

Improved Extraction of the V_{ud} Matrix Element

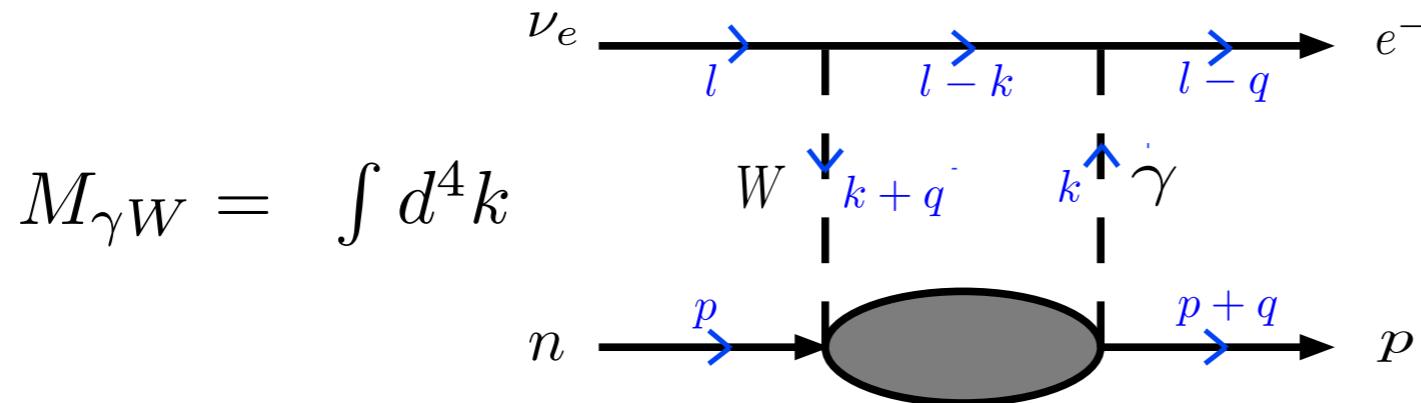
Kyle Shiells, PhD Candidate

University of Manitoba

June 6, 2019

In collaboration with Peter Blunden (UM, supervisor) and Wally Melnitchouk (Jefferson Lab)

Dispersion Relation for the γW Box



For on-shell states, hadronic tensor involves structure functions:

$$MW_{\gamma W}^{\mu\nu} = -g^{\mu\nu} F_1^{\gamma W} + \frac{p^\mu p^\nu}{p \cdot q} F_2^{\gamma W} - i\epsilon^{\mu\nu\lambda\rho} \frac{p_\lambda q_\rho}{2p \cdot q} F_3^{\gamma W}$$

Only the axial part is relevant:

$$\Rightarrow \text{Im}M_{\gamma W}^{(A)} = 2\sqrt{2}G_F(4\pi\alpha)\frac{1}{8\pi p \cdot l} \int \frac{dW^2 dQ^2}{(1+Q^2/M_W^2)} 2\left(\frac{2ME}{W^2-M^2+Q^2} - \frac{1}{2}\right) F_3^{\gamma W}$$

Depends on knowledge of the F_3 structure function at all W^2 and Q^2 .

The axial part of the gW box is odd with respect to the electron's incident energy E :

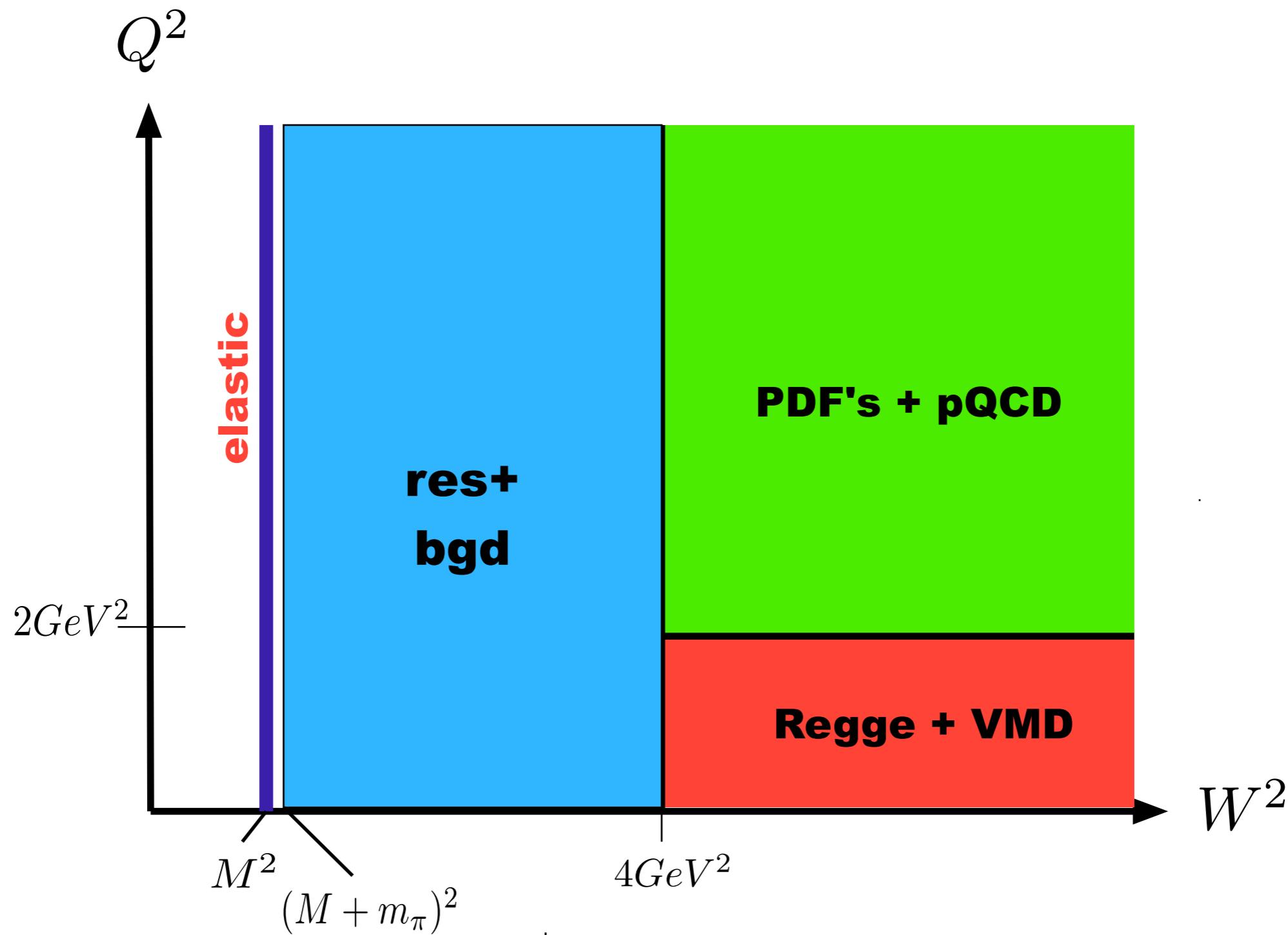
$$\text{Im} \square_{\gamma W}^{(A)}(-E) = -\text{Im} \square_{\gamma W}^{(A)}(E)$$

