Chats 2019
Applied Superconductivity

ELÚ

A-formulation method for full 3D FEM computation of the superconductor magnetization

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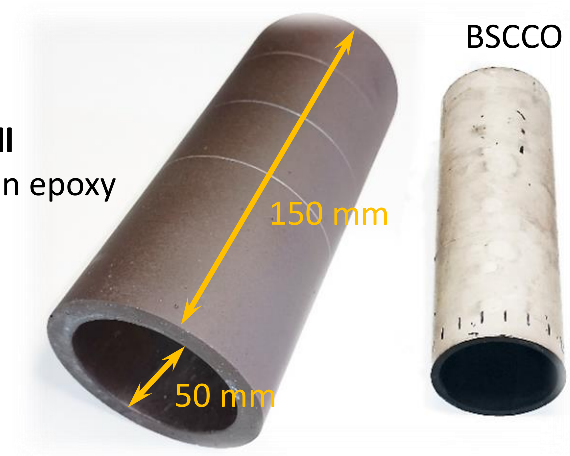
Motivation

Magnetic cloak:

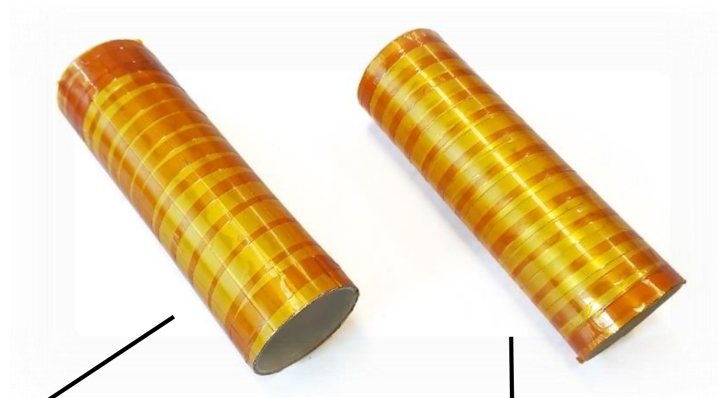
SC inserts

FM shell
Ferrite in epoxy

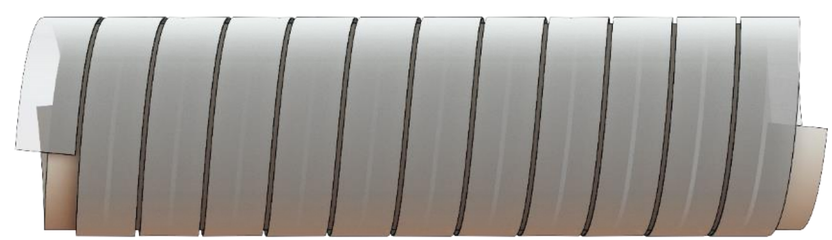
BSCCO



ReBCO



Straight tapes



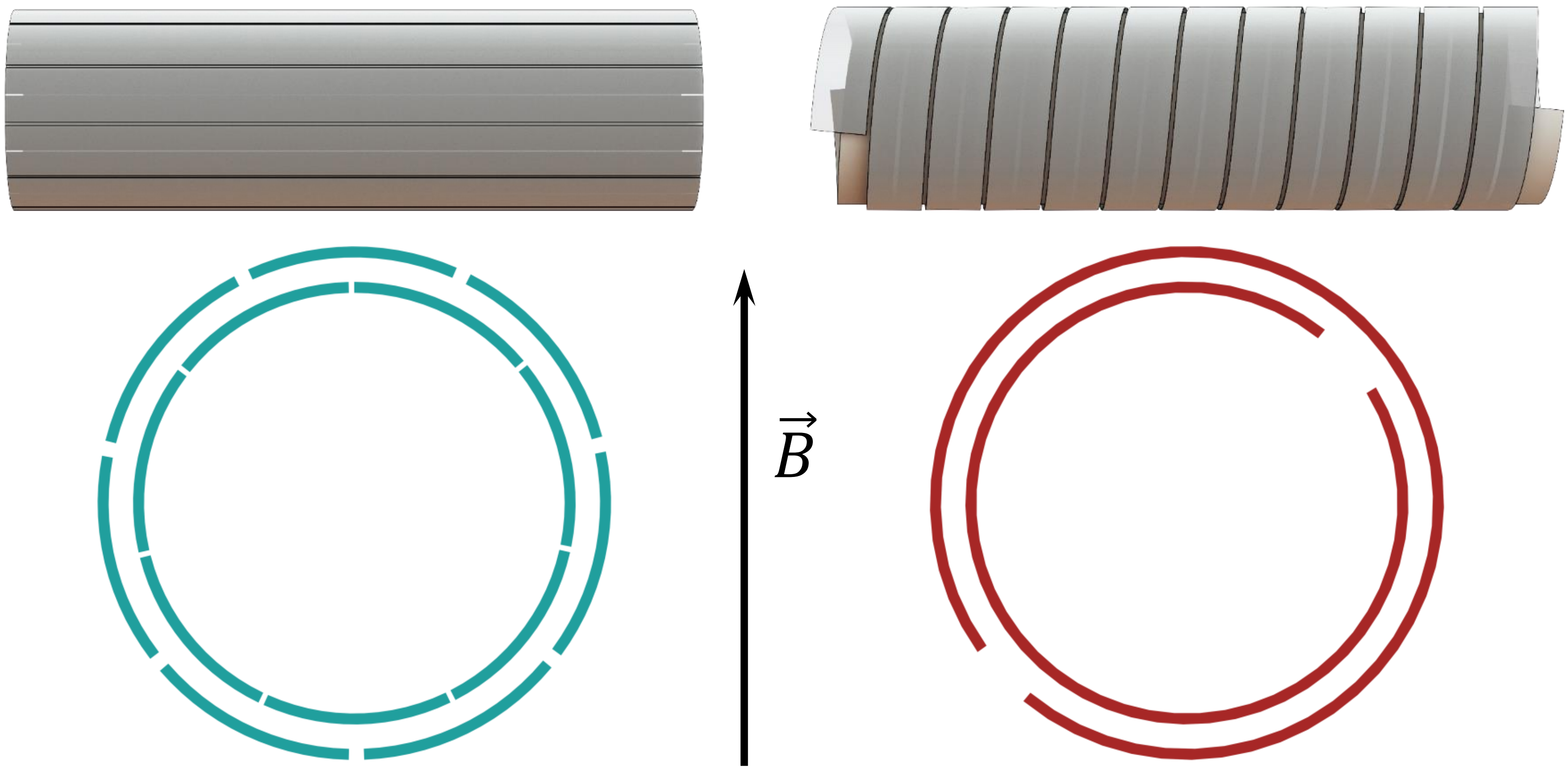
Helicoidally wound tapes

Motivation

How to build a numerical model?

2d model works well

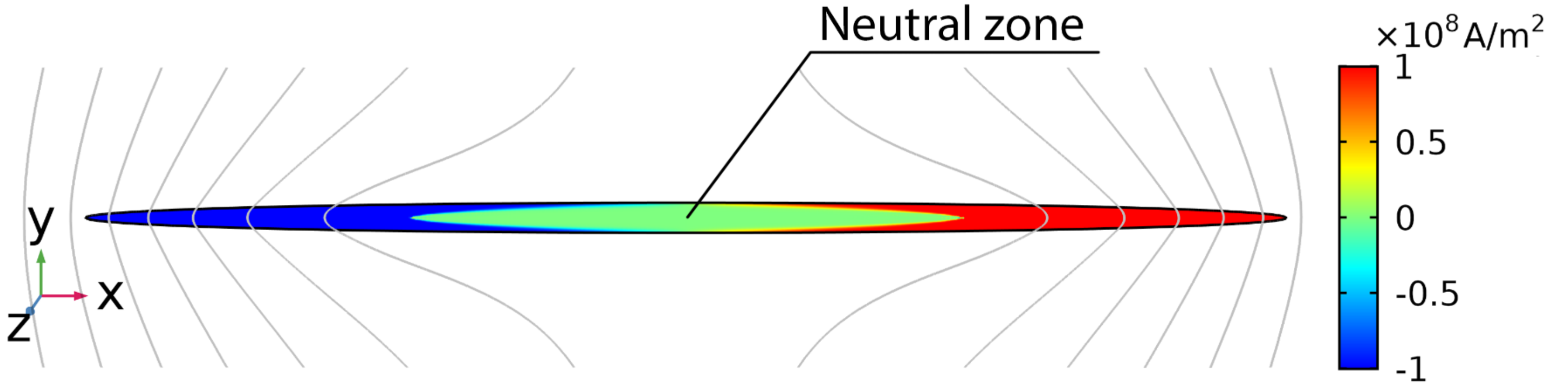
Full 3d modelling is necessary



Numerical modelling. From 2d to 3d.

A-formulation, time domain

2D: the neutral zone is beneficial for the correct solution[1]



1. Gömöry, F., Vojenčiak, M., Pardo, E., Solovyov, M. & Šouc, J. AC losses in coated conductors. *Superconductor Science & Technology* **23**, (2010).

