Fundamental description of (field induced) Josephson junctions coupling with semiconductor position based electrostatic qubits

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Overview

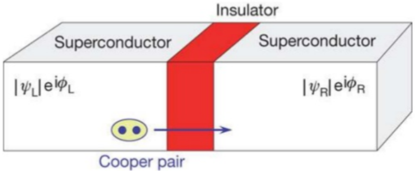
1 First Section
- Essence of Josephson junction
- Andreev Bound State

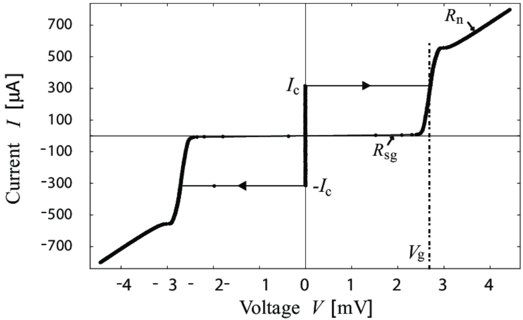
2 Field induced Josephson junction

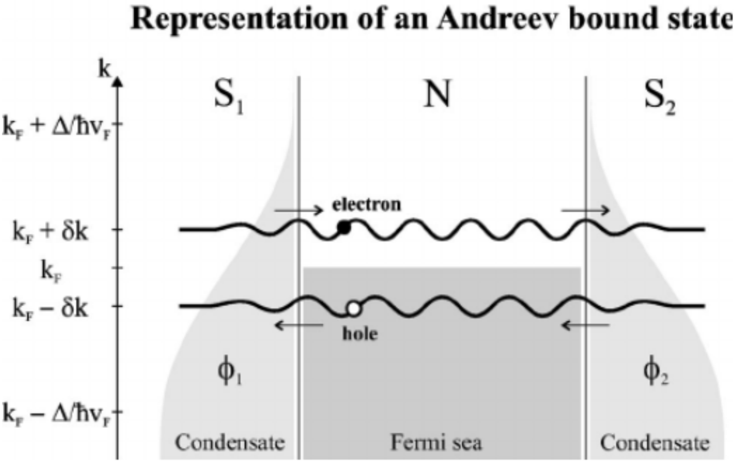
3 Position based semiconductor devices

4 Interface between semiconductor qubit and Josephson junction

Essence of Josephson effct






Representation of an Andreev bound state
k
$k_F + \Delta/\hbar v_F$
$k_F + \delta k$
k_F
$k_F - \delta k$
$k_F - \Delta/\hbar v_F$
S_1
N
S_2
electron
hole
ϕ_1
ϕ_2
Condensate
Fermi sea
Condensate

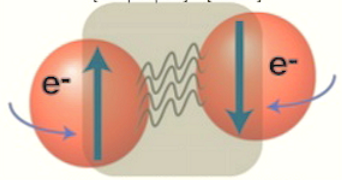
Josephson Junction

1962~

✓ Superconductors (SC)

(1973)

[Cooper pair] = [Boson]



B. D. Josephson, [Phys. Lett. 1, 251 (1962)]

Fig by [Fa Wang and Dung-Hai Lee, Science, 332 (2011) 200]

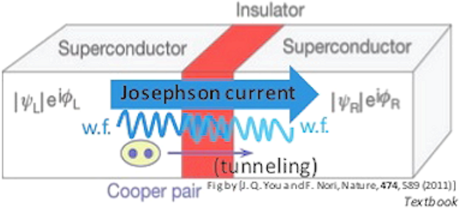


Fig by [J. Q. You and F. Nori, Nature, 474, 589 (2011)]

Textbook by Leggett

Josephson equations in SC

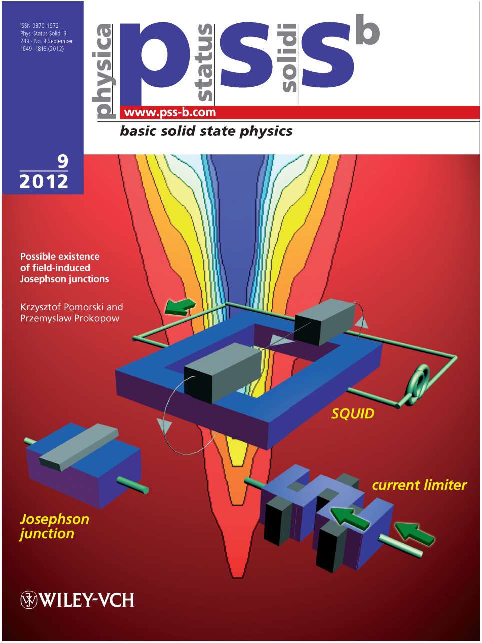
$$\begin{cases} I = I_c \sin\phi \\ \frac{d}{dt}\phi(t) = \frac{2eV(t)}{\hbar} = \frac{2\pi V}{\Phi_0}, \quad \Phi_0 = h/2e \end{cases}$$

; Josephson current
→"charge current"

(quantized magnetic flux)

✓ dc Josephson effect; $\frac{d}{dt}\phi(t) \propto V(t) = 0$

→ Relative phase is time-independent; $\frac{d}{dt}\phi(t) = 0$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Possible existence of field-induced Josephson junctions
Krzysztof Pomorski and Przemyslaw Prokopow
SQUID
current limiter
Josephson junction
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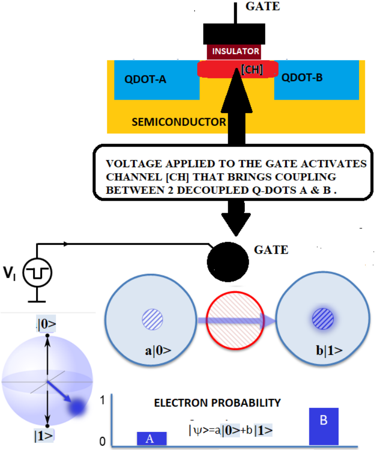
Concept of position based semiconductor qubit



The Hamiltonian of this system is given as

$$\hat{H}(t) = \begin{pmatrix} E_{p1}(t) & t_{s12}(t) \\ t_{s12}^{\dagger}(t) & E_{p2}(t) \end{pmatrix}_{[x=(x_1,x_2)]} = (E_1(t) \left| E_1 \right\rangle_t \left\langle E_1 \right|_t + E_2(t) \left| E_2 \right\rangle \left\langle E_2 \right|)_{[E=(} \quad (1)$$

The Hamiltonian $\hat{H}(t)$ eigenenergies $E_1(t)$ and $E_2(t)$ with $E_2(t) > E_1(t)$ are given as

$$E_1(t) = \left(-\sqrt{\frac{(E_{p1}(t) - E_{p2})^2}{4} + |t_{s12}(t)|^2} + \frac{E_{p1}(t) + E_{p2}(t)}{2} \right),$$

$$E_2(t) = \left(+\sqrt{\frac{(E_{p1}(t) - E_{p2})^2}{4} + |t_{s12}(t)|^2} + \frac{E_{p1}(t) + E_{p2}(t)}{2} \right), \quad (2)$$

Having Hermitian matrix $\hat{A}$ with real valued coefficients $a_{11}(t)$, $a_{22}(t)$, $a_{12r}(t)$, $a_{12i}(t)$ we observe that

$$\hat{A}_{2\times 2} = \begin{pmatrix} a_{11} & a_{12r} + ia_{12i} \\ a_{12r} - ia_{12i} & a_{22} \end{pmatrix}, \quad (3)$$