$$\Rightarrow \text{Re} \square_{\gamma W}^{(A)}(E) = \frac{2}{\pi} \int_{\nu_\pi}^{\infty} dE' \frac{E'}{E'^2 - E^2} \text{Im} \square_{\gamma W}^{(A)}(E')$$

$$\square_{\gamma W}^{(A)} = \frac{\alpha}{2\pi} \int_{W_\pi^2}^{\infty} dW^2 \int_0^{\infty} dQ^2 \frac{F_3^{\gamma W}(W^2, Q^2)}{1 + Q^2/M_W^2} \frac{1}{ME_{min}} \left(\frac{2}{\chi} - \frac{1}{4ME_{min}} \right)$$

$$\chi = W^2 - M^2 + Q^2 \quad E_{min} = \frac{\chi + \sqrt{\chi^2 + 4M^2Q^2}}{4M}$$

Kinematical Regions of $F_3^{\gamma W}$



Elastic Contribution:

$$\square_{\gamma W, el}^{(A)} = \frac{\alpha}{\pi} \int_0^\infty dQ^2 \frac{[\mu_p G_M^p(Q^2) + \mu_n G_M^n(Q^2)]}{(1+Q^2/M_V^2)^2} \frac{G_A}{(1+Q^2/M_A^2)^2} \frac{1+2\sqrt{1+4M^2/Q^2}}{Q^2(1+\sqrt{1+4M^2/Q^2})^2}$$

$$G_A = 1.2723$$

$$M_A = 1.35 \pm 0.17 GeV$$

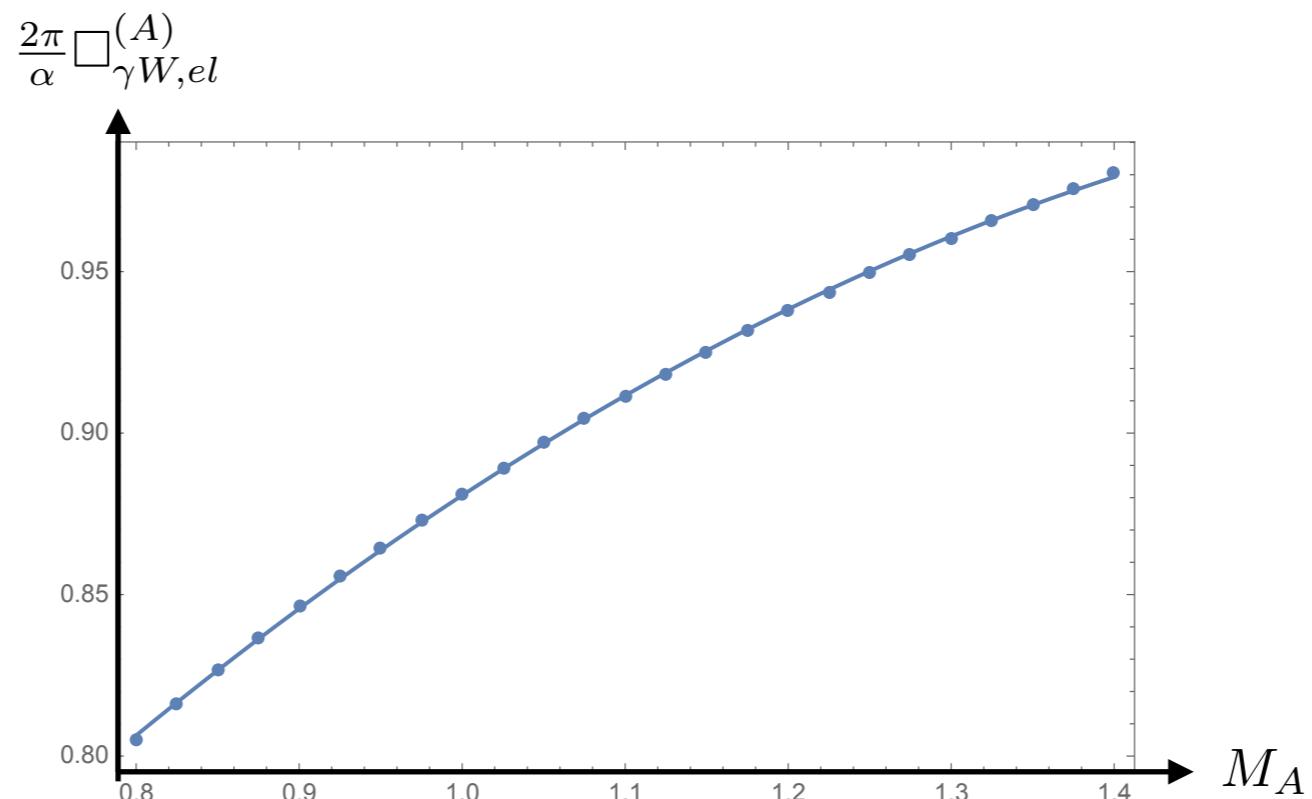
(MiniBOONE 2018)

$$M_V = 0.8426$$

$$G_M^p, G_M^n$$

(Arlington's 2018 data)

$$\square_{\gamma W, el}^{(A)} = (0.9707 \pm 0.0684) \frac{\alpha}{2\pi} = .001213(82)$$



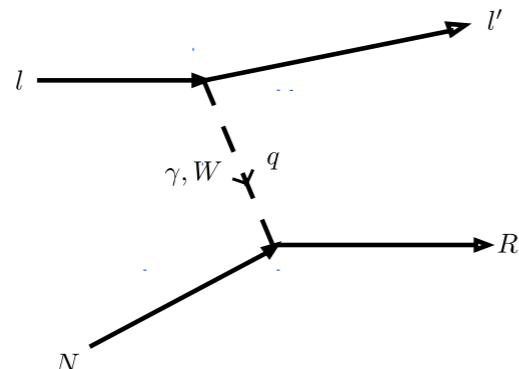
Resonance Contributions:

Fact: the photon has an isovector (V) and an isoscalar (S) component
the W boson only has isovector (V)

Adding the crossed box + box \Rightarrow photon (V) $\rightarrow 0$

\Rightarrow only the (S) component of the photon contributes

\Rightarrow only $|l|=1/2$ resonances are allowed



$$R = P_{11}(1440), D_{13}(1520), S_{11}(1535), \dots$$

We use the Lalakulich parametrizations to find F_3 for these resonances.

example: D_{13}

$$\begin{aligned} F_3^{\gamma W}(D_{13}) &= -\frac{4\nu}{3M} \left[-C_4^S(Q^2 - \nu M) + C_5^S \nu M \right. \\ &\quad \left. + C_3^S \frac{M}{M_R} (2M_R^2 - 2MM_R + Q^2 - \nu M) \right] C_5^A \Gamma_R(W, M_R) \end{aligned}$$

$C_i^{A,V}$ are experimentally determined FF's

$$\Rightarrow \square_{\gamma W}^{(A)}(D_{13}) = 0.00023$$

the most dominant resonance is small but non-negligible

Background Contribution:

The background is a smoothly decreasing curve which goes to 0 at the pion threshold and matches the DIS and Regge regions at $W^2 = 4 \text{ GeV}^2$

As a consequence of Isospin symmetry and the Wigner-Eckart theorem, we know that $F_3(0)$ is proportional to $F_3(\text{gZ})$ of the deuteron:

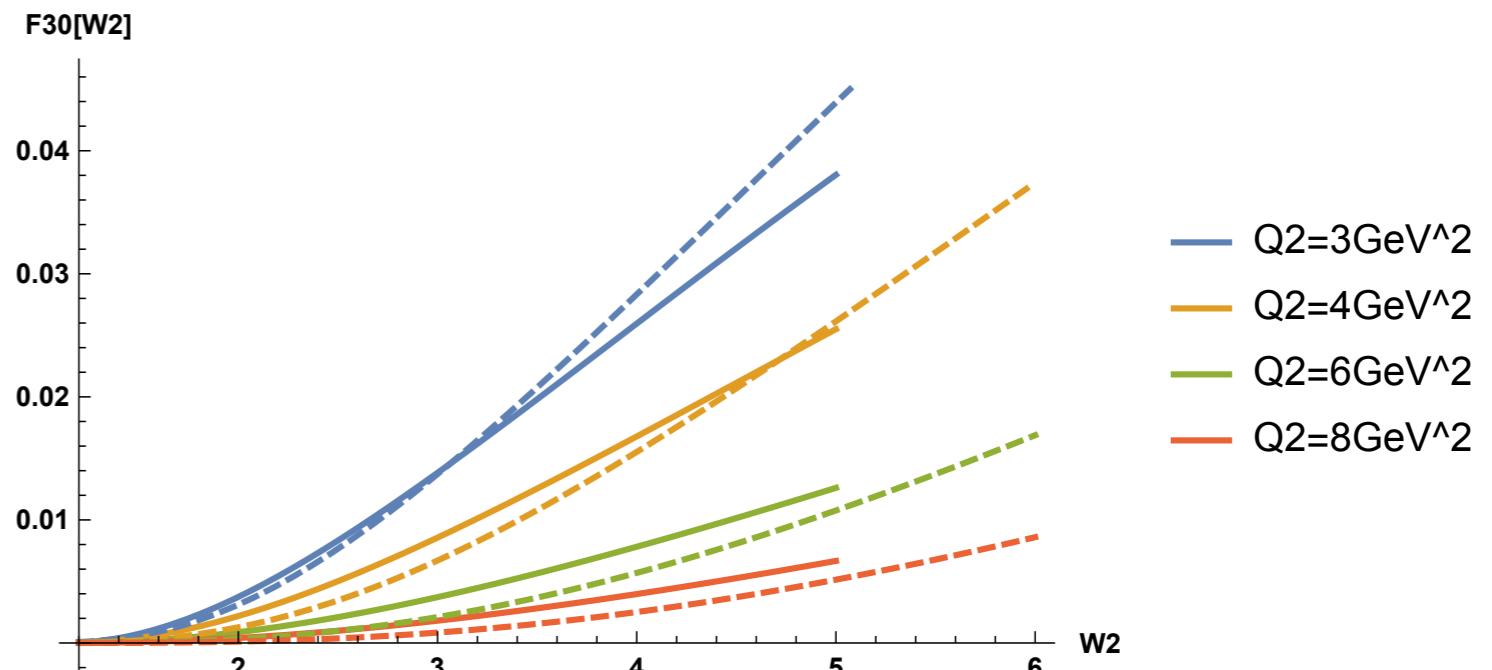
$$\left. \begin{aligned} F_3^{(0)} &\approx \frac{1}{24} F_{3,d}^{\gamma Z} \\ F_{3,d}^{\gamma Z} &\approx \frac{18}{5} F_{1,\text{bgd}}^{\gamma\gamma} \end{aligned} \right\} \Rightarrow F_{3,\text{bgd}}^{(0)} \approx \frac{3}{20} F_{1,\text{bgd}}^{\gamma\gamma}$$

Taken from Bosted-Christy parametrization:

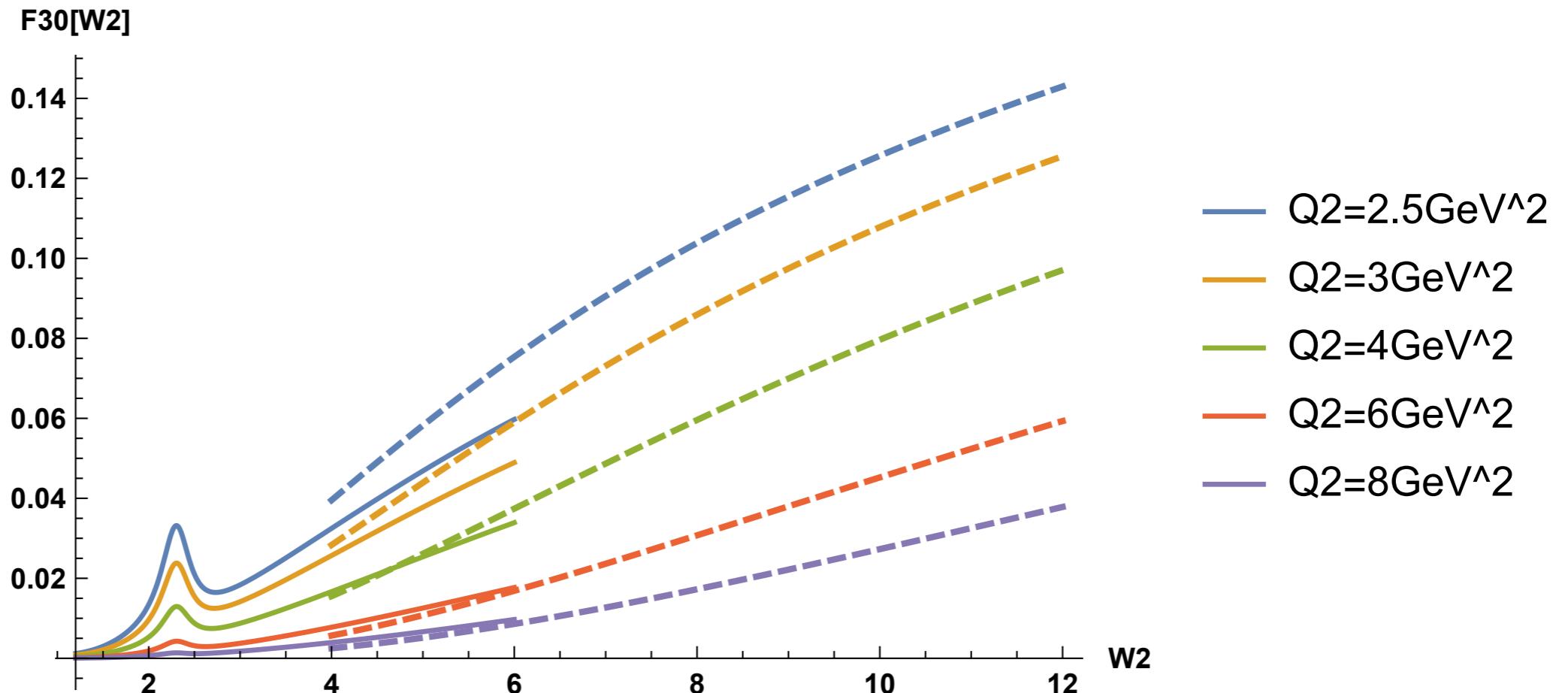
$$F_{1,\text{bgd}}^{\gamma\gamma} = \frac{W^2 - M^2}{8\pi^2\alpha} \left(1 + \frac{W^2 - (M+m_\pi)^2}{Q^2+Q_0^2}\right)^{-1} \sum_{i=1}^2 \frac{\sigma_T^{NR,i}(0) [W - (M+m_\pi)]^{(i+1)/2}}{(Q^2+a_i^T)^{(b_i^T+c_i^T Q^2+d_i^T Q^4)}}$$

$$\square^{\gamma W}_{A,\text{bgd}}(\text{DIS}) = .07 \times 10^{-3}$$

$$\square^{\gamma W}_{A,\text{bgd}}(\text{BC}) = .08 \times 10^{-3}$$



Resonance + Background:



Total contribution from the region $(M + m_\pi)^2 \leq W^2 \leq 4 \text{ GeV}^2$

$$\square_{A, \text{bgd+res}}^{\gamma W} = .31 \times 10^{-3}$$

(we are still verifying this)

DIS Contribution:

High Q^2 means the hadron looks like individual free quarks - so we use the parton model

$$\square_{\gamma W}^{A(\text{DIS})} = \frac{2\alpha}{\pi} \int_{2GeV^2}^{\infty} dQ^2 \frac{v_e(Q^2)}{Q^2(1+Q^2/M_W^2)} \times \int_0^{x_{\max}} dx F_3^{(0),\text{DIS}}(x, Q^2) \frac{(2r-1)}{r^2},$$

$$r \equiv 1 + \sqrt{1 + 4M^2x^2/Q^2}$$

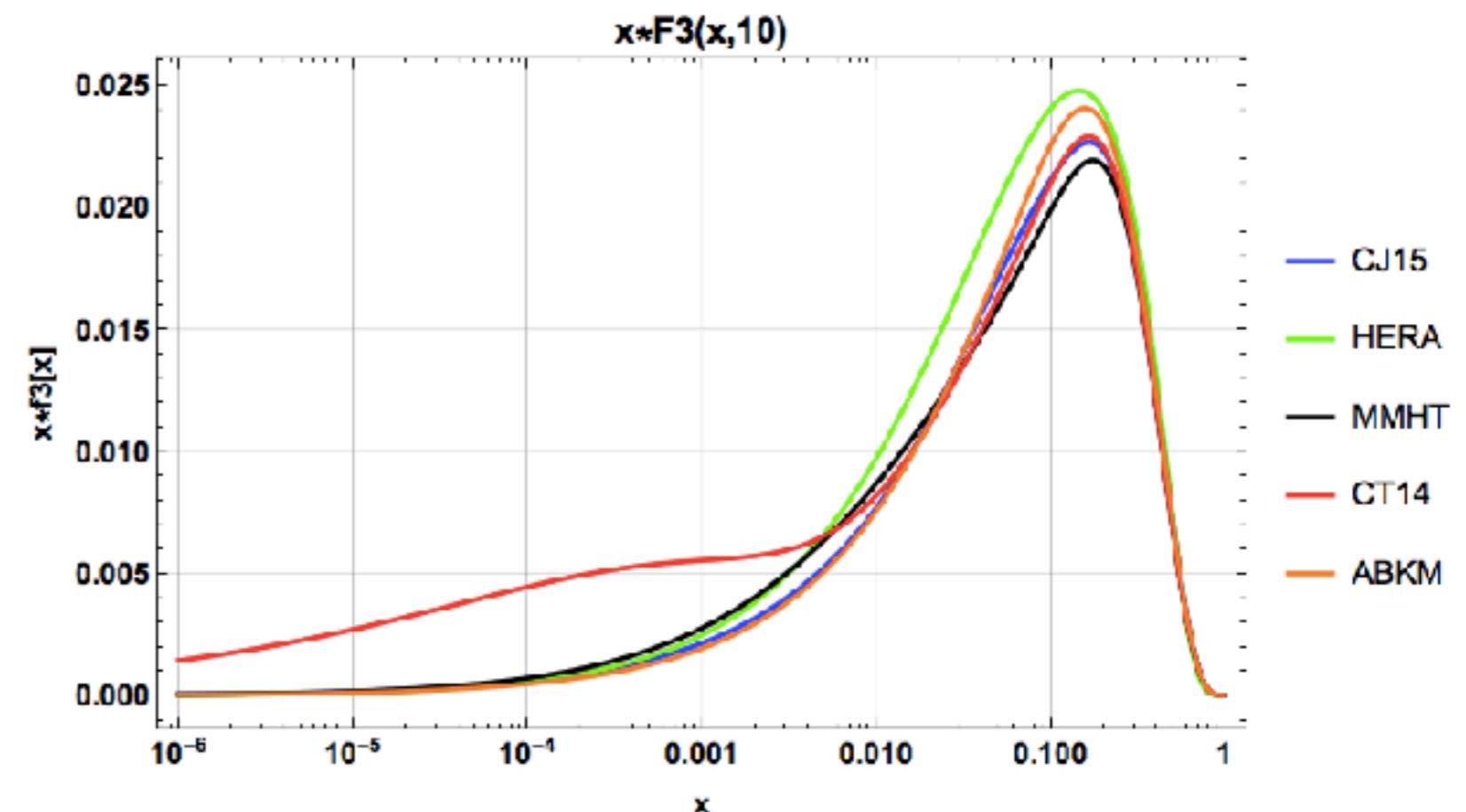
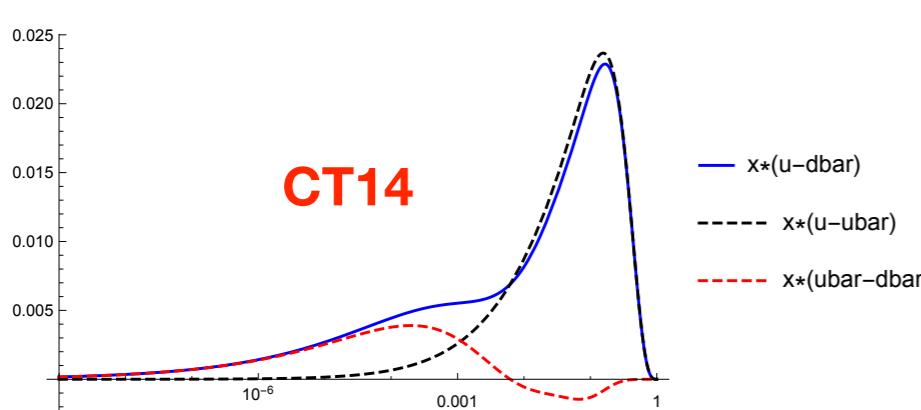
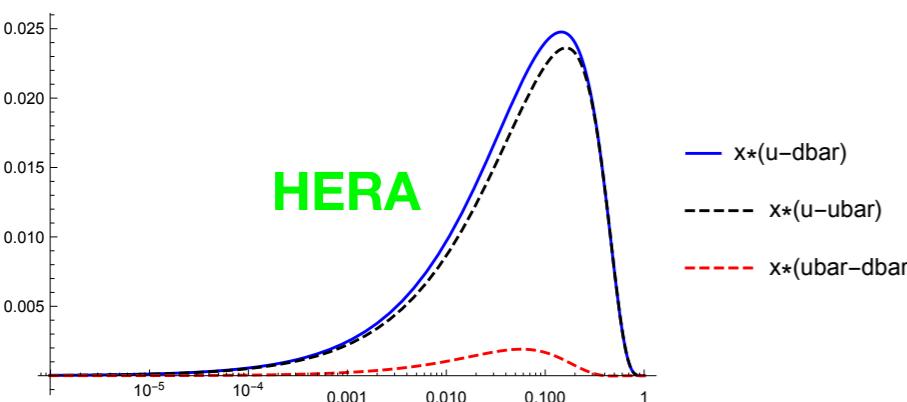
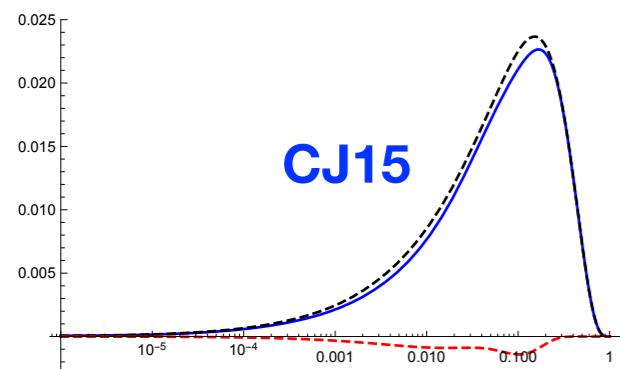
$$O(\alpha_S^0) : \quad F_{3,LO}^{(0)(\text{DIS})}(x, Q^2) = \frac{(u-\bar{d})}{24}$$

$$O(\alpha_S^1) : \quad F_{3,NLO}^{(0)(\text{DIS})}(x, Q^2) = \int_x^1 \frac{dz}{z} C_3^{(1)}(z) F_{3,LO}^{(0)(\text{DIS})}(x/z, Q^2)$$

In order to take into account the effects of the strong interaction, we take the convolution of the leading order $F_3^{(0)}$ valence distribution with the $O(\alpha_s)$ Wilson coefficient function.

PDF Curves are in general agreement between different groups for u_v and d_v
However, $\bar{u} - \bar{d}$ can vary significantly between LHA groups!

$$u - \bar{d} = (u - \bar{u}) + \bar{u} - \bar{d}$$



Gives our following results for the gW box contributions from the DIS region:

PDF set	u_v	$\bar{u} - \bar{d}$	higher power	total
MMHT14	.002356	-.000113	-.000007	.002073(50)
CJ15	.002359	-.000144	-.000004	.002040(25)
HERA20	.002361	+.000180	-.000008	.002348(40)
CT14	.002361	+.000664	-.000004	.002800(927)
ABKM09	.002359	-.000117	-.0000052	.002074(30)

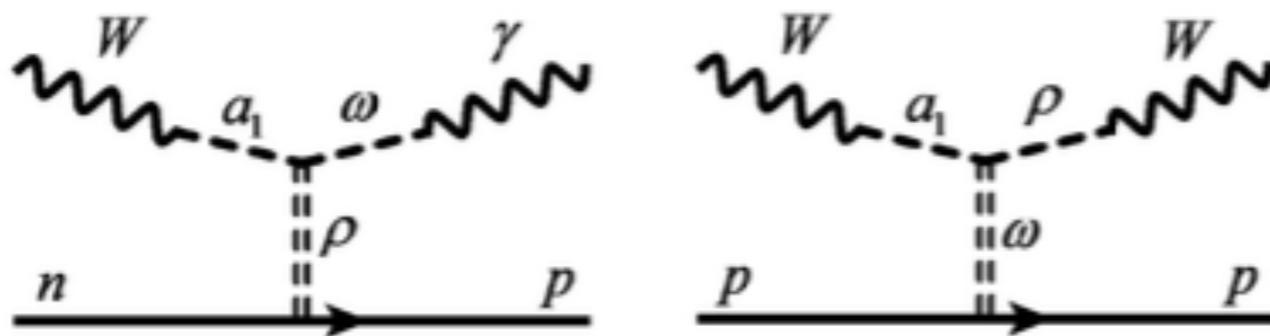
7. Regge Contribution:

Our model for F3 at $Q^2 \leq 2\text{GeV}^2$:

$$F_{3,Reg}^{(0)}(W^2, Q^2) = \frac{C(Q^2)}{(1+Q^2/m_\rho^2)(1+Q^2/m_{a_1}^2)} f_{th}(W) \left(\frac{\nu}{\nu_0}\right)^{\alpha_0}$$

$$f_{th}(W) = \Theta(W^2 - W_{th}^2) (1 - e^{(W_{th}^2 - W^2)/\Lambda_{th}^2})$$

$C(Q^2)$ is an unknown function with a Q^2 dependence that is not well-determined by theory



Diagrams: Seng et. al. Phys. Rev. Lett. **121**, 241804 (2018)

Idea: match this function to the well-known $F3(0)$ in the DIS region around $Q^2 = 2\text{GeV}^2$ AND constrain it from available data on $F3(\text{neutrino})$

VMD Theory:

$$F_i^{VDM} \sim \Sigma_V r_V \left(\frac{m_V^2}{Q^2 + m_V^2} \right)^2$$

GVMD Theory:

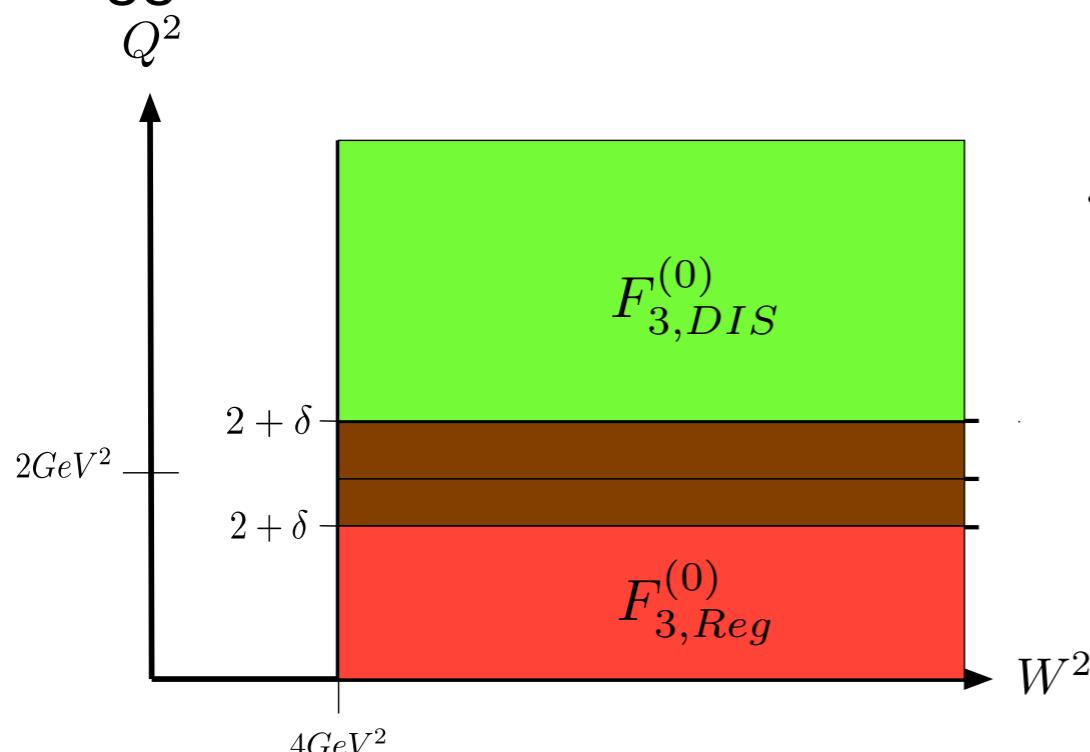
$$\begin{aligned} F_i^{GVDM} &\sim \Sigma_V r_V \left(\frac{m_V^2}{Q^2 + m_V^2} \right)^2 \left(1 + \xi_V \frac{Q^2}{m_V^2} \right) \\ &+ r_C \left[(1 - \xi_C) \frac{m_0^2}{Q^2 + m_0^2} + \xi_C \frac{m_0^2}{Q^2} \ln \left(1 + \frac{Q^2}{m_0^2} \right) \right] \\ &\approx 0 \end{aligned}$$

Based on this simplified GVDM model we take the 2 parameter ansatz:

$$C(Q^2) = d + eQ^2$$

Constraints from DIS data:

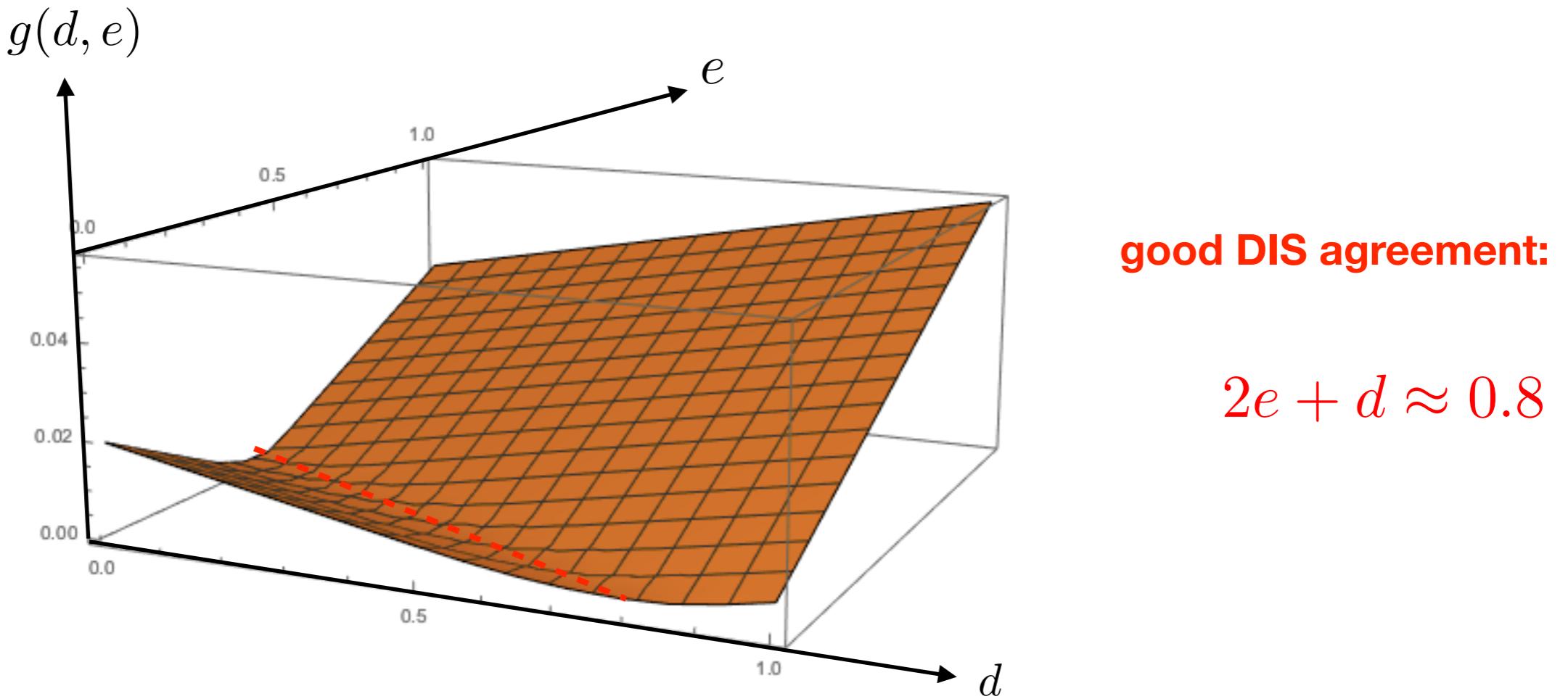
We then want to minimize the weighted area difference between the DIS and Regge structure functions on a narrow strip around $Q^2 = 2\text{GeV}^2$:



$$g(d, e) = \int_{2-\delta}^{2+\delta} dQ^2 \int_{4\text{GeV}^2}^{\infty} dW^2 \frac{1}{1+Q^2/M_W^2} \frac{1}{E_{min}} \left(\frac{2}{\chi} - \frac{1}{4ME_{min}} \right) [F_{3,DIS}^{(0)} - F_{3,Reg}^{(0)}(d, e)]$$

minimize g:

$$\begin{aligned} \nabla g(d, e) &= 0 \\ \text{Hess}(g) &> 0 \end{aligned}$$



Constraints from Neutrino scattering data:

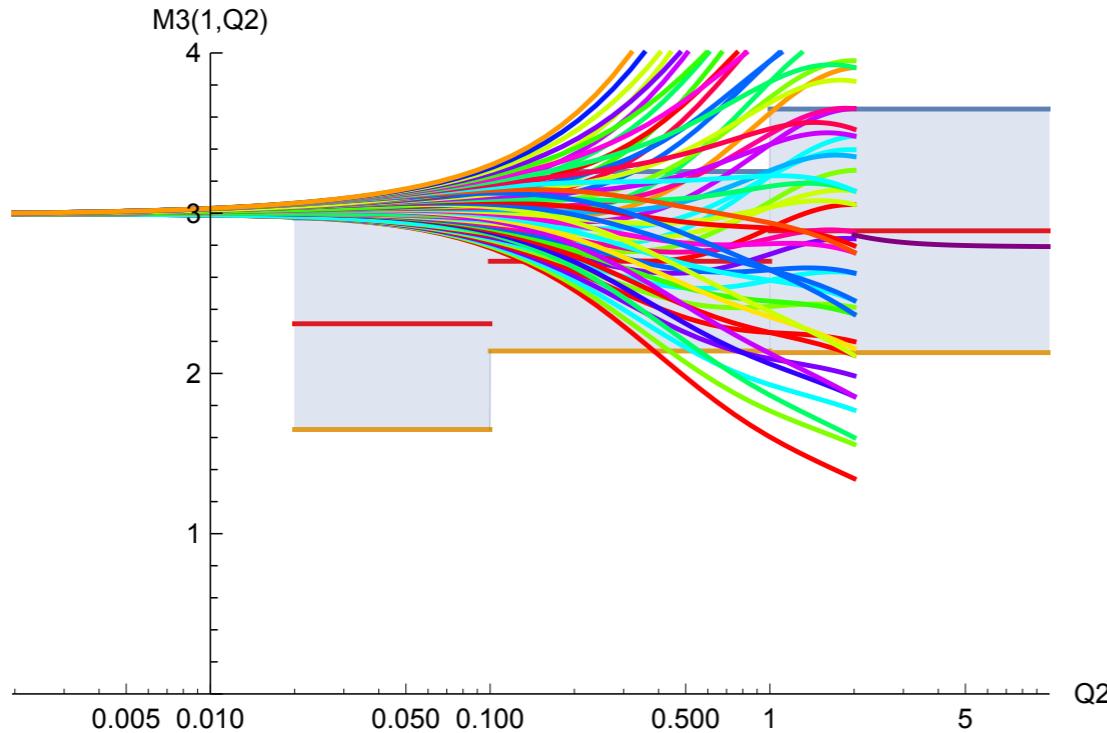
In the Regge region: $F_3^{(0)} \approx \frac{1}{36} F_3^{\nu p + \bar{\nu} p}$

Some data exists on the 1st Nachtmann moment of $F_3^{\nu p + \bar{\nu} p}$

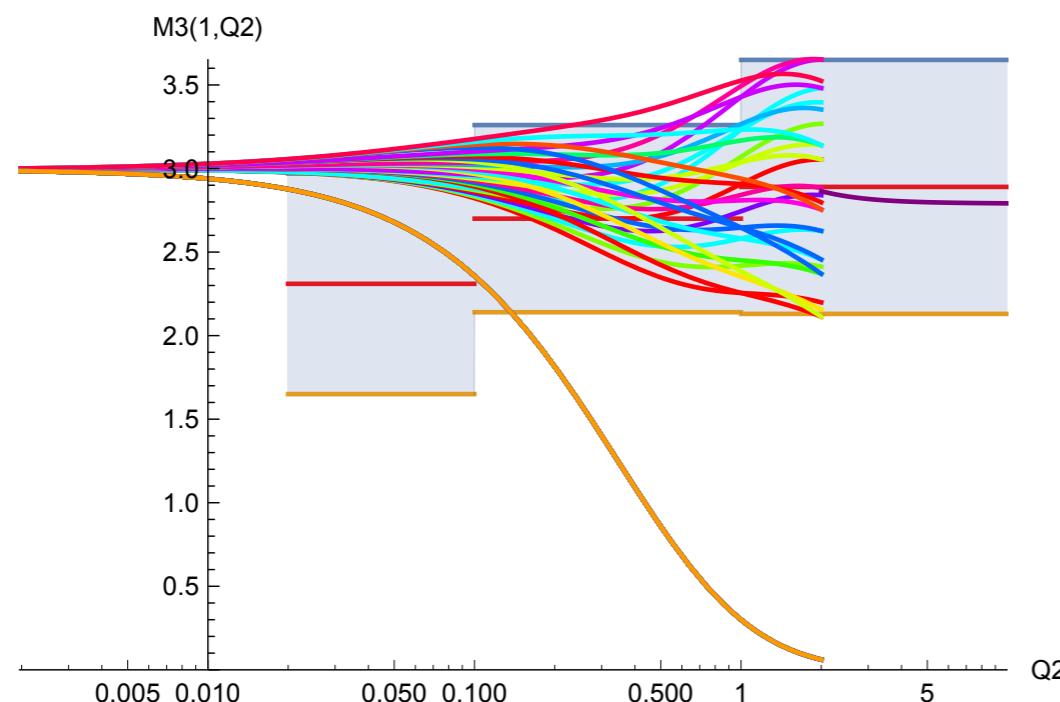
$$M_3^{\nu p + \bar{\nu} p}(1, Q^2) = \frac{2}{3} \int_0^1 dx \frac{\xi}{x^2} \left(2x - \frac{\xi}{2}\right) [F_{3,el}^{\nu p + \bar{\nu} p} + F_{3,res}^{\nu p + \bar{\nu} p} + 36 F_{3,Reg}^{(0)}]$$

and this term $\sim d, e$

Reparametrize: $C(Q^2) = [d + eQ^2] = f[1 + gQ^2]$



$$0 \leq f(1 + g) \leq 1$$

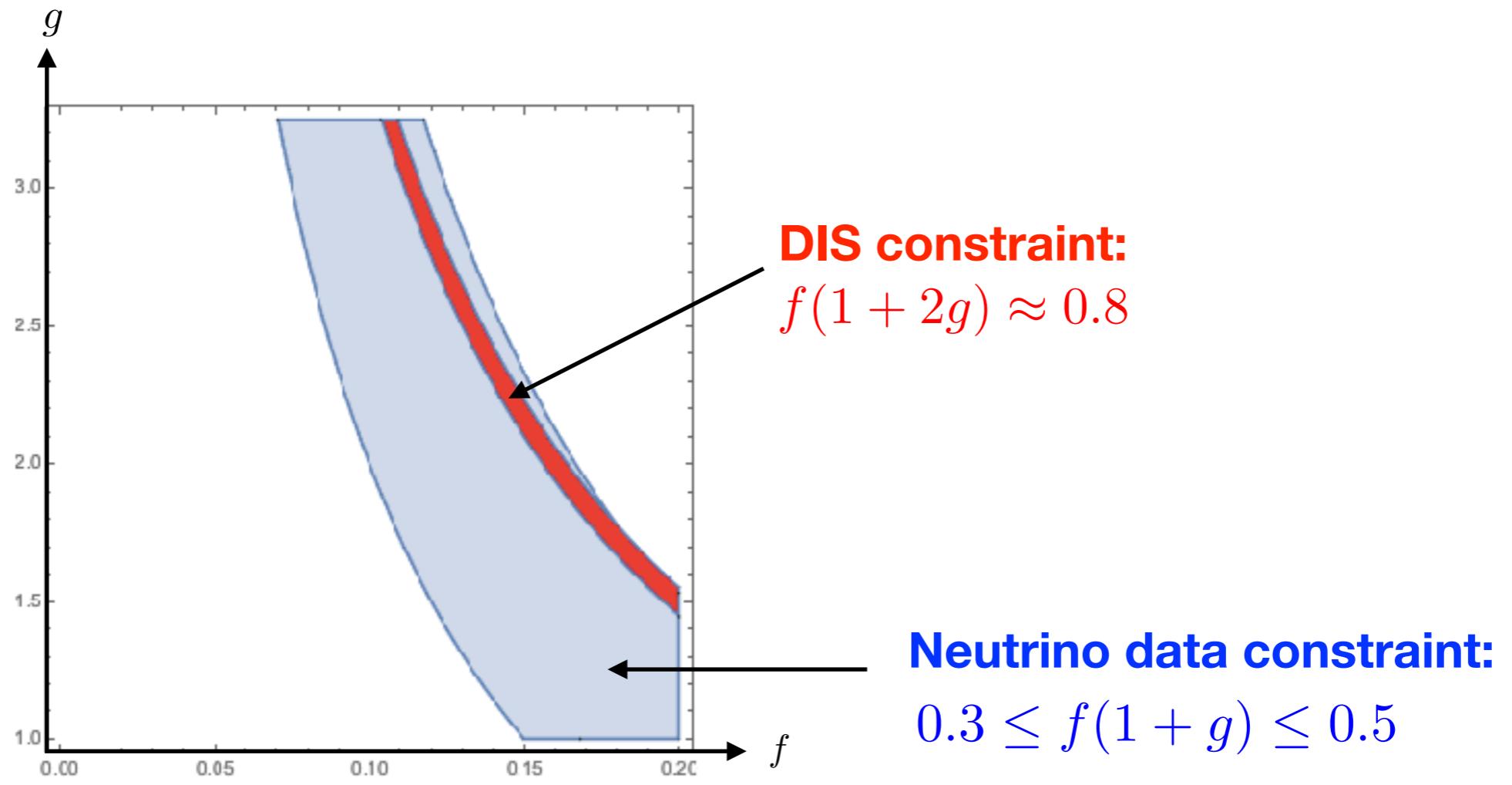


$$0.3 \leq f(1 + g) \leq 0.5$$

Also:

$$g \geq 1$$

$$f \leq .2$$



$$F_{3,Reg}^{(0)}(W^2, Q^2) = \frac{f[1+gQ^2]}{(1+Q^2/m_\rho^2)(1+Q^2/m_{a_1}^2)} f_{th}(W) \left(\frac{\nu}{\nu_0}\right)^{\alpha_0}$$

Regge Region contributions: $4GeV^2 \leq W^2, Q^2 \leq 2GeV^2$

- $(f, g) = (.16, 1) \rightarrow \square_{A,min}^{\gamma W} = 0.46 \times 10^{-3}$
- $(f, g) = (.1, 2.75) \rightarrow \boxed{\square_{A,central}^{\gamma W} = 0.51 \times 10^{-3}}$ **(still deciding on this)**
- $(f, g) = (.16, 2) \rightarrow \square_{A,max}^{\gamma W} = 0.69 \times 10^{-3}$

Conclusion:

Region	$\square_A^{\gamma W} (\times 10^{-3})$
elastic	1.12(8)
res + bgd	0.31(?)
DIS(CJ15)	2.04(3)
Regge	$0.51^{+.18}_{-.07}$
TOTAL	$3.98^{+.22}_{-.11}$



$$\Delta_R^V = 0.01709 + 2\square_A^{\gamma W}$$



$$|V_{ud}|^2 = \frac{2984.43s}{\bar{F}t(1+\Delta_R^V)}$$



$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = .998447^{+.001059}_{-.001309}$$