Interacting Gravitational Waves

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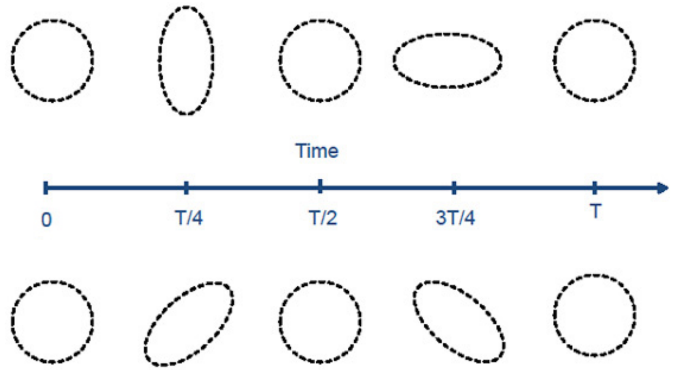
Siddhartha Morales, ADG `Scalar and Fermion Field Interaction with a Gravity Wave' (to appear)

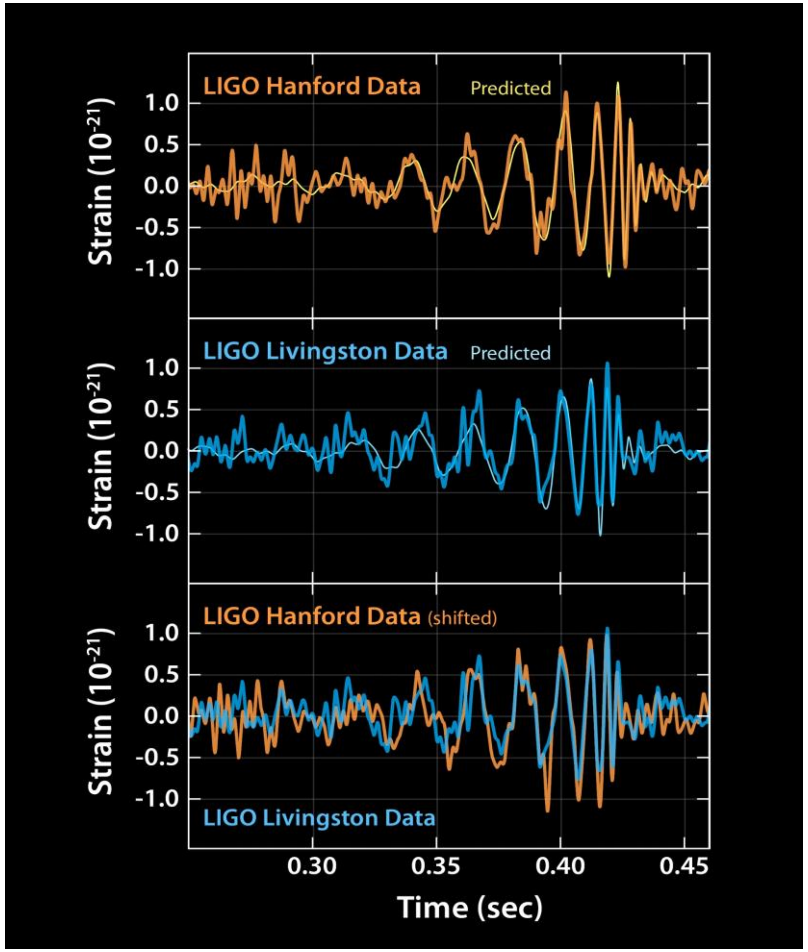
Gravitational Waves

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1+h_+ & h_\times & 0 \\ 0 & h_\times & 1-h_+ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_\times \cos(\omega(z-t)+\delta)$$

$$A_+ \cos(\omega(z-t))$$





Gravity Waves as the window to the Universe

- Gravity waves from Black Hole binary mergers/Neutron Star mergers have already been detected
- Gravity waves are expected to be emitted during the initial formation of the Universe. These are known as early Universe gravity waves, or primordial gravity waves
- One expects them to have a stochastic distribution
- They have not been detected yet
- Could there be interactions/scatterings we have yet to search for?

How does a scalar field (maybe the inflaton) react to the wave?

$$\partial_\nu \left(g^{\mu\nu} \partial_\mu \phi \right) = 0.$$

$$\Box\phi - \left[h_+ (\partial_{xx} - \partial_{yy}) + 2h_\times \partial_y \partial_x \right] \phi = 0$$

$$\phi \approx \phi_0 + A_+ \phi_+ + A_\times \phi_\times$$

a Perturbative solution

$$\Box\phi_+ = \cos(\omega(z-t))(\partial_{xx} - \partial_{yy})\phi_0$$

$$\Box\phi_\times = 2\cos(\omega(z-t)+\delta)(\partial_x \partial_y)\phi_0$$

Inhomogeneous differential equations

Kirchoff’s Laws and Duhamel’s principle

$$\phi_+(\mathbf{x},t) = \frac{1}{4\pi}(k_{0x}^2 - k_{0y}^2)\int_{\bar{B}(\mathbf{x},t)} d^3\mathbf{x}' \frac{\phi_0(\mathbf{x}',t')}{r}\cos(w(z'-t'))$$

$$\phi_0(\mathbf{x},t) = \Re\left(A_0 \mathrm{e}^{i(\mathbf{k}_0\cdot\mathbf{x}-w_0 t)}\right)$$

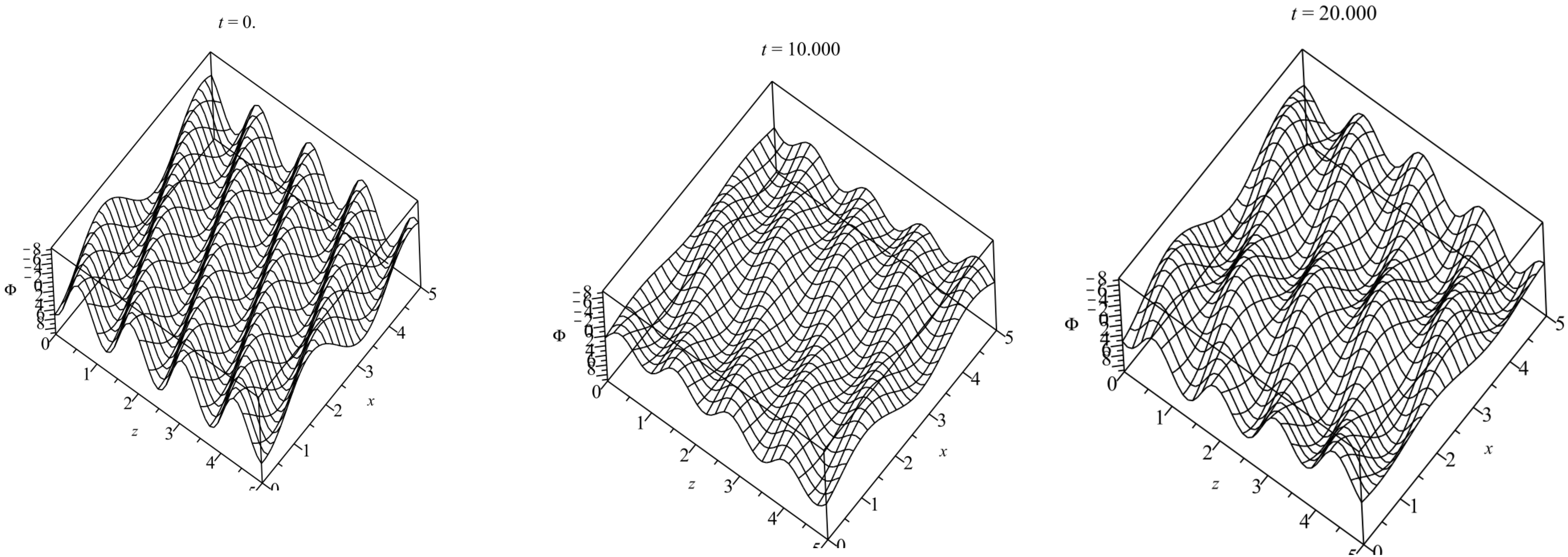
$$\phi_\times(\mathbf{x},t) = -\frac{1}{4\pi}(2k_{0x}k_{0y})\int_{\bar{B}(\mathbf{x},t)} d^3\mathbf{x}' \frac{\phi_0(\mathbf{x}',t')}{r}\cos(w(z'-t')+\delta)$$

Solutions exist if the scalar wave has transverse components to the gravitational wave

$$\frac{4\pi}{2\omega\omega_0(1-\cos\beta)} e^{\left(i\left(\vec{k}_0+\vec{k}\right)\cdot\vec{x}\right)}\left[-1+e^{i(\omega+\omega_0)t}\left[\cos(||\vec{\omega}+\vec{\omega_0}||\ t) - i\frac{(\omega+\omega_0)}{||\vec{\omega}+\vec{\omega_0}||}\sin(||\vec{\omega}+\vec{\omega_0}||\ t)\right]\right] + (\omega\rightarrow-\omega)$$

No resonance, but an additional mode

The scalar wave perturbation



In addition to the usual frequency, there is a new wave mode with dispersion

$$\bar{\omega}^2 = (\vec{k} + \vec{k}_0)^2.$$

Massive scalar perturbation

$$G(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^2}} \int_{\mathbb{R}^4} \frac{\mathrm{e}^{ik_\mu x^\mu}}{k_\mu k^\mu + m^2}$$

$$\phi(\mathbf{x}) = -\frac{A_0}{2}(k_y^2 - k_x^2) \lim_{\epsilon \to 0} \int_{\mathbb{R}^4} d^4k \; \mathrm{e}^{i(k_\mu x^\mu)}.$$

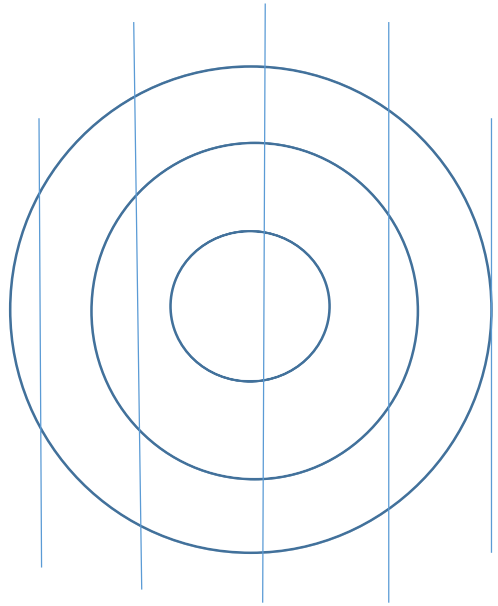
$$\cdot \frac{(\delta(\omega\text{-}(\omega_0 + \omega_g))\delta(k_{0x}\text{-}k_x)\delta(k_{0y}\text{-}k_y)\delta(k_{0z} + \omega_g\text{-}k_z) + \delta(\omega\text{-}(\omega_0\text{-}\omega_g))\delta(k_{0x}\text{-}k_x)\delta(k_{0y}\text{-}k_y)\delta(k_{0z}\text{-}\omega_g\text{-}k_z))}{-k_\mu k^\mu\text{-}m^2 + i\epsilon}$$

$$= \frac{A_0}{2}(k_x^2 - k_y^2)\left(\frac{\mathrm{e}^{i((\vec{k}_0+\vec{\omega}_g)\cdot\vec{x}-(w_0+w_g)t)}}{2\omega_g(\omega_0 - k_0\cos\beta)} - \frac{\mathrm{e}^{i((\vec{k}_0-\vec{\omega}_g)\cdot\vec{x}-(w_0-w_g)t)}}{2\omega_g(\omega_0 - k_0\cos\beta)}\right)$$

Usual `beats’ pattern if one uses the linear propagator; and no resonance

Spherical propagator

$$G_m(\vec{r},\vec{r}',t-t') = \frac{mJ_1(m\sqrt{(t-t')^2-|\vec{x}-\vec{x}'|^2})}{\sqrt{(t-t')^2-|\vec{x}-\vec{x}'|^2}}$$



Similar perturbation to the massive scalar wave as the massless wave

$$4\pi(k_x^2-k_y^2)\frac{e^{-i\tilde{\omega}t}}{||\vec{k}_0+\vec{\omega}||}\frac{e^{i(\vec{k}+\vec{k}_0)\cdot\vec{x}}}{2\omega(\omega_0-k_0\cos\beta)}\Big[||\vec{k}_0+\vec{\omega}||\cos(||\vec{k}_0+\vec{\omega}||t)\cos(\sqrt{\tilde{\omega}^2-m^2}\ t)$$
$$+\sqrt{\tilde{\omega}^2-m^2}\sin(||\vec{k}_0+\vec{\omega}||\ t)\sin(\sqrt{\tilde{\omega}^2-m^2}\ t)-||\vec{k}_0+\vec{\omega}||\Big]$$

Neutrino or massless fermions

$$i\gamma^a e_a^\mu D_\mu \psi = 0. \qquad D_\mu = \partial_\mu - \frac{i}{4}\omega_\mu^{ab}\sigma_{ab}$$

$$\bar{\sigma}^\mu \partial_\mu \psi_L = -\frac{h_\times}{2}\sigma^1 \partial_v \psi_L - \frac{h_\times}{2}\sigma^2 \partial_u \psi_L - i\frac{1}{8}\partial_t h_\times \sigma^3 \psi_L - \frac{3}{8} i \partial_z h_\times \psi_L$$

$$\sigma^\mu \partial_\mu \psi_R = \frac{h_\times}{2}\sigma^1 \partial_v \psi_R + \frac{h_\times}{2}\sigma^2 \partial_u \psi_R - i\frac{1}{8}\partial_t h_\times \sigma^3 \psi_R + \frac{3}{8} i \partial_z h \psi_R$$

↔ Cross Polarization

$$\bar{\sigma}^\mu \partial_\mu \psi_L = \sigma^2 \frac{h_+}{2}\partial_y \psi_L - \sigma^1 \frac{h_+}{2}\partial_x \psi_L$$

$$\sigma^\mu \partial_\mu \psi_R = \sigma^1 \frac{h_+}{2}\partial_x \psi_R - \sigma^2 \frac{h_+}{2}\partial_y \psi_R$$

↔ Plus polarization

The plus polarization causes a perturbation in the Neutrino wave

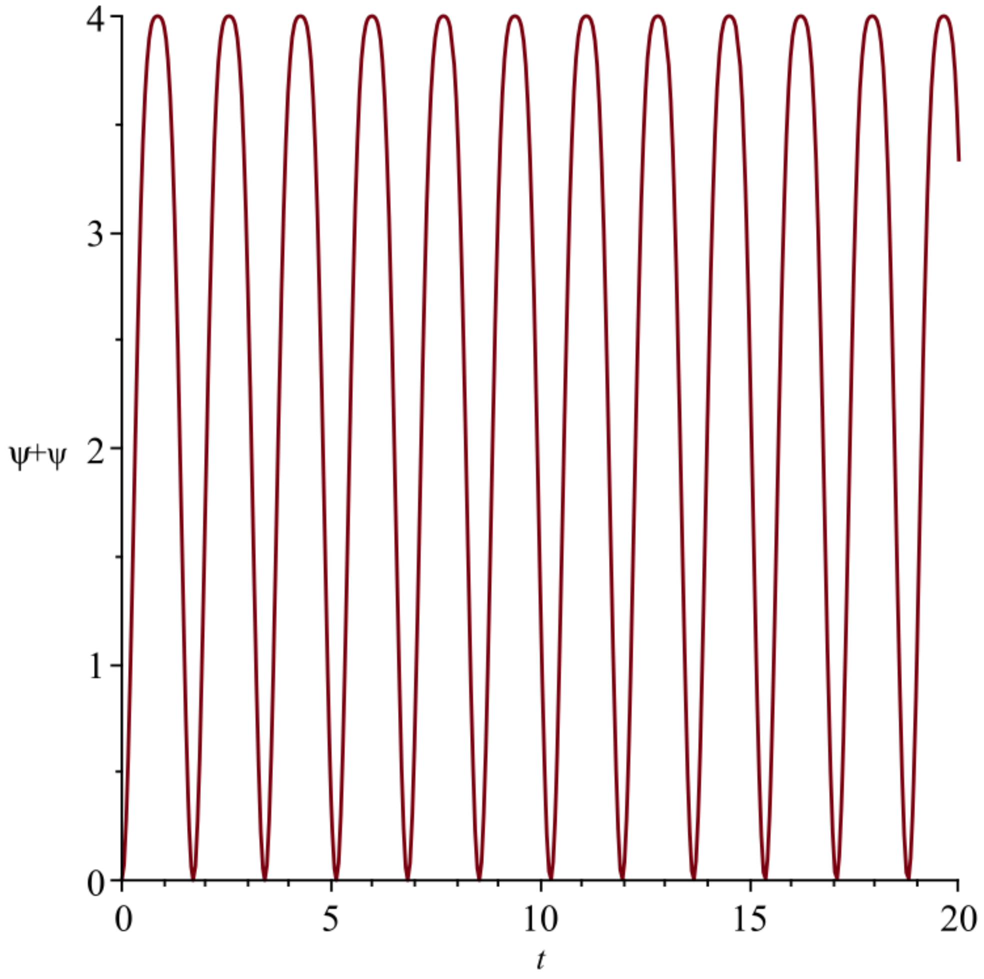
$$\Box\tilde{\psi}_{L1} = \omega_0^2 e^{i\omega_0(x-t)} \cos\omega(z-t)$$

$$\Box\tilde{\psi}_{L2} = e^{i\omega_0(x-t)} \left(\omega_0^2 \cos(\omega(z-t)) - i\omega\omega_0 \sin\omega(z-t)\right)$$

$$\psi_{L1+\omega} = \frac{4\pi\omega_0}{\omega} e^{i\omega_0(x-t)} e^{i\omega(z-t)} e^{i(\omega+\omega_0)t} \left[\cos\left(\sqrt{\omega_0^2+\omega^2}\, t\right) + \frac{i(\omega+\omega_0)}{\sqrt{\omega^2+\omega_0^2}} \sin\left(\sqrt{\omega^2+\omega_0^2}\, t\right) - 1\right]$$