A-formulation, time domain

Maxwell equation says

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla\varphi \qquad (1)$$

In the cross-section of superconducting wire

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} \qquad (2)$$

Power law definition

$$\vec{E} = E_c\left(\frac{\vec{J}}{J_{c0}}\right)^n \qquad (3)$$

Bean model

$$\vec{J} = \begin{cases} +J_{c0}, & for \quad {}^{\partial A}/_{\partial t} < 0 \\ -J_{c0}, & for \quad {}^{\partial A}/_{\partial t} > 0 \\ 0, & for \quad {}^{\partial A}/_{\partial t} = 0 \end{cases} \qquad (4)$$

$$\vec{J} = J_{c0}\sqrt[n]{-\frac{\partial \vec{A}}{\partial t E_c}} \qquad (5)$$

$$\vec{J} = J_{c0} sign\left(-\frac{\partial \vec{A}}{\partial t E_c}\right) \qquad (6)$$

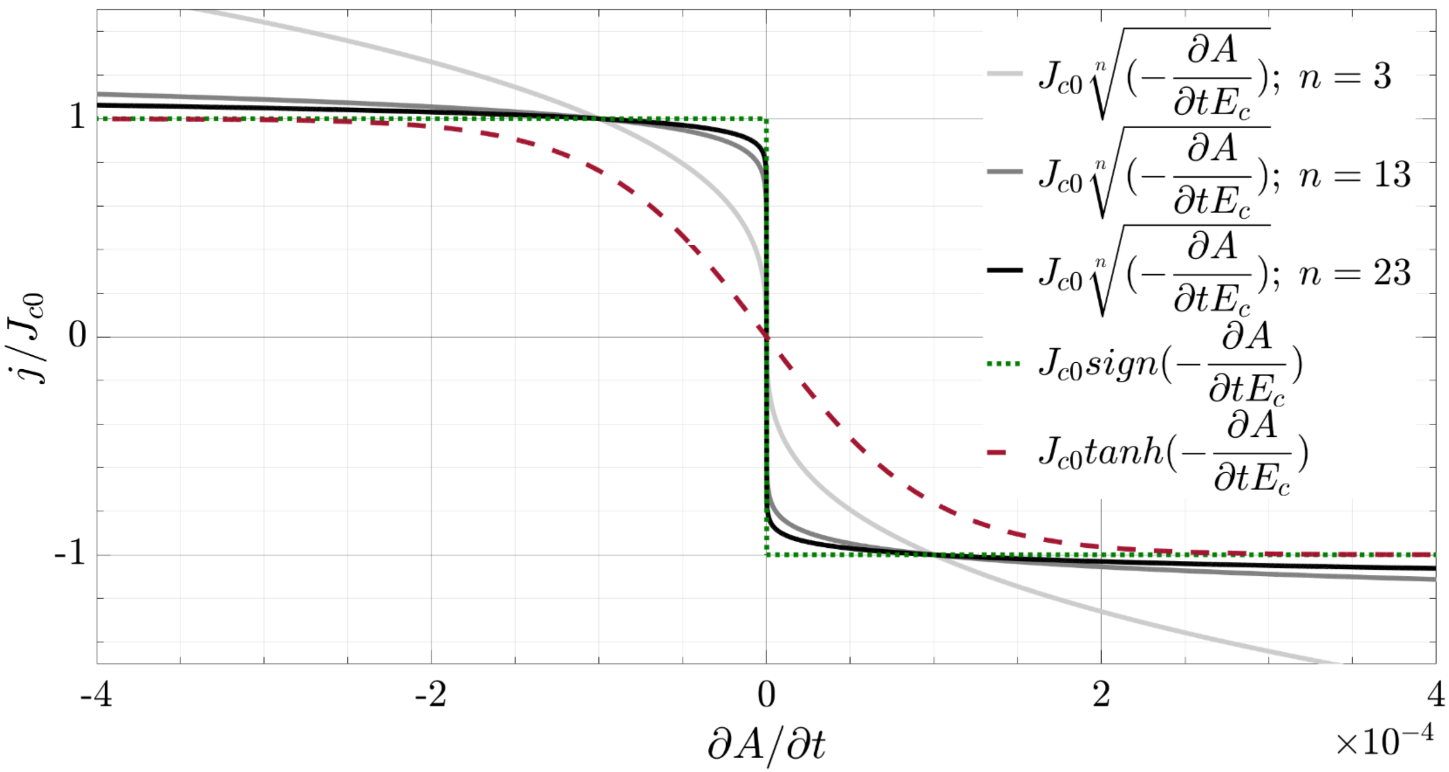
A-formulation, time domain

$$\vec{j} = J_{c0}\sqrt[n]{-\frac{\partial \vec{A}}{\partial t E_c}}$$

$$\vec{j} = J_{c0} sign\left(-\frac{\partial \vec{A}}{\partial t E_c}\right)$$

The alternative function proposed by A. Campbell[2]

$$\boldsymbol{\vec{j} = J_{c0} tanh\left(-\frac{\partial \vec{A}}{\partial t E_c}\right)} \quad \textbf{(7)}$$



2. Campbell, A. M. A new method of determining the critical state in superconductors. *Superconductor Science & Technology* **20**, 292–295 (2007).

$$\vec{j} = J_{c0} tanh\left(\frac{\vec{E}}{E_c}\right) \qquad (7)$$

In 3D problem is assumed that the relation (7) is valid for each component of the vectors $\vec{j}$ and $\vec{E}$ $(\vec{A})$ separately.

Correct solution requires also collinearity between $\vec{j}$ and $\vec{E}$ $(\vec{A})$

$$E_x : E_y : E_z = j_x : j_y : j_z \qquad (8)$$

And, if model assumes the isotropic J_c

$$\sqrt{j_x^2 + j_y^2 + j_z^2} \le J_{c0} \qquad (9)$$

After combining eq. (7), (8) and (9) the final expression for current density is:

$$\vec{J} = \frac{J_{c0}}{|E|}\left(|E_x| tanh\left(\frac{E_x}{E_c}\right)\hat{\imath} + |E_y| tanh\left(\frac{E_y}{E_c}\right)\hat{\jmath} + |E_z| tanh\left(\frac{E_z}{E_c}\right)\hat{k}\right) \qquad (10)$$

Model verification

Numerical verification

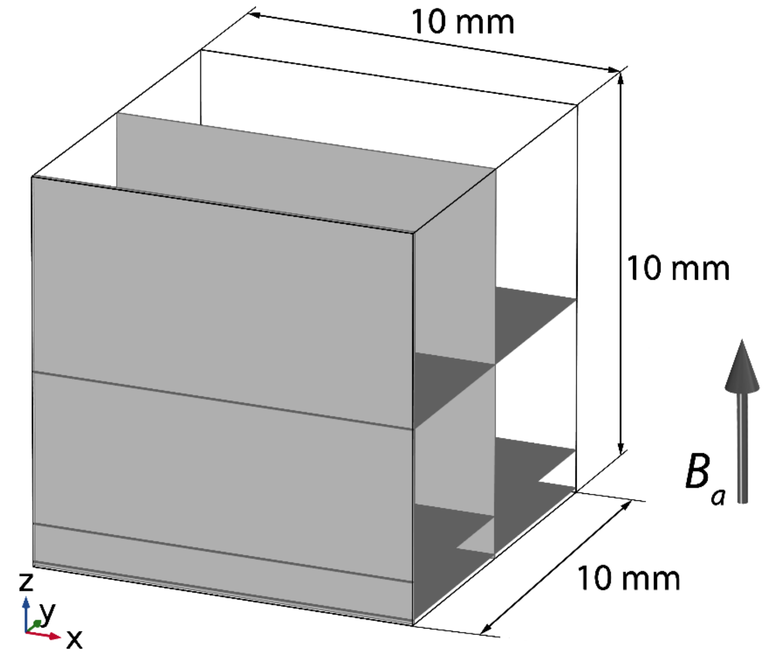
www.htsmodelling.com

Cubic bulk as benchmark for 3D modelling of superconductors under slowly varying magnetic fields

M. Kapolka, E. Pardo, V. M. R. Zermeño, F. Grilli, A. Morandi, P. L. Ribani

$J_{c0} = 10^8 \mathrm{A/m^2}$

B_a = 200 mT / 50 Hz

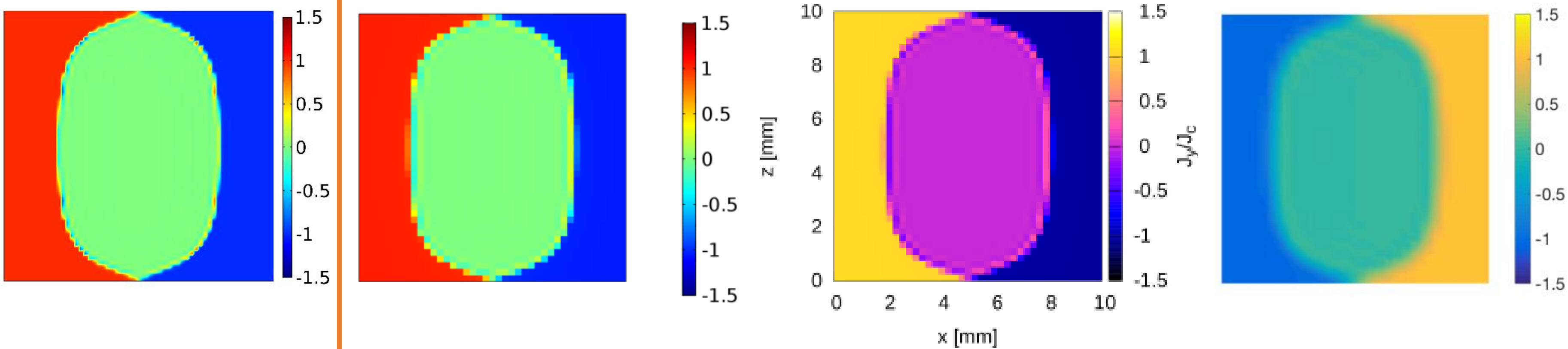


- cube was placed into the sphere with 60 mm of diameter (free tetrahedral mesh);
- cubic regular mesh 216 000 domain elements (60x60x60) for the superconductor;
- linear mesh discretization gave **1 357 645** DOF;
- **64** hours (1¼ T) on PC on Intel® Core™ i7-7740X CPU @ 4.3 GHz (8 cores) and 64GB RAM;
- Comsol Multiphysics© version 5.4 running under Windows 10 64-bit.

