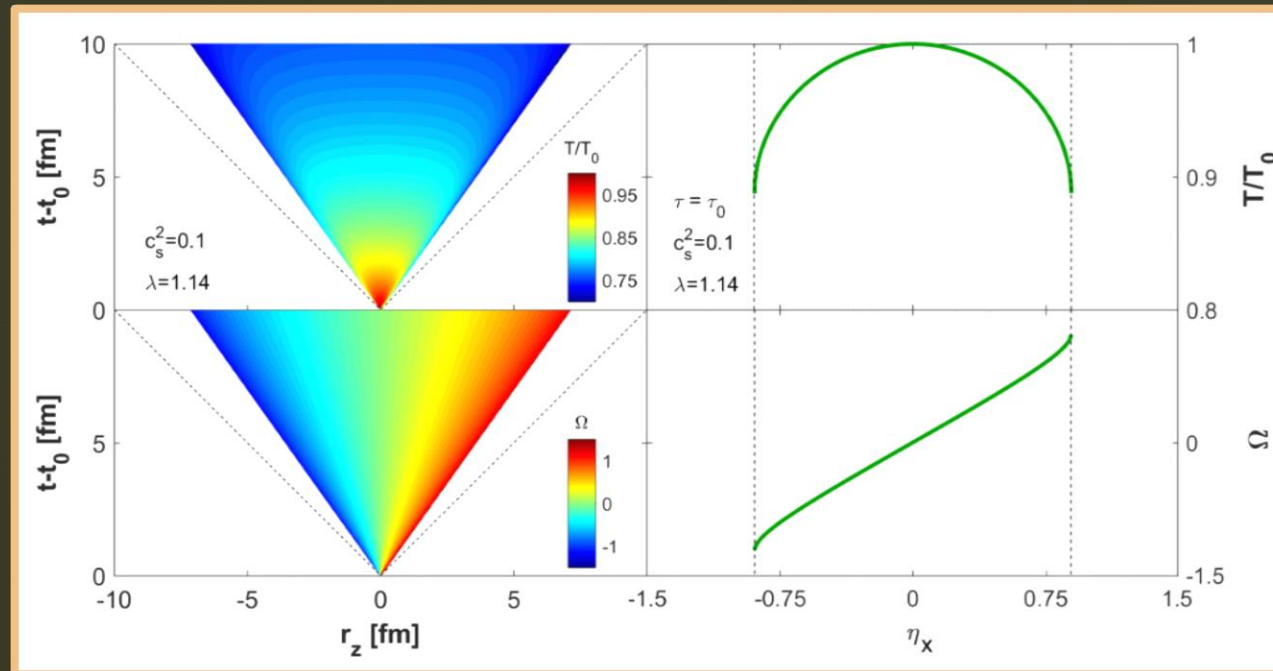
New, exact solutions of perfect hydro

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$$(\tau, \eta_x) = \left(\sqrt{t^2 - r_z^2}, \frac{1}{2} \ln \left[\frac{t + r_z}{t - r_z} \right] \right)$$

