Asymptotic Safety in the Conformal Hidden Sector

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References

Collaborators

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- Wang, Sage, Steele, Mann, J. Phys. G45 (2018) 095002, 1511.0253

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Asymptotic Safety: Overview

- A fundamental theory requires a UV fixed point in coupling(s) $\beta(g^*) = 0$ where $\mu \frac{dg}{d\mu} = \beta(g)$ see Wilson (1971)
- Amongst other issues Standard Model does not meet Wilson's criterion: e.g., U(1) coupling has Landau pole in UV
- UV fixed points can be classified in two ways:
  - Asymptotically free (Gaussian fixed point): $g(\mu) \to 0, \mu \to \infty$ (e.g., QCD)
  - *Asymptotically Safe* (interacting fixed point): $g(\mu) \to g^* \neq 0, \mu \to \infty$
- Original ideas of asymptotic safety introduced by Weinberg (1979)

  Weinberg, GR: Einstein Centenary Survey, Eds. Hawking & Israel
- c.1980: Weinberg's motivation for asymptotic safety was a different perspective on renormalizability of quantum gravity
- c. 2010: Asymptotic safety may provide a powerful principle for extending the Standard Model

Asymptotic Safety: UV Critical Surface

- Focus on *essential couplings* of theory: e.g., anomalous field dimensions are inessential and ignored in asymptotic safety See Weinberg for definition of inessential
- Asymptotic safety has two ingredients:
  - A fixed point $g^* = \{g_1, g_2, \ldots\}$ for essential couplings: $\beta_i(g^*) = 0$, $\mu \frac{dg_i}{d\mu} = \beta_i(g)$
  - Couplings lie on a renormalization-group (RG) trajectory that reaches the fixed point
- The surface formed by these asymptotic safety renormaliztion-group trajectories is the *UV critical surface*

Asymptotic Safety and Model Building

- Number of free coupling parameters in asymptotically-safe theory is the dimension of the UV critical surface
- If UV critical surface has dimension C then require $C - 1$ (dimensionless) coupling parameters defining a trajectory and one dimensionful scale identifying a point
- Worst case: $C = 0$ no asymptotic safety
- Best case: $C = 1$ only a scale needs to be specified

Key Message

Asymptotic safety constrains the parameter space of a theory through its UV critical surface

Dimension of UV Critical Surface

- Dimension of UV critical surface can be determined by behaviour near critical point

$$\mu \frac{dg_i}{d\mu} \approx B_{ij}\left(g_j - g_j^*\right) \ , \ B_{ij} = \left.\frac{\partial \beta_i}{\partial g_j}\right|_{g=g^*}$$

$$g_i - g_i^* = \sum_k C_k V_i^{(k)} \mu^{\lambda_k}$$

  where $V^{(k)}$ is an eigenvector of B with eigenvalue λ_k
- If $\lambda_k > 0$ then $C_k = 0$ so that $g \to g^*$ as $\mu \to \infty$
- Dimension of UV critical surface is the number of negative eigenvalues of B, C_k corresponds to the parameters
- See e.g., Weinberg (1979); Litim & Sannino JHEP 1406.2337; Hansen, Mann, Wang, TGS et al, PRD 1706.06402

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Conformal Symmetry: Gravity Motivation

- Conformally symmetric theories of gravity with couplings to scalars (Higgs H, additional scalar S) such as see e.g., Stelle (1977), Shapiro (1997), Einhorn & Jones (2015)

$$S = \int d^4x \sqrt{|\det g|} \left[\frac{R^2}{6 f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} - \xi_H |H|^2 R - \xi_S |S|^2 R \right]$$

have a number of interesting features including:

  - Induced Einstein-Hilbert gravity at low-energy via symmetry breaking in scalar sector see Zee (1979), Adler (1982), Holdom & Ren (2016), Salvio & Strumia (2014)
  - Renormalizable and asymptotically free see e.g., Stelle (1977), Fradkin & Tseytlin (1982)
  - Various attempts to solve problems with ghost fields see e.g., Einhorn & Jones (2015), Holdom & Ren (2016), Donoghue & Menezes (2018)

Conformal Symmetry: Standard Model Motivation

- Conventional EW symmetry breaking leads to unnaturally large separation between unification and EW scale Georgi, Quinn, Weinberg (1974)
- Massless scalars allow a natural scale hierarchy for embedding of SM below a unification scale Weinberg, (1979)
- Bardeen (1995) suggests that Lagrangian conformal invariance would protect Higgs mass from approaching unification scale: idea similar to dimensional transmutation in QCD
- Explosion of interest in Bardeen's idea Hill 1401.4185 literature survey and superb overview, CFT arguments in Tavares et al 1308.0025
- Coleman-Weinberg mechanism (1973): effective potential used to realize EW symmetry-breaking via loop effects
- Can find a Higgs mass consistent with 125 GeV but some tension from larger predicted value of Higgs self-coupling Wang, TGS, PRL (2013)
- Connection to Asymptotic safety: softly broken conformal symmetry restored with $\beta(g^*) = 0$

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Extending the Standard Model: Asymptotic Safety and Conformal Symmetry

Basic ideas for our work

Wang, Sage, Steele, Mann, J. Phys. G45 (2018) 095002, 1511.0253

- Consider an extension of the standard model: leptophobic $U'_B(1)$ with an extended scalar sector
- Asymptotic Safety: predictive power to connect UV and IR physics, constraining parameters of theory
- Conformal Symmetry: constrain structure of theory, including gravity
- Gravity: treat gravity contributions to $\beta(g)$ using effective field theory analysis
- Purpose is not to promote this particular model but to illustrate how the ingredients of asymptotic safety and conformal symmetry can guide UV extensions of the Standard Model

Leptophobic $U'_B(1)$ Model

- Lagrangian in Scalar Sector of Standard Model with $U'_B(1)$ extension see e.g., Hashimoto, Iso, Orikasa (2014)

$$L = D_\mu H^\dagger D^\mu H + D_\mu S^\dagger D^\mu S - \lambda_2 |S|^2 H^\dagger H - \lambda_3 |S|^4 - \lambda_1 \left(H^\dagger H\right)^2$$

- S is a complex singlet and in basis where kinetic gauge terms diagonal (g_m comes from $U(1)$ kinetic mixing)

$$D_\mu = \partial_\mu - i g_3 \frac{\lambda_a}{2} G^a_\mu - i g_2 \frac{\tau_i}{2} W^i_\mu - iY \left(g_Y B_\mu + g_m B'_\mu\right) - i g' Q'_B B'_\mu$$

- Assume $U'_B(1)$ couples only to third generation (FCNC constraints); spectator fermions needed for anomaly cancellation

Charge Assignments

Table: Standard Model fermion charge assignments

Gauge group	$q_L^{u,d}$	u_R	d_R	$q_L^{c,s}$	c_R	s_R	$q_L^{t,b}$	t_R	b_R	l_L (all gens)	e_R (all gens)
$SU(3)_c$	3	3	3	3	3	3	3	3	3	1	1
$SU(2)_L$	2	1	1	2	1	1	2	1	1	2	1
$U(1)_Y$	1/3	4/3	-2/3	1/3	4/3	-2/3	1/3	4/3	-2/3	-1	-2
$U(1)'$	0	0	0	0	0	0	1/3	1/3	1/3	0	0

Table: Scalar charge assignments

Gauge group	S	H
$SU(3)_c$	1	1
$SU(2)_L$	1	2
$U(1)_Y$	0	1/2
$U(1)'$	2	0

Table: Exotic fermion charge assignments

Gauge group	ν_R	ψ_L^l	ψ_R^l	ψ_L^e	ψ_R^e
$SU(3)_c$	1	1	1	1	1
$SU(2)_L$	1	2	2	1	1
$U(1)_Y$	0	-1	-1	-2	-2
$U(1)'$	-1	-1	0	-1	0

Symmetry Breaking

- Coleman-Weinberg mechanism generates vev v_1 for S, which qualitatively requires λ_3 to change sign from positive to negative near v_1
- S mass generated dynamically by Coleman-Weinberg mechanism (determined by effective potential V): V has minimum at vev and mass determined by second derivative at vev
- Since S couples to Z' , vev v_1 generates Z' mass
- S vev triggers conventional EW symmetry breaking at lower EW scale v, generating Higgs mass: requires negative λ_2

$$V = \lambda_2 \left|S\right|^2 H^\dagger H + \lambda_3 \left|S\right|^4 + \lambda_1 \left(H^\dagger H\right)^2$$

- Four symmetry-breaking constraints:
  - $\left.\frac{dV}{d\varphi}\right|_{\varphi=v_1} = 0\,,\ S = \frac{1}{\sqrt{2}}\left(\varphi_1 + i\varphi_2\right)\,,\ \varphi^2 = \sum_i \varphi_i^2$
  - $2v^2\lambda_1\left(v_1\right) = -v_1^2\lambda_2\left(v_1\right) = M_H^2$
  - $M_{Z'} = 2g'\left(v_1\right)v_1$

Gravity Contributions to β Functions

- Gravity treated as effective theory: leading order result for gravity contribution to β functions for coupling x_j see Shaposhnikov and Wetterich PLB (2010)

$$\beta_j^{\text{grav}} = \frac{a_j}{8\pi}\frac{k^2}{M_p^2}x_j$$

- Key question is the sign of a_j
  - $a_j < 0$ implies gravity will make coupling asymptotically free, stabilizing UV behaviour
  - $a_j > 0$ requires $x_j = 0$ to be an asymptotically safe fixed point to control UV behaviour from gravity
- All gauge couplings have $a_j < 0$ Shaposhnikov & Wetterich PLB (2010), Robinson & Wilczek PRL (2006)
- All SM Yukawa couplings have $a_j < 0$ Shaposhnikov & Wetterich PLB (2010), Shapiro CQG (1989)
- Quartic scalar couplings have $a_j > 0$ Percacci & Perini PRD (2003), Narain & Percacci CQG (2010)
- Situation for Higgs portal coupling λ_2 unknown: assume $a_j < 0$

Summary: Constraints of Asymptotic Safety

- Taking into account the leading-order contributions of gravity, require a UV boundary condition at a scale M_{UV}

$$\beta_{\lambda_1}(M_{UV}) = \beta_{\lambda_3}(M_{UV}) = \lambda_1(M_{UV}) = \lambda_3(M_{UV}) = 0$$

- Have to make a choice for the UV boundary condition for λ_2: interesting choice is $\lambda_2(M_{UV}) = 0$ so that the Higgs portal interaction is generated radiatively
- There could be other realizations of asymptotic safety for $\beta_{\lambda_3} < 0$ (non-perturbative gravity contributions) which would allow greater freedom in parameters of model, but highly speculative

$$\beta_{\lambda_1}(M_{UV}) = \lambda_1(M_{UV}) = 0; \quad \lambda_2(M_{UV}), \lambda_3(M_{UV}) \neq 0$$
$$\beta_{\lambda_1}(M_{UV}) = \lambda_1(M_{UV}) = \lambda_2(M_{UV}) = 0; \quad \lambda_3(M_{UV}) \neq 0$$

Renormalization-Group Functions

Hashimoto, Iso, Orikasa PRD (2014); Iso, Okada, Orikasa PRD (2009); Buttazzo et al JHEP (2013)

$$
\begin{aligned}
16\pi^2\beta_{\lambda_1} &= \lambda_2^2 - 3\lambda_1 g_m^2 + \frac{3}{8}g_m^4 + \beta_{\lambda_1}^{SM} \\
16\pi^2\beta_{\lambda_2} &= 12 g_m^2 g'^2 + 6Y_t^2\lambda_2 - 24 g'^2\lambda_2 + 4Y_M^2\lambda_2 + 4\lambda_2^2 + 12\lambda_1\lambda_2 + 8\lambda_2\lambda_3 \\
&\quad - \frac{3}{2}\lambda_2\left(g_m^2 + 3g_2^2 + g_1^2\right) \\
16\pi^2\beta_{\lambda_3} &= 96 g'^4 - 16Y_M^4 + 2\lambda_2^2 - \lambda_3\left(48 g'^2 + 8Y_M^2 - 20\lambda_3\right) \\
16\pi^2\beta_{Y_t} &= -\frac{17}{12}Y_t g_m^2 - \frac{2}{3}Y_t g'^2 - \frac{5}{3}Y_t g' g_m + \beta_{Y_t}^{SM} \\
16\pi^2\beta_{g'} &= \frac{1}{18}g'\left(76 g'^2 + 64 g' g_m + 123 g_m^2\right) \\
16\pi^2\beta_{g_m} &= g_m\left(\frac{41}{6}\left(g_m^2 + 2g_1^2\right) + \frac{38}{9}g'^2\right) + \frac{32}{3}g'\left(g_m^2 + g_1^2\right) \\
16\pi^2\beta_{Y_M} &= -6Y_M g'^2 + 6Y_M^3; \quad 32\pi^2\gamma_\varphi = Y_M^2 - 24 g'^2
\end{aligned}
$$

- Y_t is top Yukawa, Y_M is Majorana-Yukawa coupling of S to the spectator RH neutrino $m_{\nu_R} = \sqrt{2}v_1 Y_M\ (v_1)$

Analysis

- Reduce system of nine unknowns $(\lambda_1,\, \lambda_2,\, \lambda_3,\, g',\, g_m,\, Y_M,\, Y_t,\, v_1,\, M_{UV})$ to that of five $(g_m,\, v_1,\, Y_M,\, Y_t,\, M_{UV})$ using the four symmetry-breaking constraints
- Solve RG equations subject to the four UV boundary conditions
- One-parameter family of solutions for different choices of $M_{Z'}$
- Example solution shown in Figure 1

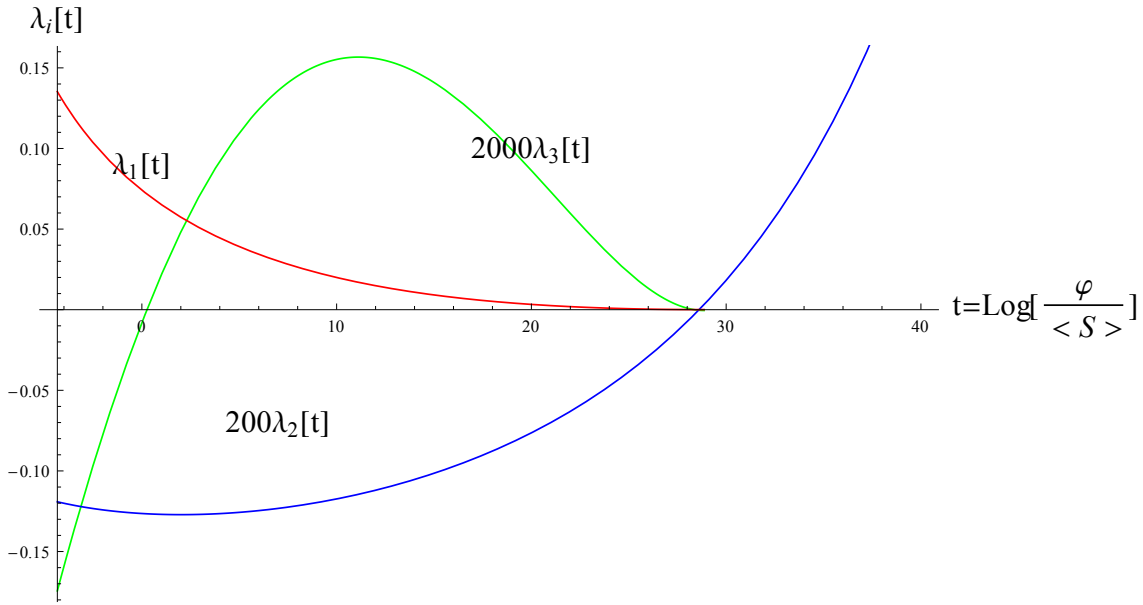

Figure: Running scalar couplings are shown as a function of the scale $t = \log(\varphi/\langle S\rangle)$. The red, blue, and green curves represent $\lambda_1(t)$, $200\lambda_2(t)$, $2000\lambda_3(t)$ respectively.

Results

Wang, Sage, Steele, Mann

J. Phys. G45 (2018) 095002, 1511.0253

- Results for three different choices of $M_{Z'}$ shown in Table
- Couplings referenced to scale v_1
- Find $M_{UV} \sim M_{pl}$ and the predicted $Y_t(v) = 0.93$ very close to the current experimental central value $Y_t(v) = 0.936$ Buttazzo et al JHEP (2013)
- Implicitly assumed spectator fermions active in RG equations: masses must be less than v_1

$M_{Z'}$ (TeV)	$10^6\lambda_2$	$10^6\lambda_3$	g'	g_m	Y_M	Y_t	m_{ν_R} (TeV)	v_1 (TeV)	m_S (GeV)
1.9	-6	-4.05	0.18	0.042	0.28	0.77	2.02	5.1	20
3.4	-0.59	-0.13	0.1	0.023	0.16	0.74	3.72	16.9	11.5
6.8	-0.036	-0.002	0.05	0.011	0.08	0.70	7.59	68.8	6

Other Phenomenology

- Investigated several of collider signals related to Z', see Table for example
- Other observables discussed in paper Wang, Sage, Steele, Mann J. Phys. G45 (2018) 095002, 1511.0253

Table: Z' predicted production cross sections $\sigma_{pp\to Z'} \times BR$ (units of fb)

$M_{Z'}$	Channel	$\sqrt{s} = 8$ TeV	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 14$ TeV
1.9 TeV	Diboson	0.013	0.058	0.069
	Dilepton	4.27	18.59	22.14
	Dijet	19.47	84.79	100.98
	Ditop	5.67	24.71	29.42
3.8 TeV	Diboson	9.15×10^{-5}	1.62×10^{-3}	2.19×10^{-3}
	Dilepton	0.029	0.518	0.701
	Dijet	0.110	1.95	2.64
	Ditop	0.028	0.491	0.665
6.8 TeV	Diboson	7.50×10^{-13}	4.90×10^{-7}	1.22×10^{-6}
	Dilepton	2.40×10^{-10}	1.57×10^{-4}	3.90×10^{-4}
	Dijet	9.41×10^{-10}	6.14×10^{-4}	1.53×10^{-3}
	Ditop	2.48×10^{-10}	1.62×10^{-4}	4.03×10^{-4}

Related Work

- Mann, Meffe, Sannino, Steele, Wang, Zhang, PRL 119 (2017) 261802
- Extend Standard Model with N_f vector-like fermions with various charge assignments: (i) $N_F\ (3,2,1/6)$; (ii) $N_{F3}\ (3,1,0) \oplus\ N_{F2}\ (1,2,1/2)$; (iii) $N_{F3}\ (3,1,0) \oplus N_{F2}\ (1,3,0) \oplus N_{F1}\ (1,1,1)$
- Assume fermions all become active at 2 TeV
- Large N_f methods lead to poles in RG functions that can drive fixed points
- Can realize asymptotic safety if Higgs quartic coupling is enhanced beyond its SM expectation (say factor of 2 at EW scale)

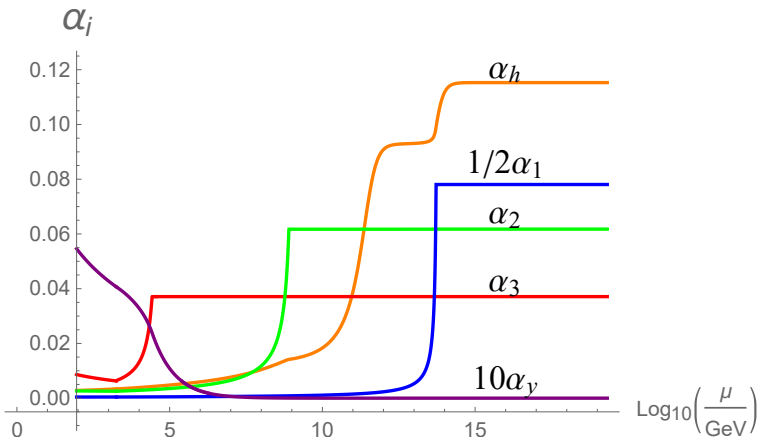

Figure: Model (ii) $N_{F3} = 40, N_{F2} = 24$. Higgs quartic coupling is chosen to be 0.0034.

Summary: Key Ideas

- Asymptotic safety provides a valuable principle for constraining structure and parameters of theory from UV behaviour
- Effective theory approach to gravity contributions to β functions can help achieve asymptotic safety by identifying couplings that require UV stabilization and stabilizing some that may otherwise be uncontrolled
- Lagrangian conformal symmetry not only guides structure of Lagrangians but adds dynamical symmetry-breaking constraints to constrain parameters of theory

Thank you!

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