Laws of Black Hole Thermodynamics in Semiclassical Gravity

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Historical Background

1973: Laws of black hole mechanics resemble the laws of thermodynamics

1975: Black holes have positive temperature

1976: The similarities between the laws of black hole mechanics and thermodynamics have thus physical significance. Desirable to interpret black hole's entropy

Semiclassical Gravity

Classical spacetime geometry

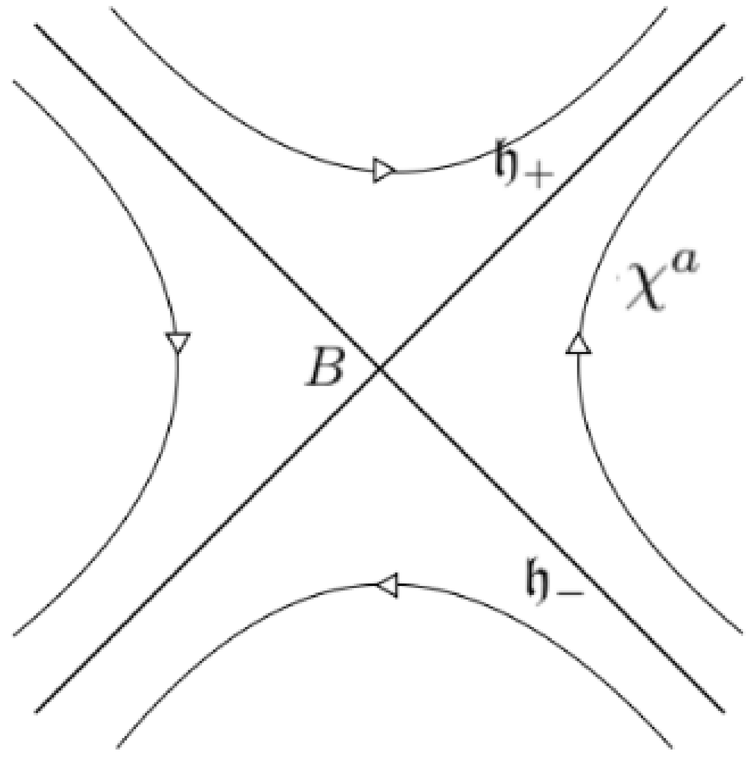
$$G_{ab} = 8\pi \langle T_{ab} \rangle_{\mathrm{reg}}$$

Quantum matter + Regularisation

Zeroth Law

Originally assumed GR + dominant energy condition

Independent formulation works as a **kinematic** property of bifurcate Killing horizons



Zeroth Law

Surface gravity is constant throughout a Killing horizon

$$\nabla^a(\chi^b\chi_b) = -2\kappa\chi^a$$

Existence of Thermal States

The restriction of the "vacuum" with respect to a globally defined timelike Killing field to a wedge is thermal with temperature

$$T = \frac{\kappa}{2\pi}$$

First Law

Hamiltonian description for the semiclassical theory splits into vacuum and matter parts for stationary spacetimes

$$\delta H_\chi = \int_\Sigma (\delta \mathbf{j}^g_\chi + \langle \delta \mathbf{j}^\psi_\chi \rangle) + \int_{\partial\Sigma} \chi \cdot (\mathbf{\Theta}^g + \langle \mathbf{\Theta}^\psi \rangle)$$

First Law

Relationship between changes in expectation value of Hamiltonian and quantities at infinity.

$$\langle \delta H_\chi \rangle = -\int_B \mathbf{Q}^g + \int_\infty [\mathbf{Q}^g + \chi \cdot \mathbf{B}^g]$$

For perturbations around a thermal state

$$\frac{\kappa}{2\pi}\delta(S_{\mathrm{vN}} + S_{\mathrm{NC}}) = \delta\mathcal{E}_\chi$$