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accelerating solution

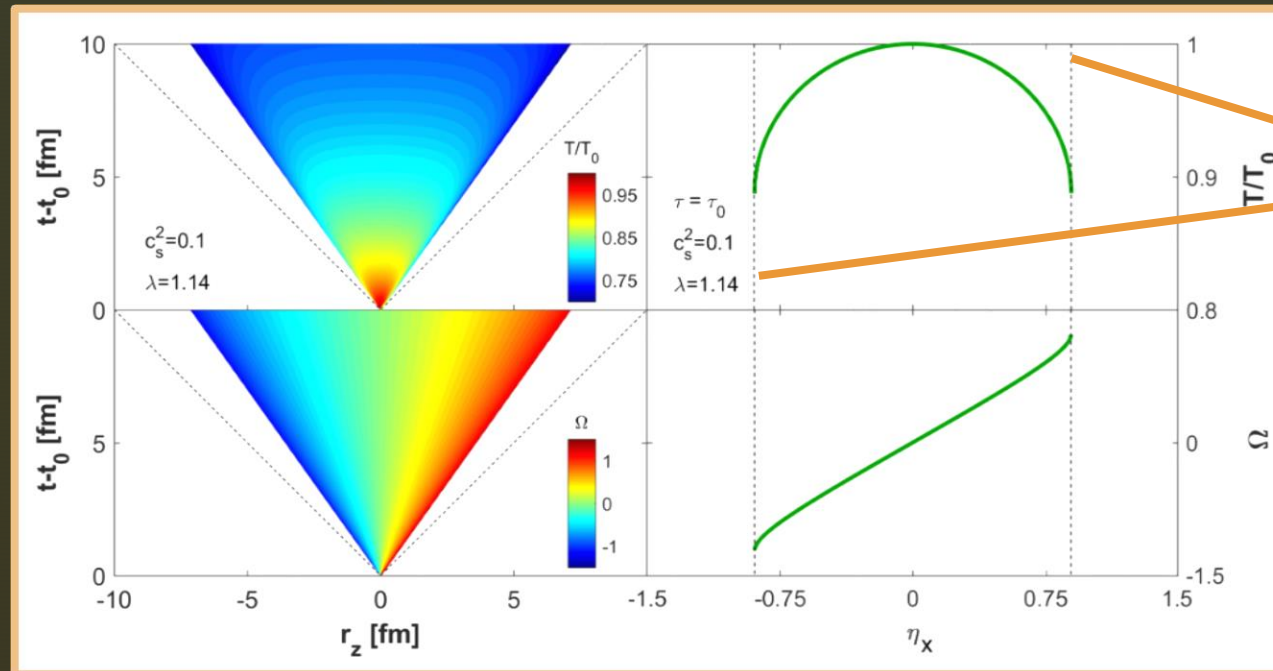
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- 1+1 dimensional, parametric solution (in $\kappa \rightarrow 1$ limit: CNC):



Finite solution,
should be extended
with shockwaves

λ : acceleration
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accelerating
solution

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Observables – Pseudorapidity density

- Rapidity density (embedded to 1+3 d space):

$$\frac{dn}{dy} \approx \left. \frac{dn}{dy} \right|_{y=0} \cosh^{-\frac{1}{2}\alpha(\kappa)-1} \left(\frac{y}{\alpha(1)} \right) \exp \left(-\frac{m}{T_{\text{eff}}} \left[\cosh^{\alpha(\kappa)} \left(\frac{y}{\alpha(1)} \right) - 1 \right] \right)$$

T_{eff} : effective temperature,

$$\alpha(\kappa) = \frac{2\lambda - \kappa}{\lambda - \kappa}$$

- Explicit relation between the average \bar{p}_T , T_{eff} and m :

$$\bar{p}_T(y) \approx \sqrt{T_{\text{eff}}^2 + 2mT_{\text{eff}}} \left(1 + \frac{\alpha(\kappa)}{2\alpha(1)^2} \frac{T_{\text{eff}} + m}{T_{\text{eff}} + 2m} y^2 \right)^{-1}$$

- Pseudorapidity density: parametric curve

$$\left(\eta_p(y), \frac{dn}{d\eta_p}(y) \right) = \left(\frac{1}{2} \log \left[\frac{\bar{p}(y) + \bar{p}_z(y)}{\bar{p}(y) - \bar{p}_z(y)} \right], \frac{\bar{p}(y)}{\bar{E}(y)} \frac{dn}{dy} \right)$$

- Four fit parameters: κ , λ , T_{eff} , $dn/dy|_{y=0}$

Observables – R_L

- For a 1+1 d relativistic source (LCMS):

$$R_L^2 = \cosh^2(\eta_x^s) \tau_f^2 \Delta\eta_x^2 + \sinh^2(\eta_x^s) \Delta\tau^2$$

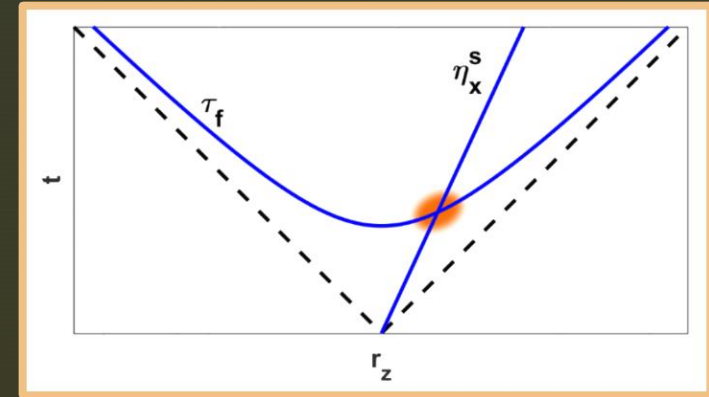
- (η_x^s, τ_f) : saddle point
- CKCJ solutions limited to a narrow rapidity interval around $\eta_x \approx 0$
- At midrapidity:

$$R_L = \tau_f \Delta\eta_x$$

with the new solution

$$R_L = \tau_f \Delta\eta_x \approx \frac{\tau_f}{\sqrt{\lambda(2\lambda - 1)}} \sqrt{\frac{T_f}{m_T}}$$