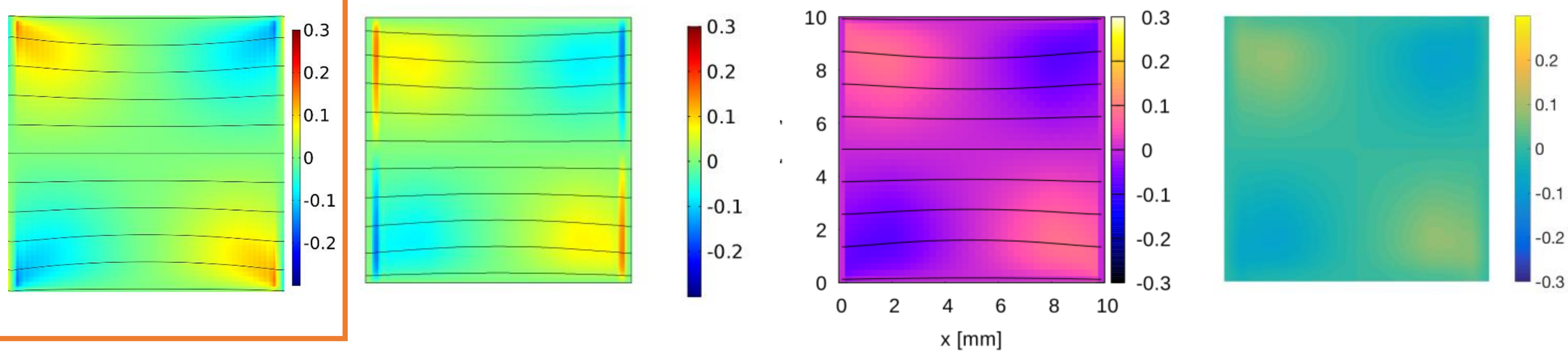
Numerical verification

A-formulation | **H-formulation** | **Minimum Electro-Magnetic Entropy Production (MEMEP)** | **Volume Integral Method (VIM)**

J_y component at plane y = 5 mm



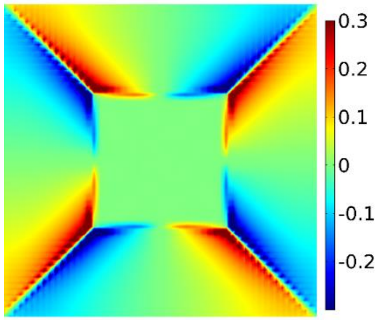
J_z component and current flux lines at plane y = 0.12 mm

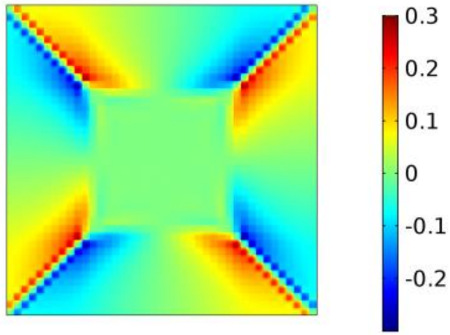


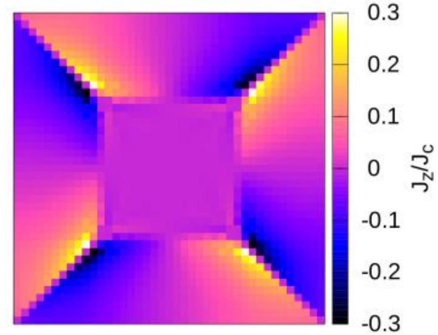
Numerical verification

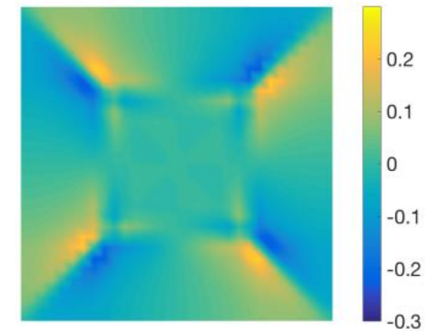
A-formulation	H-formulation	Minimum Electro-Magnetic Entropy Production (MEMEP)	Volume Integral Method (VIM)

J_z component at plane y = 1.1 mm

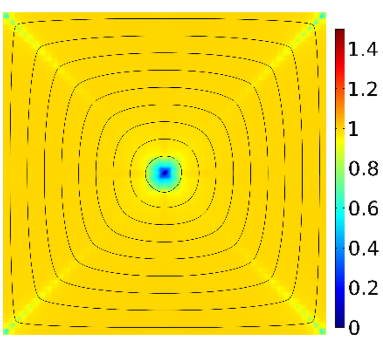


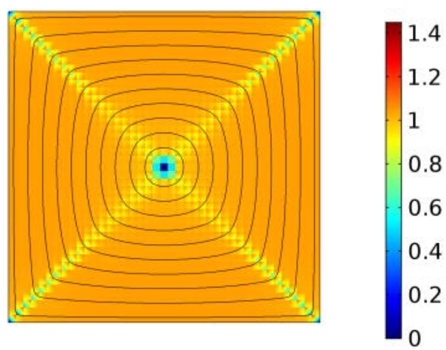


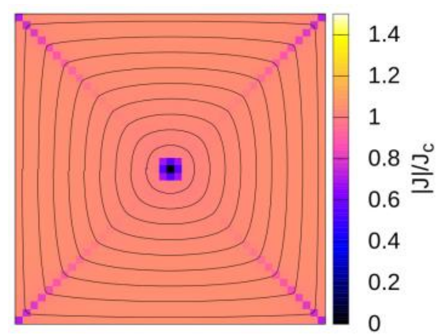


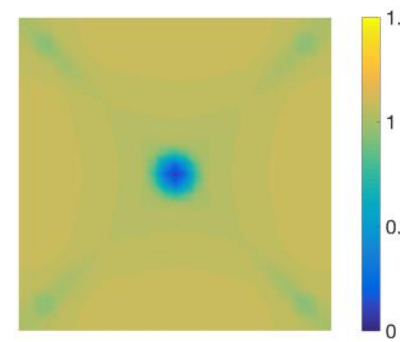


modulus of current density $|J|$ with current flux lines at plane z = 0.12 mm









Numerical verification

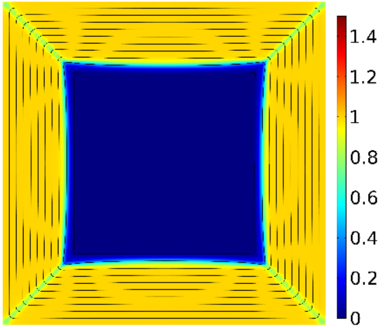
A-formulation

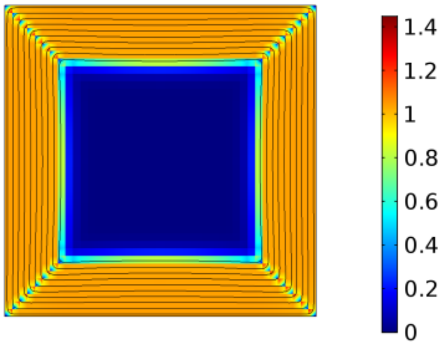
H-formulation

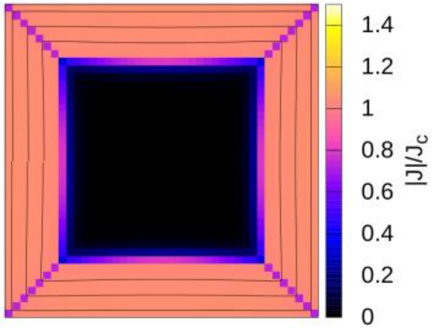
Minimum Electro-Magnetic Entropy Production (MEMEP)

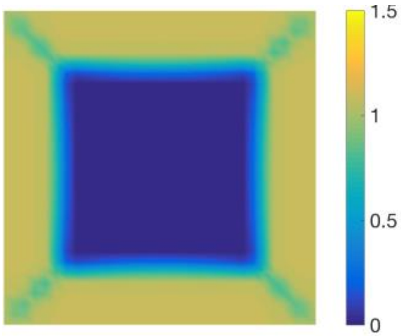
Volume Integral Method (VIM)