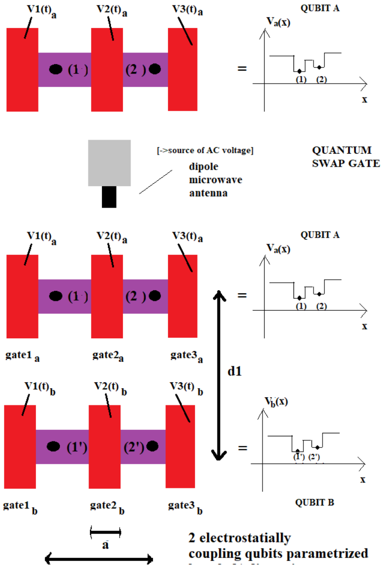
$$\exp(\frac{1}{\hbar i}\hat{A}_{2\times 2}) = \begin{pmatrix} e^{\frac{1}{\hbar}(a_{11})} & e^{(\frac{1}{\hbar}a_{12r}+ia_{12i})} \\ e^{\frac{1}{\hbar}(a_{12r}-ia_{12i})} & e^{\frac{1}{\hbar}(a_{22})} \end{pmatrix} \quad (4)$$

For $\hat{A}_{N\times N}$ we obtain

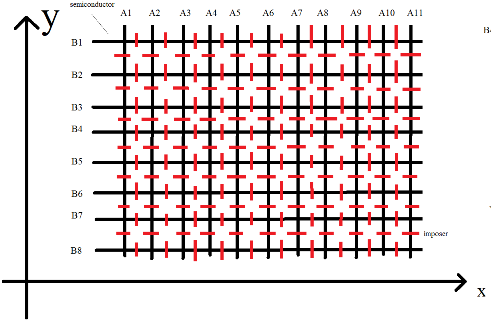
$$\exp(\frac{1}{\hbar i}\hat{A}_{N\times N}) = \begin{pmatrix} e^{\frac{1}{\hbar}(a_{11})} & e^{(\frac{1}{\hbar}a_{12r}+ia_{12i})} & .. & e^{(\frac{1}{\hbar}a_{1Nr}+ia_{1Ni})} \\ e^{\frac{1}{\hbar}(a_{12r}-ia_{12i})} & e^{\frac{1}{\hbar}(a_{22})} & .. & e^{(\frac{1}{\hbar}a_{2Nr}+ia_{2Ni})} \\ \dots & \dots & .. & .. \\ e^{(\frac{1}{\hbar}a_{N1r}-ia_{N1i})} & e^{(\frac{1}{\hbar}a_{N2r}-ia_{N2i})} & .. & e^{(\frac{1}{\hbar}a_{N,N})} \end{pmatrix} \quad (5)$$

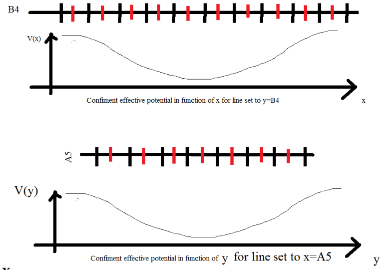
Using the above property for matrix of size 2 by 2 we obtain

$$e^{\frac{1}{i\hbar}\int_{t_0}^{t}\hat{H}(t_1)dt_1} = \hat{U}(t,t_0) = \tag{6}$$
$$\begin{pmatrix} e^{\frac{1}{i\hbar}(\int_{t_0}^{t} E_{p1}(t_1)dt_1)} & e^{\frac{1}{i\hbar}(\int_{t_0}^{t} t_{sr}(t_1)dt_1 + i\int_{t_0}^{t} t_{si}(t_1)dt_1)} \\ e^{\frac{1}{i\hbar}(\int_{t_0}^{t} t_{sr}(t_1)dt_1 - i\int_{t_0}^{t} t_{si}(t_1)dt_1)} & e^{\frac{1}{i\hbar}(\int_{t_0}^{t} E_{p2}(t_1)dt_1)} \end{pmatrix}$$


V1(t)a
V2(t)a
V3(t)a
(1)
(2)
QUBIT A
Va(x)
x
[->source of AC voltage]
dipole
microwave
antenna
QUANTUM
SWAP GATE
gate1 a
gate2a
gate3a
d1
V1(t)b
V2(t) b
V3(t) b
(1')
(2')
Vb(x)
QUBIT B
gate1 b
gate2 b
gate3 b
a
2 electrostatially
coupling qubits parametrized

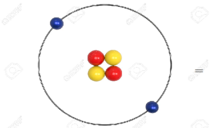
Concept of programmable quantum matter





Concept of programmable Quantum Matter:

Having 2 electrons in cofiment potential created by imposers we can replicate dynamics of He atom and many body systems .



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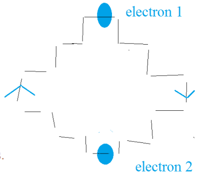


2 Electrons can move on 2 dimensional lattice simulating Helium atom etc , ...

We can replicate N-body dynamics having N electrons in effective potential created by imposers.
In particular we can simulate vortices of magnetic field and many other phenomena

The state of Josephson junction is well described by Bogoliubov-de Gennes equation

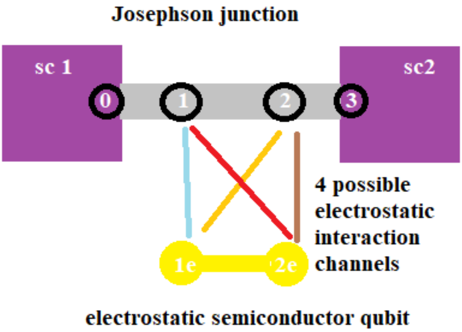
$$\begin{pmatrix} H_0 & \Delta(x) \\ \Delta(x) & -H_0^\dagger \end{pmatrix} \begin{pmatrix} u_n(x) \\ v_n(x) \end{pmatrix} = E_n \begin{pmatrix} u_n(x) \\ v_n(x) \end{pmatrix}. \tag{7}$$

Semiconductor single electron line with 2 nodes can be regarded as electrostatic position dependent qubit and can be described by

$$H_{semi} = ts_{1,2}|1><2| + ts_{2,1}|2><1| + E_{p1}|1><1| + E_{p2}|2><2|,$$

We can express coupling of 2 systems assuming 4 nodes for electron or hole and 2 nodes for electron confined in semiconductor so we have eigenvector having 16 components $|0_{Ee}>|Es_1>, |0_{Ee}>|Es_2>, |1_{Ee}>|Es_1>, |1_{Ee}>|Es_2>, .., |3_{Ee}>|Es_1>, |3_{Ee}>|Es_2>, |0_{Eh}>|Es_1>, |0_{Ee}>|Es_2>, .., |3_{Eh}>|Es_1>, |3_{Eh}>|Es_2>$.

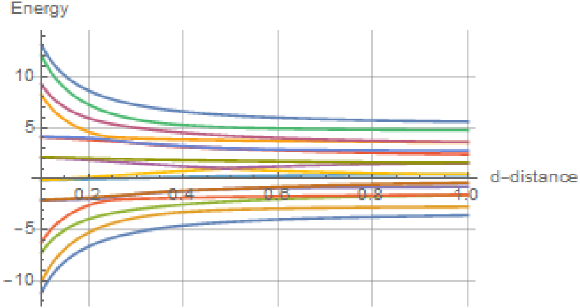
Interace betweem semiconductor qubit and Josephson junction