- $\Delta\eta_x$: gaussian width of rapidity distribution



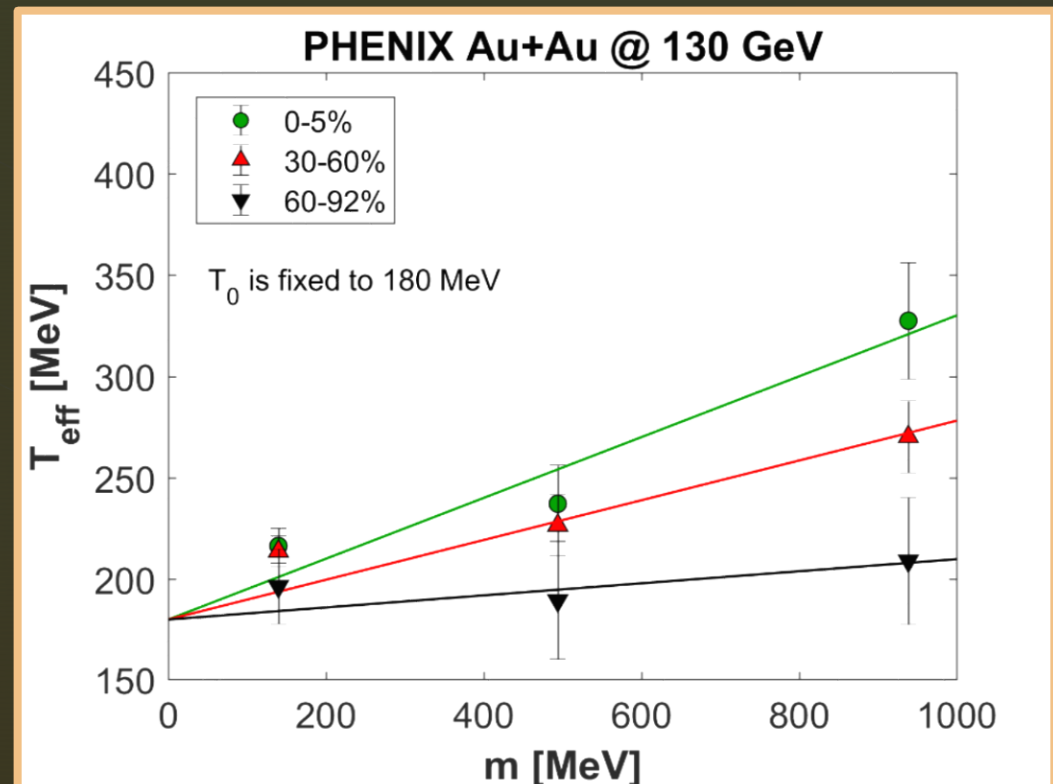
Inverse slope fits – Au+Au @ 130 GeV

- Effective temperature: need centrality dependence
- Linear model is fitted to m_0 vs T_{eff} PHENIX data
- T_f is fixed to 180 MeV

$$T_{eff}(m) = T_f + m \langle u_T \rangle^2$$

[arXiv:1801.05716](https://arxiv.org/abs/1801.05716)

[arXiv:1811.09990](https://arxiv.org/abs/1811.09990)



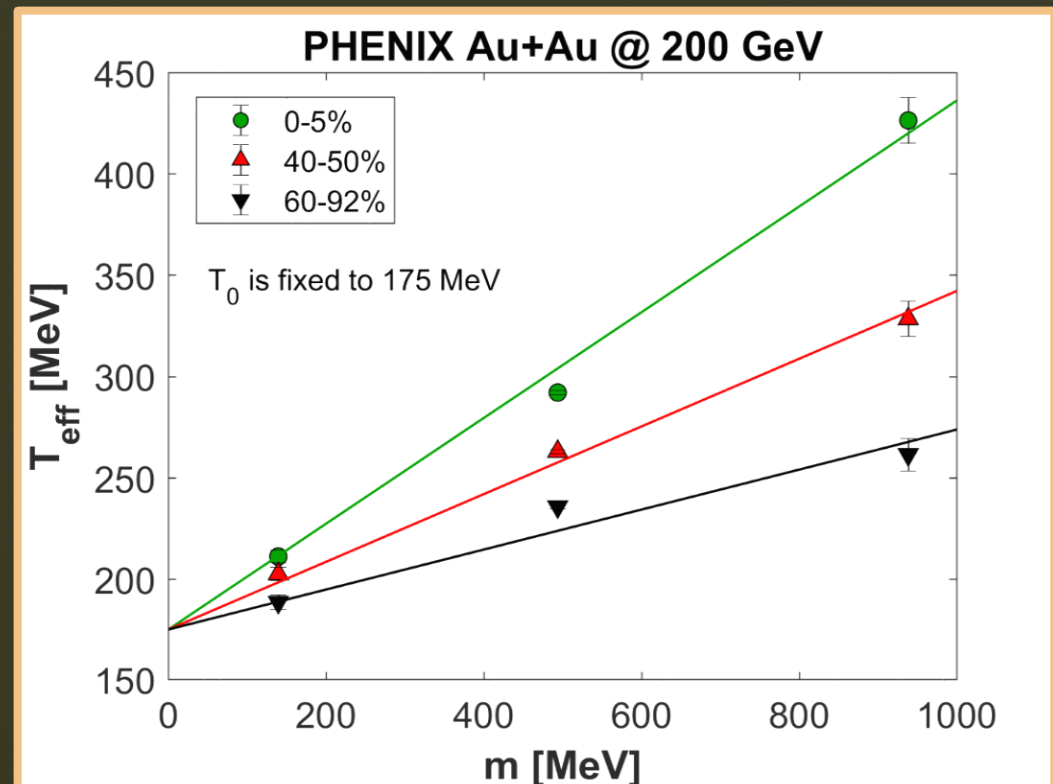
Inverse slope fits – Au+Au @ 200 GeV

- Effective temperature: need centrality dependence
- Linear model is fitted to m_0 vs T_{eff} PHENIX data
- T_f is fixed to 175 MeV

$$T_{eff}(m) = T_f + m \langle u_T \rangle^2$$

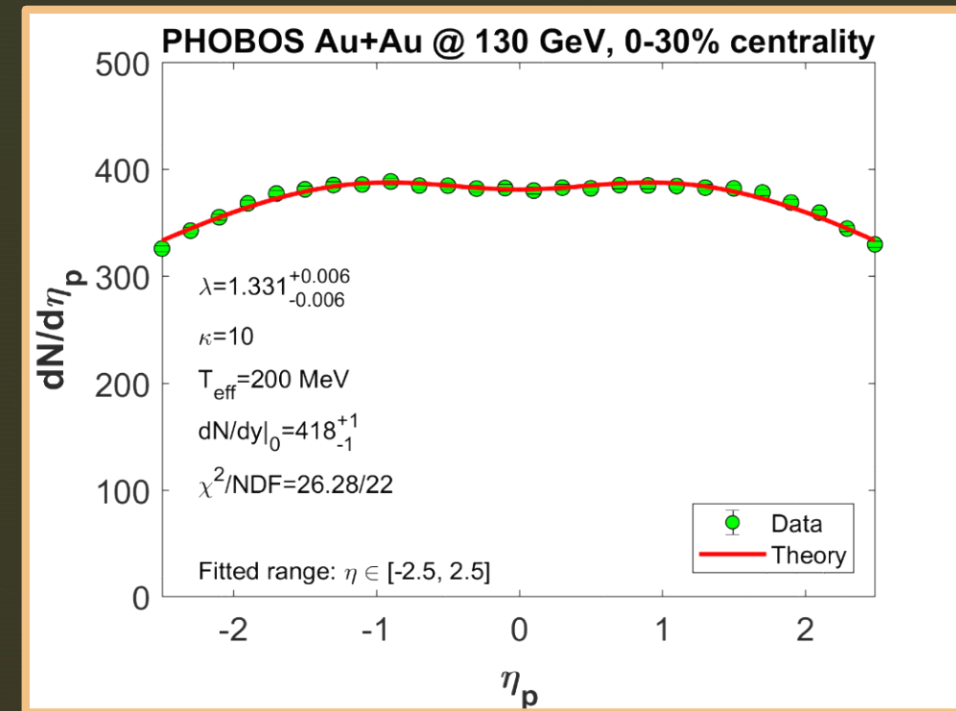
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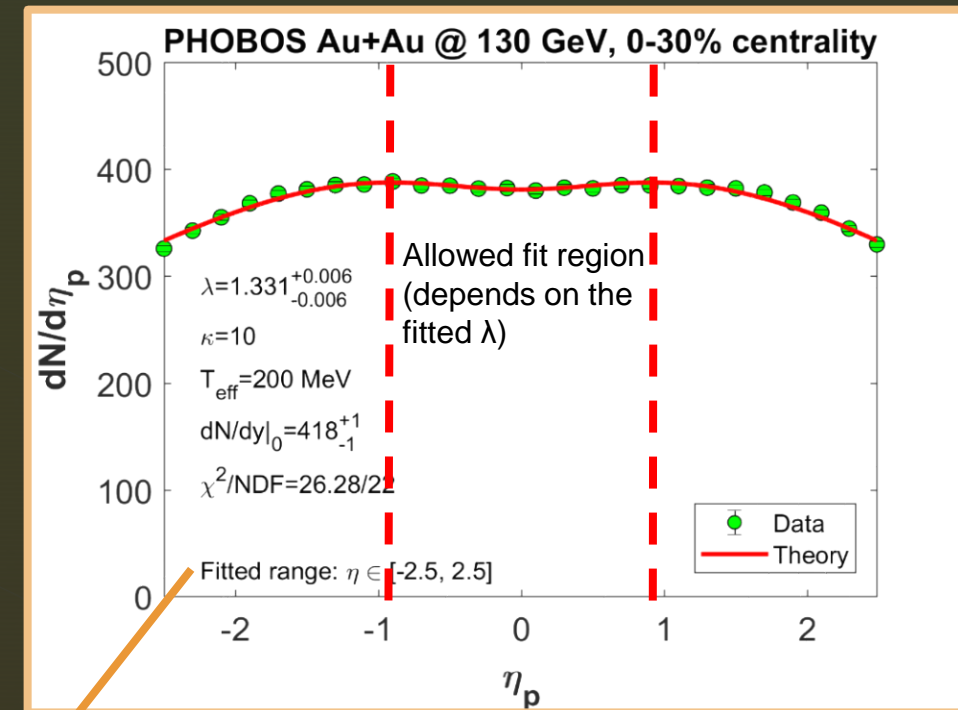
$dN/d\eta$ fits – Au+Au @ 130 GeV

- Fit the CKCJ model to $dN/d\eta$ PHOBOS data
- Fit range limited (because of the finiteness CKCJ model)
- c_s^2 is fixed to 0.1 ($\kappa=10$)
- R_L datas of 130 GeV Au+Au are published with 0-30% centrality
- No T_{eff} data with 0-30% centrality \rightarrow fit to $dN/d\eta$, result seems consistent with the centrality dependence



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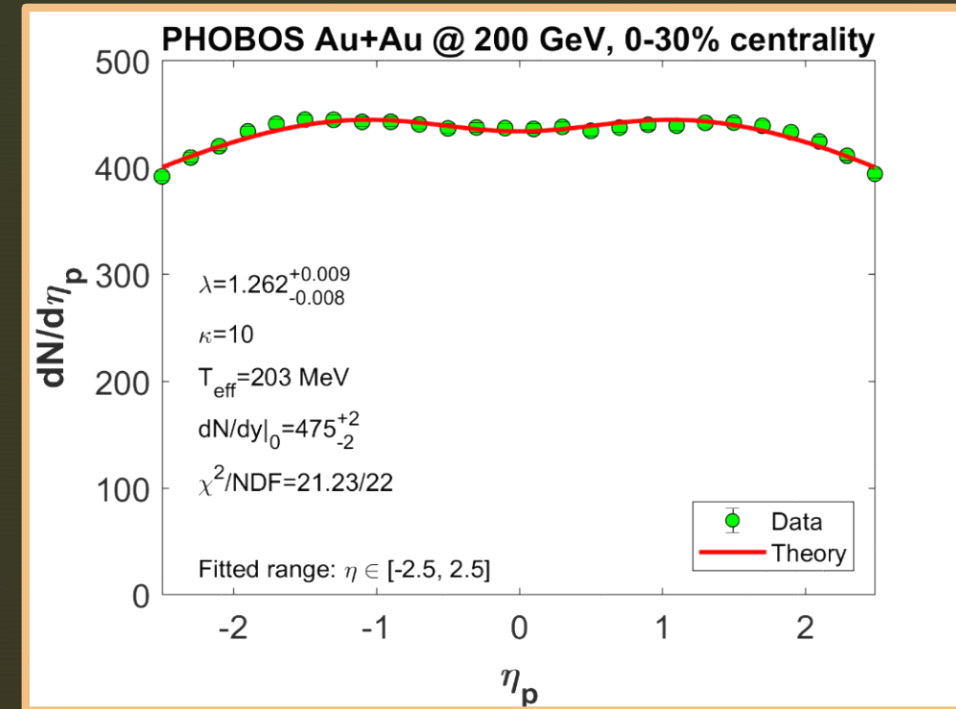
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Fit range is λ dependent, but λ determined by the fit \rightarrow iteration is needed (postponed)