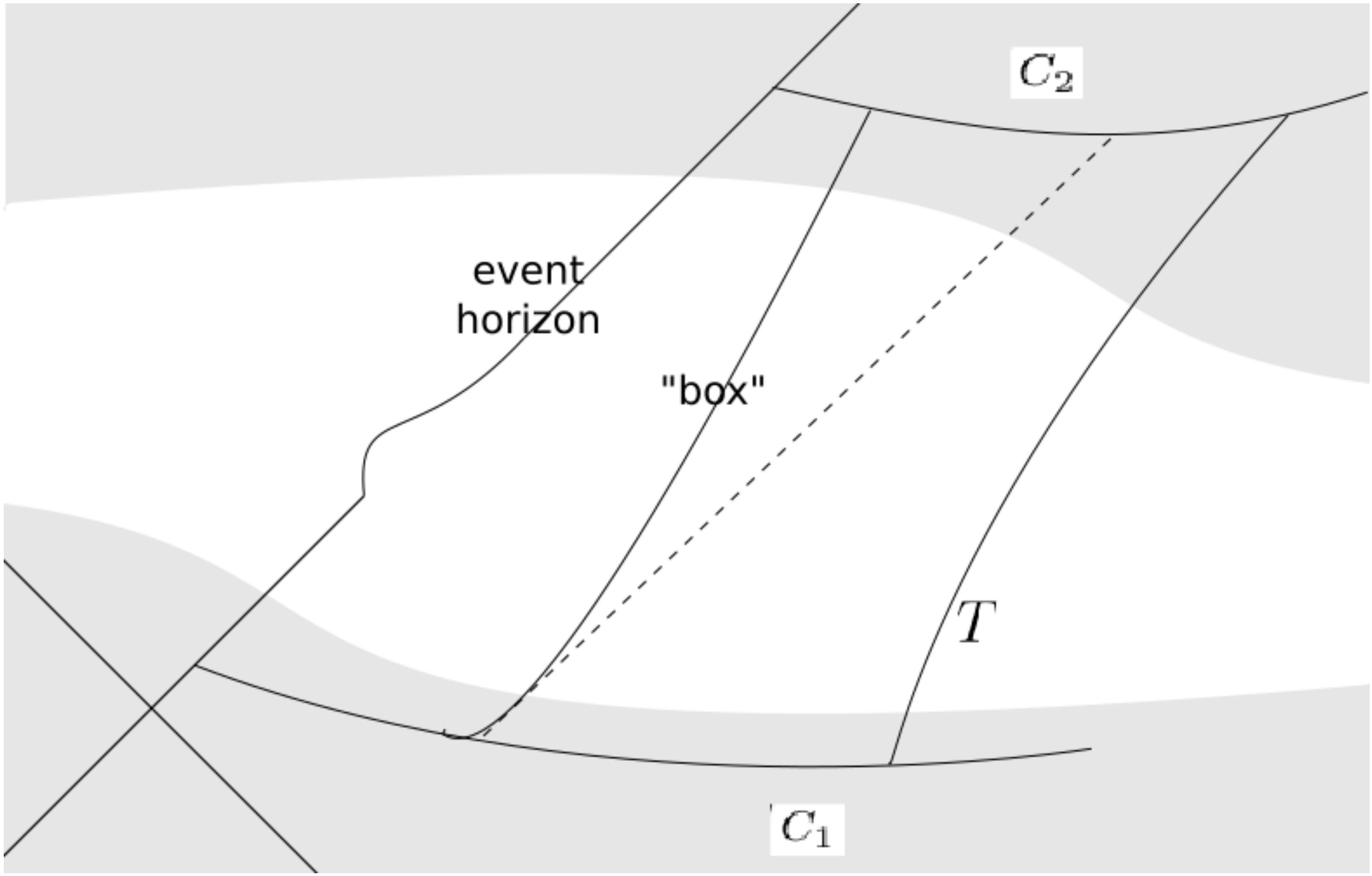
Second Law

Spacetime that evolves from stationary in the past to a different stationary state in the future

Equilibrium cannot be stable unless the fields are confined into finite volume

Fixed boundary term in the action, e.g. Hawking-Gibbons-York

Second Law



$$\Delta\langle\delta H_\chi\rangle = -\Delta\int \delta\mathbf{Q}^g[\chi]$$

Second Law

Property of the relative entropy

$$S(\rho|\sigma) \equiv \mathrm{Tr}(\rho \log \rho) - \mathrm{Tr}(\rho \log \sigma)$$

$$S(\rho_{\mathrm{red}}|\sigma_{\mathrm{red}}) \leq S(\rho|\sigma)$$

For carefully chosen reference state

$$\Delta\delta(S_{\mathrm{vN}} + S_{\mathrm{NC}}) \geq 0$$

$$S_{\mathrm{NC}\ 1,2} = \frac{2\pi}{\kappa_{1,2}} \int \mathbf{Q}^g$$

Second Law

For thermal state itself

$$\Delta S_{\mathrm{vN}} = -\Delta S_{\mathrm{NV}}$$

Generically

$$\Delta(S_{\mathrm{vN}} + S_{\mathrm{NC}}) \geq 0,$$

Origin of Entropy

Derivation suggests that B-H entropy is accounting for the information hidden behind horizon.

Thank you

Submitted to PRD. Pre-print available at arXiv: