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Einstein-Maxwell-dilaton Solutions Based on Bianchi type IX geometry

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We construct new classes of cosmological solution to the five-dimensional Einstein-Maxwell-dilaton theory that are non-stationary and almost conformally regular everywhere. The base geometry for the solutions is four-dimensional Bianchi type IX (BIX) geometry.
In the theory, the dilaton field couples to both electromagnetic field and the cosmological constant term with two different coupling constants. We consider all possible solutions with different values of the coupling constants as well as the cosmological constant, as a positive, negative or zero -valued constant.
In the ansatzes for the metric, dilaton and electromagnetic fields, we consider dependence on one time and two spatial directions. We discuss the physical properties of the five-dimensional space-time. We also consider the special case of the Bianchi type IX geometry in which the geometry reduces to that of Eguchi-Hanson space.

The Bianchi type IX metric is locally given by the following metric with an SU(2) or SO(3) isometry group:

$$ds^2_{tri.\ B.\ IX} = \frac{dr^2}{\sqrt{F(r)}} + \frac{r^2}{4}\sqrt{F(r)}\left(\frac{\sigma_1^2}{1-\frac{a_1^4}{r^4}} + \frac{\sigma_2^2}{1-\frac{a_2^4}{r^4}} + \frac{\sigma_3^2}{1-\frac{a_3^4}{r^4}}\right)$$

where:

$$F(r) = \prod_{i=1}^{3}\left(1-\frac{a_i^4}{r^4}\right)$$

a_1, a_2 and a_3 are three parameters that can be chosen such that

$$0 = a_1 \le a_2 = 2kc \le a_3 = 2c$$

where 0<k<1 is the square root of modulus of different types of Jacobi elliptic functions. BIX metric will be Eguchi-Hanson metric when k=1. The solutions for this specific case are known.

The action for five dimensional Einstein-Maxwell-dilaton theory, where the dilaton field is coupled to both the electromagnetic field with the coupling constant a, and to the cosmological term with the coupling constant b, is as below:

$$S = \int d^5x\sqrt{-g}\ \{R - \frac{4}{3}(\nabla\phi)^2 - e^{-4/3a\phi}F^2 - e^{4/3b\phi}\Lambda\}$$

For the case that the coupling constants are not equal to each other and are non-zero, we consider the following ansatzes for the five-dimensional metric, the electromagnetic gauge field and the dilaton field

$$ds_5^2 = -\frac{1}{H^2(r,\theta)}dt^2 + R^2(t)H(r,\theta)ds_{EH}^2$$

$$A_t(t,r,\theta) = \alpha R^X(t)H^Y(r,\theta)$$

$$\phi(t,r,\theta) = -\frac{3}{4a}\ln(H^U(r,\theta)R^V(t))$$

where H(r,θ) and R(t) are two metric functions and depend on two coordinates, r, θ and t, respectively. X, Y, U, V and α are constants.

The Maxwell's equations and also Einstein's equations yield to the following relationship for the constants:

$$X = 2,\quad Y = -1-\frac{a^2}{2},\ U = a^2,\ \ V = -4,\ \alpha^2 = \frac{3}{a^2+2}.$$

The t component of Maxwell's equation gives H(r,θ) the following form:

$$H(r,\theta) = (g_+ r^2\cos(\theta) + g_-)^{2/(2+a^2)}$$

where g_+ and g_- are two constants.

By considering the following form for R(t), all of the equations would be satisfied.

$$R(t) = (\xi\, t + \zeta)^{a^2/4}$$

Where ξ and ζ are two arbitrary constants.

And the relation between the coupling constants would be:

$$b = -\frac{2}{a}$$

By these assumptions, we find the cosmological constant as:

$$\Lambda = \frac{3}{8}\xi^2 a^2(a^2-1).$$

Conclusions:

We found new exact solutions for the five dimensional Einstein-Maxwell-dilaton theory in Bianchi type IX metric and also found the relation between the coupling constants. The next step in this research is to find the solutions for the case that the coupling constants are equal to each other. It is worth noting that the relations between constants are similar to the Eguchi-Hanson case.

This poster is based on papers:

New class of exact solutions in Einstein-Maxwell-dilaton theory
A.M. Ghezelbash

Supergravity solutions without Tri-holomorphic U(1) isometries
A.M Ghezelbash