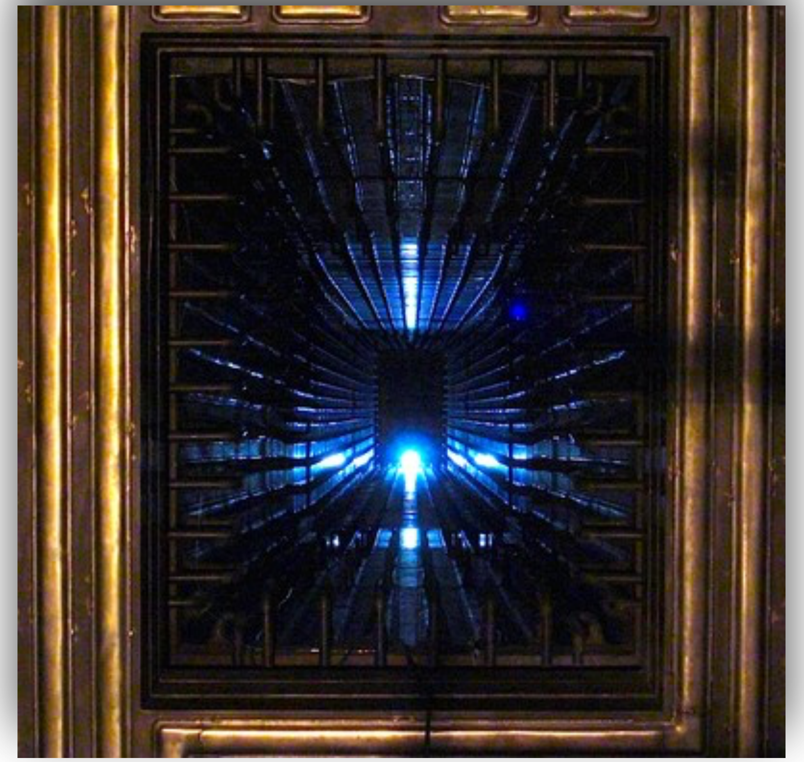
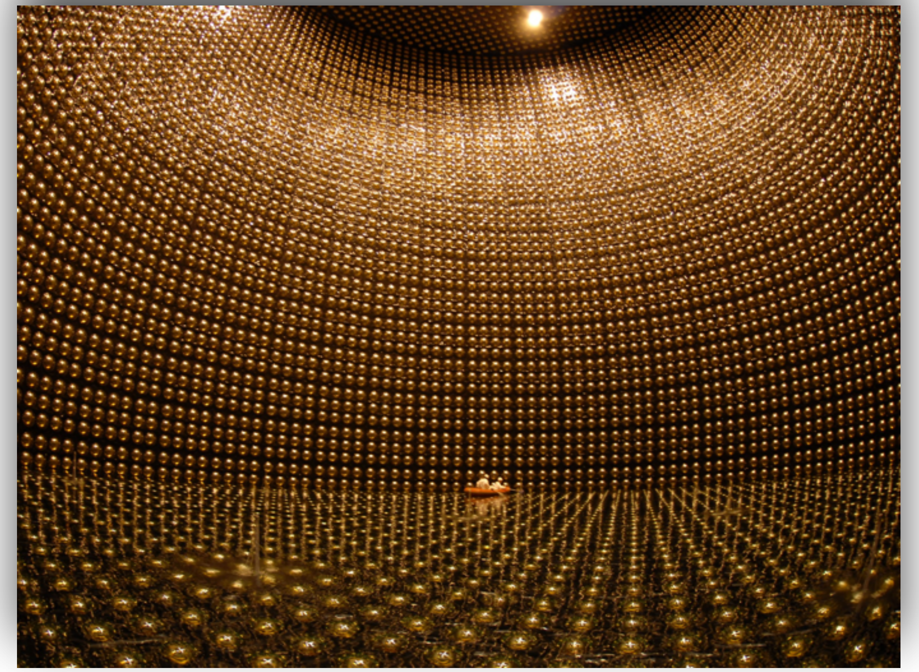
Neutrino Oscillations at T2K and Hyper-K





Mark Hartz
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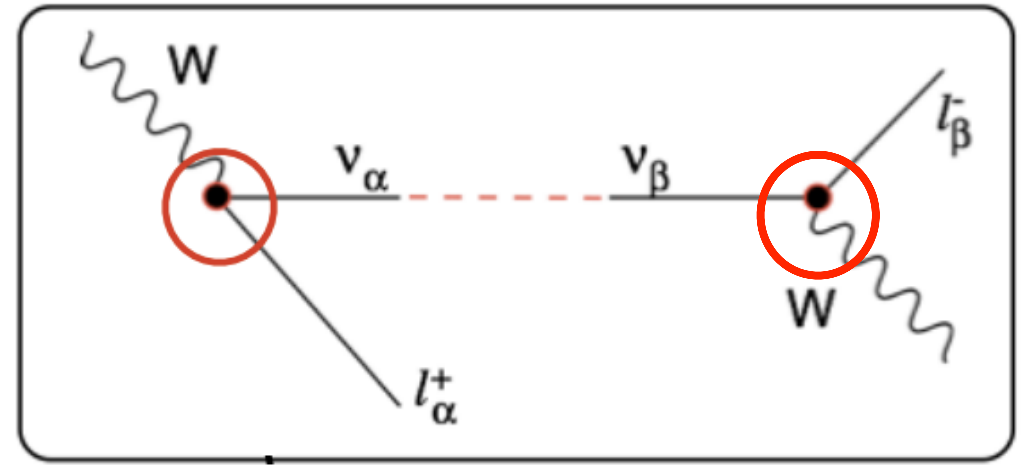
2019 CAP Congress
Simon Fraser University
June 6, 2019

Introduction to Neutrino Oscillations

- Neutrinos interact via the weak force
- Have states of definite flavor associated with charged leptons



$\alpha = e, \mu, \tau$

- Will propagate as states of definite mass
- Weak states and mass state need not be the same

$$|\nu_\alpha> = \sum_{i=1}^{i=3} U^*_{\alpha i} |\nu_i>$$

Flavor states   Mass states

- Mass states propagate with a relative phase
- After propagation, the composition of flavor state is changed

$$P_{\alpha\to\beta} = |\langle \nu_\beta(L)|\,\nu_\alpha\rangle|^2 = \left| \sum_i U^*_{\alpha i} U_{\beta i} e^{-i\frac{m_i^2 L}{2E}} \right|^2$$

3-Flavor Oscillations

Pontecorvo-Maki-Nakagawa-Sakata Mixing Matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s^{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

Unitary mixing matrix oscillations ($s_{12} = \sin\theta_{12}$, etc.) *Majorana phases*

δ, α_{21} and α_{31} introduce new sources of CP violation

- 3-flavor oscillations: unitary matrix parametrized with three mixing angles and a phase

- Oscillations also depend on the mass squared differences of the mass states, distance traveled (L) and neutrino energy (E)

$$P_{\alpha\rightarrow\beta} = \delta_{\alpha\beta} - 4\sum_{i>j}\mathrm{Re}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right)$$
$$+2\sum_{i>j}\mathrm{Im}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right)$$

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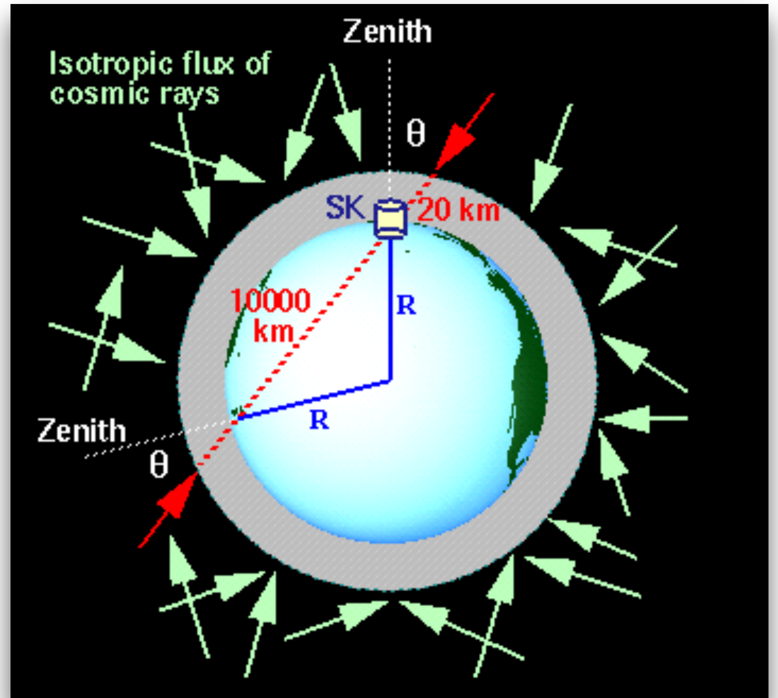
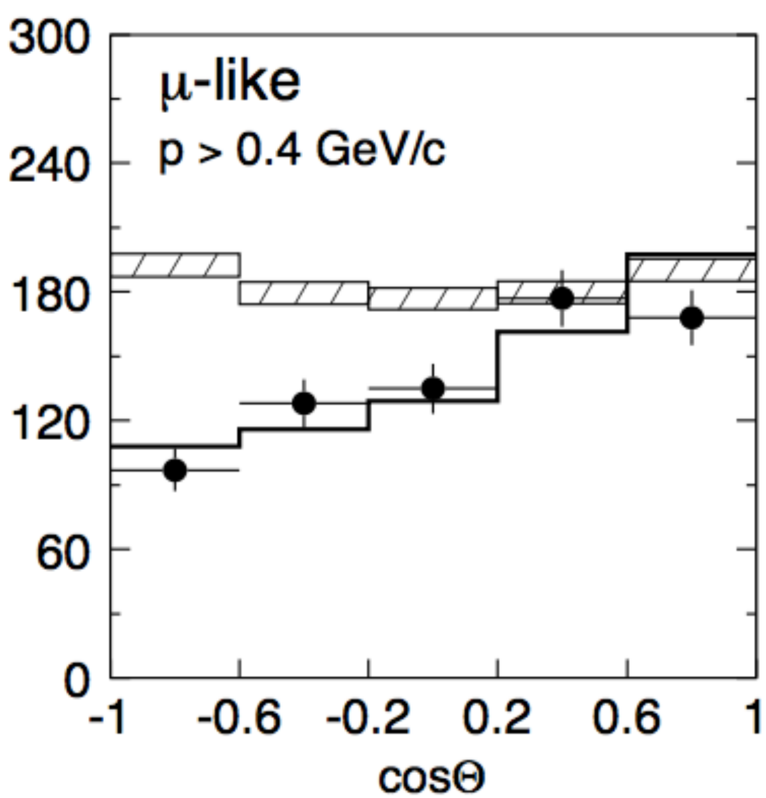
How do we know neutrinos oscillate?

Evidence of Oscillations: Super-Kamiokande



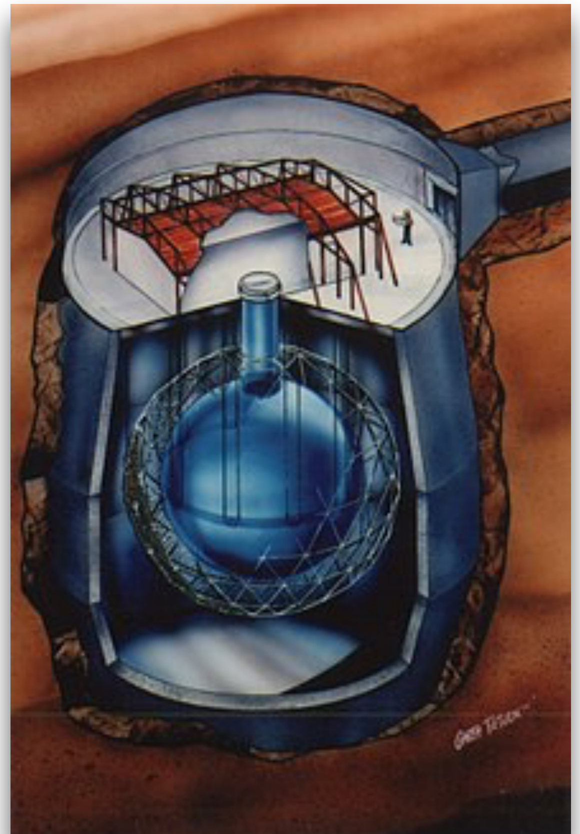


Phys.Rev.Lett. 81 (1998) 1562-1567

- By the late 90's deficits observed in solar and atmospheric neutrino data
- Super-Kamiokande: 22.5 kton fiducial mass water Cherenkov detector
- Measure neutrino oscillations as a function of zenith angle
    - **Downward events = short baseline (L)** and no oscillations
    - **Upward events = long baseline (L)** and significant oscillations
- This was a smoking gun for the deficits to be explained by neutrino oscillations

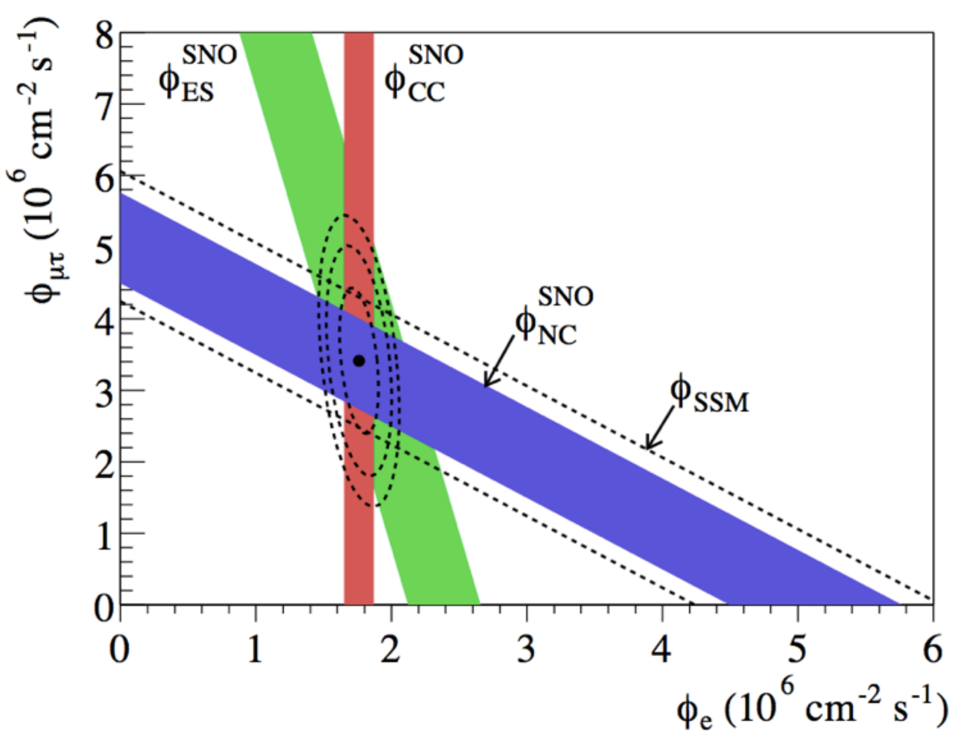
2015 Nobel Prize

Evidence of Oscillations: SNO



- With heavy water target, SNO was sensitive to 3 types of neutrino interactions:

$$\begin{aligned} \nu_e + \mathrm{d} &\rightarrow \mathrm{p} + \mathrm{p} + \mathrm{e}^- && \text{(CC)}, \\ \nu_x + \mathrm{d} &\rightarrow \mathrm{p} + \mathrm{n} + \nu_x && \text{(NC)}, \\ \nu_x + \mathrm{e}^- &\rightarrow \nu_x + e^- && \text{(ES)}. \end{aligned}$$



Phys.Rev.Lett. 89 (2002) 011301

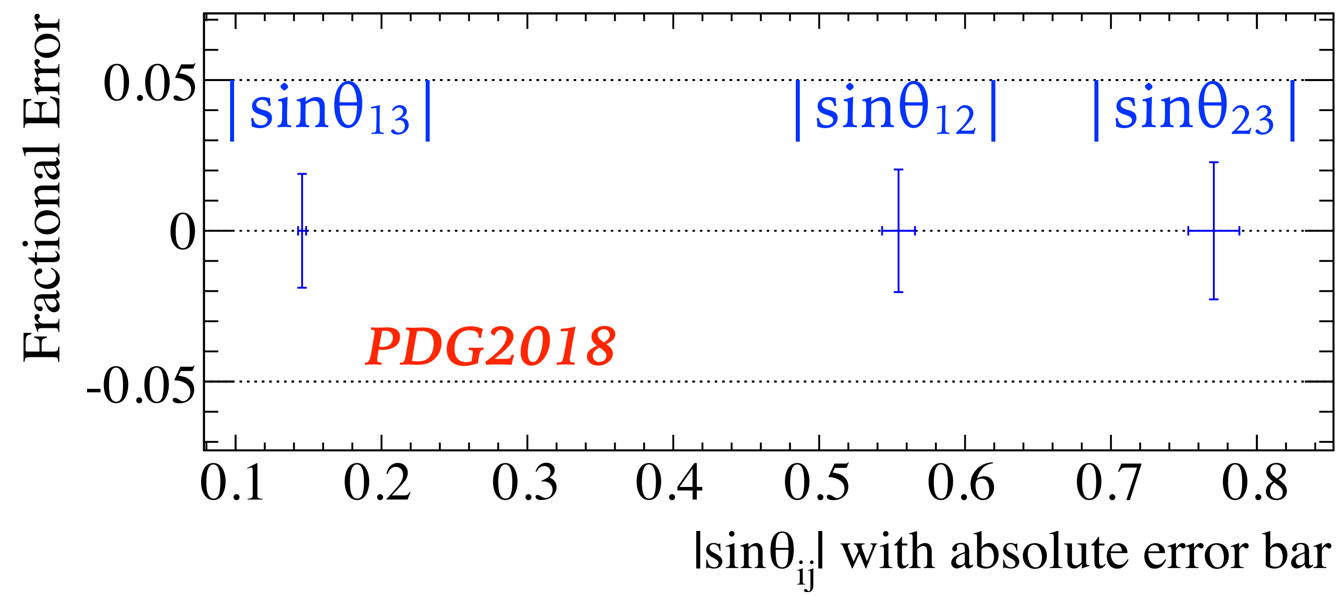
- By combining all three modes, showed that electron neutrino flux was 1/2 of the muon+tau neutrino flux

- Showed for the first time that neutrinos were changing flavor, confirming the oscillation

2015 Nobel Prize

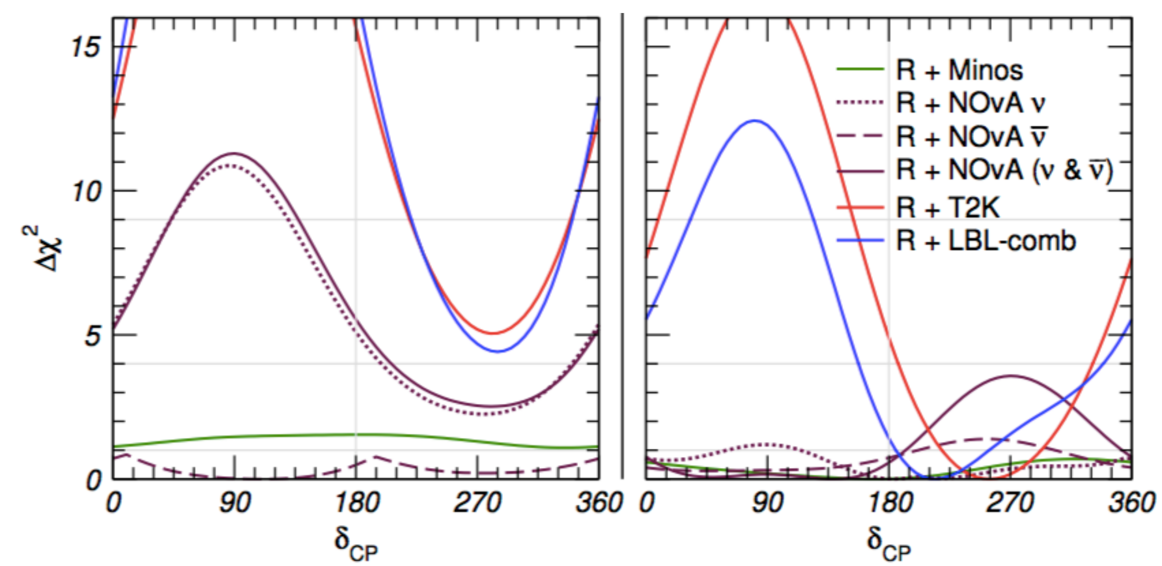
State of Oscillation Parameter Measurements

- Sine of mixing angles measured to 5% precision or better
  - Maximal mixing for $\sin^2(2\theta_{23})=1$, $\sin(\theta_{23}) = 1/\sqrt{2}$



- Weak global preference for δ_{cp} near $3\pi/2$ ($-\pi/2$) driven by T2K+SK+Reactors+NOvA

JHEP 01 (2019) 106



- m_3 state is heaviest (normal hierarchy) or lightest (inverted hierarchy) undetermined
  - The matter effect in long baseline experiments gives sensitivity to hierarchy

$$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\,\mathrm{eV}^2$$

$$\Delta m^2_{32} = (2.45 \pm 0.05) \times 10^{-3}\,\mathrm{eV}^2$$

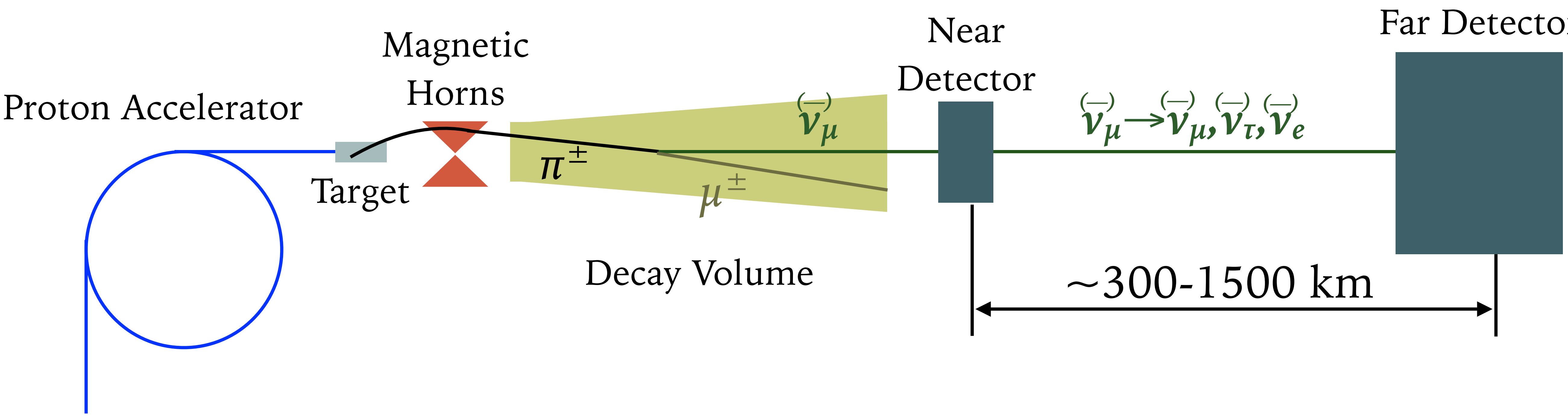
or

$$\Delta m^2_{32} = (-2.52 \pm 0.05) \times 10^{-3}\,\mathrm{eV}^2$$

Neutrino oscillations in long baseline experiments

Human-made Neutrino Beams

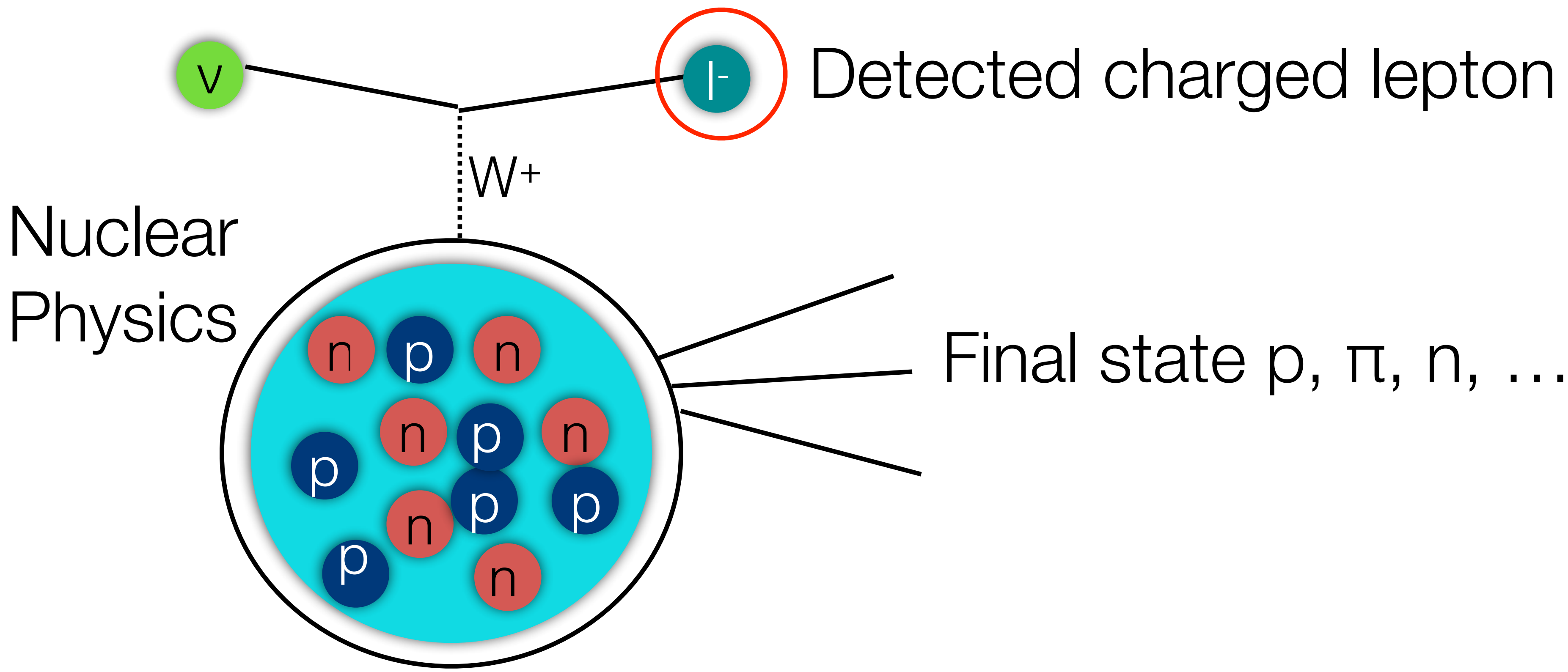
- Produce controlled sources of neutrinos
    - Reactor produced electron antineutrinos
    - **Accelerator production muon (anti)neutrinos**



- Control baseline (L) for oscillations
- Tune neutrino spectrum for desired energies
- Select muon neutrinos or muon antineutrinos by focusing π^+ or π^-
- **Measure neutrinos before oscillations with near detectors**

Neutrino Interactions in Detectors

- Select neutrino interactions in the charged current interaction mode:



- Final state includes hadrons that may not be detected:
  - Water Cherenkov: most protons below Cherenkov threshold
  - Tracking detectors: neutrons may not be detected
- Need neutrino energy to calculate $P(\theta_{23},\Delta m^2_{32},\ldots,E_\nu)$
  - **Since we don't fully reconstruct the event, we rely on models to infer energy**

The T2K Experiment

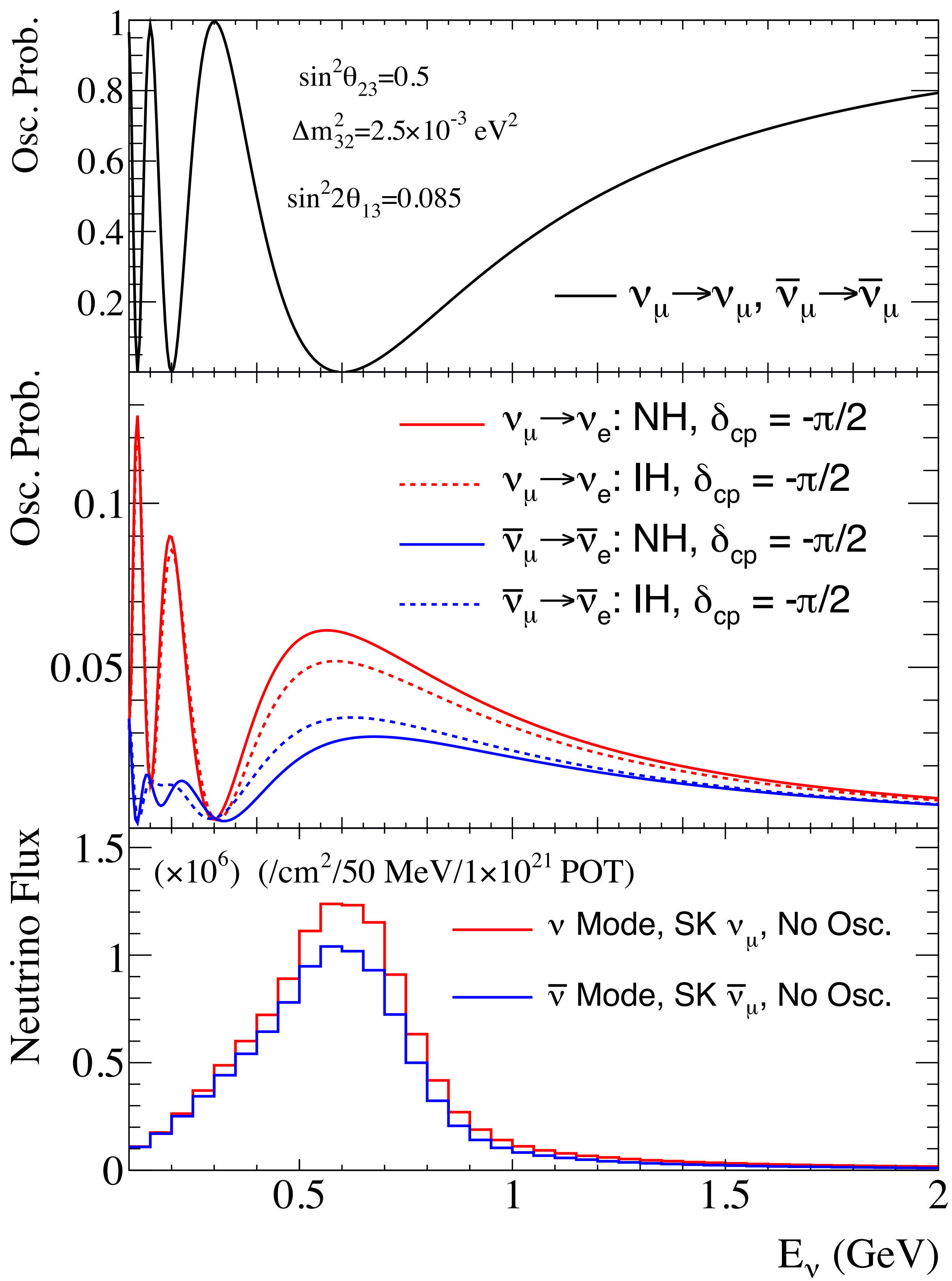
~500 researchers, 62 institutes, 11 countries

Muon (anti)neutrino beam generated at J-PARC and detected at Super-Kamiokande

In 2013 T2K made the first discovery of an appearance mode: $\nu_\mu \rightarrow \nu_e$

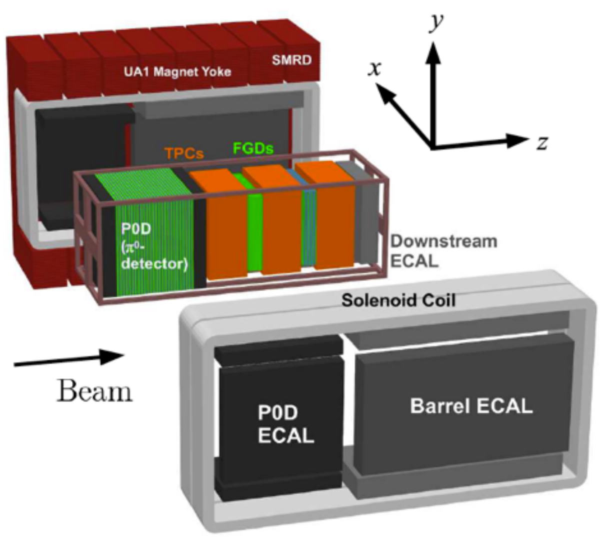
(Phys. Rev. Lett. 112, 061802 (2014))

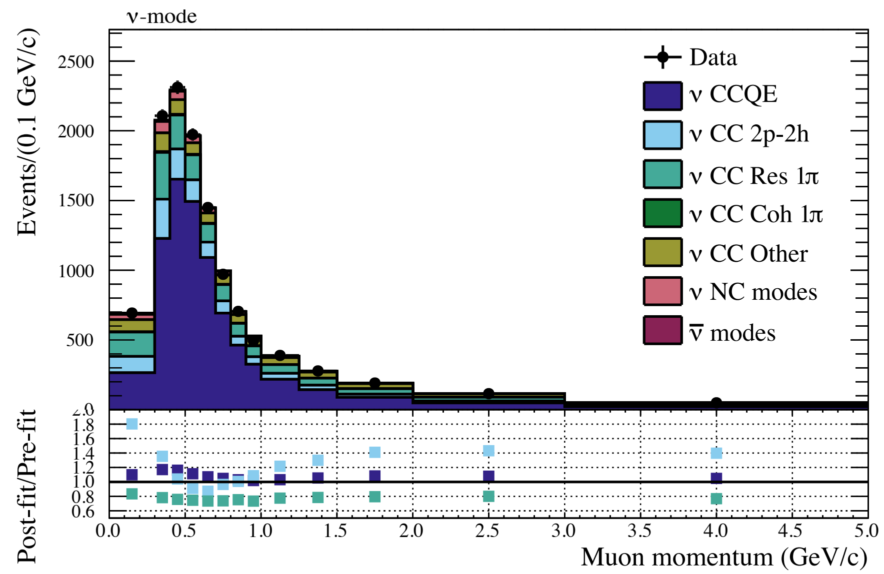
Oscillation Physics at T2K



- **Muon (anti)neutrino survival** depends on $\sin^2(2\theta_{23})$ and Δm^2_{32}
- **Electron (anti)neutrino appearance**
  - $\sin^2(\theta_{23})$, $\sin^2(2\theta_{13})$ and Δm^2_{32} in leading term
  - Sub-leading dependence on δ_{cp}
    - CP conservation at $\delta_{cp}=0,\pi$
    - Maximal CP violation at $\delta_{cp}=-\pi/2,\pi/2$
  - Matter effect → dependence on the mass hierarchy
    - Normal Hierarchy (NH): enhanced rate for neutrinos, decreased for antineutrinos

T2K Oscillation Analysis (In 1 Slide)

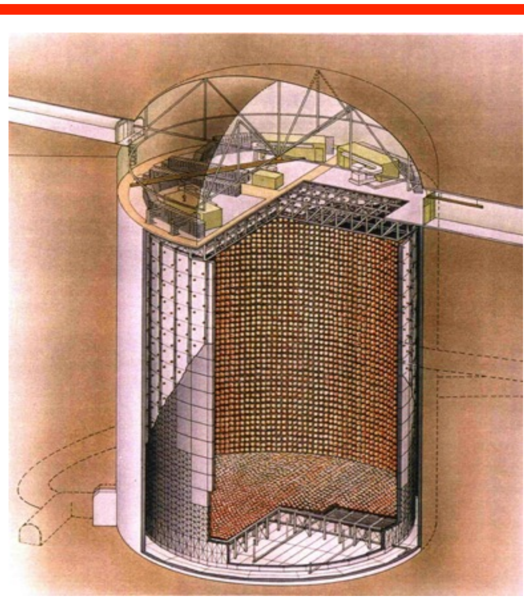


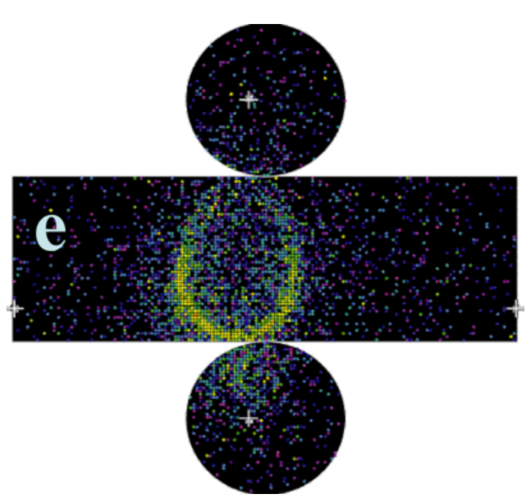


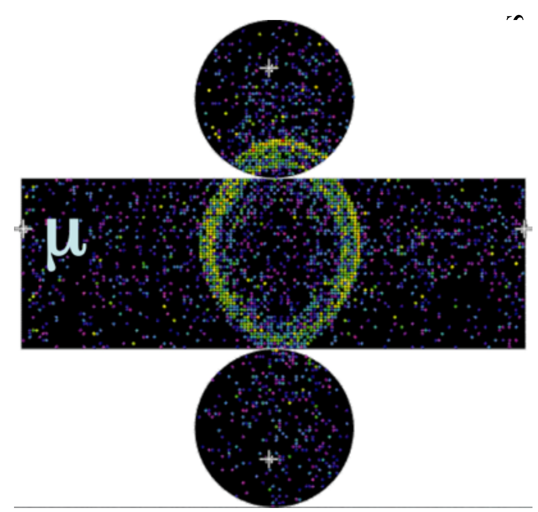
- Fit flux and interaction models to ND280 data
  - Update and constrain model parameters
- Fit μ and e neutrino candidates in Super-K to extract oscillation parameter constraints

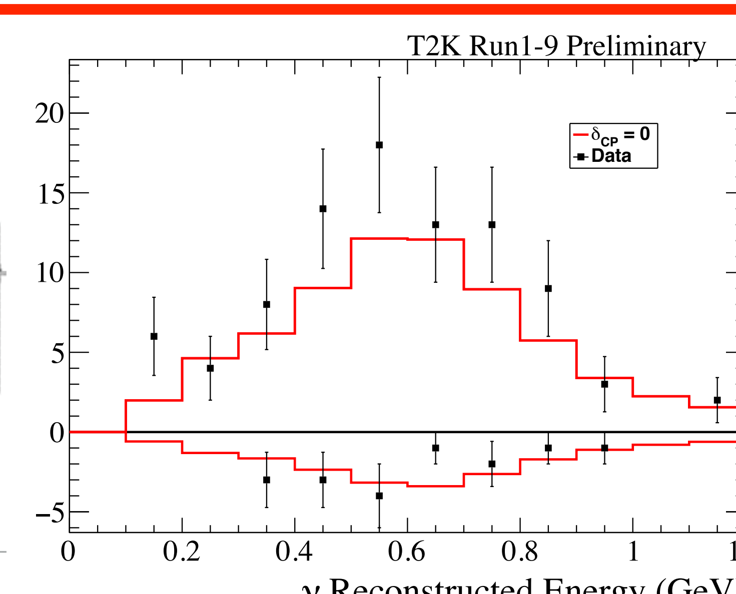
Neutrino Production Model

Neutrino Interaction Model

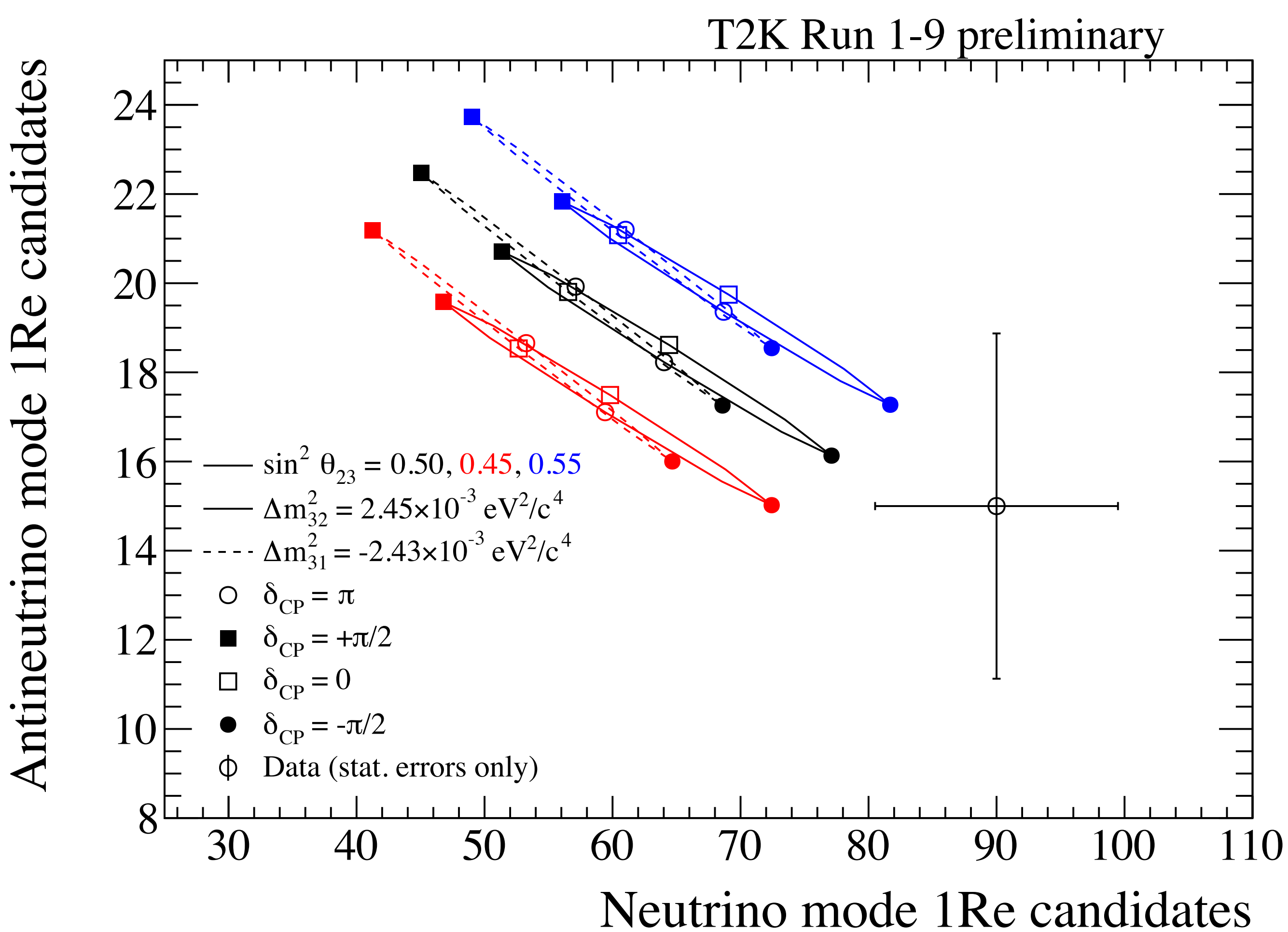








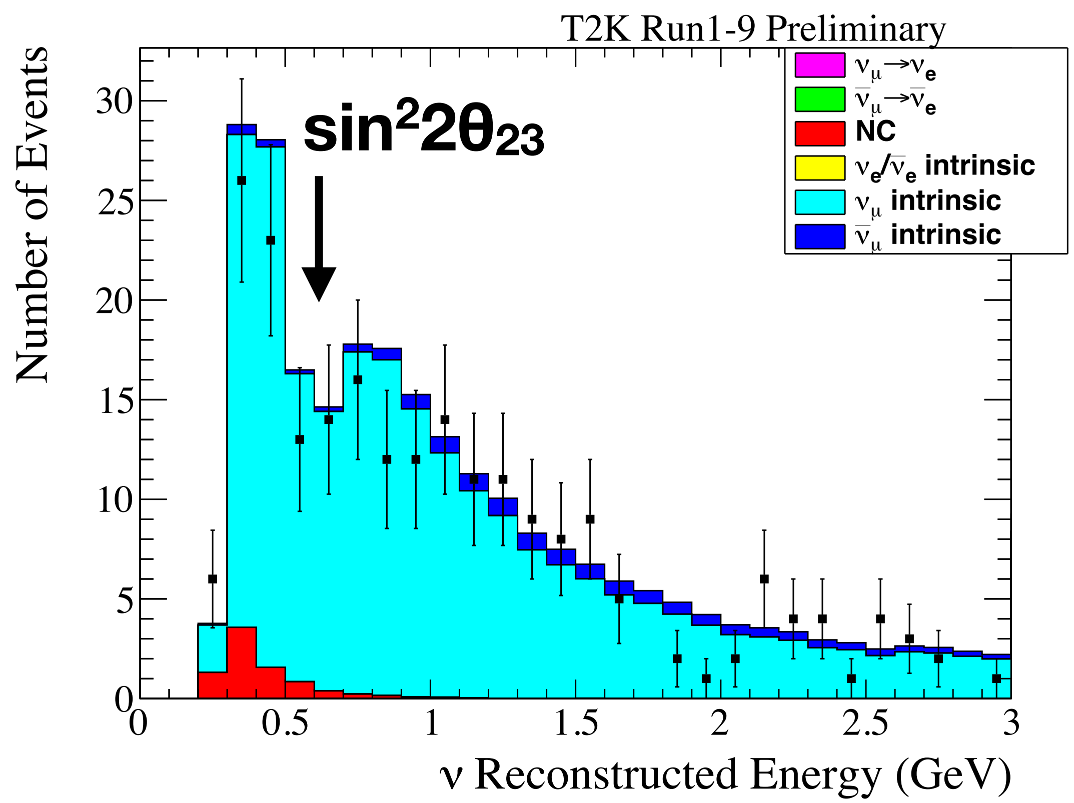
CP Preference at T2K

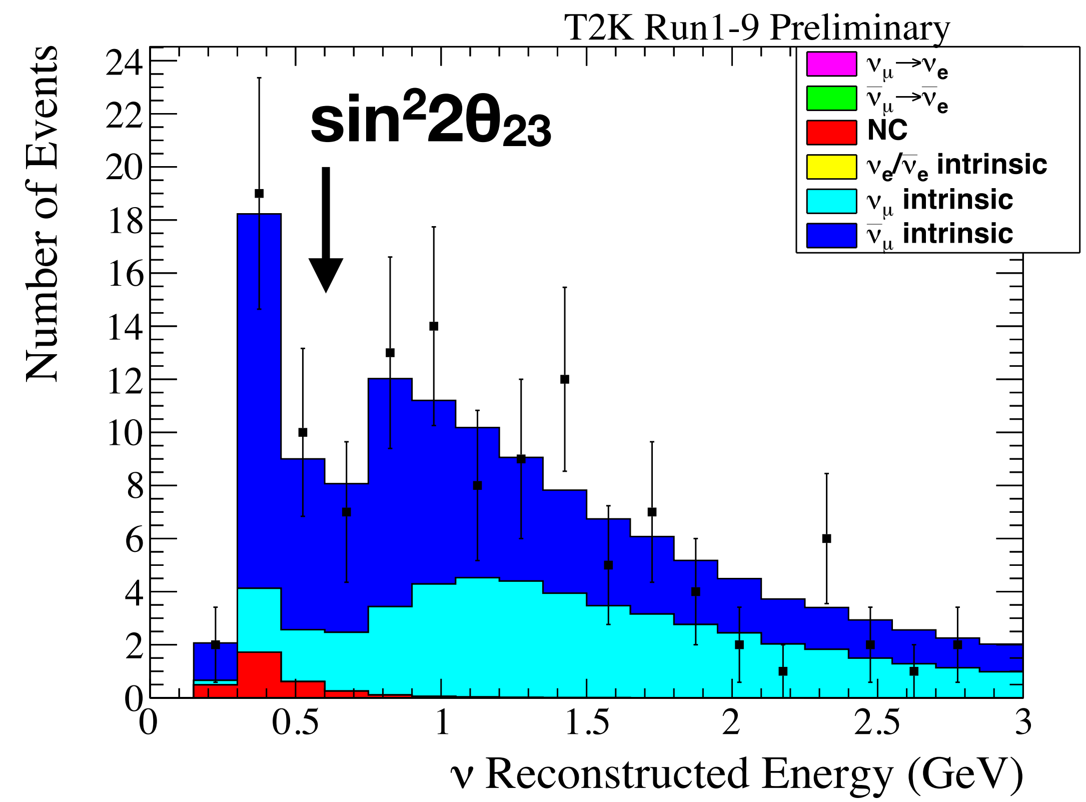


Error Source	% Error on neutrino/ antineutrino rate
Pion Interactions	1.58
Neutral Current Background	1.50
Electron (anti)neutrino cross section	3.03
Extrapolation from near detector	2.31
Removal Energy	3.74
Far Detector model	1.47
Total	5.87

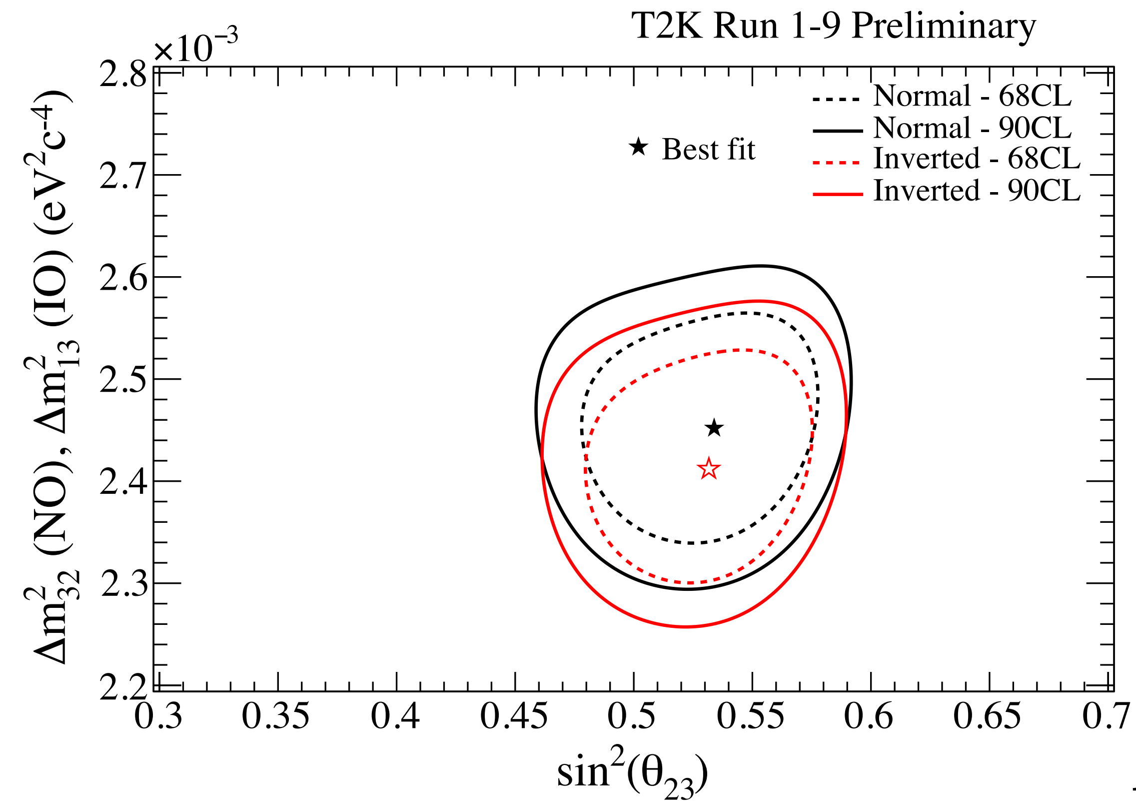
- T2K observes what appears to be a statistical fluctuation in the data
- T2K has a weak preference for the normal hierarchy with a posterior probability of 0.89
- **The 2σ interval for δ_{cp}: [–2.965, –0.628] (NH), [–1.799, –0.979] (IH)**
- T2K's systematic uncertainty is 6%

Muon Neutrino Survival

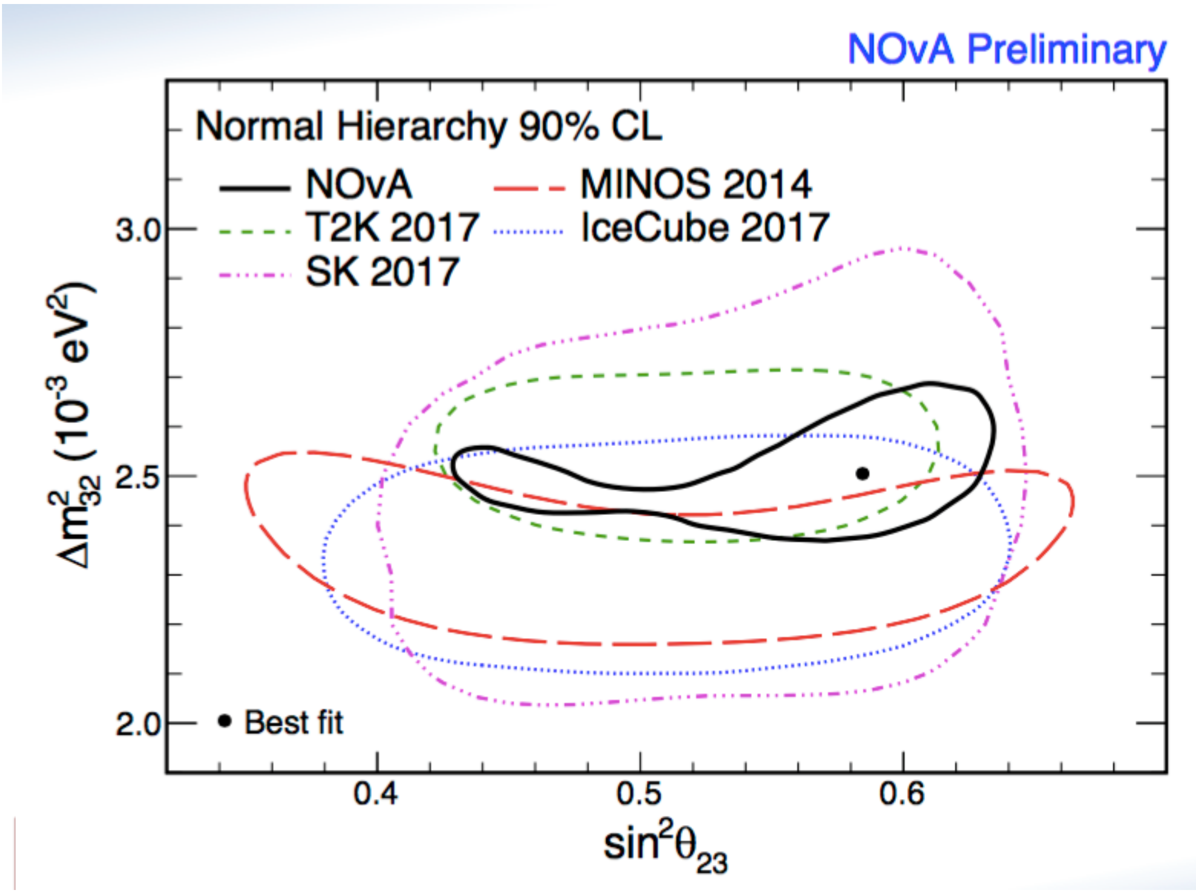




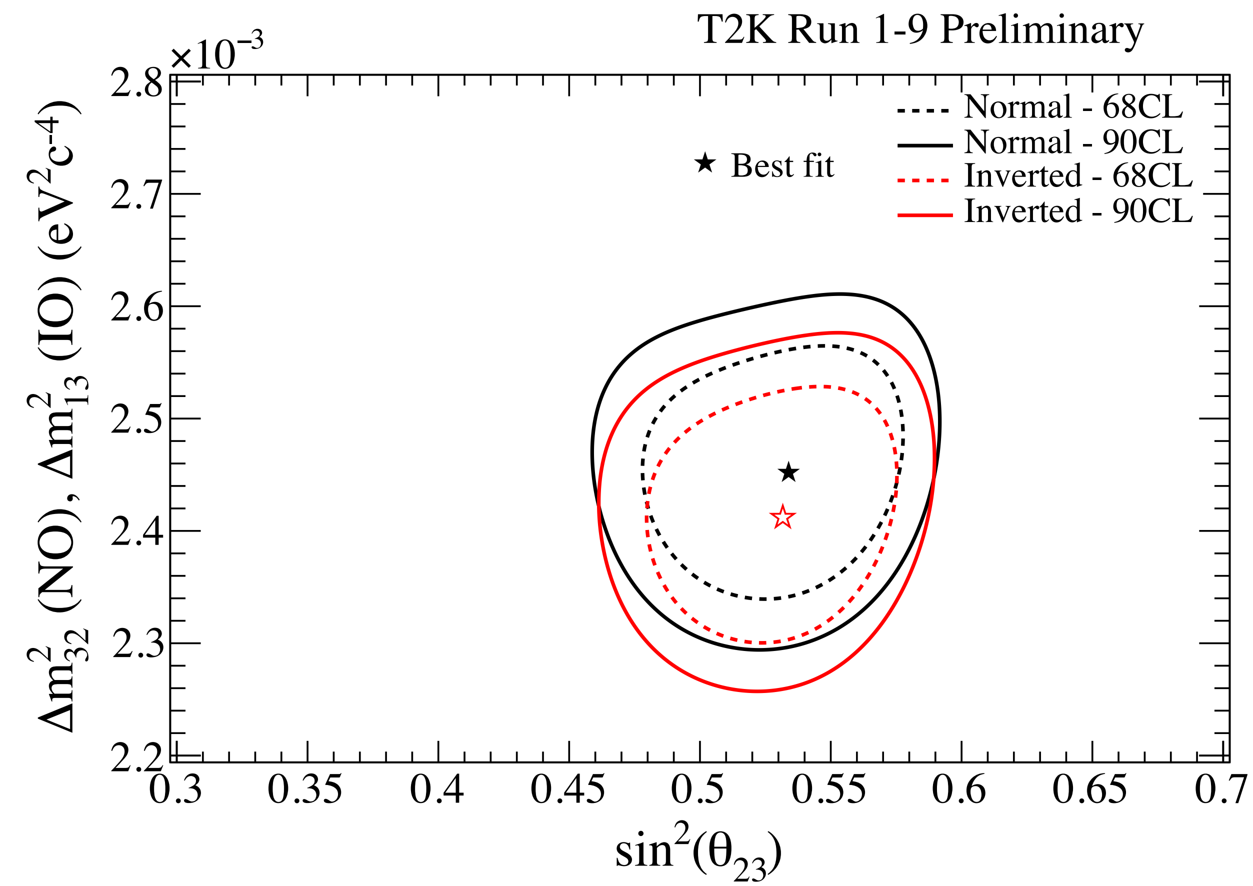
- T2K is consistent with maximal mixing ($\sin^2\theta_{23}$=0.5)
- Small preference for the upper octant



Muon Neutrino Survival



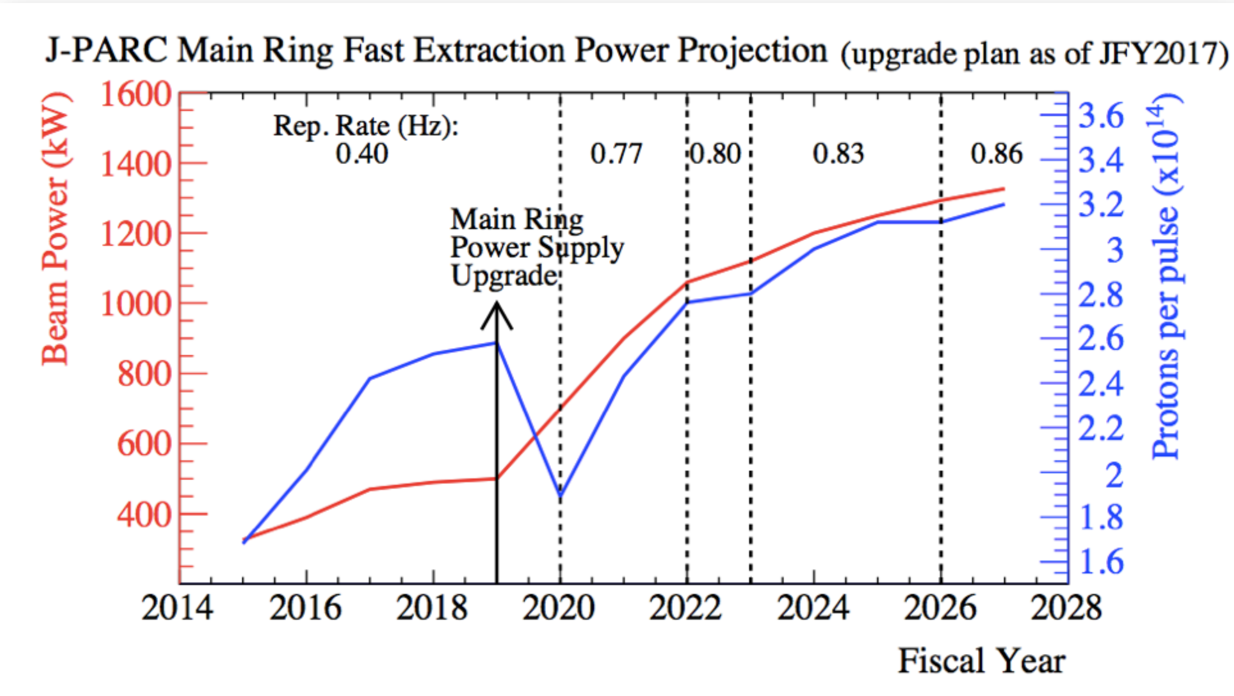
FNAL JTEP Seminar, June 2018

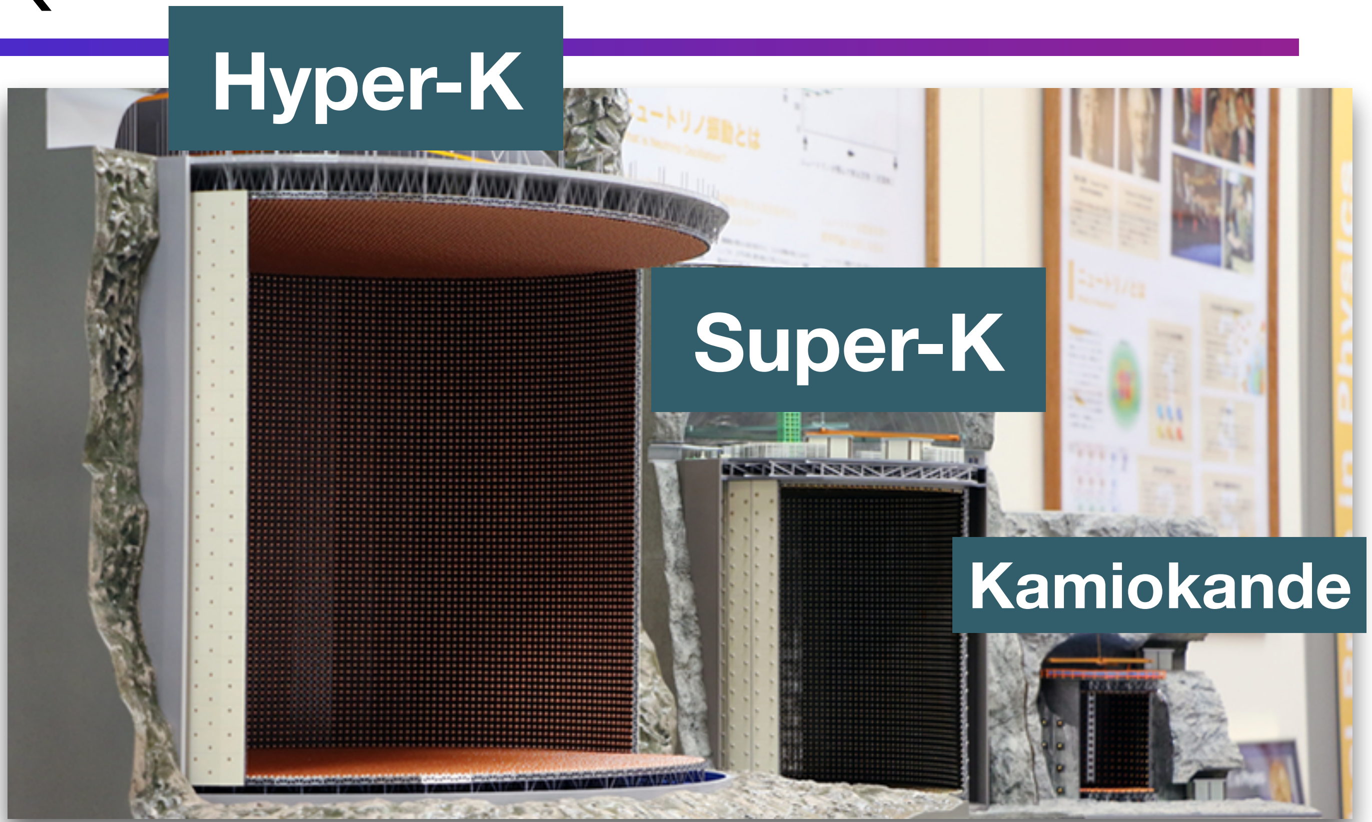


- T2K and NOvA measure consistent values of Δm^2_{32}
- T2K is consistent with maximal mixing ($\sin^2\theta_{23}$=0.5), while NOvA prefers non-maximal mixing
- Maximal mixing could point to underlying symmetry
    - Important measurement for the future

Hyper-K, The Next Generation

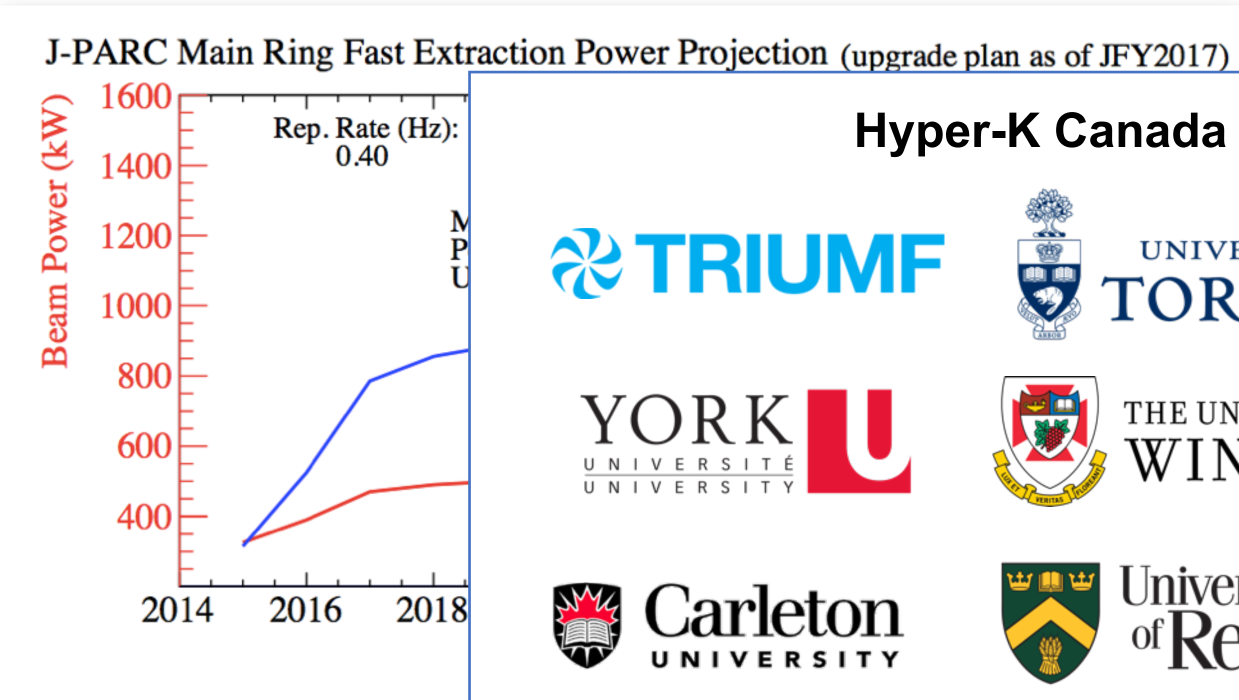
Improvements for Hyper-K





- Beam will be upgraded from current power of ~500 kW to 1.3 MW
  - New main ring power supplies, upgrade of RF
- Hyper-K will have an 8 times larger fiducial mass than Super-K
  - Same off-axis angle
- New photo-detector technologies -> improved photon detection
- **New near/intermediate detectors to control systematic uncertainties**
- Hyper-K will accumulate stats for accelerator based program 20x faster than T2K currently does

Improvements for Hyper-K



Hyper-K Canada group is formed

TRIUMF

University of Toronto

University of Victoria

York University / Université York

The University of Winnipeg

BCIT British Columbia Institute of Technology

Carleton University

University of Regina

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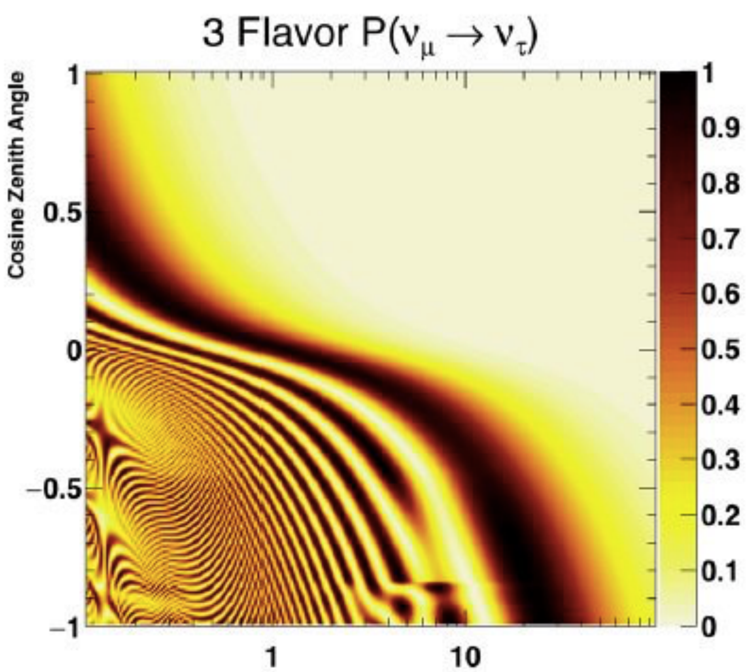
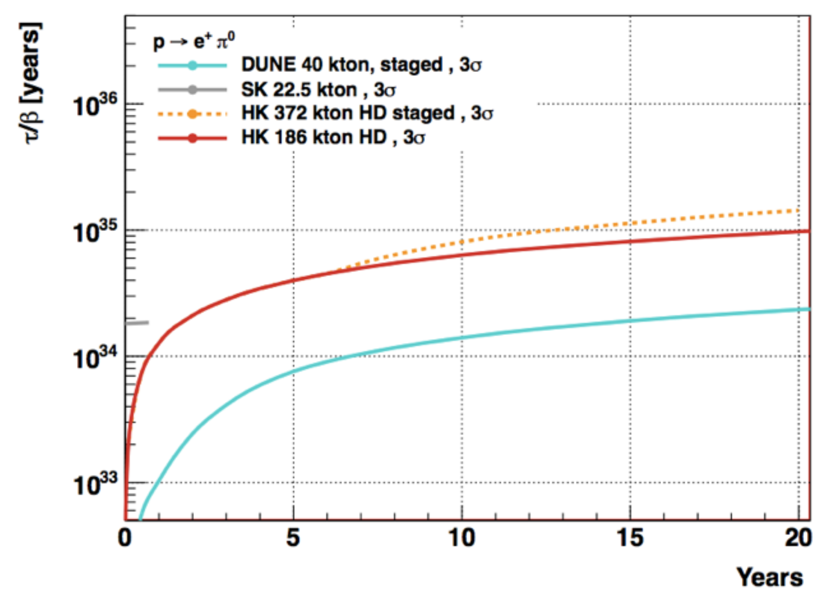
Broad Physics Programs

- Hyper-K has a broad physics programs beyond long baseline neutrino oscillations

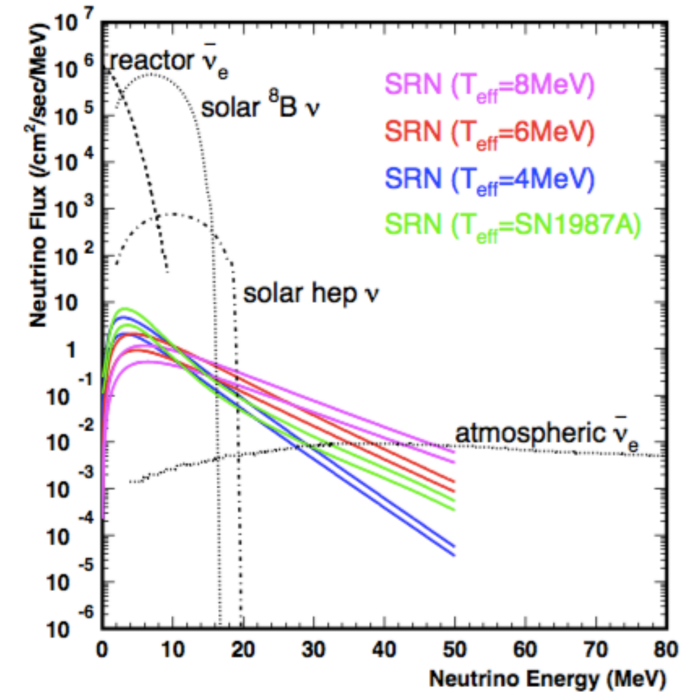
Atmospheric neutrinos



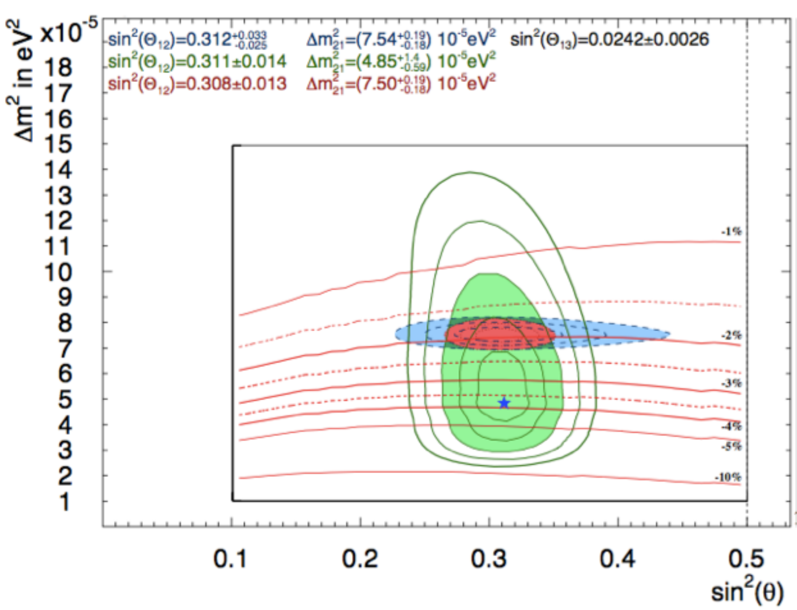
Nucleon decay



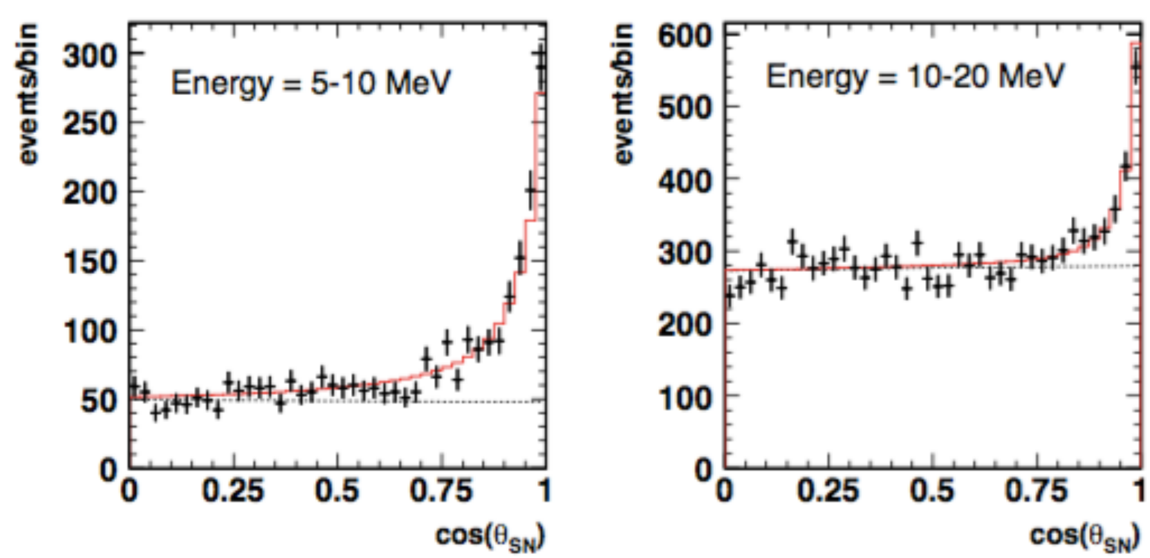
Supernova relic neutrinos



Solar neutrinos

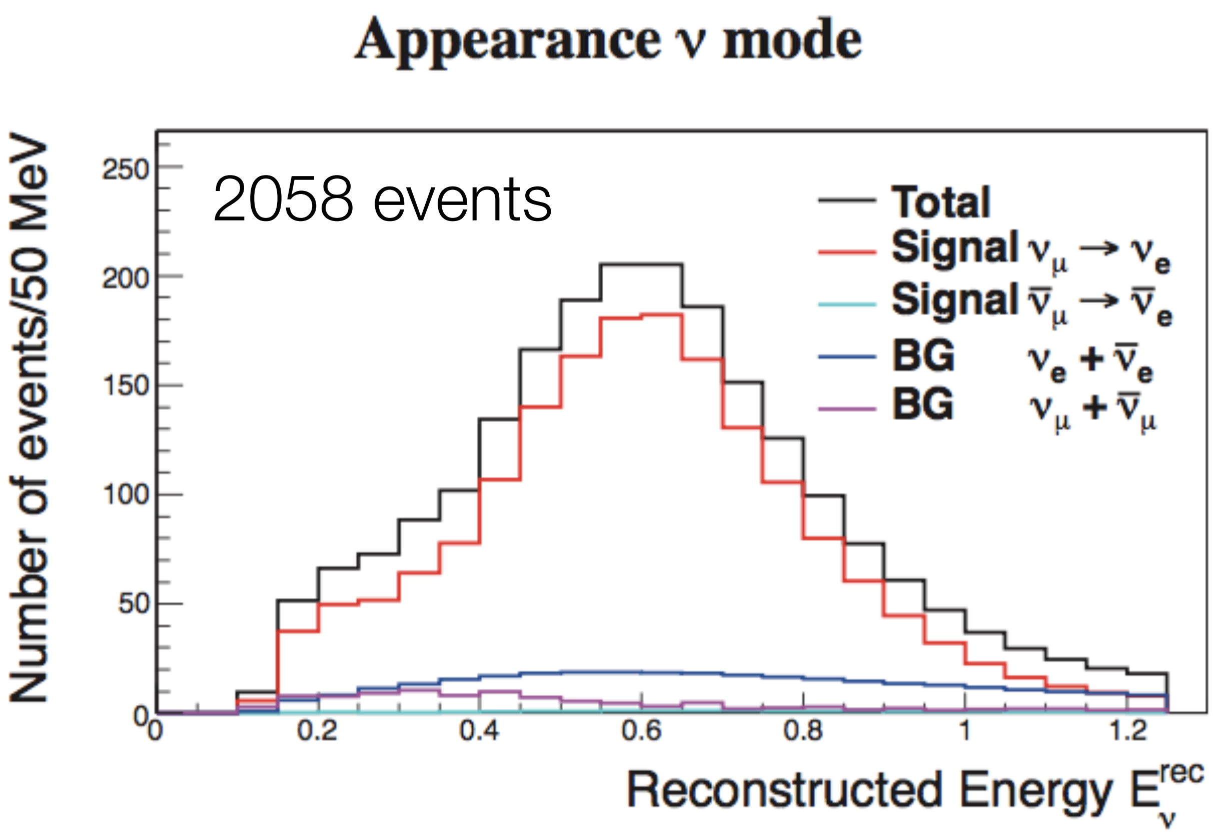


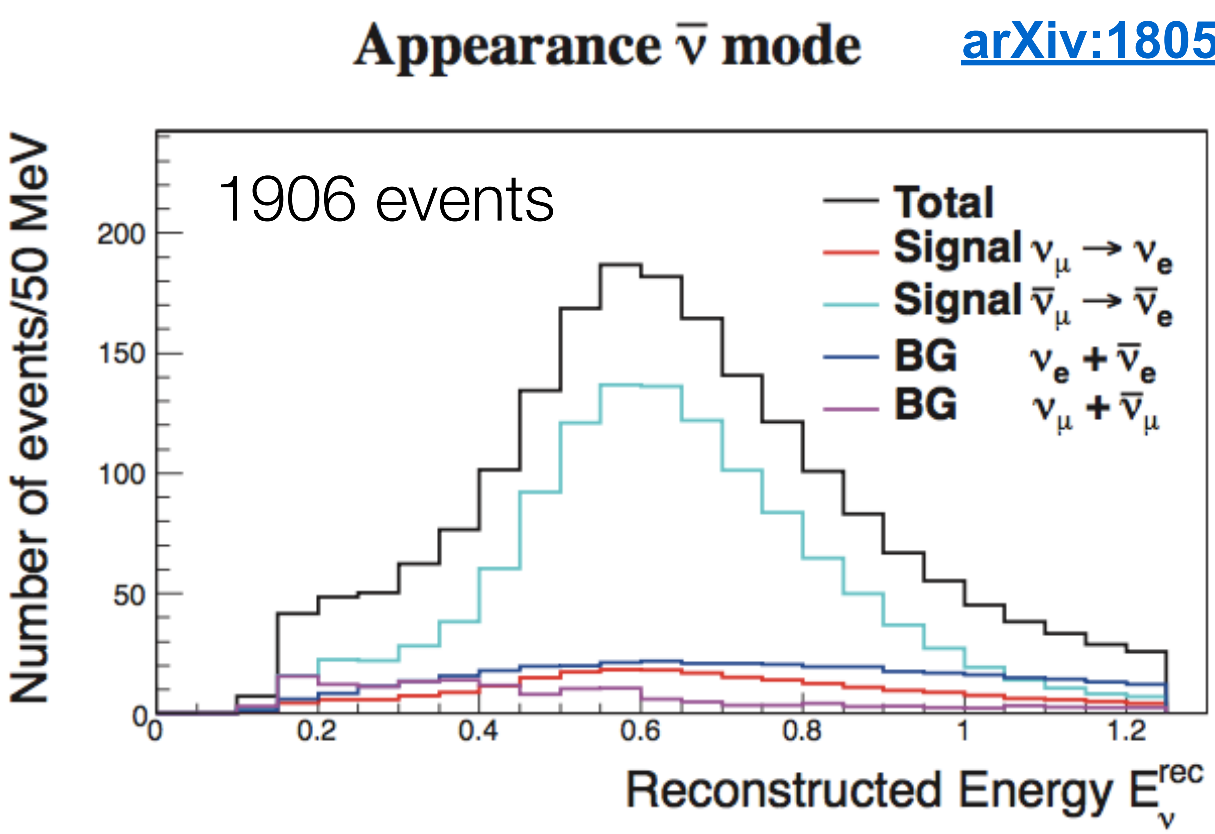
Supernova burst



CP Violation Search at Hyper-K

[arXiv:1805.04163](https://arxiv.org/abs/1805.04163)





- Recall that T2K and NOvA are observing 10's of candidate events
- Hyper-K will observe ~2000 electron neutrino and electron antineutrino candidates each
  - 3% statistical error on the CP violation measurement is achieved
  - **Controlling systematic errors is critical: T2K's current errors are ~6%**

CP Violation Search at Hyper-K

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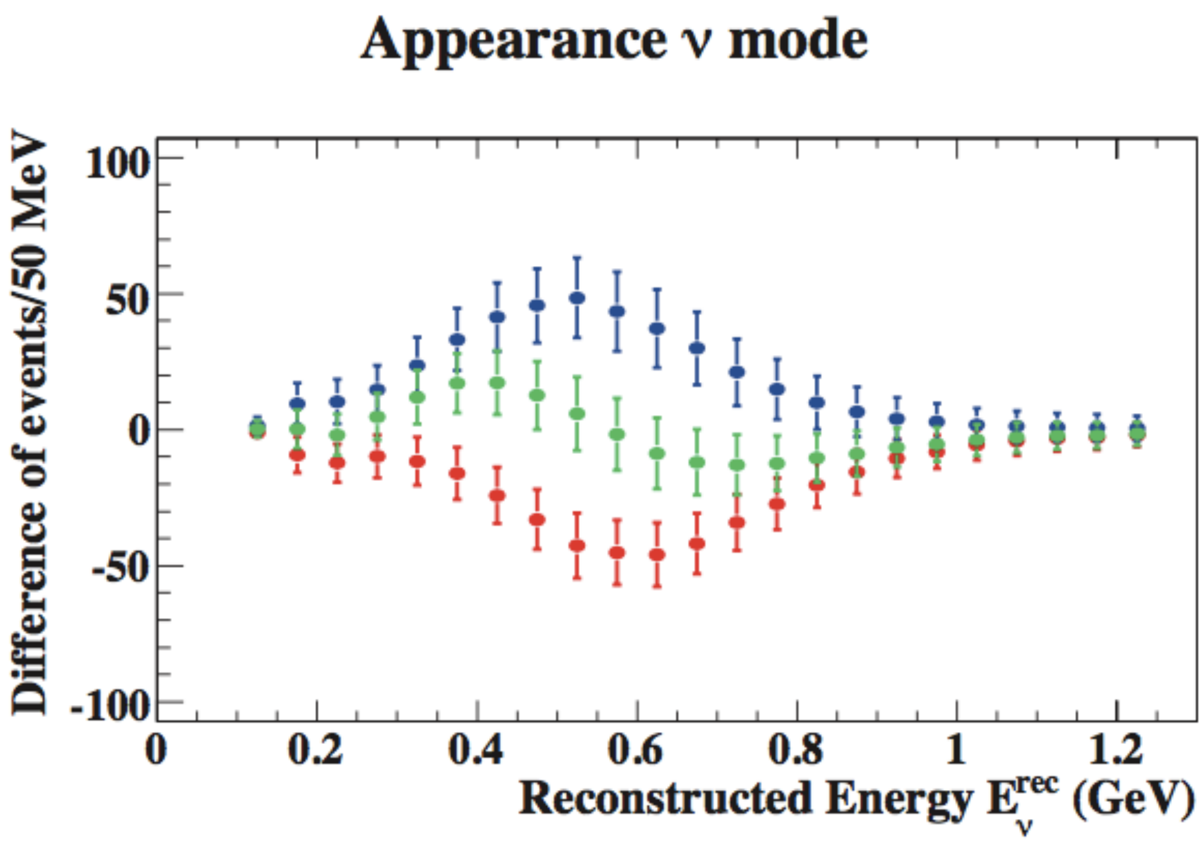

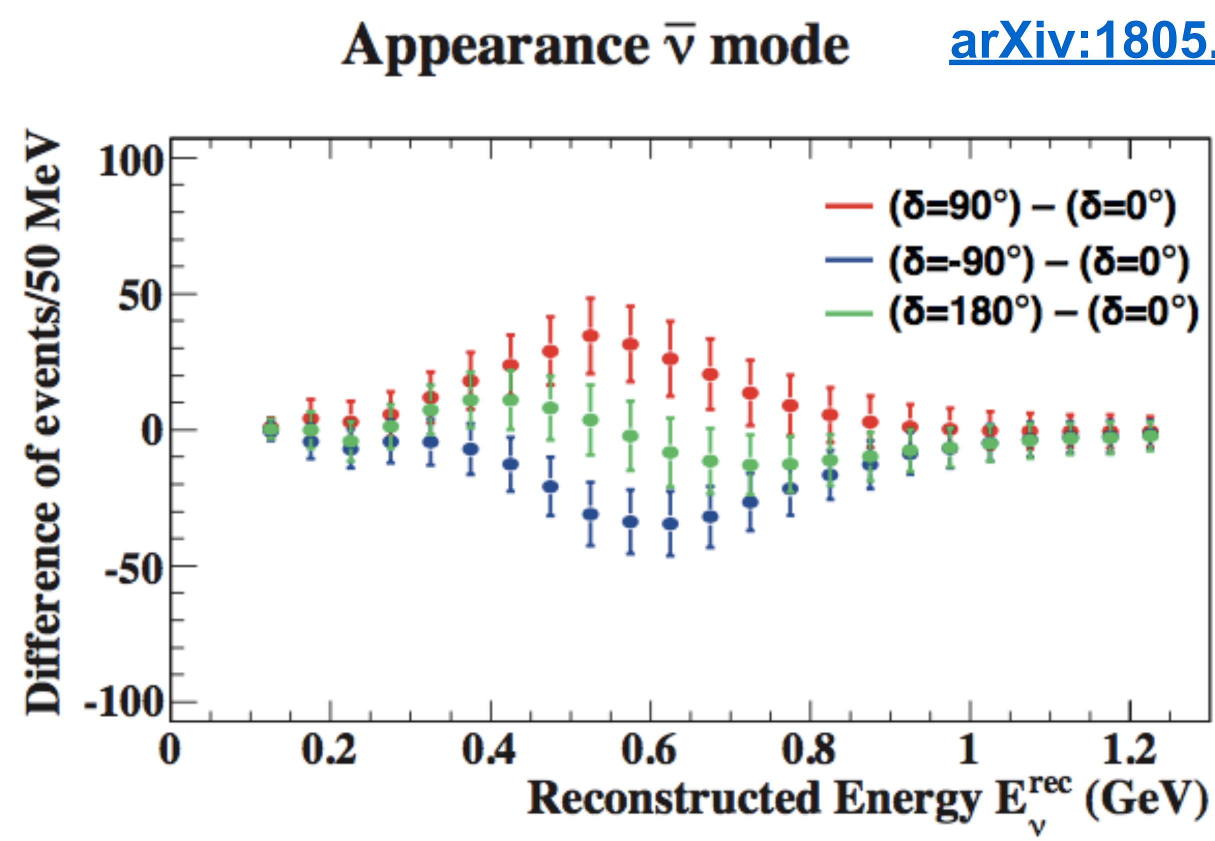

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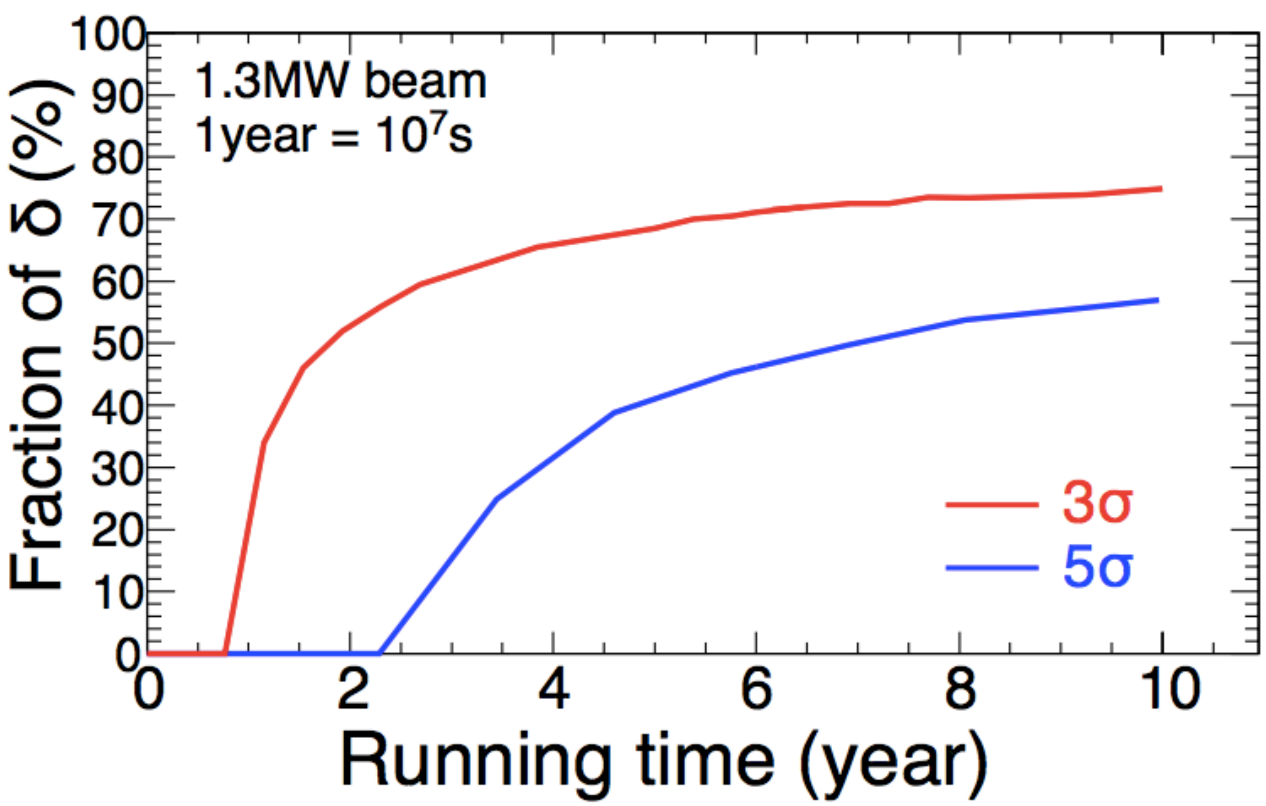
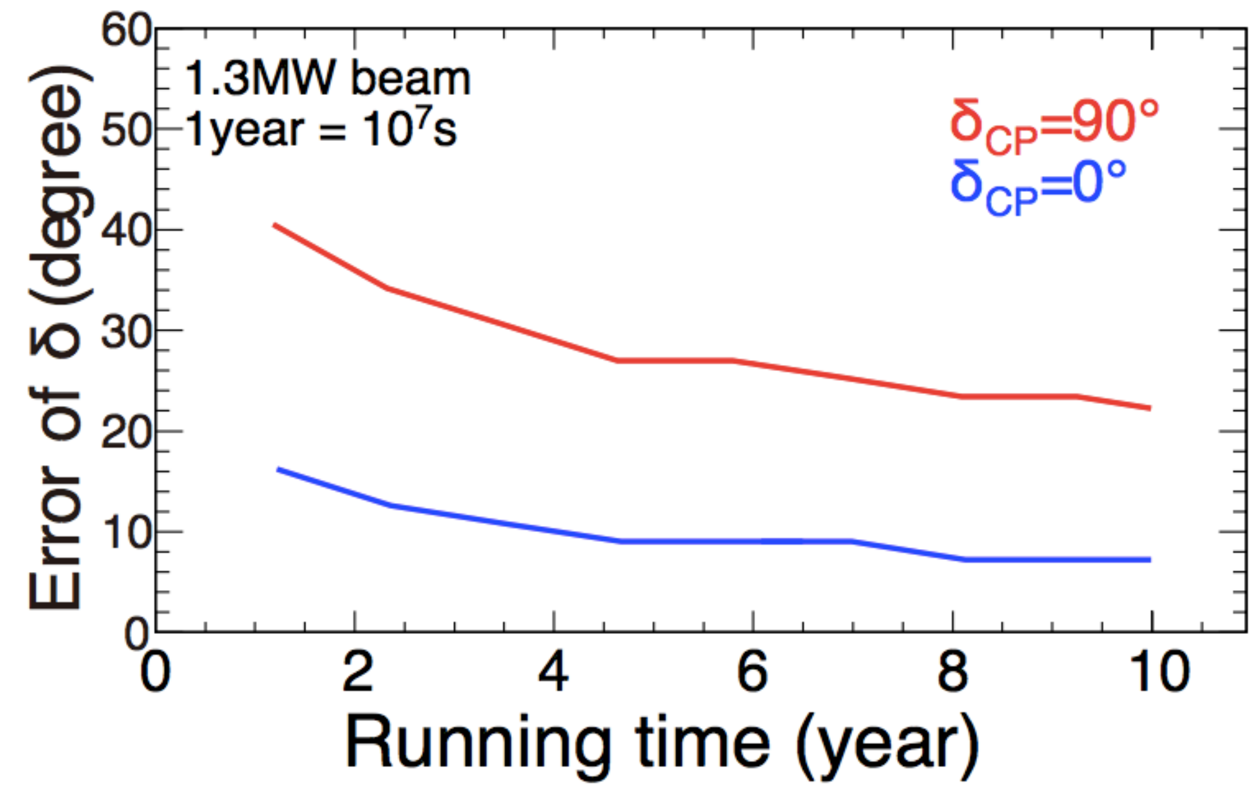
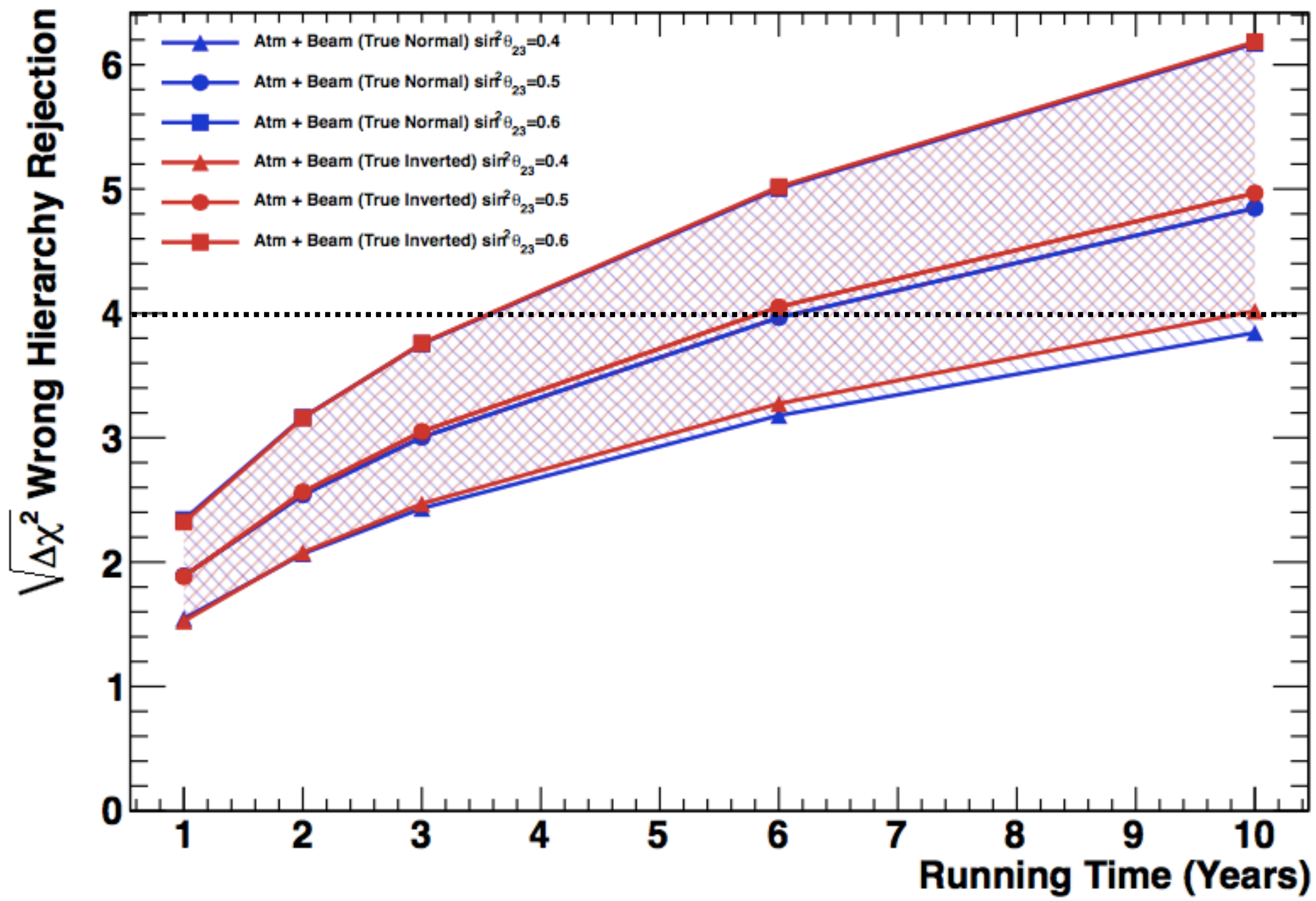
CP Phase Sensitivity





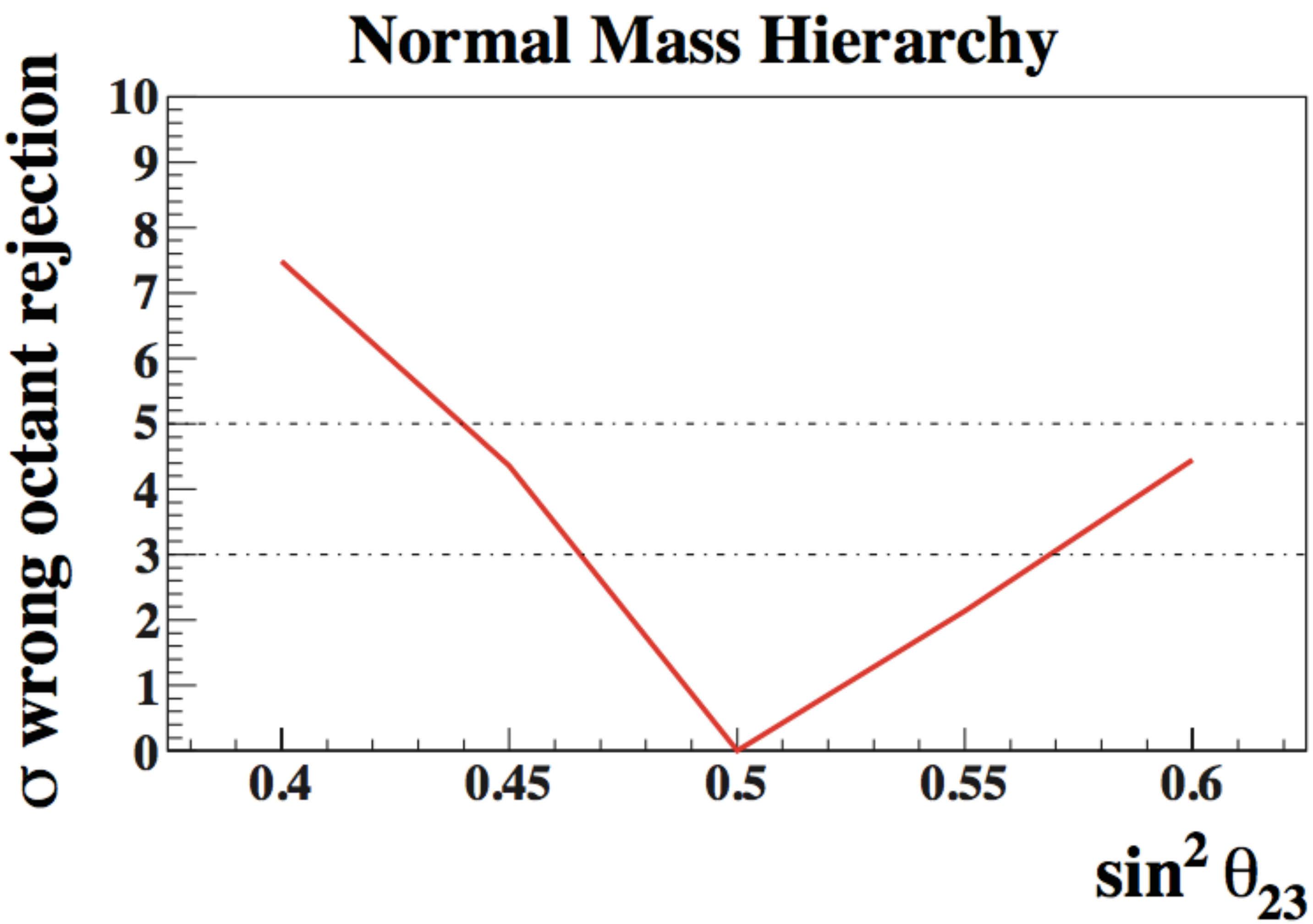
- After 10 years of operation, Hyper-K has 5(3)σ sensitivity for discovery of CP violation for 57%(76%) of δ_{cp} values
- Hyper-K can achieve 10-20° precision on δ_{cp}, depending on the true value

Mass Hierarchy at Hyper-K



- Short baseline - little hierarchy sensitivity from Hyper-K accelerator neutrinos
- Large sample of atmospheric neutrinos
  - Determine the hierarchy with 4σ significance in 10 years of operation

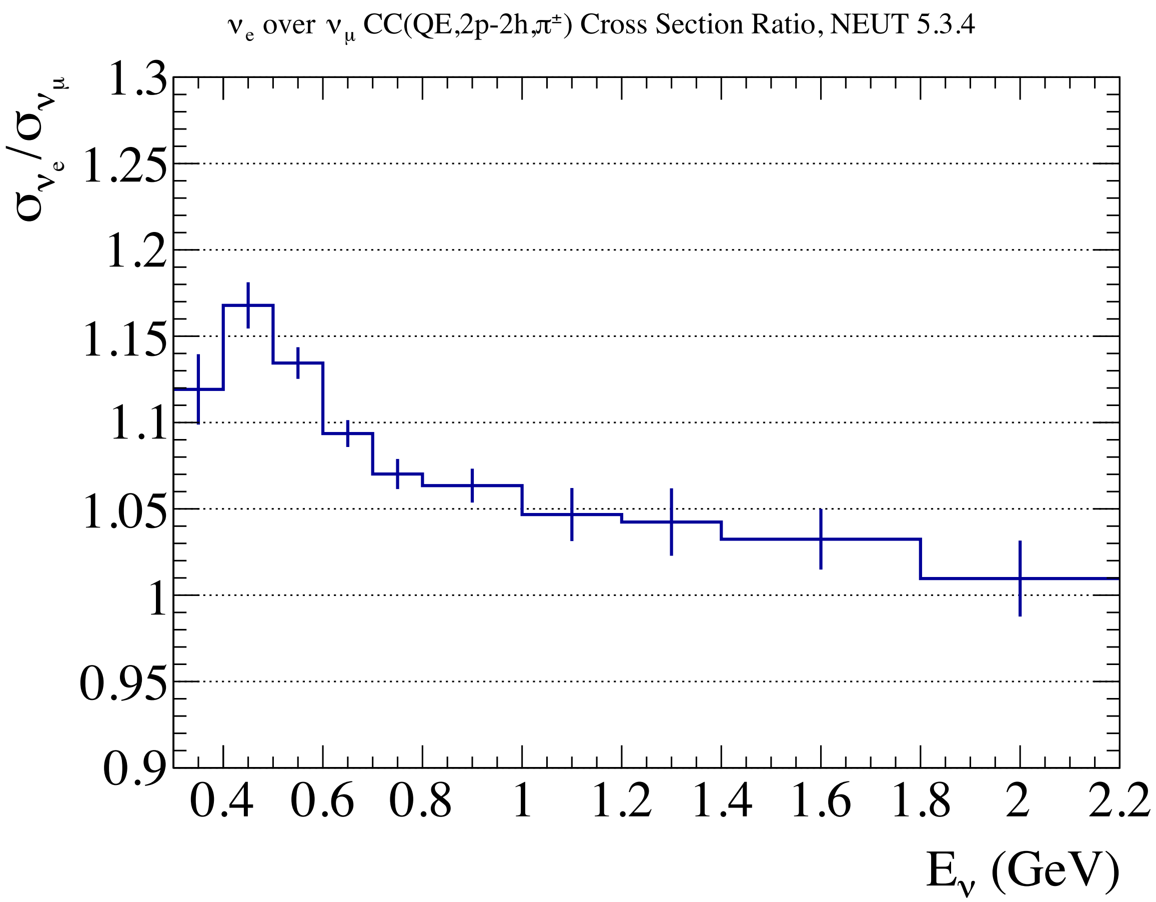
Maximal Mixing?

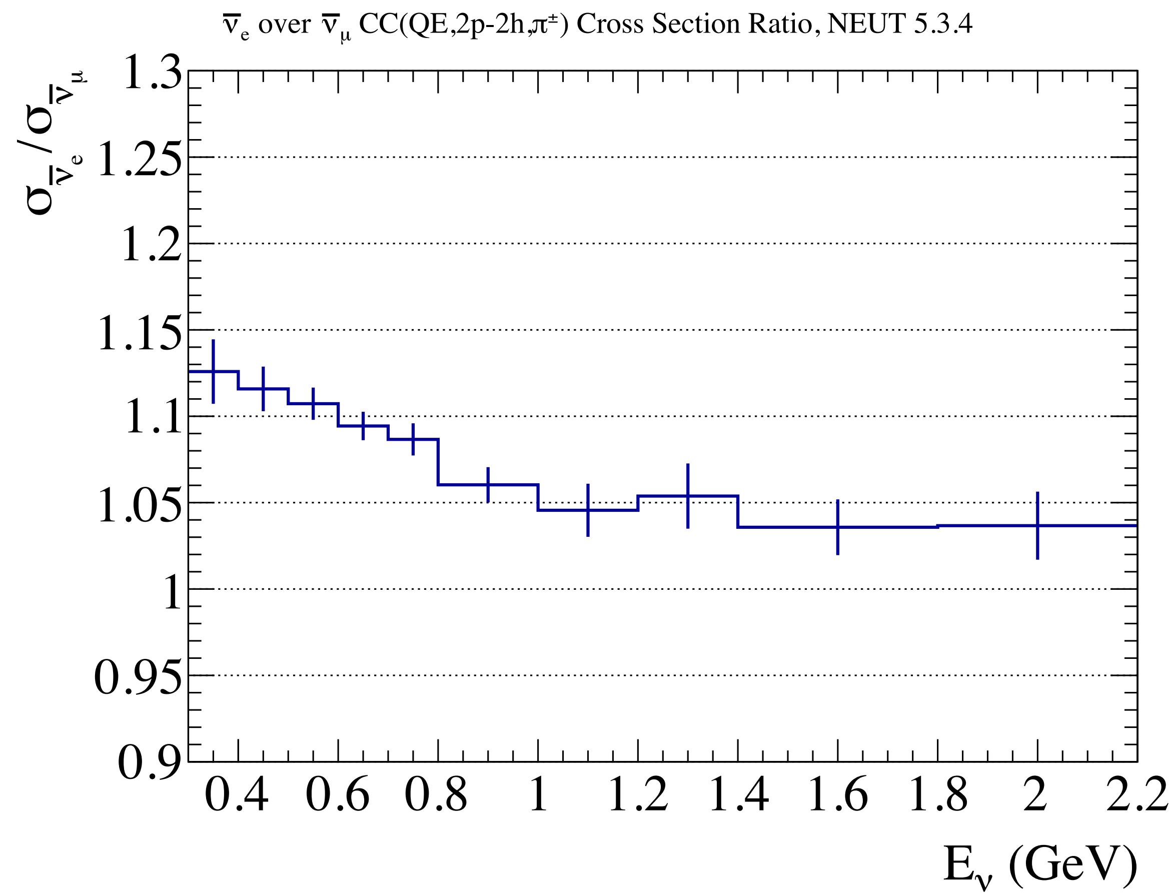


- Hyper-K will have world leading sensitivity to $\sin^2\theta_{23}$
- Open questions: θ_{23} = 45°, >45° or <45°
- Controlling systematic uncertainties for this measurement will be a challenge

Systematic Uncertainties for Hyper-K (1)

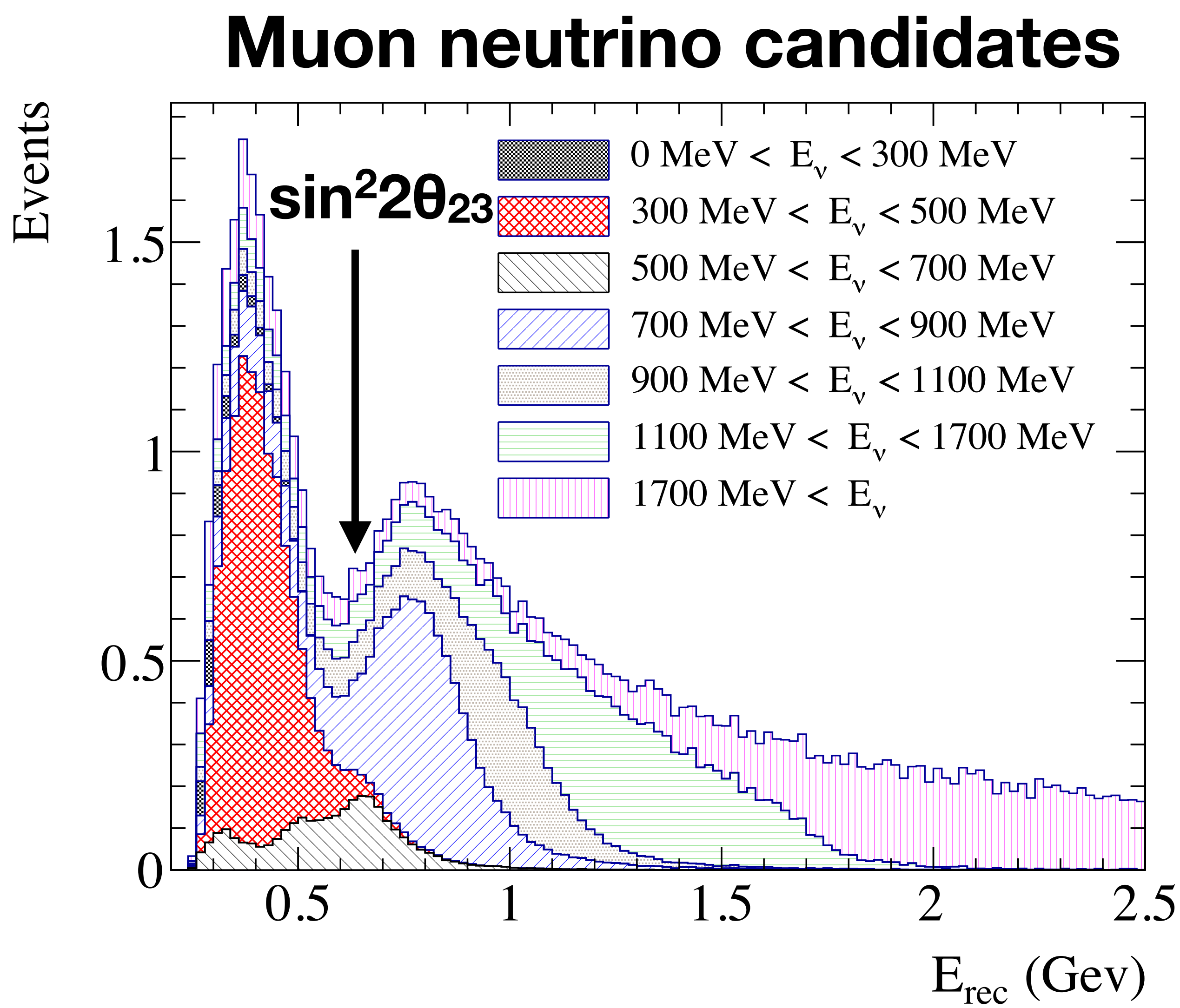
- CP violation measurement in the electron (anti)neutrino appearance modes
- Beam at near detectors is primarily muon (anti) neutrinos





- Electron (anti)neutrino cross sections differ from muon (anti)neutrino cross section
- Theoretical errors on difference are at least 3%
- No precise measurements in the sub-GeV region

Systematic Uncertainties for Hyper-K (2)

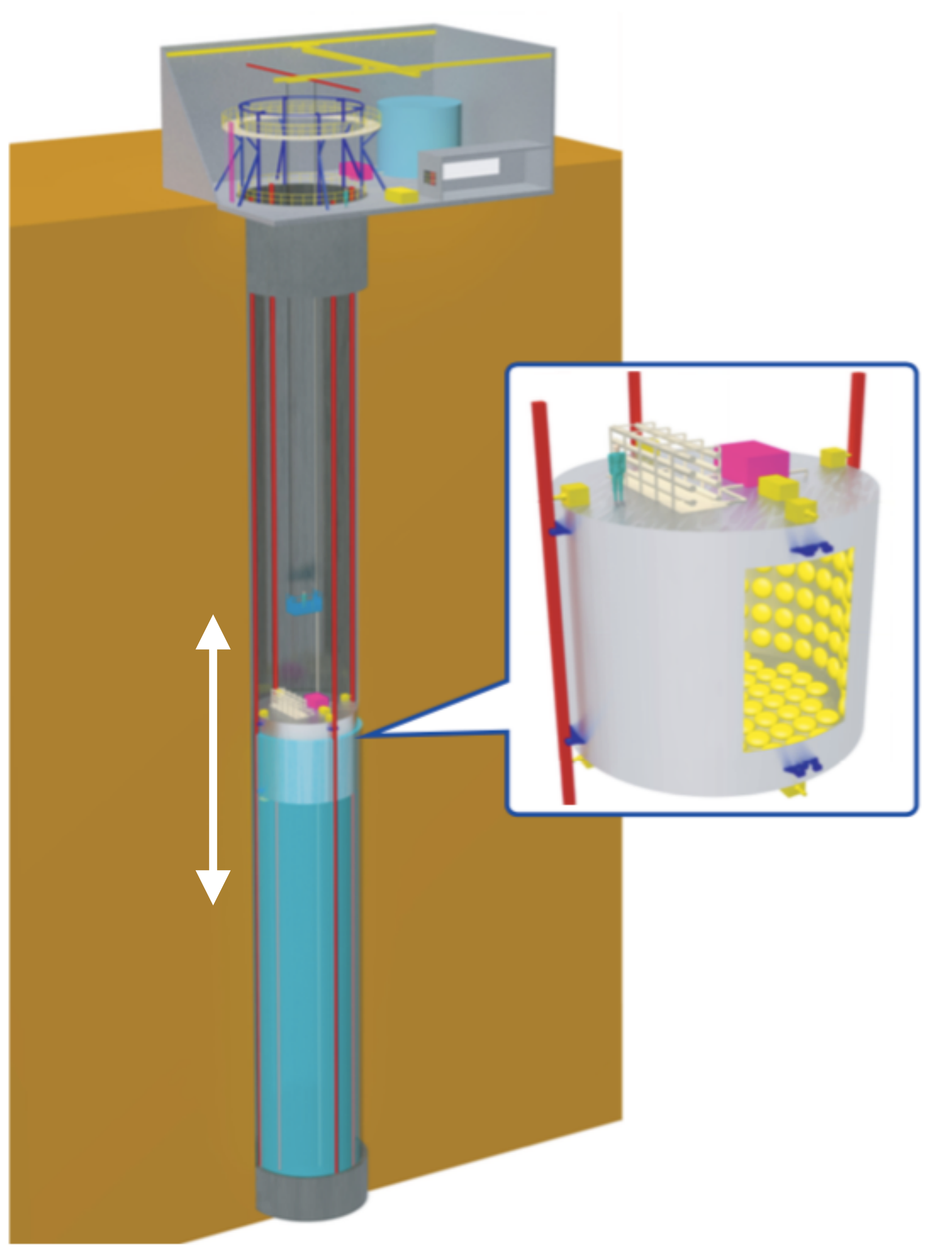


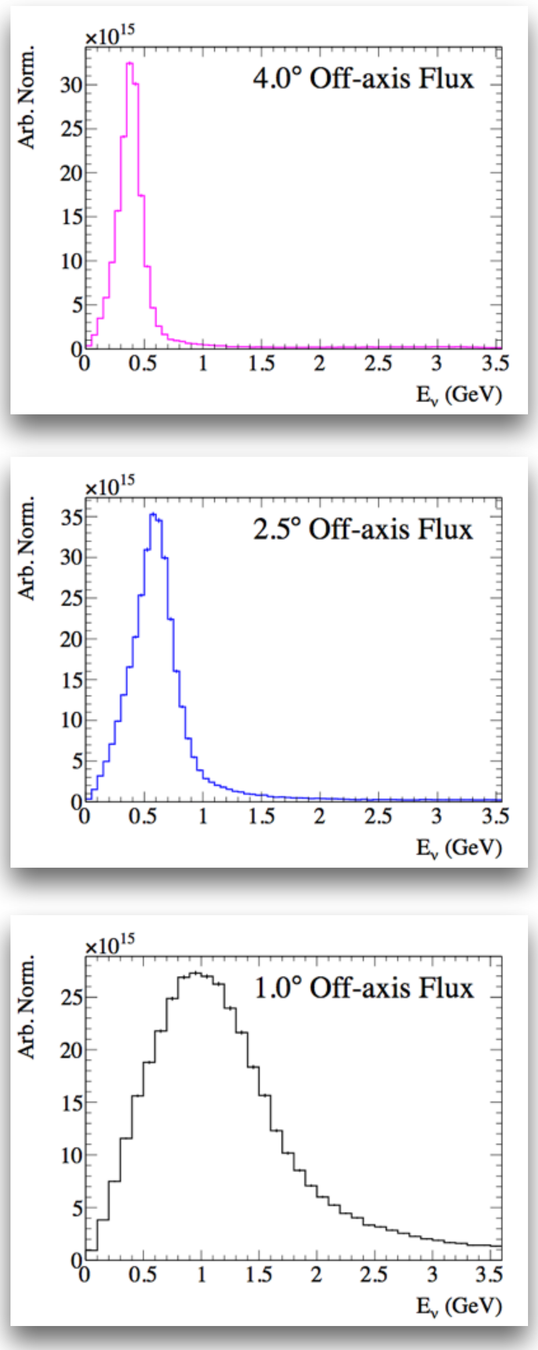
- Oscillation maximum region filled with high energy events that feed down to lower reconstructed energy
  - From difficult to model processes (2p-2h, pion production/absorption) with large systematic uncertainties
- Measuring this feed down is critical for the measurement of θ_{23}

NuPRISM (IWCD)

- New intermediate water Cherenkov detector (IWCD) for Hyper-K
- Detector position is movable, probing different neutrino spectra due to the off-axis angle effect

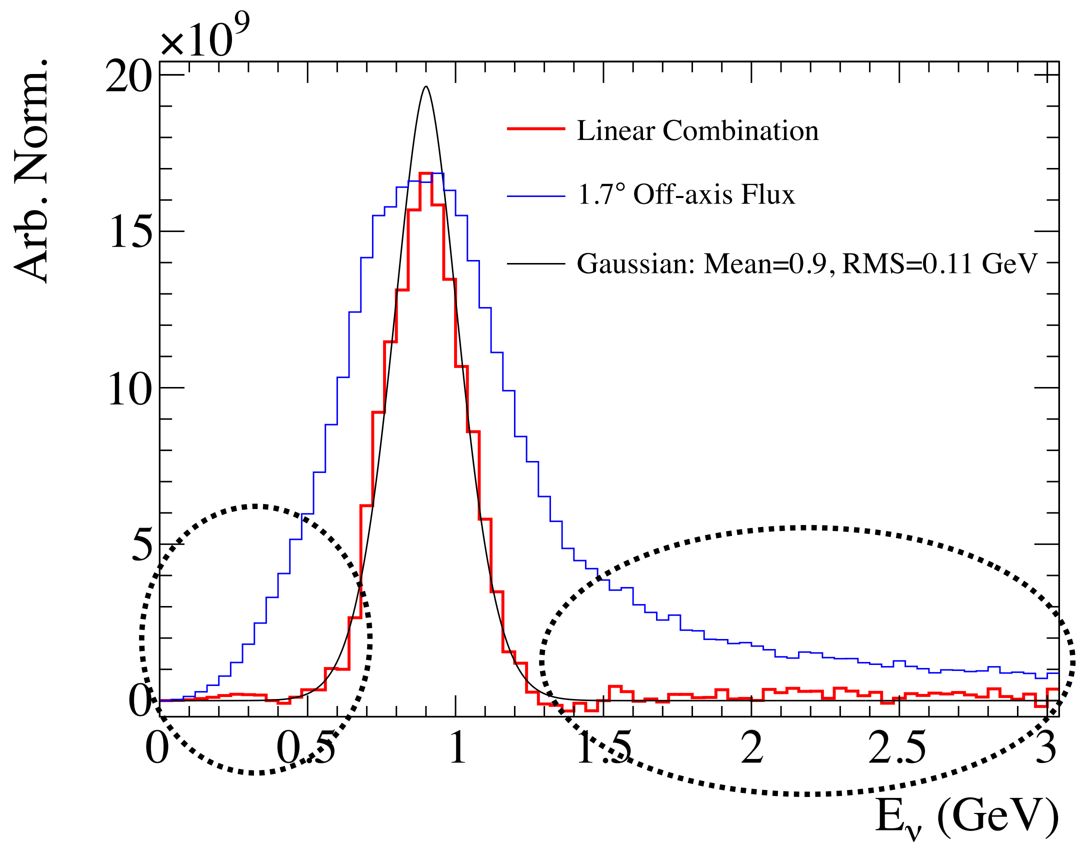
NuPRISM (IWCD) moves vertically to probe range of off-axis angles

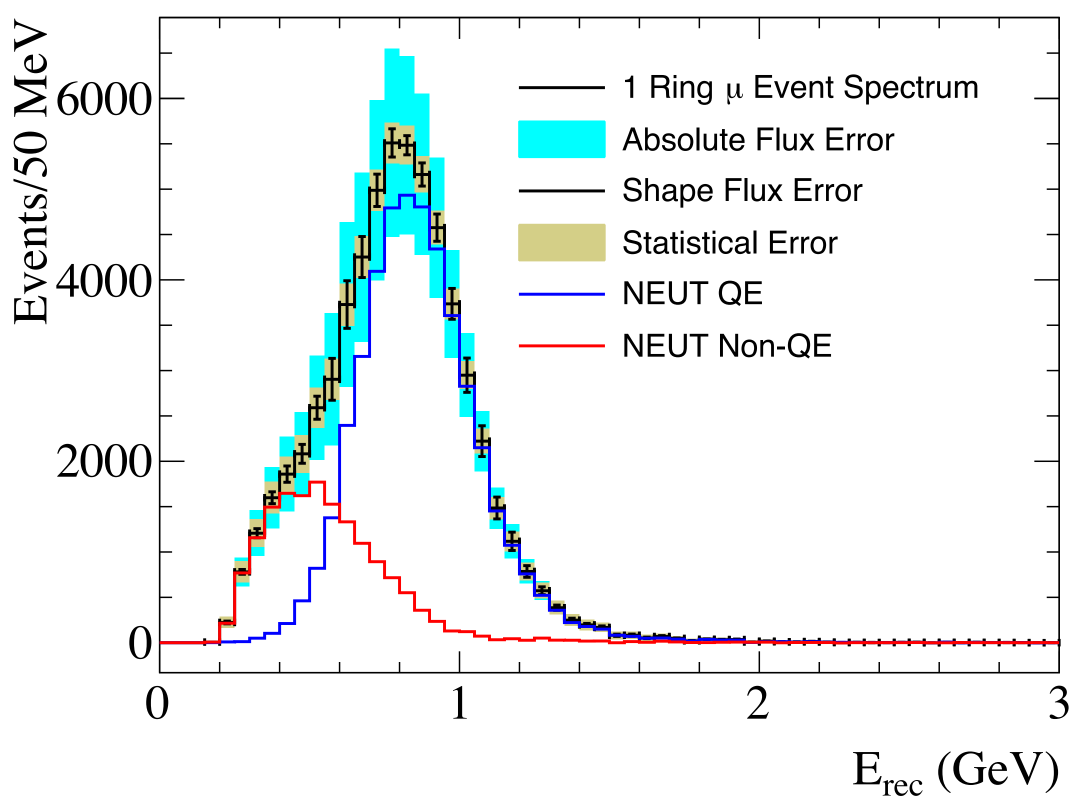




See talk by J. Walker in this session

Measuring the Feed Down



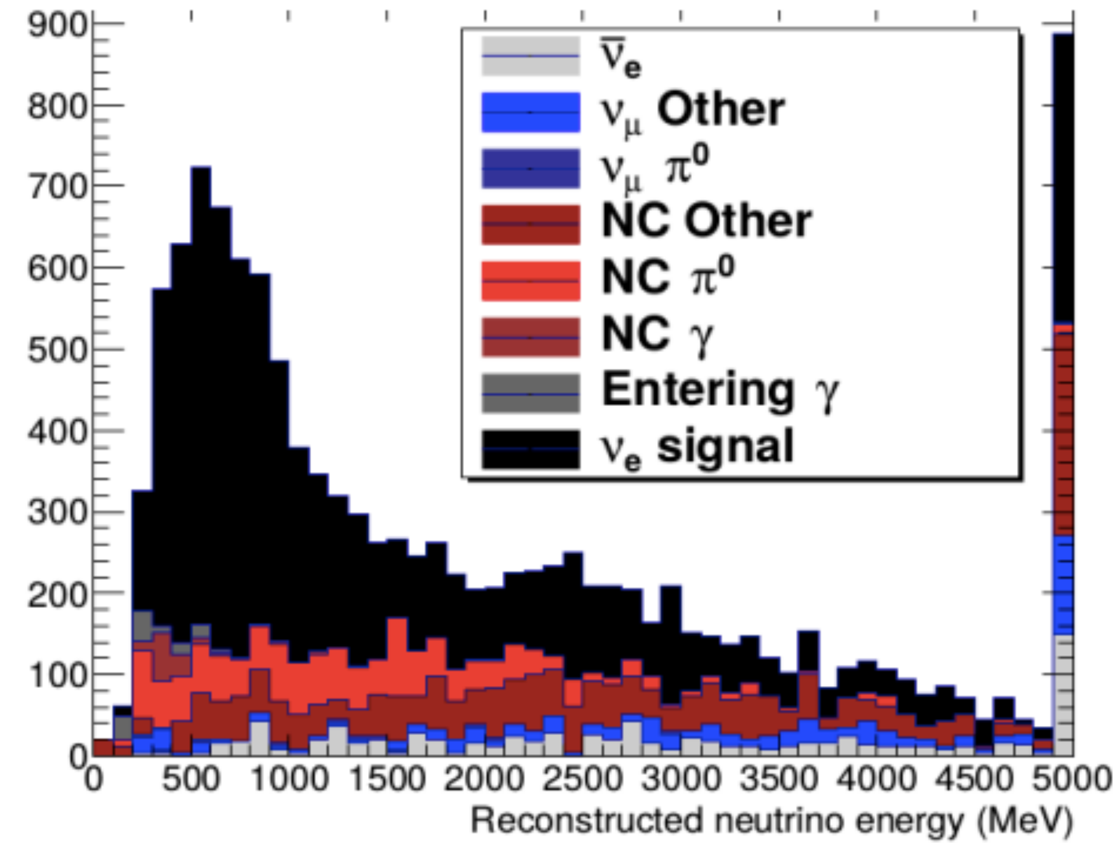


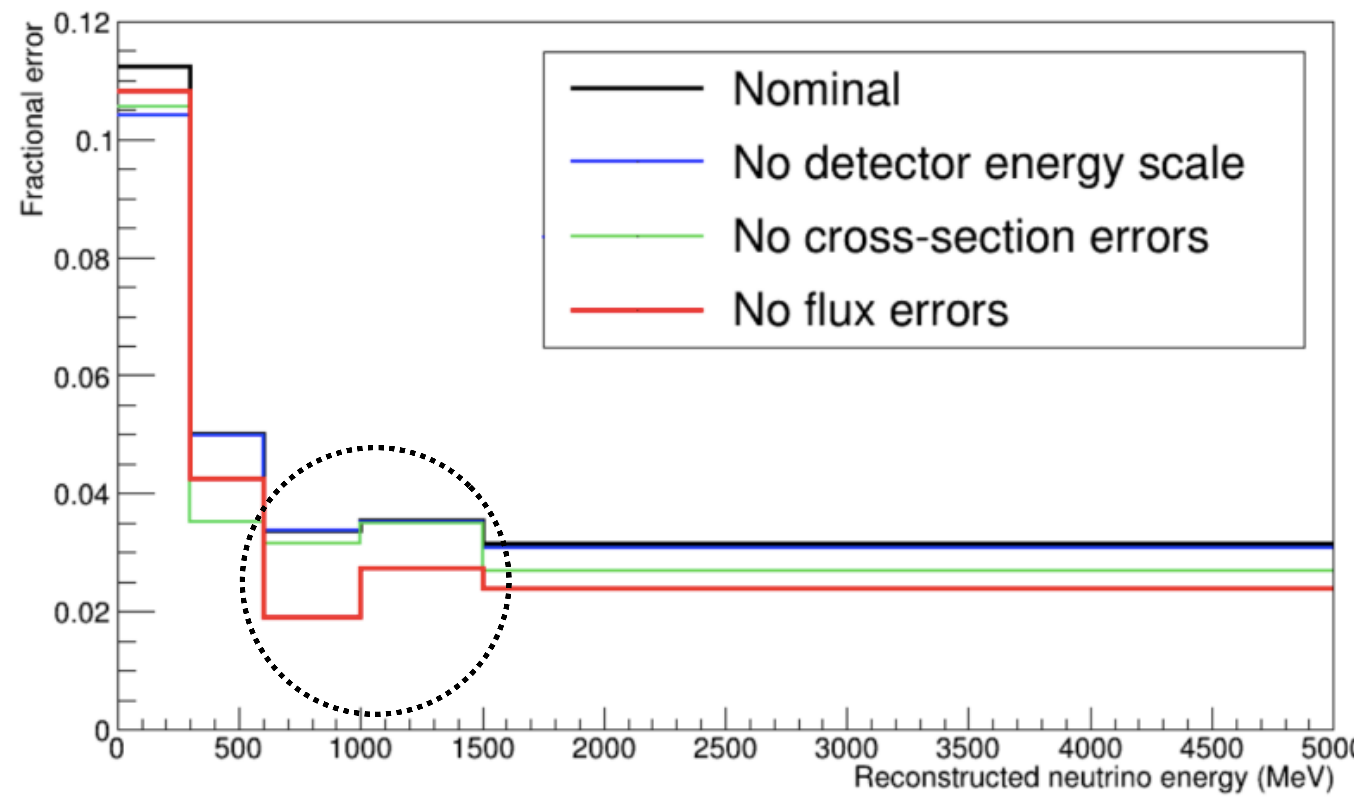
- Start with off-axis spectrum (blue)
- More on-axis measurements subtract high energy tail
- More off-axis measurements subtract low energy tail
- Left with spectrum for narrow range of energies (red)
- Can directly measure the reconstructed energy resolution function (left) including the feed down

Measuring the Electron (anti)Neutrino X-sec

- A pure, high-statistics sample of electron neutrinos can be selected in the IWCD
- Can measure $\sigma(\nu_e)/\sigma(\nu_\mu)$ with ~3% accuracy or better
  - Updating for antineutrinos
- The error on the measurement can best be reduced by reducing the flux model uncertainty
- Benefit from improvements in understanding kaon production and interactions (EMPHATIC)

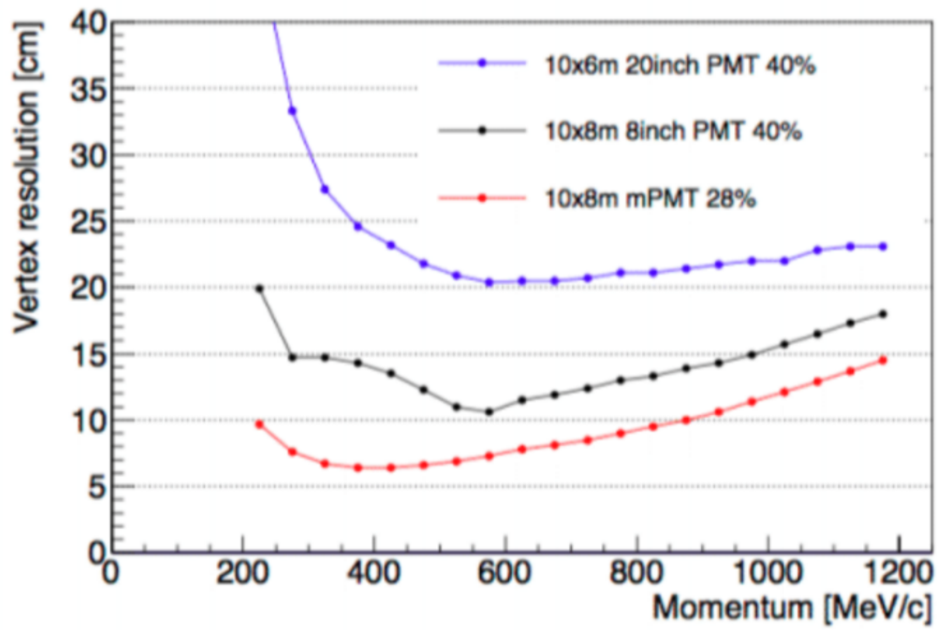
See talk by M. Pavin in this session





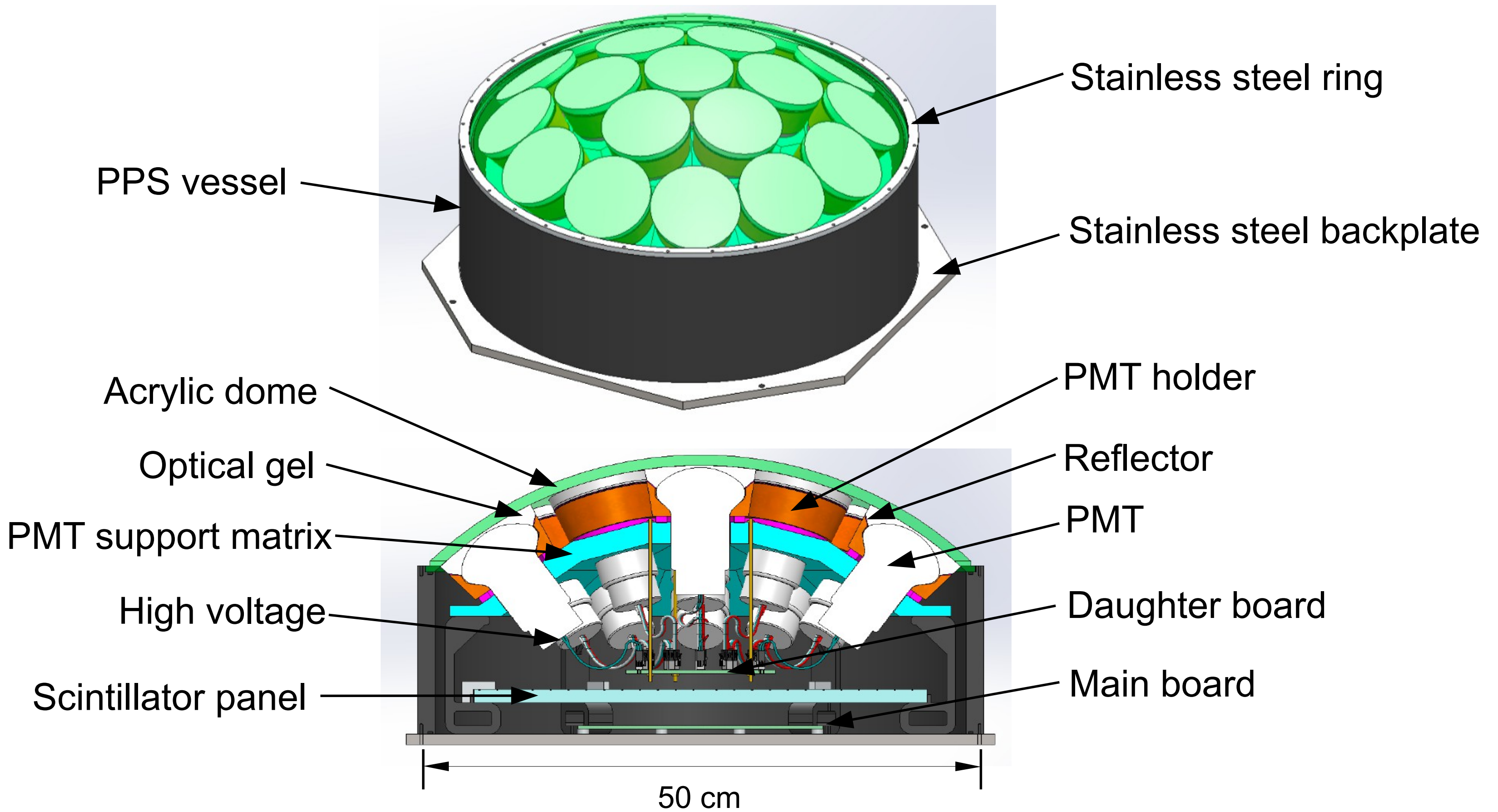
Photosensors for the IWCD (NuPRISM)

- IWCD detector size requires small and fast photosensors
- Have developed a multi-PMT photodetector design with many small sensors integrated in modules



See talk by T. Lindner in session W1-7 for more details on the mPMT

See talk by N. Prouse in this session for machine learning applied to event reconstruction with mPMTs



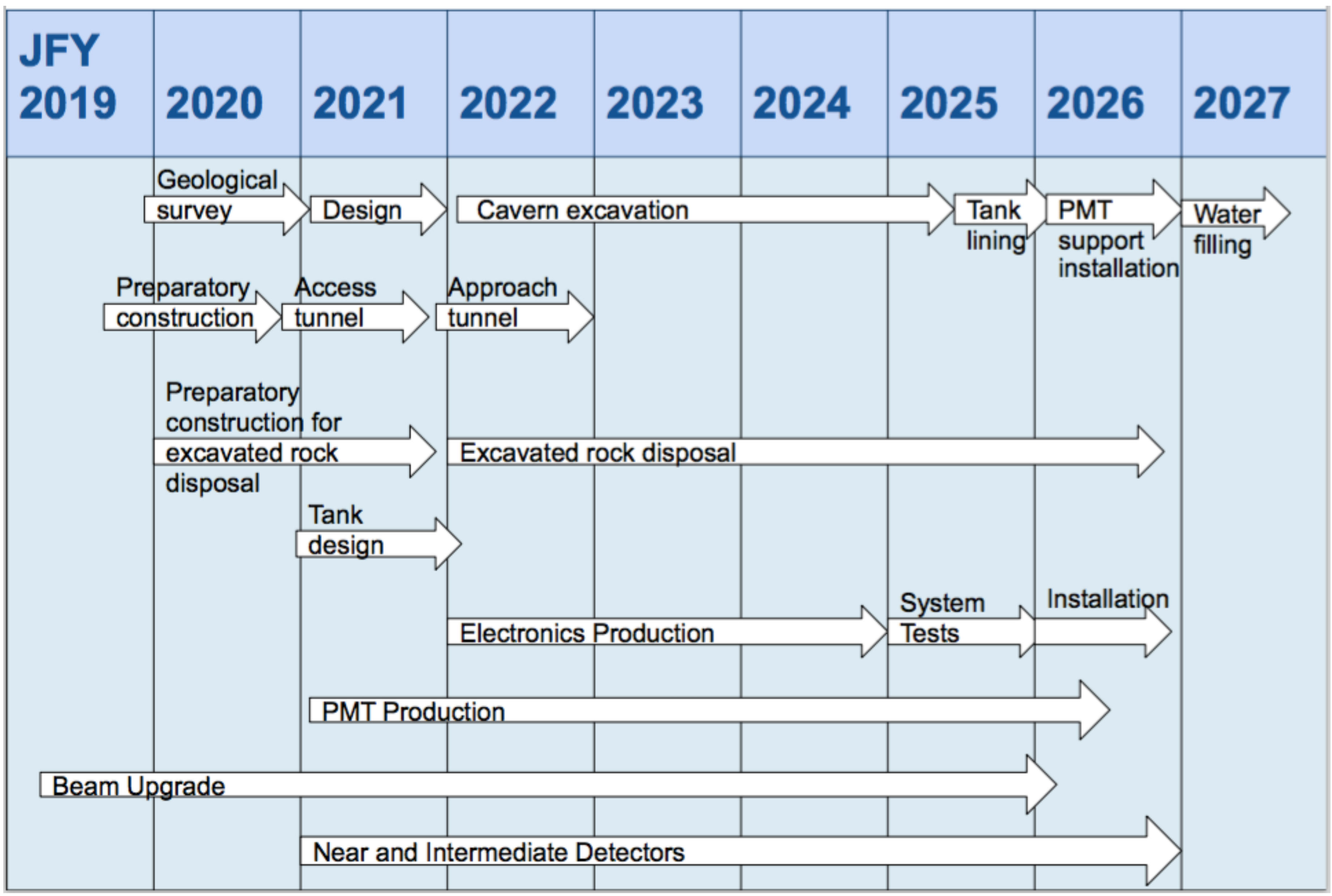
Conclusion

- Neutrino oscillations probe physics beyond the Standard model, including a potential new source of CP violation
- T2K shows a hint of CP violation and preference for maximal mixing through θ_{23}
- Hyper-K will deliver 20 times the sensitivity of T2K to make precision oscillation measurements
- Controlling systematic uncertainties is one of the primary challenges of Hyper-K
  - Exciting solutions such as the IWCD (NuPRISM) are being pursued hear in Canada
  - Success promises great rewards with a world-leading physics program at Hyper-K

Thank You!

Schedule for Hyper-K

- Seed funding for Hyper-K was approved for 2019
    - Typical for large projects in Japan to start with seed funding
- University of Tokyo has committed to the start of construction in 2020



Appearance Measurements at T2K & NOvA

$$P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) \approx \sin^2\theta_{23}\sin^2 2\theta_{13}\frac{\sin^2(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)^2}\Delta_{31}^2$$
$$+ \sin 2\theta_{23}\sin 2\theta_{13}\sin 2\theta_{12}\cos\theta_{13}\frac{\sin(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)}\Delta_{31}\frac{\sin(aL)}{aL}\Delta_{21}\cos(\Delta_{32})\cos\delta$$
$$\mp \sin 2\theta_{23}\sin 2\theta_{13}\sin 2\theta_{12}\cos\theta_{13}\frac{\sin(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)}\Delta_{31}\frac{\sin(aL)}{aL}\Delta_{21}\sin(\Delta_{32})\sin\delta$$
$$+ \cos^2\theta_{13}\cos^2\theta_{23}\ \sin^2 2\theta_{12}\frac{\sin^2(aL)}{(aL)^2}\Delta_{21}^2. \qquad (2)$$

$$\Delta_{ji} = \frac{\Delta m_{ji}^2 L}{4E} \qquad aL/\Delta_{31} = 2\sqrt{2}G_F N_e E/\Delta m_{31}^2$$

Appearance Measurements at T2K & NOvA

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\begin{aligned}
P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) \approx\ & \boxed{\sin^2\theta_{23}\sin^2 2\theta_{13}\frac{\sin^2(\Delta_{31}\mp aL)}{(\Delta_{31}\mp aL)^2}\Delta_{31}^2} \\
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\end{aligned}
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$$\Delta_{ji} = \frac{\Delta m^2_{ji}L}{4E} \qquad aL/\Delta_{31} = 2\sqrt{2}G_F N_e E/\Delta m^2_{31}$$

- Leading term probes mixing angle and mass splitting

Appearance Measurements at T2K & NOvA

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- CP odd interference term can introduce CP violation (sign flips for neutrinos/antineutrinos)

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- Matter effect introduces a dependence on the mass ordering