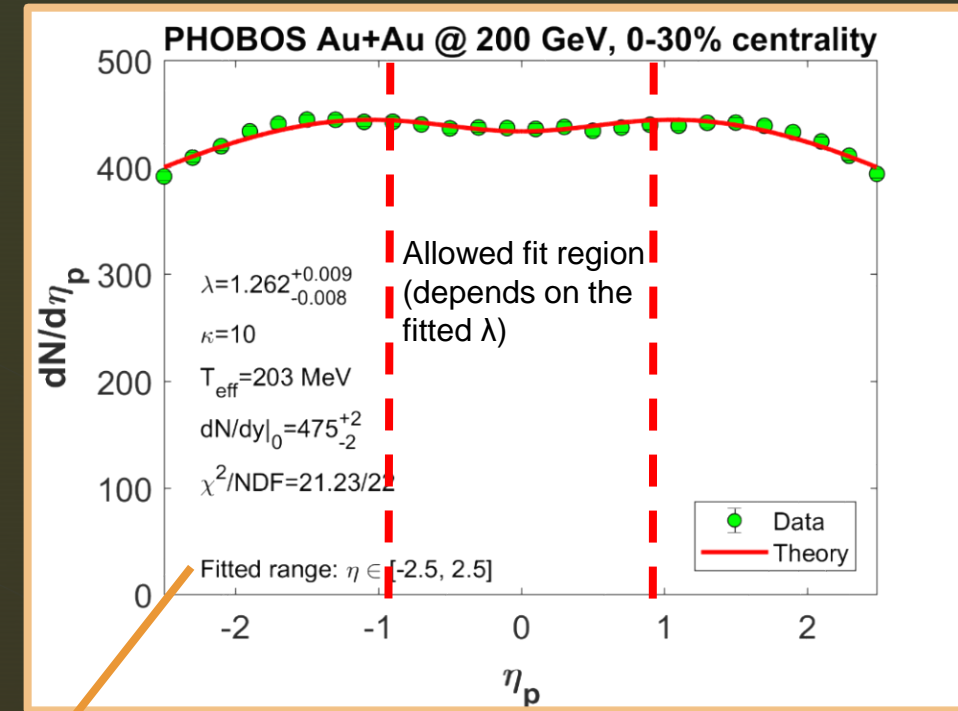
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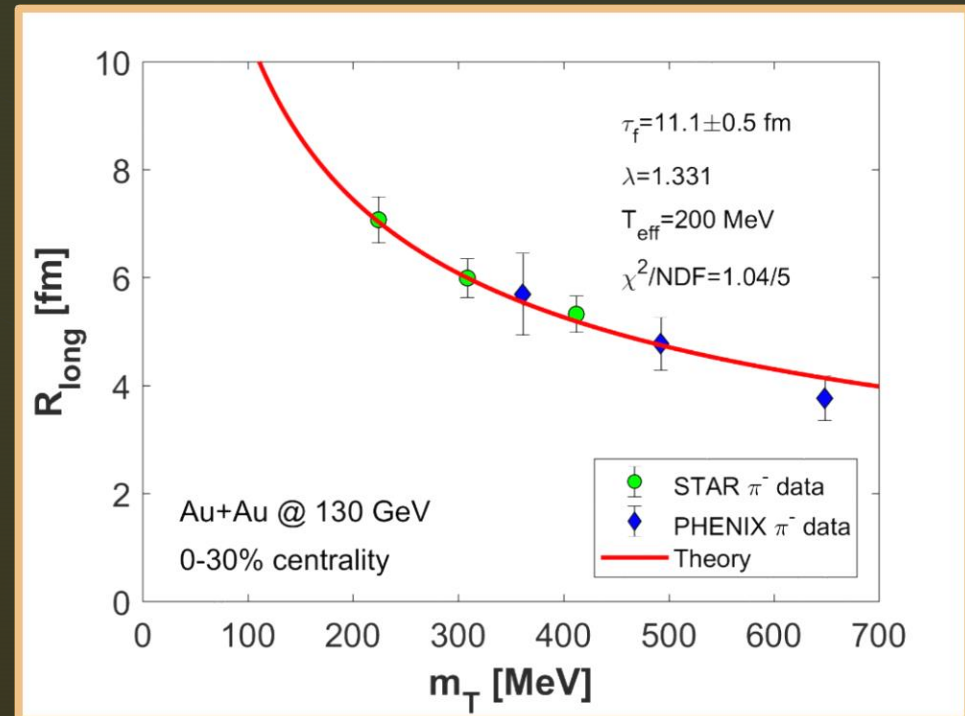
R_L fits – Au+Au @ 130 GeV

- The new formula is fitted to PHENIX and STAR R_L data
- From fits to $dN/d\eta$: $\lambda=1.33$, $T_{\text{eff}}=200$ MeV
- τ_f is fit parameter

[arXiv:1811.09990](https://arxiv.org/abs/1811.09990)

[arXiv:nucl-ex/0201008](https://arxiv.org/abs/nucl-ex/0201008)

$$R_L = \tau_f \Delta\eta_x \approx \frac{\tau_f}{\sqrt{\lambda(2\lambda - 1)}} \sqrt{\frac{T_f}{m_T}}$$



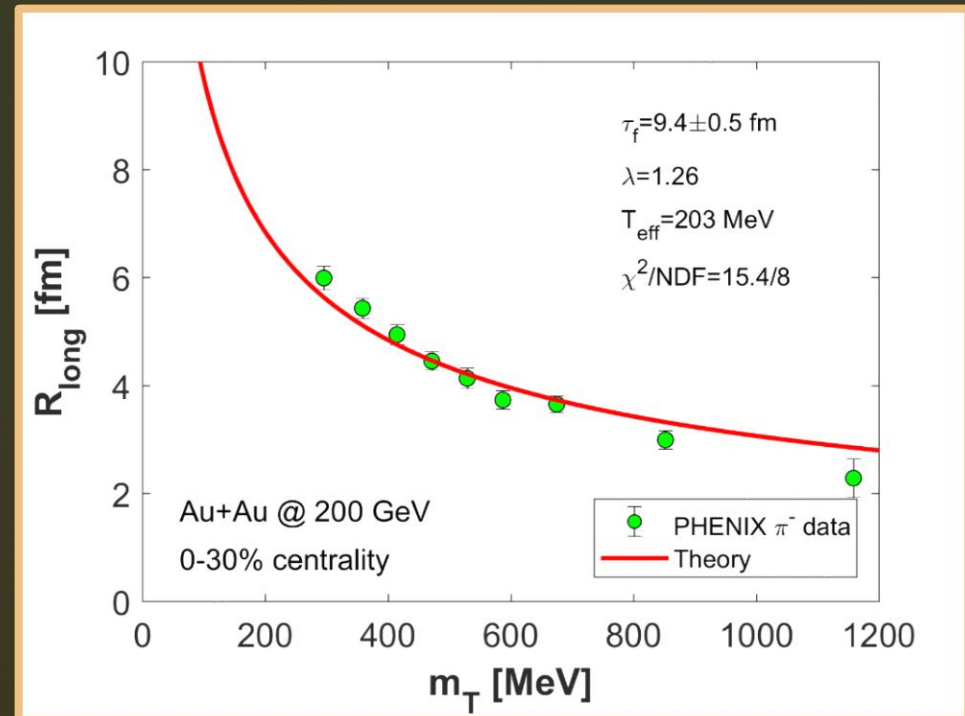
R_L fits – Au+Au @ 200 GeV

- The new formula is fitted to PHENIX R_L data
- From fits to $dN/d\eta$: $\lambda=1.26$, $T_{\text{eff}}=203$ MeV
- τ_f is fit parameter

[arXiv:1811.09990](https://arxiv.org/abs/1811.09990)

[arXiv:nucl-ex/0401003](https://arxiv.org/abs/nucl-ex/0401003)

$$R_L = \tau_f \Delta\eta_x \approx \frac{\tau_f}{\sqrt{\lambda(2\lambda - 1)}} \sqrt{\frac{T_f}{m_T}}$$



Exact result of initial energy density

- From the CKCJ model the ε_0 is exactly calculated:

$$\varepsilon_0(\kappa, \lambda) = \varepsilon_0^{Bj} (2\lambda - 1) \left(\frac{\tau_f}{\tau_0} \right)^{\lambda(1 + \frac{1}{\kappa}) - 1}$$

- $\lambda \rightarrow 1$ limit: EoS dependence is not vanishing
- Bjorken's formula can be reobtained only in the case of dust (thus we find the fundamental correction to Bjorken's initial energy estimate)
- The new, EoS dependent correction takes into account the work of the pressure
- λ and τ_f are obtained by $dN/d\eta$ and R_L fits $\rightarrow \tau_0$ dependence

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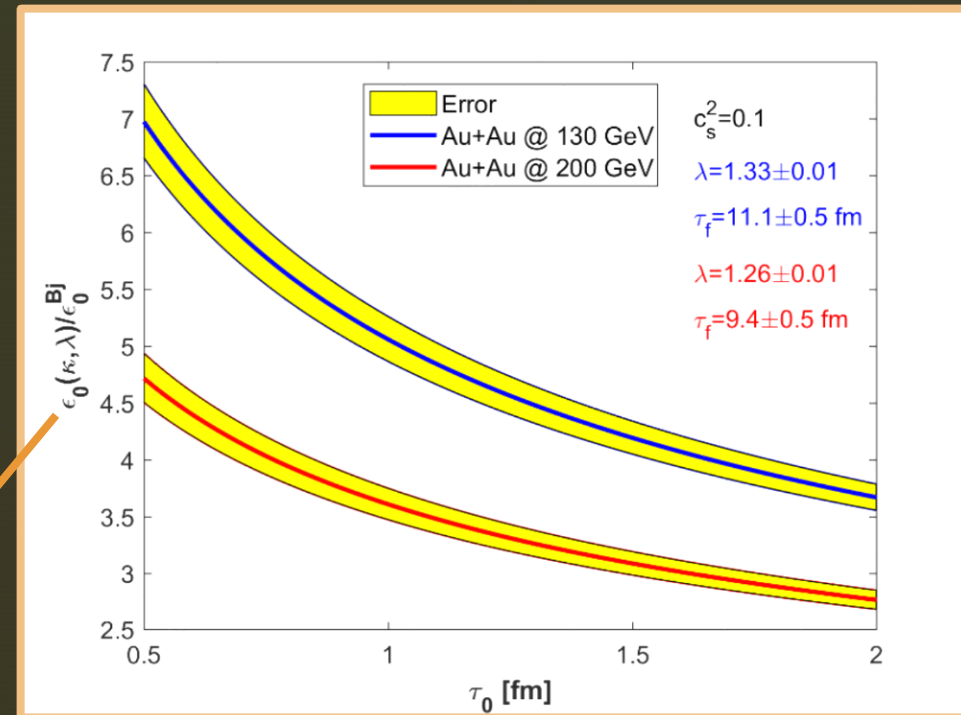
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Exact result of initial energy density

- Indication for an increasing initial energy density with decreasing colliding energy?
- Suggest a non-monotonic behaviour of the initial energy density with \sqrt{s} .
- Need to extend the studies to lower energies (RHIC BES I, BES II, SPS and LHC data)
- Need to extend the CKCJ solution to 1+3 dimensions
- Need to investigate the fit range iteration

the corrections to Bjorken's initial energy estimate



Summary

- New, exact solutions of hydrodynamics: CKCJ model
- Initial energy density is calculated exactly \rightarrow Bjorken's estimate ignores the first law of thermodynamics (except for dust, $p=0$)
- Fit to PHOBOS $dN/d\eta$ data: obtain λ
- Fit to PHENIX and STAR R_L data: obtain τ_f
- τ_0 dependence of ε_0 can be calculated \rightarrow surprising result
- Further investigations are needed: lower energies, 1+3 dimensional extension of CKCJ model, fit range iteration

Summary

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Thank you for your attention!

Exact result of initial energy density

- At both collision energies the correction is significant
- Only the thermalized energy is included
- Greater energy density on lower collision energy??