$$\psi_{L2+\omega} = \frac{\omega_0-\omega}{\omega_0} \psi_{L1+\omega}$$

The Dirac Bilinear



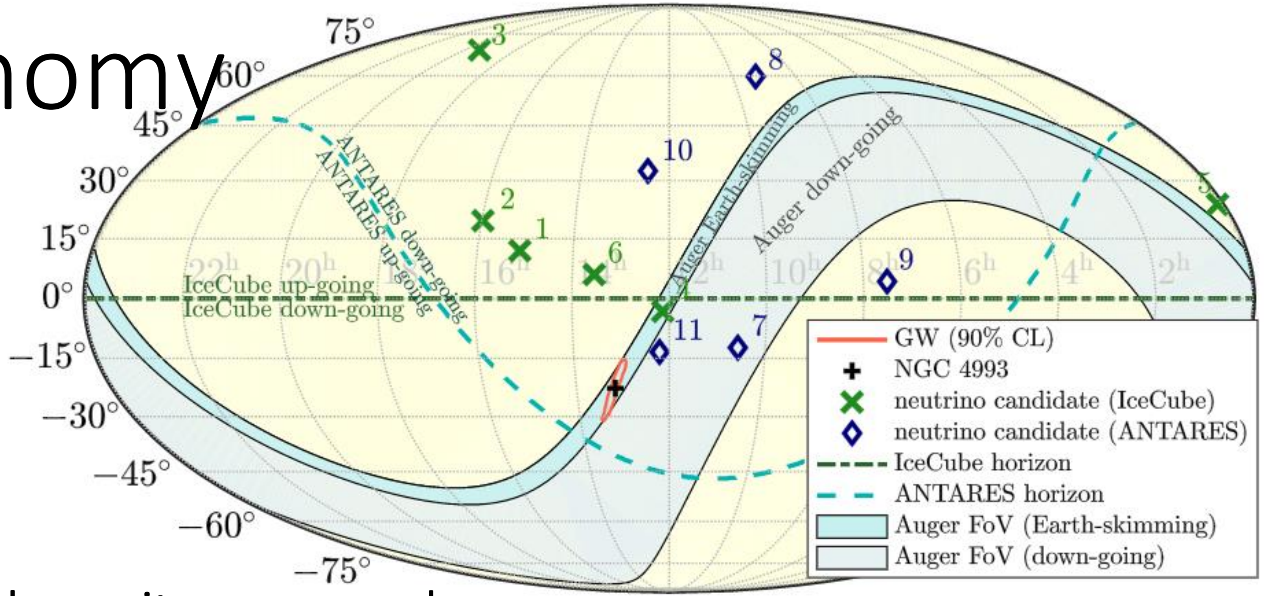
Can we use these oscillatory modes to detect gravity waves?

$$\Delta\phi \approx A_+\phi_+ \sim h\phi_+$$
$$\sim h\frac{A_0 k_0^2}{2\omega\omega_0} \sim hA_0\left(\frac{\omega_0}{\omega}\right)$$

$$\omega_0 = 160.23Ghz$$
$$\omega \in (10^{-7}Hz - 100Hz)$$

~10^{-3} -10^{-13}

Multi-messenger astronomy



- Neutron star mergers are expected to release both gravity waves and neutrinos.
- The above calculation could have important implications for neutrinos from the event GW170817
- This event did not detect any neutrinos by Ice Cube and ANTARES, but that is expected from the prediction that the outflow jet was not pointing towards the earth
- We expect the neutrino flux to be perturbed when interacting with gravity waves

Early Universe Cosmology

- Neutrinos from the early universe constitute what is known as the cosmic neutrino background
- From the cosmological data it seems there is a slight discrepancy in the number of degrees of freedom, pointing towards small fluctuations
- It could be that what we are observing is the regular neutrino flux+ perturbations due to interactions with gravity waves
- N=3.13 +/- 0.31 (CMB data)
- = 3.01+/- 0.15 (95% Planck+ BBN)

$$N_{eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_\nu}{\rho_\gamma}$$

Neutrino density

Photon density

Detection on Earth

- scalar waves can be used to detect gravitational waves
- A normal stream of fermion waves will interfere with the gravitational waves to produce tiny oscillations, which can be detected.
- Electromagnetic waves will also see such an oscillatory perturbation, it is not difficult to see, that a change in frequency of the Electromagnetic wave would be easy to detect on earth (work in progress)