Asymptotic Safety in the Conformal Hidden Sector

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1. Asymptotic Safety

2. Conformal Symmetry

3. Putting the Ingredients Together: Asymptotic Safety and Conformal Symmetry
References

Collaborators
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1. Asymptotic Safety

2. Conformal Symmetry

3. Putting the Ingredients Together: Asymptotic Safety and Conformal Symmetry
A fundamental theory requires a UV fixed point in coupling(s) \( \beta(g^*) = 0 \) where \( \mu \frac{dg}{d\mu} = \beta(g) \) see Wilson (1971)

Amongst other issues Standard Model does not meet Wilson’s criterion: e.g., U(1) coupling has Landau pole in UV

UV fixed points can be classified in two ways:
- Asymptotically free (Gaussian fixed point): \( g(\mu) \rightarrow 0, \mu \rightarrow \infty \) (e.g., QCD)
- Asymptotically Safe (interacting fixed point): \( g(\mu) \rightarrow g^* \neq 0, \mu \rightarrow \infty \)

Original ideas of asymptotic safety introduced by Weinberg (1979)

Weinberg, GR: Einstein Centenary Survey, Eds. Hawking & Israel

c.1980: Weinberg’s motivation for asymptotic safety was a different perspective on renormalizability of quantum gravity

c. 2010: Asymptotic safety may provide a powerful principle for extending the Standard Model
Focus on *essential couplings* of theory: e.g., anomalous field dimensions are inessential and ignored in asymptotic safety

Asymptotic safety has two ingredients:

- A fixed point \( g^* = \{g_1, g_2, \ldots \} \) for essential couplings: \( \beta_i(g^*) = 0 \),
  \[ \mu \frac{dg_i}{d\mu} = \beta_i(g) \]
- Couplings lie on a renormalization-group (RG) trajectory that reaches the fixed point

The surface formed by these asymptotic safety renormalization-group trajectories is the *UV critical surface*
Number of free coupling parameters in asymptotically-safe theory is the dimension of the UV critical surface.

If UV critical surface has dimension $C$ then require $C - 1$ (dimensionless) coupling parameters defining a trajectory and one dimensionful scale identifying a point.

Worst case: $C = 0$ no asymptotic safety

Best case: $C = 1$ only a scale needs to be specified

**Key Message**

Asymptotic safety constrains the parameter space of a theory through its UV critical surface.
Dimension of UV critical surface can be determined by behaviour near critical point

\[ \mu \frac{dg_i}{d\mu} \approx B_{ij} (g_j - g_j^*) , \quad B_{ij} = \left. \frac{\partial \beta_i}{\partial g_j} \right|_{g=g^*} \]

\[ g_i - g_i^* = \sum_k C_k V_i^{(k)} \mu^{\lambda_k} \]

where \( V^{(k)} \) is an eigenvector of \( B \) with eigenvalue \( \lambda_k \)

- If \( \lambda_k > 0 \) then \( C_k = 0 \) so that \( g \to g^* \) as \( \mu \to \infty \)
- Dimension of UV critical surface is the number of negative eigenvalues of \( B \), \( C_k \) corresponds to the parameters

See e.g., Weinberg (1979); Litim & Sannino JHEP 1406.2337; Hansen, Mann, Wang, TGS et al, PRD 1706.06402
Asymptotic Safety

Conformal Symmetry

Putting the Ingredients Together: Asymptotic Safety and Conformal Symmetry
Conformally symmetric theories of gravity with couplings to scalars (Higgs $H$, additional scalar $S$) such as see e.g., Stelle (1977), Shapiro (1997), Einhorn & Jones (2015)

$$S = \int d^4 x \sqrt{|\det g|} \left[ \frac{R^2}{6 f_0^2} + \frac{1}{3} \frac{R^2 - R_{\mu\nu}^2}{f_2^2} - \xi_H |H|^2 R - \xi_S |S|^2 R \right]$$

have a number of interesting features including:

- Induced Einstein-Hilbert gravity at low-energy via symmetry breaking in scalar sector see Zee (1979), Adler (1982), Holdom & Ren (2016), Salvio & Strumia (2014)
- Renormalizable and asymptotically free see e.g., Stelle (1977), Fradkin & Tseytlin (1982)
- Various attempts to solve problems with ghost fields see e.g., Einhorn & Jones (2015), Holdom & Ren (2016), Donoghue & Menezes (2018)
Conformal Symmetry: Standard Model Motivation

- Conventional EW symmetry breaking leads to unnaturally large separation between unification and EW scale (Georgi, Quinn, Weinberg, 1974)
- Massless scalars allow a natural scale hierarchy for embedding of SM below a unification scale (Weinberg, 1979)
- Bardeen (1995) suggests that Lagrangian conformal invariance would protect Higgs mass from approaching unification scale: idea similar to dimensional transmutation in QCD
- Explosion of interest in Bardeen’s idea (Hill 1401.4185 literature survey and superb overview, CFT arguments in Tavares et al 1308.0025)
- Coleman-Weinberg mechanism (1973): effective potential used to realize EW symmetry-breaking via loop effects
- Can find a Higgs mass consistent with 125 GeV but some tension from larger predicted value of Higgs self-coupling (Wang, TGS, PRL, 2013)
- Connection to Asymptotic safety: softly broken conformal symmetry restored with $\beta(g^*) = 0$
Consider an extension of the standard model: leptophobic $U'_B(1)$ with an extended scalar sector.

Asymptotic Safety: predictive power to connect UV and IR physics, constraining parameters of theory.

Conformal Symmetry: constrain structure of theory, including gravity.

Gravity: treat gravity contributions to $\beta(g)$ using effective field theory analysis.

Purpose is not to promote this particular model but to illustrate how the ingredients of asymptotic safety and conformal symmetry can guide UV extensions of the Standard Model.
Leptophobic $U_B'(1)$ Model

- Lagrangian in Scalar Sector of Standard Model with $U_B'(1)$ extension see e.g., Hashimoto, Iso, Orikasa (2014)

$$L = D_\mu H^\dagger D^\mu H + D_\mu S^\dagger D^\mu S - \lambda_2 |S|^2 H^\dagger H - \lambda_3 |S|^4 - \lambda_1 (H^\dagger H)^2$$

- $S$ is a complex singlet and in basis where kinetic gauge terms diagonal ($g_m$ comes from $U(1)$ kinetic mixing)

$$D_\mu = \partial_\mu - ig_3 \frac{\lambda_a}{2} G^a_\mu - ig_2 \frac{\tau_i}{2} W^i_\mu - iY (g_Y B_\mu + g_m B'_\mu) - ig' Q'_B B'_\mu$$

- Assume $U_B'(1)$ couples only to third generation (FCNC constraints); spectator fermions needed for anomaly cancellation
### Table: Standard Model fermion charge assignments

<table>
<thead>
<tr>
<th>Gauge group</th>
<th>$q^u,d_{L,R}$</th>
<th>$u_R$</th>
<th>$d_R$</th>
<th>$q^c,s_{L,R}$</th>
<th>$c_R$</th>
<th>$s_R$</th>
<th>$t^{,b}_{L,R}$</th>
<th>$t_R$</th>
<th>$b_R$</th>
<th>$l_L$ (all gens)</th>
<th>$e_R$ (all gens)</th>
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<tr>
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<tr>
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<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
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<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>$U(1)_Y$</td>
<td>1/3</td>
<td>4/3</td>
<td>-2/3</td>
<td>1/3</td>
<td>4/3</td>
<td>-2/3</td>
<td>1/3</td>
<td>4/3</td>
<td>-2/3</td>
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<td>-2</td>
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<tr>
<td>$U(1)'$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>0</td>
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</table>

### Table: Scalar charge assignments

<table>
<thead>
<tr>
<th>Gauge group</th>
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<th>$H$</th>
</tr>
</thead>
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<td>$SU(3)_c$</td>
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<td>1</td>
</tr>
<tr>
<td>$SU(2)_L$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$U(1)_Y$</td>
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<td>1/2</td>
</tr>
<tr>
<td>$U(1)'$</td>
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<td>0</td>
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### Table: Exotic fermion charge assignments

<table>
<thead>
<tr>
<th>Gauge group</th>
<th>$\nu_R$</th>
<th>$\psi^l_{L,R}$</th>
<th>$\psi^t_{R,L}$</th>
<th>$\psi^e_{L,R}$</th>
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<tbody>
<tr>
<td>$SU(3)_c$</td>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$SU(2)_L$</td>
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<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$U(1)_Y$</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>$U(1)'$</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>
Symmetry Breaking

- Coleman-Weinberg mechanism generates vev $v_1$ for $S$, which qualitatively requires $\lambda_3$ to change sign from positive to negative near $v_1$
- $S$ mass generated dynamically by Coleman-Weinberg mechanism (determined by effective potential $V$): $V$ has minimum at vev and mass determined by second derivative at vev
- Since $S$ couples to $Z'$, vev $v_1$ generates $Z'$ mass
- $S$ vev triggers conventional EW symmetry breaking at lower EW scale $v$, generating Higgs mass: requires negative $\lambda_2$

$$V = \lambda_2 |S|^2 H^\dagger H + \lambda_3 |S|^4 + \lambda_1 (H^\dagger H)^2$$

- Four symmetry-breaking constraints:
  - $\frac{dV}{d\varphi} \bigg|_{\varphi=v_1} = 0$, $S = \frac{1}{\sqrt{2}} (\varphi_1 + i\varphi_2)$, $\varphi^2 = \sum_i \varphi_i^2$
  - $2v^2 \lambda_1 (v_1) = -v_1^2 \lambda_2 (v_1) = M_H^2$
  - $M_{Z'} = 2g' (v_1) v_1$
Gravity Contributions to $\beta$ Functions

- Gravity treated as effective theory: leading order result for gravity contribution to $\beta$ functions for coupling $x_j$. See Shaposhnikov and Wetterich PLB (2010)

$$\beta_{\text{grav}}^j = \frac{a_j}{8\pi} \frac{k^2}{M_p^2} x_j$$

- Key question is the sign of $a_j$
  - $a_j < 0$ implies gravity will make coupling asymptotically free, stabilizing UV behaviour
  - $a_j > 0$ requires $x_j = 0$ to be an asymptotically safe fixed point to control UV behaviour from gravity

- All gauge couplings have $a_j < 0$. Shaposhnikov & Wetterich PLB (2010), Robinson & Wilczek PRL (2006)
- All SM Yukawa couplings have $a_j < 0$. Shaposhnikov & Wetterich PLB (2010), Shapiro CQG (1989)
- Quartic scalar couplings have $a_j > 0$. Percacci & Perini PRD (2003), Narain & Percacci CQG (2010)
- Situation for Higgs portal coupling $\lambda_2$ unknown: assume $a_j < 0$
Taking into account the leading-order contributions of gravity, require a UV boundary condition at a scale $M_{UV}$

\[ \beta \lambda_1 (M_{UV}) = \beta \lambda_3 (M_{UV}) = \lambda_1 (M_{UV}) = \lambda_3 (M_{UV}) = 0 \]

Have to make a choice for the UV boundary condition for $\lambda_2$: interesting choice is $\lambda_2 (M_{UV}) = 0$ so that the Higgs portal interaction is generated radiatively

There could be other realizations of asymptotic safety for $\beta \lambda_3 < 0$ (non-perturbative gravity contributions) which would allow greater freedom in parameters of model, but highly speculative

\[ \beta \lambda_1 (M_{UV}) = \lambda_1 (M_{UV}) = 0; \quad \lambda_2 (M_{UV}), \lambda_3 (M_{UV}) \neq 0 \]

\[ \beta \lambda_1 (M_{UV}) = \lambda_1 (M_{UV}) = \lambda_2 (M_{UV}) = 0; \quad \lambda_3 (M_{UV}) \neq 0 \]
\begin{align*}
16\pi^2 \beta_{\lambda_1} &= \lambda_2^2 - 3\lambda_1 g_m^2 + \frac{3}{8} g_m^4 + \beta_{\lambda_1}^{SM} \\
16\pi^2 \beta_{\lambda_2} &= 12g_m^2 g'^2 + 6Y_t^2 \lambda_2 - 24g'^2 \lambda_2 + 4Y_M^2 \lambda_2 + 4\lambda_2^2 + 12\lambda_1 \lambda_2 + 8\lambda_2 \lambda_3 \\
&\quad - \frac{3}{2} \lambda_2 (g_m^2 + 3g_2^2 + g_1^2) \\
16\pi^2 \beta_{\lambda_3} &= 96g'^4 - 16Y_M^4 + 2\lambda_2^2 - \lambda_3 (48g'^2 + 8Y_M^2 - 20\lambda_3) \\
16\pi^2 \beta_{Y_t} &= -\frac{17}{12} Y_t g_m^2 - \frac{2}{3} Y_t g'^2 - \frac{5}{3} Y_t g' g_m + \beta_{Y_t}^{SM} \\
16\pi^2 \beta_{g'} &= \frac{1}{18} g' (76g'^2 + 64g' g_m + 123g_m^2) \\
16\pi^2 \beta_{g_m} &= g_m \left( \frac{41}{6} (g_m^2 + 2g_1^2) + \frac{38}{9} g'^2 \right) + \frac{32}{3} g' (g_m^2 + g_1^2) \\
16\pi^2 \beta_{Y_M} &= -6Y_M g'^2 + 6Y_M^3; \quad 32\pi^2 \gamma_\varphi = Y_M^2 - 24g'^2 \\

Y_t \text{ is top Yukawa, } Y_M \text{ is Majorana-Yukawa coupling of } S \text{ to the spectator RH neutrino } m_{\nu_R} = \sqrt{2} v_1 Y_M (v_1)
\end{align*}
Reduce system of nine unknowns \((\lambda_1, \lambda_2, \lambda_3, g', g_m, Y_M, Y_t, v_1, M_{UV})\) to that of five \((g_m, v_1, Y_M, Y_t, M_{UV})\) using the four symmetry-breaking constraints.

Solve RG equations subject to the four UV boundary conditions.

One-parameter family of solutions for different choices of \(M_{Z'}\).

Example solution shown in Figure 1.

Figure: Running scalar couplings are shown as a function of the scale \(t = \log \left( \frac{\varphi}{\langle S \rangle} \right)\). The red, blue, and green curves represent \(\lambda_1(t), 200\lambda_2(t), 2000\lambda_3(t)\) respectively.
Results

Wang, Sage, Steele, Mann


- Results for three different choices of $M_{Z'}$ shown in Table
- Couplings referenced to scale $\nu_1$
- Find $M_{UV} \sim M_{pl}$ and the predicted $Y_t(\nu) = 0.93$ very close to the current experimental central value $Y_t(\nu) = 0.936$ Buttazzo et al JHEP (2013)
- Implicitly assumed spectator fermions active in RG equations: masses must be less than $\nu_1$

<table>
<thead>
<tr>
<th>$M_{Z'}$ (TeV)</th>
<th>$10^6 \lambda_2$</th>
<th>$10^6 \lambda_3$</th>
<th>$g'$</th>
<th>$g_m$</th>
<th>$Y_M$</th>
<th>$Y_t$</th>
<th>$m_{\nu_R}$ (TeV)</th>
<th>$\nu_1$ (TeV)</th>
<th>$m_S$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9</td>
<td>-6</td>
<td>-4.05</td>
<td>0.18</td>
<td>0.042</td>
<td>0.28</td>
<td>0.77</td>
<td>2.02</td>
<td>5.1</td>
<td>20</td>
</tr>
<tr>
<td>3.4</td>
<td>-0.59</td>
<td>-0.13</td>
<td>0.1</td>
<td>0.023</td>
<td>0.16</td>
<td>0.74</td>
<td>3.72</td>
<td>16.9</td>
<td>11.5</td>
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<tr>
<td>6.8</td>
<td>-0.036</td>
<td>-0.002</td>
<td>0.05</td>
<td>0.011</td>
<td>0.08</td>
<td>0.70</td>
<td>7.59</td>
<td>68.8</td>
<td>6</td>
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</table>
Other Phenomenology

- Investigated several of collider signals related to $Z'$, see Table for example

Table: $Z'$ predicted production cross sections $\sigma_{pp \to Z'} \times BR$ (units of fb)

<table>
<thead>
<tr>
<th>$M_{Z'}$</th>
<th>Channel</th>
<th>$\sqrt{s} = 8$ TeV</th>
<th>$\sqrt{s} = 13$ TeV</th>
<th>$\sqrt{s} = 14$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9 TeV</td>
<td>Diboson</td>
<td>0.013</td>
<td>0.058</td>
<td>0.069</td>
</tr>
<tr>
<td></td>
<td>Dilepton</td>
<td>4.27</td>
<td>18.59</td>
<td>22.14</td>
</tr>
<tr>
<td></td>
<td>Dijet</td>
<td>19.47</td>
<td>84.79</td>
<td>100.98</td>
</tr>
<tr>
<td></td>
<td>Ditop</td>
<td>5.67</td>
<td>24.71</td>
<td>29.42</td>
</tr>
<tr>
<td>3.8 TeV</td>
<td>Diboson</td>
<td>$9.15 \times 10^{-5}$</td>
<td>$1.62 \times 10^{-3}$</td>
<td>$2.19 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>Dilepton</td>
<td>0.029</td>
<td>0.518</td>
<td>0.701</td>
</tr>
<tr>
<td></td>
<td>Dijet</td>
<td>0.110</td>
<td>1.95</td>
<td>2.64</td>
</tr>
<tr>
<td></td>
<td>Ditop</td>
<td>0.028</td>
<td>0.491</td>
<td>0.665</td>
</tr>
<tr>
<td>6.8 TeV</td>
<td>Diboson</td>
<td>$7.50 \times 10^{-13}$</td>
<td>$4.90 \times 10^{-7}$</td>
<td>$1.22 \times 10^{-6}$</td>
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<tr>
<td></td>
<td>Dilepton</td>
<td>$2.40 \times 10^{-10}$</td>
<td>$1.57 \times 10^{-4}$</td>
<td>$3.90 \times 10^{-4}$</td>
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<td>Dijet</td>
<td>$9.41 \times 10^{-10}$</td>
<td>$6.14 \times 10^{-4}$</td>
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<td>Ditop</td>
<td>$2.48 \times 10^{-10}$</td>
<td>$1.62 \times 10^{-4}$</td>
<td>$4.03 \times 10^{-4}$</td>
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</table>
Related Work

- Mann, Meffe, Sannino, Steele, Wang, Zhang, PRL 119 (2017) 261802
- Extend Standard Model with $N_f$ vector-like fermions with various charge assignments: (i) $N_F (3, 2, 1/6)$; (ii) $N_{F3} (3, 1, 0) \oplus N_{F2} (1, 2, 1/2)$; (iii) $N_{F3} (3, 1, 0) \oplus N_{F2} (1, 3, 0) \oplus N_{F1} (1, 1, 1)$
- Assume fermions all become active at 2 TeV
- Large $N_f$ methods lead to poles in RG functions that can drive fixed points
- Can realize asymptotic safety if Higgs quartic coupling is enhanced beyond its SM expectation (say factor of 2 at EW scale)

![Diagram](image)

**Figure:** Model (ii) $N_{F3} = 40, N_{F2} = 24$. Higgs quartic coupling is chosen to be 0.0034.
Asymptotic safety provides a valuable principle for constraining structure and parameters of theory from UV behaviour.

Effective theory approach to gravity contributions to $\beta$ functions can help achieve asymptotic safety by identifying couplings that require UV stabilization and stabilizing some that may otherwise be uncontrolled.

Lagrangian conformal symmetry not only guides structure of Lagrangians but adds dynamical symmetry-breaking constraints to constrain parameters of theory.
Thank you!