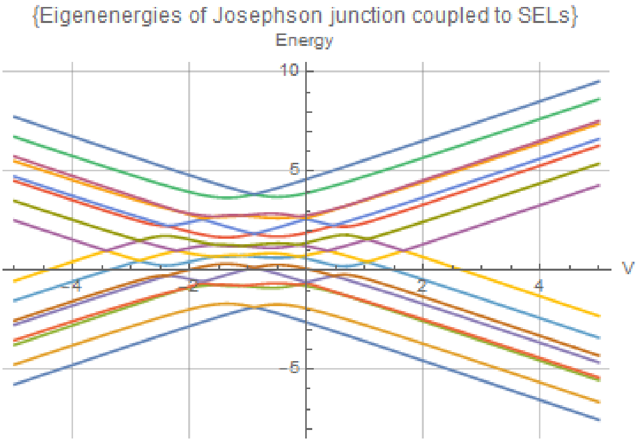


Interaction Hamiltonian between semiconductor qubit and Josephson junction

$$\begin{pmatrix}
Ep+mu+V & ts & tt & 0 & 0 & 0 & 0 & 0 & Delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
ts & Ep+mu+V & 0 & tt & 0 & 0 & 0 & 0 & 0 & Delta & 0 & 0 & 0 & 0 & 0 & 0 \\
tt & 0 & Ep+\frac{q^2}{a}+V & ts & tt & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & tt & ts & Ep+\frac{q^2}{b}+V & 0 & tt & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & tt & 0 & Ep+\frac{q^2}{b}+V & ts & tt & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & tt & ts & Ep+\frac{q^2}{a}+V & 0 & tt & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & tt & 0 & Ep+mu+V & ts & 0 & 0 & 0 & 0 & 0 & 0 & Delta & 0 \\
0 & 0 & 0 & 0 & 0 & tt & ts & Ep+mu+V & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Delta \\
Delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Ep-mu-V & ts & -tt & 0 & 0 & 0 & 0 & 0 \\
0 & Delta & 0 & 0 & 0 & 0 & 0 & 0 & ts & Ep-mu-V & 0 & -tt & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -tt & 0 & Ep-\frac{q^2}{a}-V & ts & -tt & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -tt & ts & Ep-\frac{q^2}{b}-V & 0 & -tt & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -tt & 0 & Ep-\frac{q^2}{b}-V & ts & -tt & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -tt & ts & Ep-\frac{q^2}{a}-V & 0 & -tt \\
0 & 0 & 0 & 0 & 0 & 0 & Delta & 0 & 0 & 0 & 0 & 0 & -tt & 0 & Ep-mu-V & ts \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Delta & 0 & 0 & 0 & 0 & 0 & -tt & ts & Ep-mu-V
\end{pmatrix}$$

{Eigenenergies of Josephson junction coupled to SELs($|\Delta|=1$)}




{Eigenenergies of Josephson junction coupled to SELs}
Energy
10
5
−5
−4
−2
2
4
V

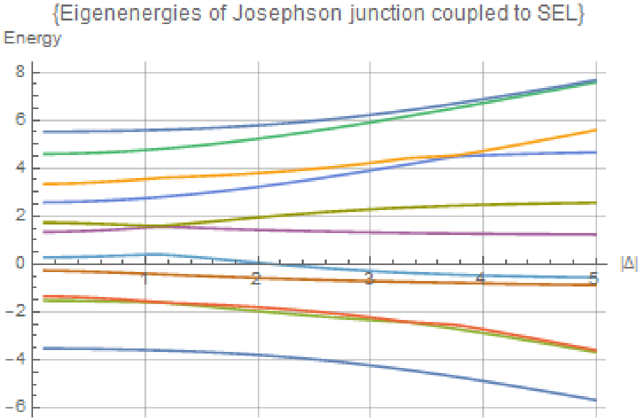
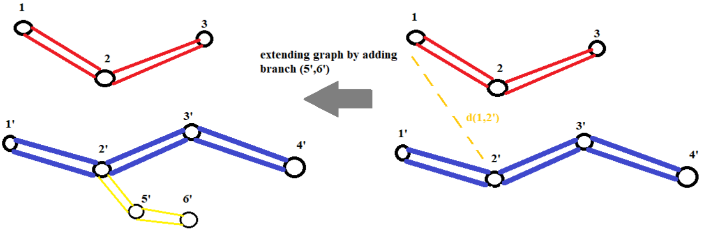


Figure: Eigenenergy spectral of semiconductor qubit coupled to Josephson junction obtained from simplistic tight-binding model (tight-binding BdGe coupled to tight-binding Schroedinger equation for semiconductor qubit).

Further extensions of tight-binding model



Conclusions

- Quantum phase transition is expected to occur both in electrostatically coupled semiconductor qubits and systems of coupled Josephson junction to superconducting qubit.
- Increase of superconducting order parameter has similar impact on energy eigenspectrum as the increase of distance between superonducting Josephson junction and semiconductor qubit.

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To be extended for JJs vs semiconductor qubit interaction:
K.Pomorski, Analytical solutions for N interacting electron system con ned in graph of coupled electrostatic quantum dots in tight-binding model, ArXiv: 1907.03180, 2019