modulus of current density $|J|$ with current flux lines at plane z = 5 mm

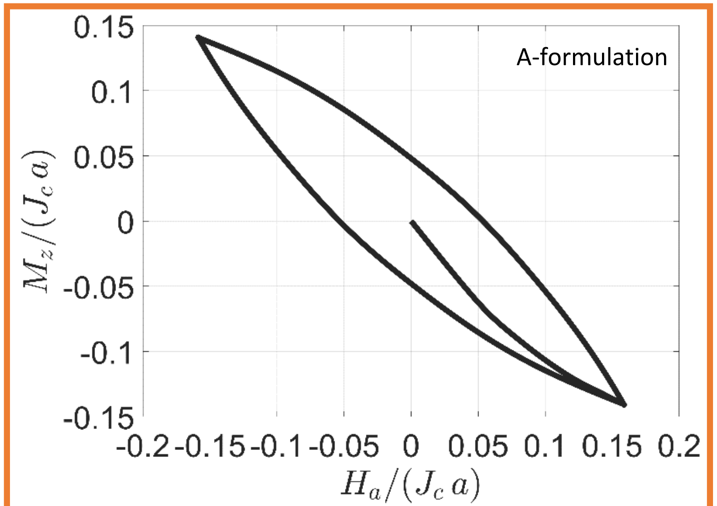


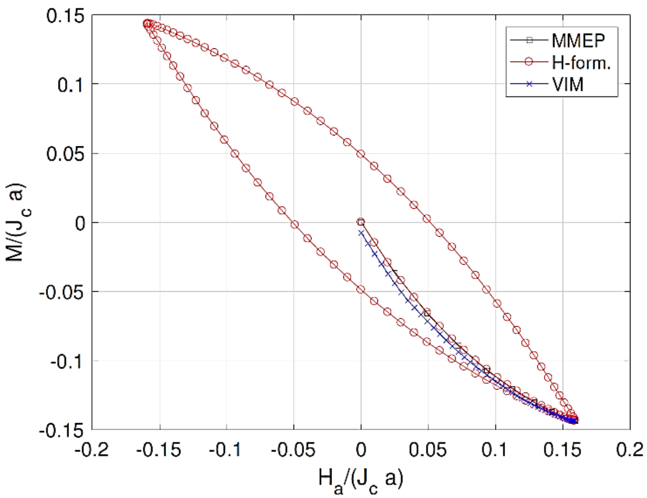




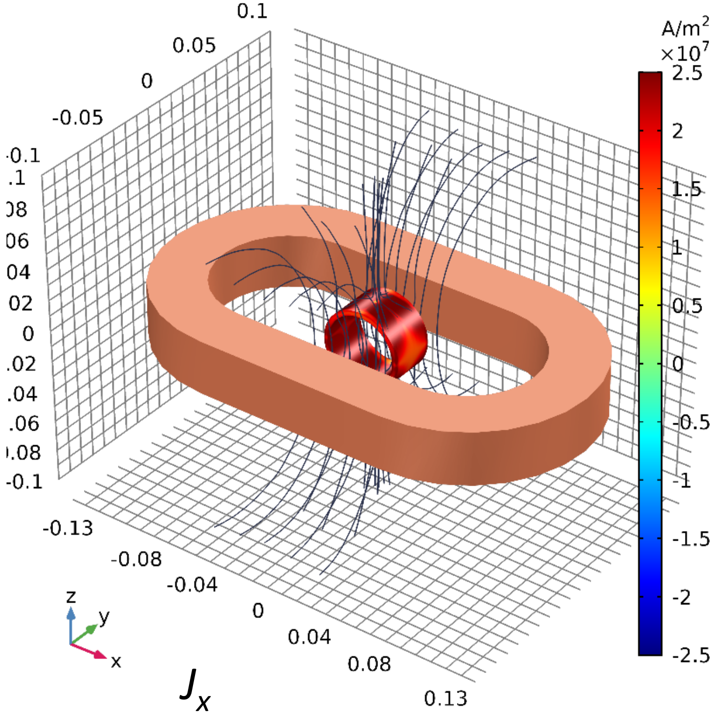


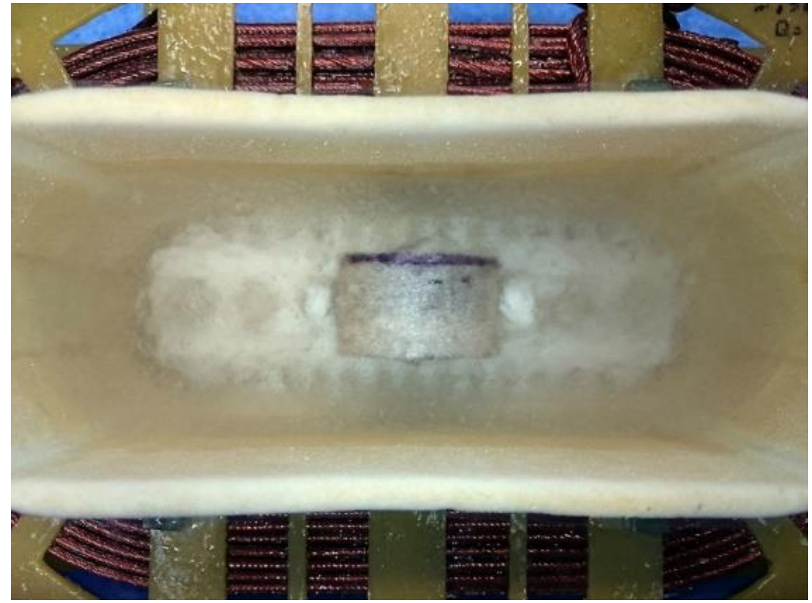
magnetization loop of the cube



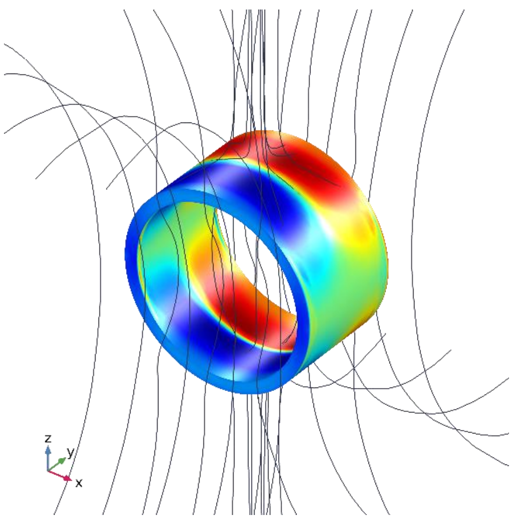


Experimental verification, BSCCO ring

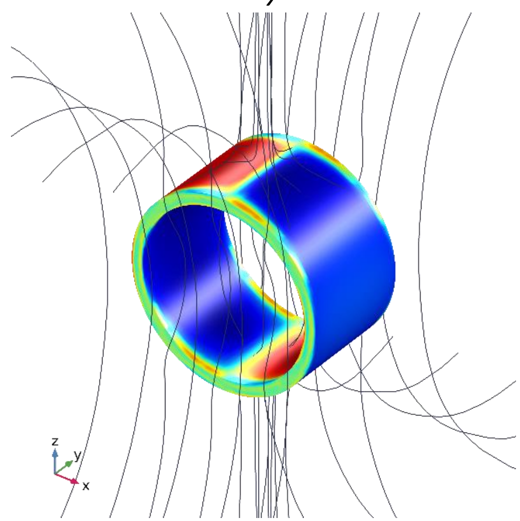




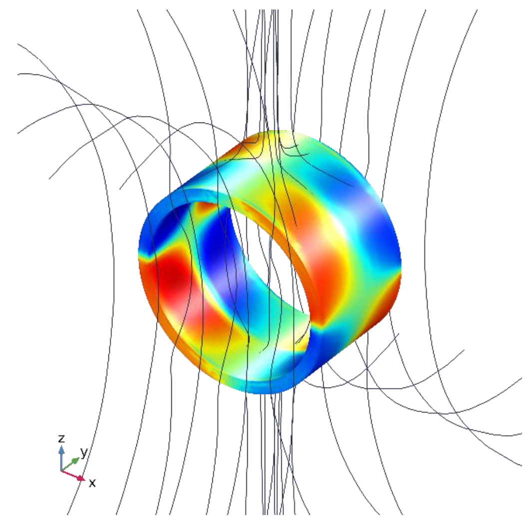
J_x



J_y

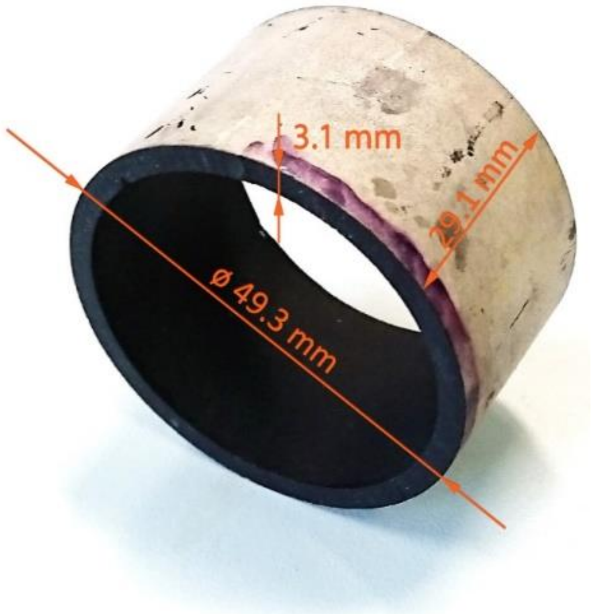


J_z



Experimental verification, BSCCO ring

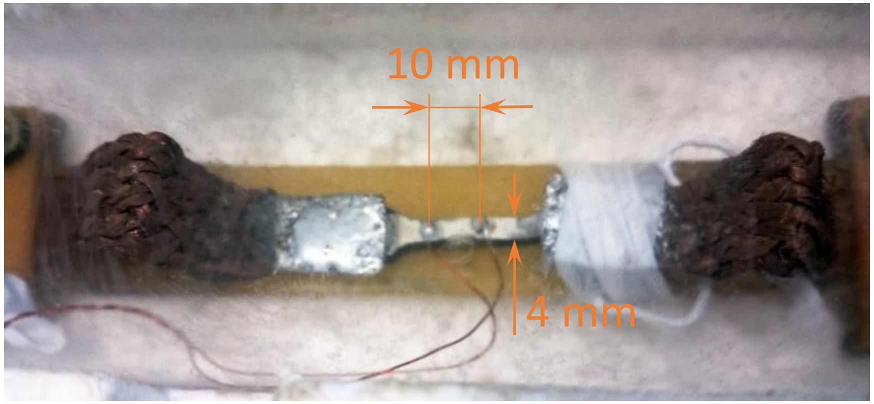
Melt-cast processed $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Nexans production for FCL program[3])

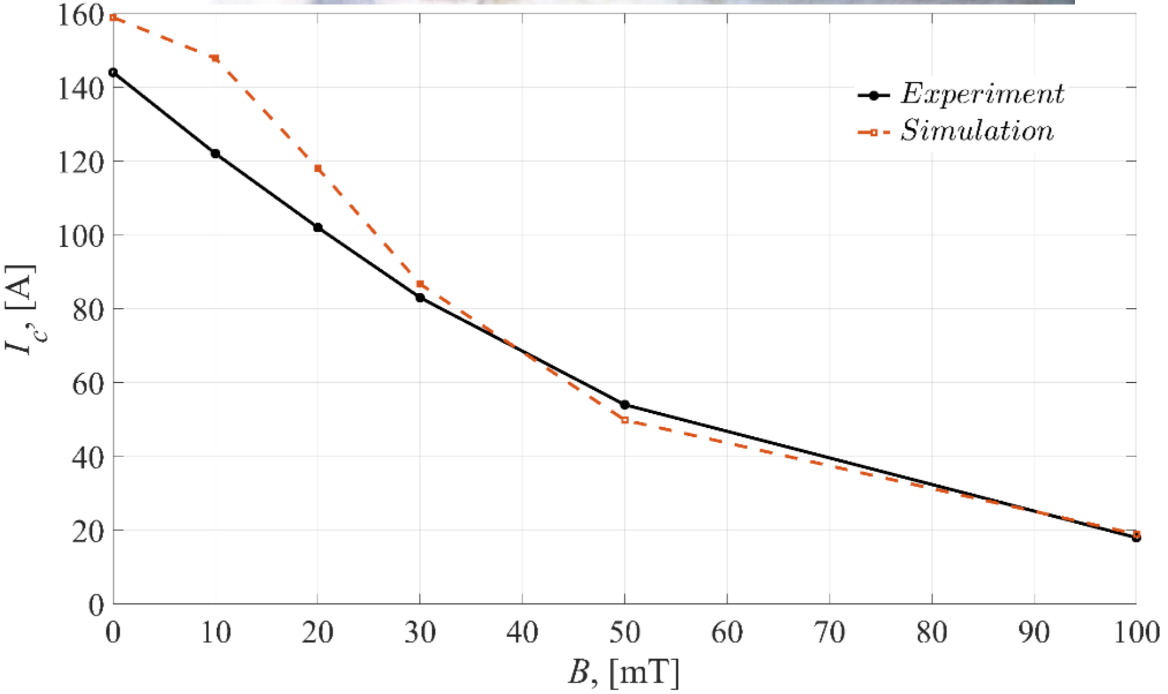


$$J_c(B) = \frac{J_{c0}}{\left(1 + {|B|}/{B_0}\right)^{\alpha}} \quad (11)$$

Selected parameters:
J_{c0} = 4.8 x 10^7 A/m^2
B_0 = 30 mT
α = 2

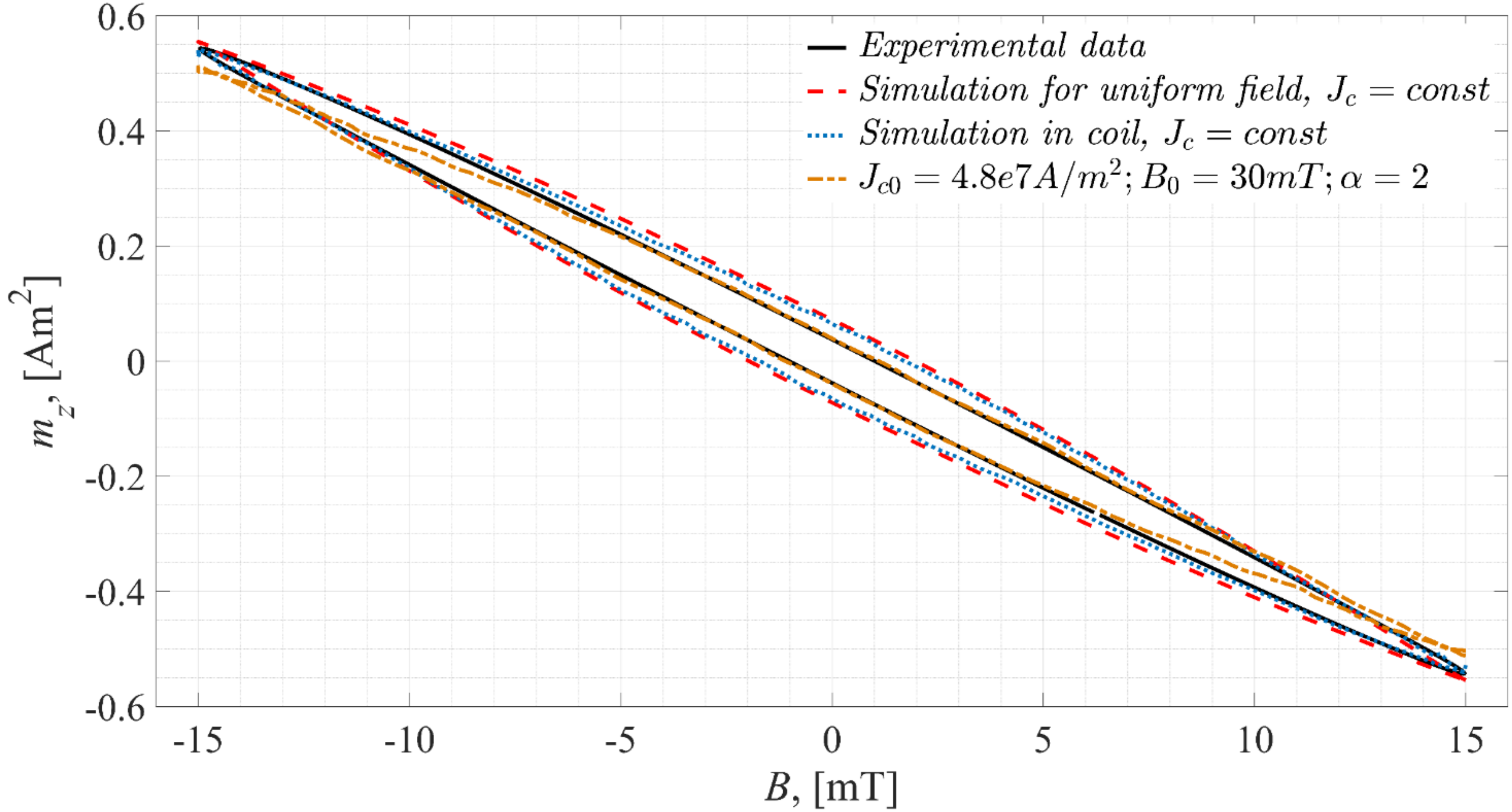
Electric measurements were performed on the small piece, cut from the tube along the main axis:





3. Bock, J. *et al.* HTS Fault Current Limiters-First Commercial Devices for Distribution Level Grids in Europe. *IEEE Trans. Appl. Supercond.* **21**, 1202–1205 (2011).

Experimental verification, BSCCO ring

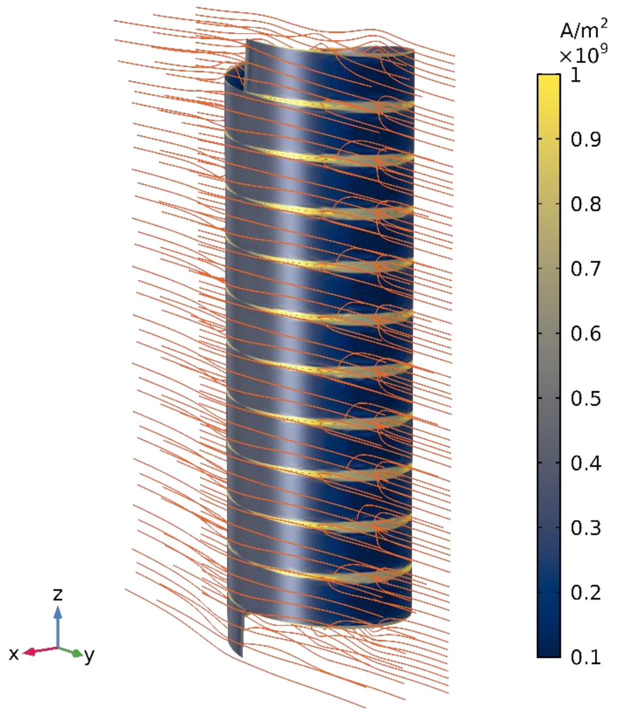


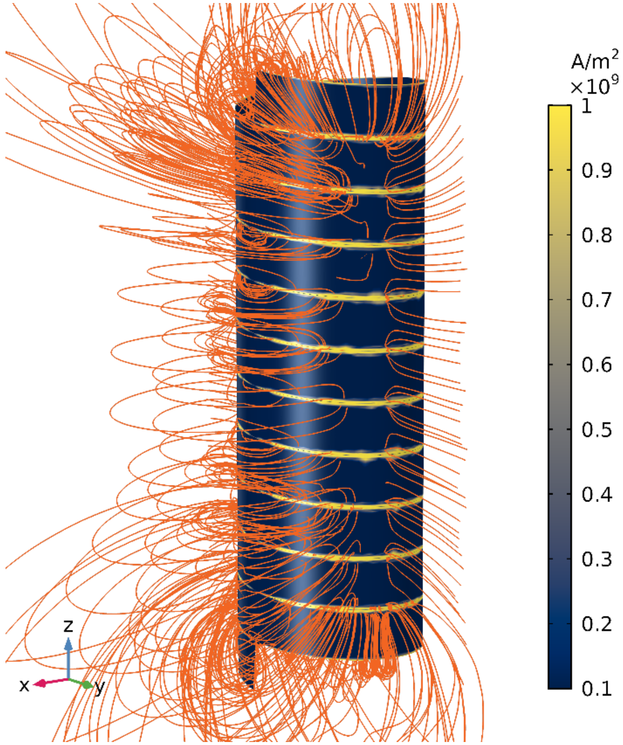
Magnetization loop measured in "calibration-free" system[4], compared with the computed ones

4. Šouc, J., Gömöry, F. & Vojenčiak, M. Calibration free method for measurement of the AC magnetization loss. Superconductor Science & Technology 18, 592–595 (2005).

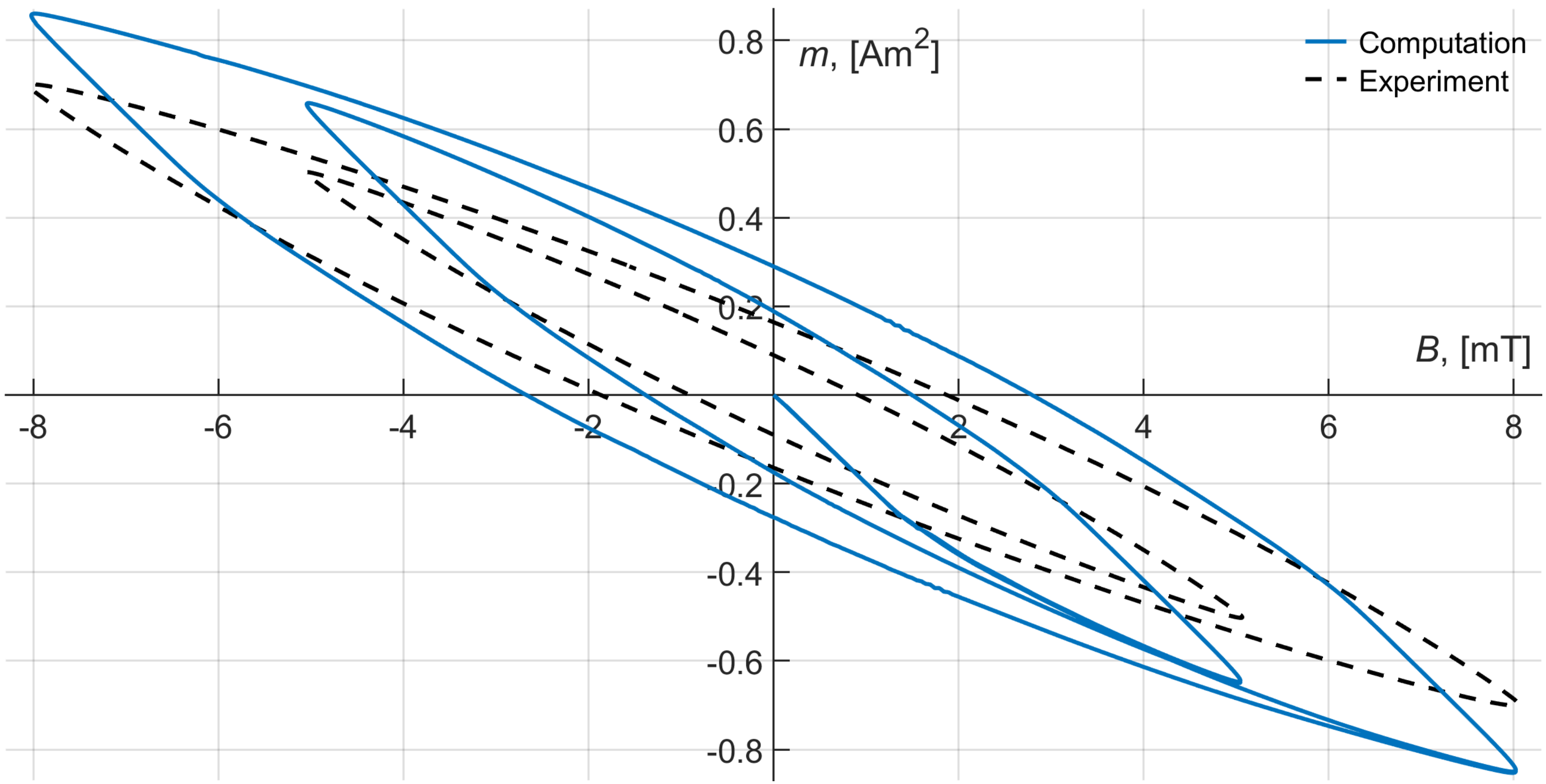
Experimental verification, helicoidally wound ReBCO tape

The SF12050AP tape on tubular former
- declared minimal critical current is 387 A
- former diameter - 45.1 mm
- former length - 145 mm





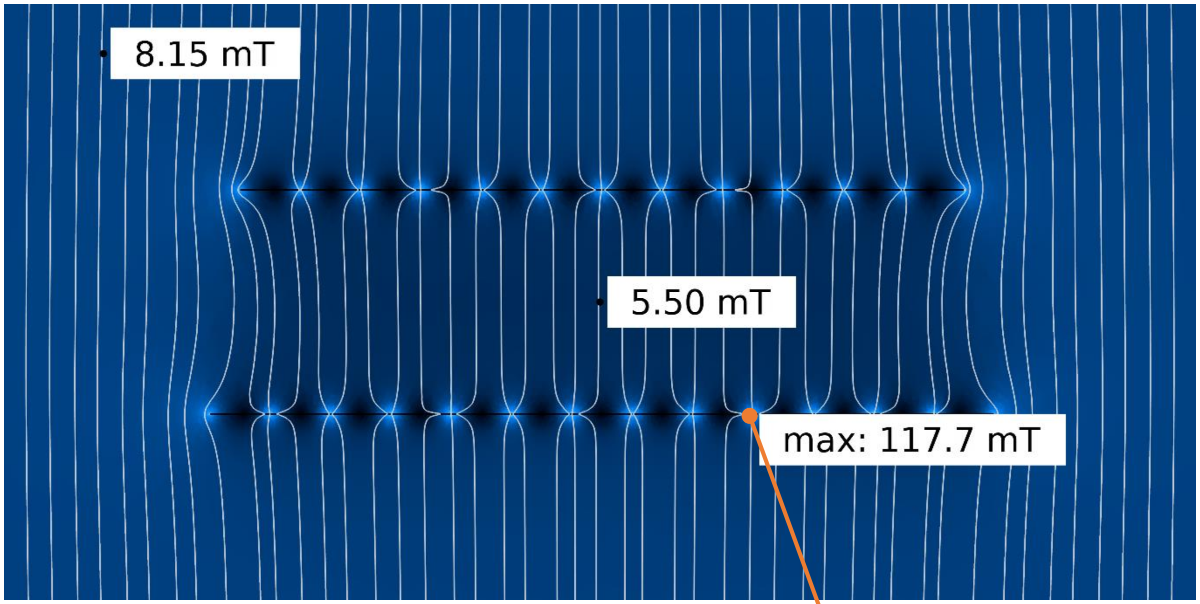
Experimental verification, helicoidally wound ReBCO tape



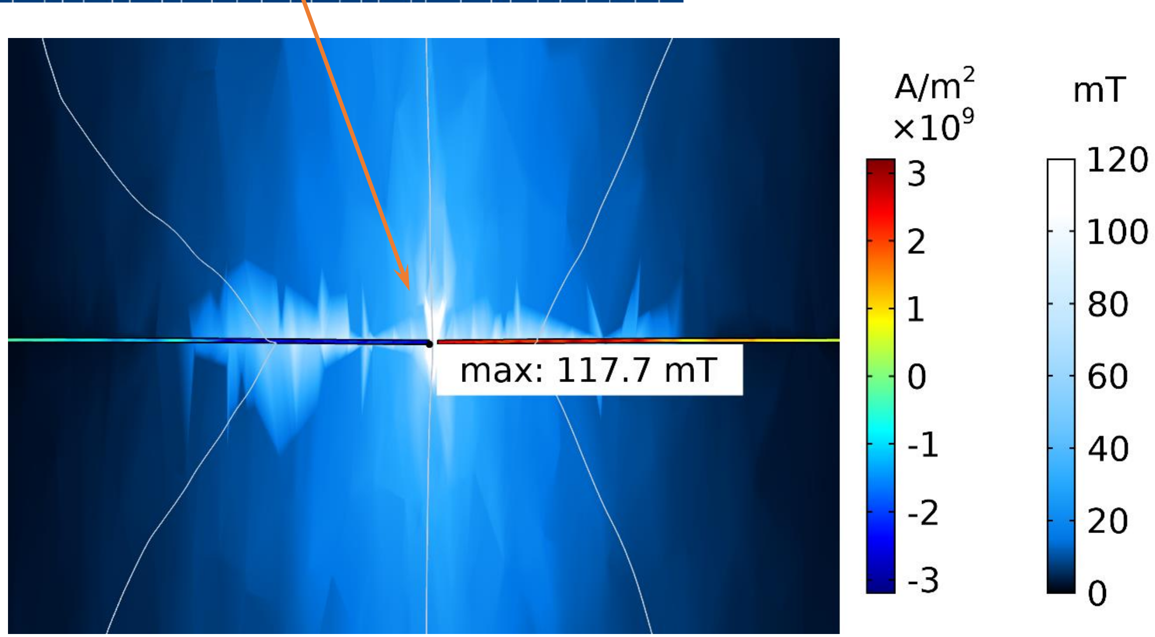
Magnetization loops obtained assuming tape width 12 mm and gap between turns 0.1 mm.

J_c value corresponds to 387 A of critical current and 10 µm layer thickness.

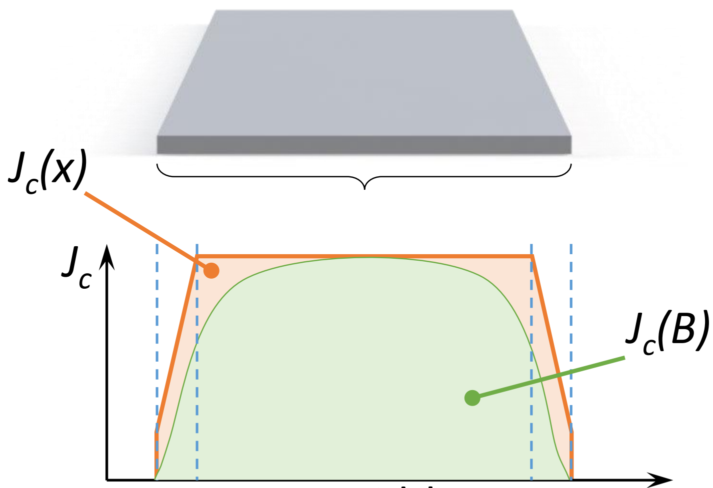
Experimental verification, helicoidally wound ReBCO tape



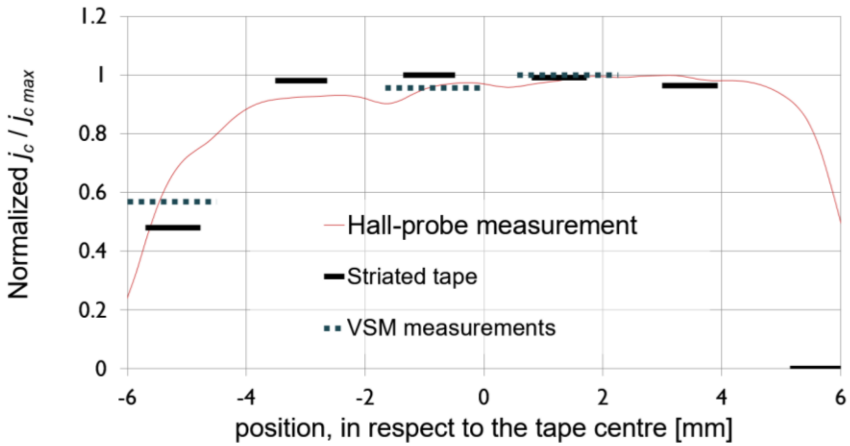
Magnetic flux density distribution in the helix cross-section



Experimental verification, helicoidally wound ReBCO tape

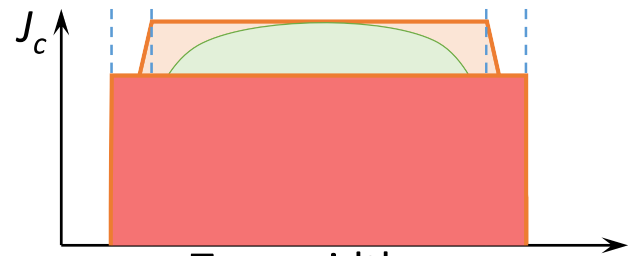


Previously measured J_c distribution across the tape width, for tape of the same manufacturer[5]



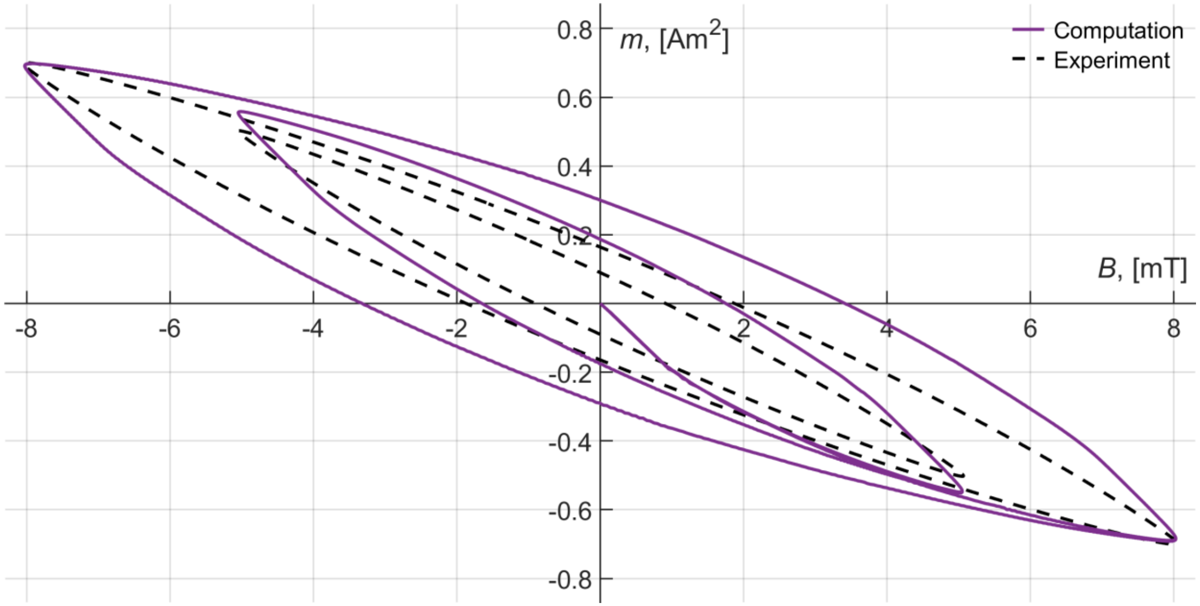
5. Solovyov, M. *et al.* Non-uniformity of coated conductor tapes. *Superconductor Science & Technology* **26**, (2013).

Using the "effective value" of critical current:

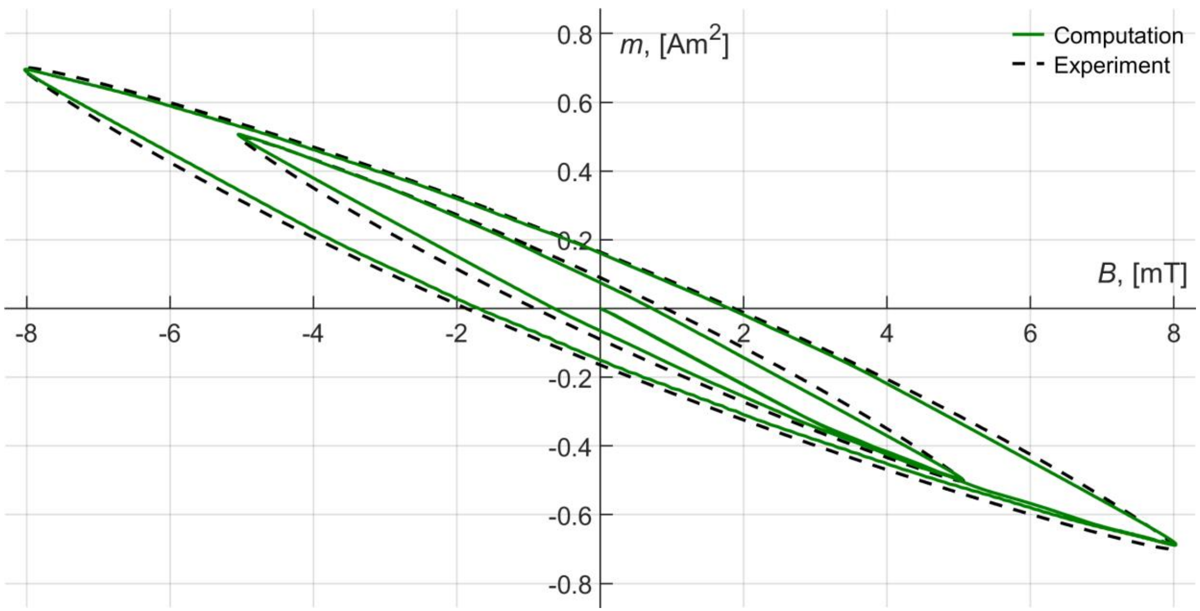


Using the "effective value" of tape width:

Experimental verification, helicoidally wound ReBCO tape

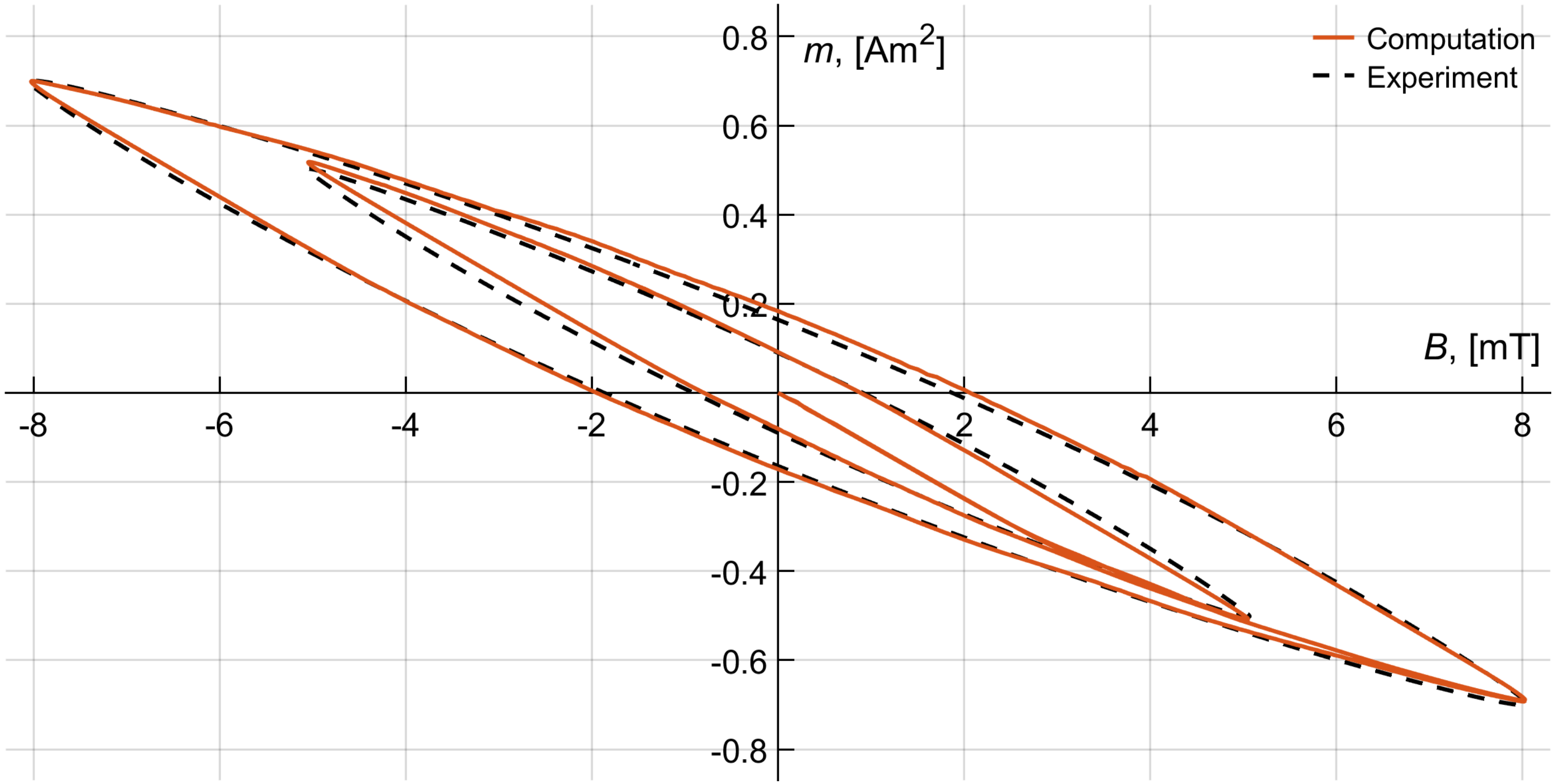


Magnetization loops for original geometry and 260 A of critical current



Original critical current (387 A) and tape width reduced by 1.2 mm

Experimental verification, helicoidally wound ReBCO tape



Combination of tape width reduction (by 1 mm) and critical current reduction (to 360 A)

Conclusion

- We developed novel numerical procedure for modeling of electromagnetic problems in superconductors in 3D geometry.

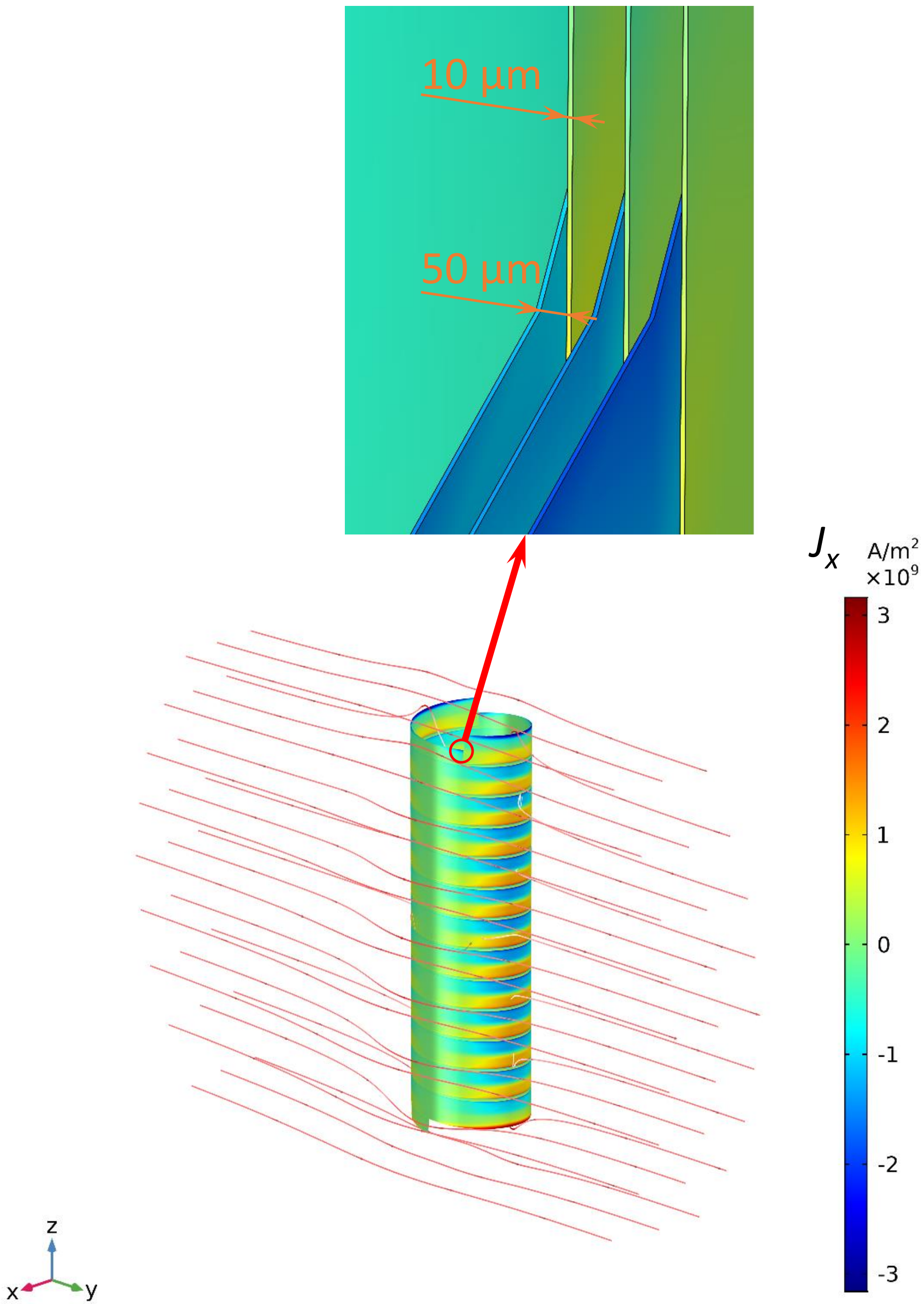
- The results were validated by numerical and experimental benchmarks.

- Our method is based on A-formulation that is the native approach for electromagnetic computation used in FEM and usually is implemented better into commercial codes than other formulations.

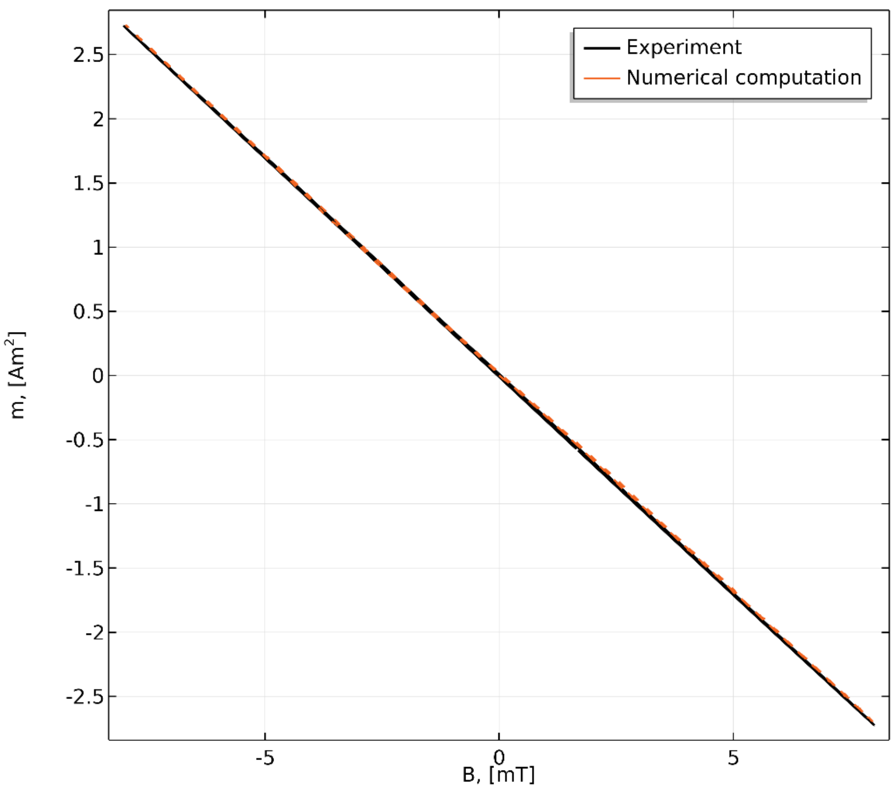
- Unfortunately, incorporation of detailed material properties rapidly increases the model complexity. As a consequence, the computation time grows, and some time it may cause the solver convergence problem.

- A general guide to be followed in search for minimal but still valid electromagnetic representation of superconducting object in 3D geometry remains and open problem.

Recent results



Six layers magnetization:



Current transport in CORC™ cable: