Probing the Strangeonium Hybrid Content of the *Y(2175)* Using Gaussian Sum-Rules

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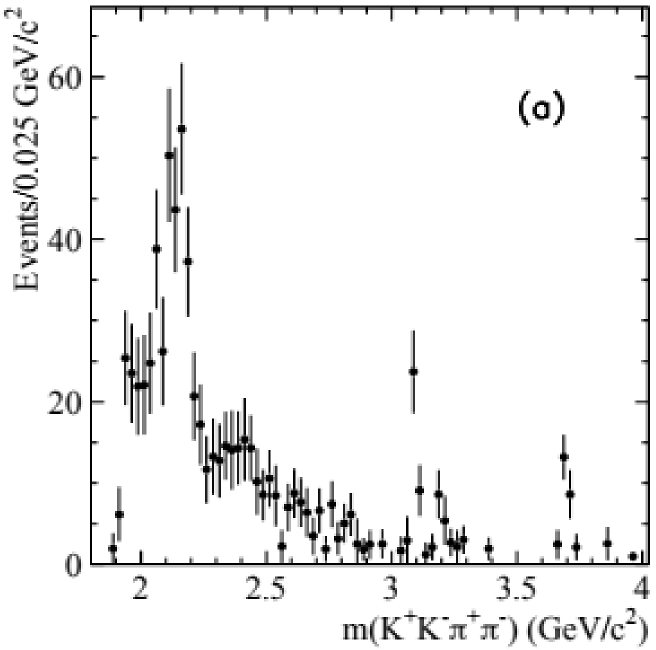
J. Ho, R. Berg, W. Chen, T. G. Steele, and D. Harnett (2019)
[arxiv: 1905.12779]

The *Y(2175) (i.e., φ(2170))* was seen by BaBar in 2006 in initial state radiation processes.

- **BaBar** [Aubert PRD74 (2006)]; **Belle** [Ablikim PRL100 (2008)]; **BES** [Shen PRD80 (2016)]; **BES III** [Ablikim PRD91 (2017)]
- $M = (2188 \pm 10)$ MeV
- $\Gamma = (83 \pm 12)$ MeV
- $I^G\,(J^{PC}) = 0^-\,(1^{--})$



$e^+e^- \to \varphi(1020) f_0(980) \to K^+K^-\pi^+\pi^-$ events vs. invariant mass [Aubert PRD74 (2006)].

The width of the *Y(2175)* does not agree with quark model meson predictions.

- $Y(2175) \to \phi(1020) f_0(980)$ implies $s\bar{s}$ component of the *Y(2175)*.
- Quark model $s\bar{s}$ mass predictions [Godfrey PRD32 (1985)]: $3\ ^3S_1$ or $2\ ^3D_1$?
- Width predictions ($^3P_0$ model) too large.
  - $3\ ^3S_1$: $\Gamma = 378$ MeV [Barnes PRD68 (2003)]
  - $2\ ^3D_1$: $\Gamma = 167$ MeV [Ding PLB657 (2007)]

Perhaps the *Y(2175)* is an outside-the-quark-model resonance.

- Hybrid meson *($sg\bar{s}$)*
  - Laplace sum-rules: $M = (2.9 \pm 0.3)$ GeV [Govaerts NPB262 (1985)]
  - flux tube model: $M = 2.1–2.2$ GeV [Barnes PRD52 (1995)]
  - lattice QCD: $M = 2.1–2.5$ GeV [Dudek PRD84 (2011)]
- Diquark-antidiquark *($[ss][\overline{ss}]$)*
  - Laplace sum-rules: $M_1 = (2.34 \pm 0.17)$ GeV & $M_2 = (2.41 \pm 0.25)$ GeV [Chen PRD98 (2018)]
- Baryon-antibaryon molecule *($\Lambda\bar{\Lambda}$)*
  - $2m_\Lambda = 2.231$ GeV
  - chromomagnetic interaction model: $M = 2.184$ GeV [Abud PRD81 (2010)]
  - one-boson exchange potential: $M = 2.149–2.177$ GeV [Zhou PRD987 (2013)]

Decay modes and rates will be crucial to determining the nature of the Y(2175).

- *$sg\bar{s}$:*
  - *KK*, *K*K** forbidden [Ding PLB 650 (2007)]
- *$[ss][\overline{ss}]$:*
  - *φη*, *φη′* dominant [Ding PLB 650 (2007)]?
  - *$\varphi f_0(980)$, $h_1\eta$, $h_1\eta'$* dominant [Ke (2018)]?
- *$\Lambda\bar{\Lambda}$:*
  - *KK* dominant [Dong PRD 96 (2017)]

φ(2170) DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	e^+e^-	seen
Γ_2	$\phi\eta$	
Γ_3	$\phi\pi\pi$	
Γ_4	$\phi f_0(980)$	seen
Γ_5	$K^+K^-\pi^+\pi^-$	
Γ_6	$K^+K^- f_0(980) \to K^+K^-\pi^+\pi^-$	seen
Γ_7	$K^+K^-\pi^0\pi^0$	
Γ_8	$K^+K^- f_0(980) \to K^+K^-\pi^0\pi^0$	seen
Γ_9	$K^{*0}K^\pm\pi^\mp$	not seen
Γ_{10}	$K^*(892)^0\overline{K}^*(892)^0$	not seen

The *Y(2175)* decay data is incomplete—can't draw definitive conclusions.

We studied the strangeonium hybrid content of the *Y(2175)* using Gaussian sum-rules.

Gaussian sum-rules

- well-suited to multi-resonance analyses

Improved field theory

- higher dimension condensate terms
- strange quark mass corrections to perturbation theory

Updated QCD parameters

- strong coupling, α_s
- four-dimensional gluon condensate $\langle \alpha_s G^2 \rangle$

We probe light strangeonium 1⁻⁻ hybrids with a diagonal two-point correlator.

correlator

$$\Pi(q^2) = \frac{i}{D-1}\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\int \mathrm{d}^D x\, e^{iq\cdot x}\langle\Omega|\tau j_\mu(x) j_\nu(0)|\Omega\rangle$$

D=4+2ε, spacetime dimension

current

$$j_\mu = \frac{g_s}{2}\bar{s}\gamma^\rho\gamma_5\lambda^a\left(\frac{1}{2}\epsilon_{\mu\rho\omega\eta}G^a_{\omega\eta}\right)s$$

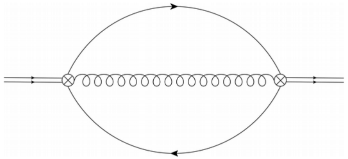
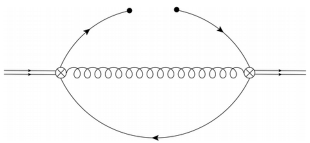
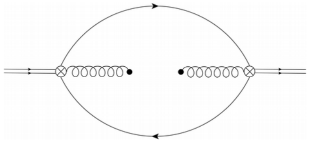
strange quark

gluon field strength

We compute the correlator, *Π(q²)*, within the operator product expansion.

perturbation theory

non-perturbative condensate corrections

$\Pi^{\mathrm{QCD}}(q^2) =$  $+$  $+$  $+\cdots$

$$= \frac{\alpha_s}{\pi}\left(-\frac{q^6}{240\pi^2} + \frac{5m_s^2 q^4}{48\pi^2} - \frac{4m_s q^2}{9}\langle \bar{s}s\rangle + \frac{q^2}{36}\langle G^2\rangle + \frac{19m_s}{72}\langle g\bar{s}\sigma G s\rangle \right)\log\left(\frac{-q^2}{\mu^2}\right)$$

renormalization scale ($\overline{\mathrm{MS}}$)

Dispersion relations relate QCD to hadron physics, *i.e.*, quark-hadron duality.

hadronic spectral function

$$\Pi\left(q^2\right) = q^8 \int_{t_0}^{\infty} \frac{\frac{1}{\pi}\mathrm{Im}\Pi(t)}{t^4\left(t-q^2\right)}\,\mathrm{d}t + \cdots \quad \text{for } q^2 < 0$$

hadron production threshold

subtraction constants

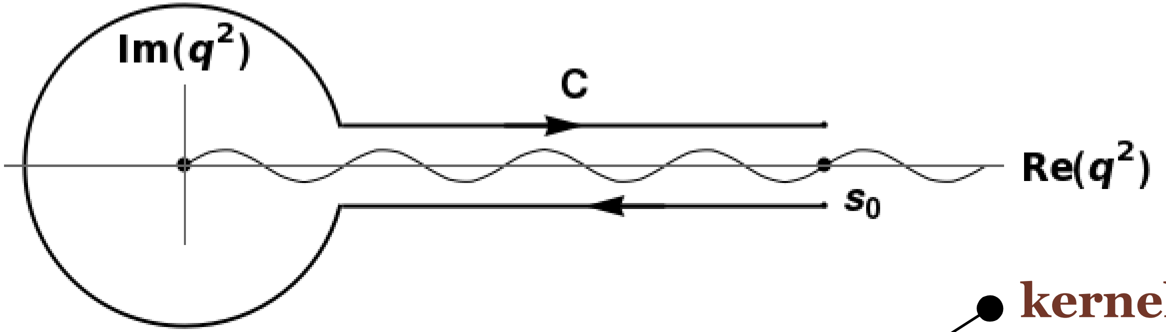
$$\Pi(q^2) \to \Pi^{\mathrm{QCD}}(q^2)$$

continuum threshold

resonance content

$$\frac{1}{\pi}\mathrm{Im}\Pi(t) \to \rho^{\mathrm{had}}(t) + \theta(t-s_0)\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{QCD}}(t)$$

QCD sum-rules are transformed dispersion relations.



kernel

$$G(\hat{s},\, \tau,\, s_0) \equiv \frac{1}{2\pi i}\int_C \frac{e^{-\frac{(\hat{s}-q^2)^2}{4\tau}}}{\sqrt{4\pi\tau}}\, \Pi^{\mathrm{QCD}}(q^2)\, \mathrm{d}q^2$$

$$= \int_{t_0}^{s_0} \frac{e^{-\frac{(\hat{s}-t)^2}{4\tau}}}{\sqrt{4\pi\tau}}\, \rho^{\mathrm{had}}(t)\, \mathrm{d}t$$

Gaussian sum-rules (GSRs)

Predictions are extracted as best-fit values between QCD and hadron physics.

hadronic couplings

$$\rho^{\text{had}}(t) = f_1^2\,\delta\left(t - m_1^2\right) + f_2^2\,\delta\left(t - m_2^2\right)$$

double narrow resonance model

m_1 = 2.188 GeV

$$G(\hat{s},\tau,\,s_0) = \frac{1}{\sqrt{4\pi\tau}}\left(f_1^2\exp\left(\frac{-(\hat{s}-m_1^2)^2}{4\tau}\right) + f_2^2\exp\left(\frac{-(\hat{s}-m_2^2)^2}{4\tau}\right)\right)$$

Extract $s_0,\ m_2,\ r \equiv \frac{f_1^2}{f_1^2+f_2^2},\ \frac{f_2^2}{f_1^2+f_2^2}.$

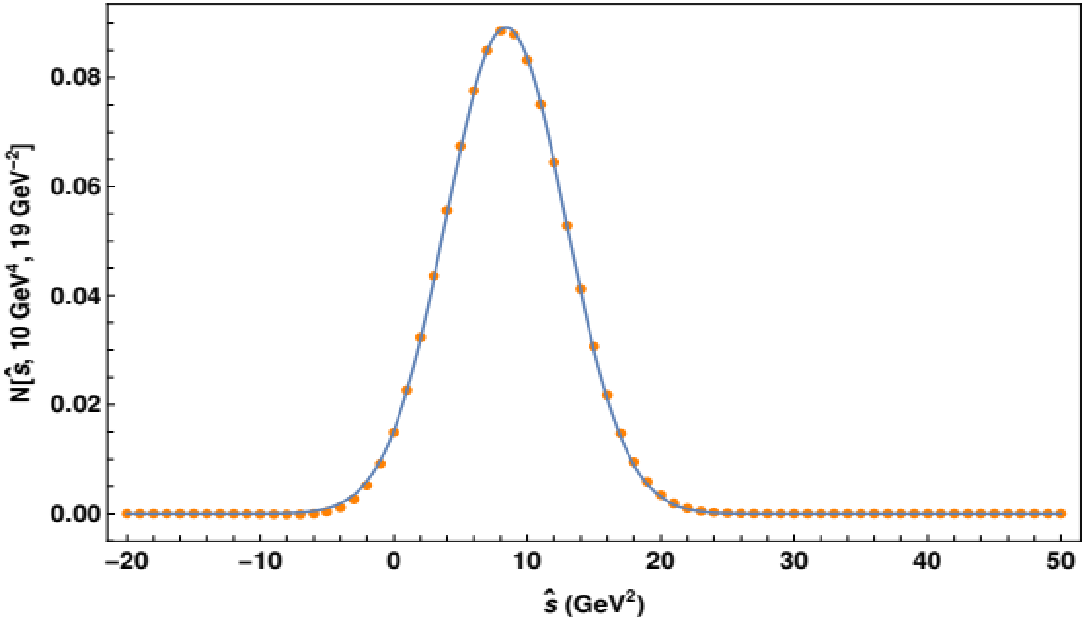
We find an essentially decoupled *Y(2175)* and a heavy second resonance.

Extracted hadronic parameters:

$$s_0 = (9.7 \pm 1.0)\ \mathrm{GeV}^2$$

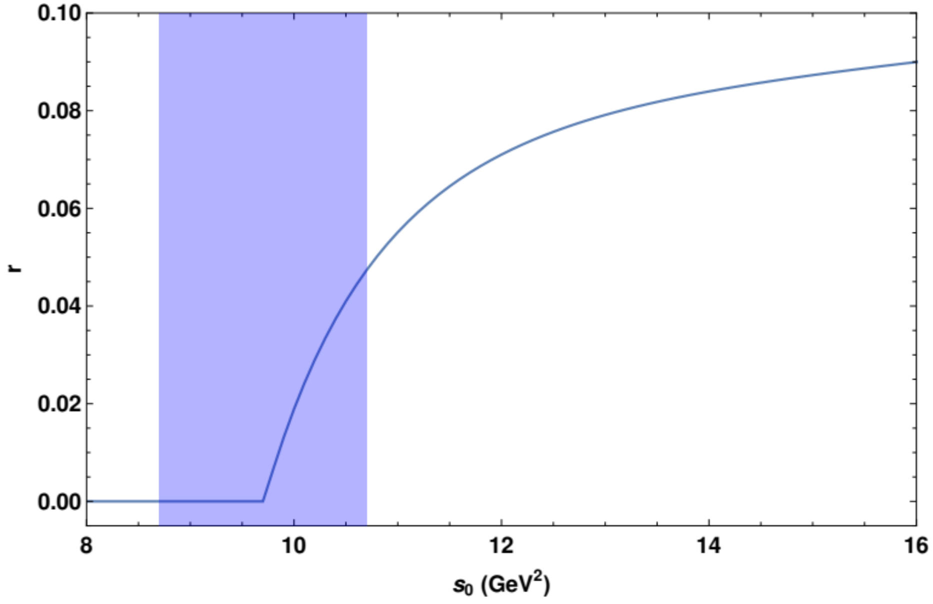
$$m_2 = (2.90 \pm 0.16)\ \mathrm{GeV}$$

$$r \equiv \frac{f_1^2}{f_1^2 + f_2^2} \leq 0.033$$

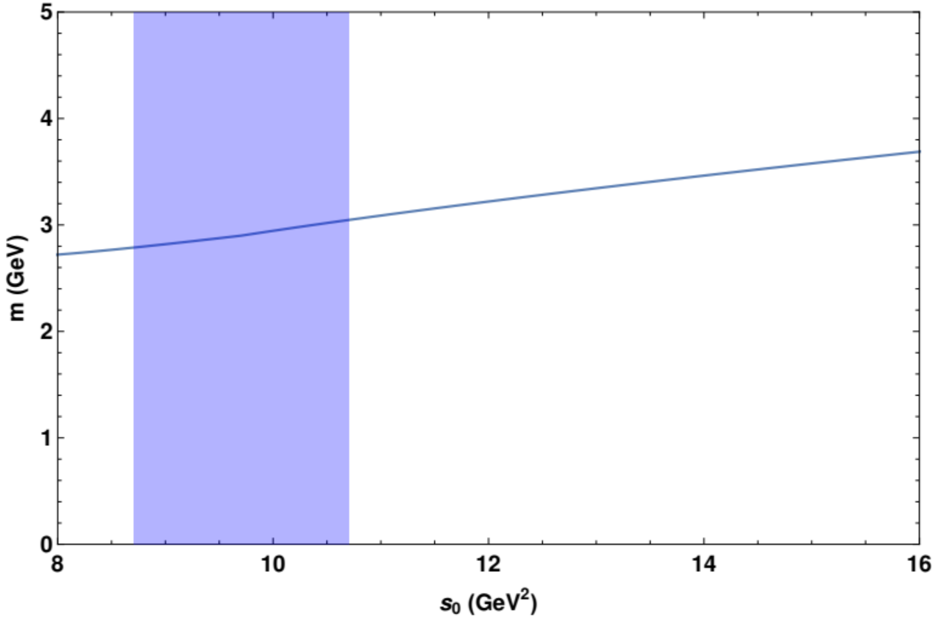


A comparison of QCD (blue curve) and fitted (orange dots) normalized Gaussian sum-rules.

As an independent check on our error bounds, we plot r & m_2 vs. s_o.



Predicted relative couping, r, vs. continuum threshold, s_o. The shaded region represents the uncertainty in s_o.



Predicted heavy resonance mass, m_2, vs. continuum threshold, s_o. The shaded region represents the uncertainty in s_o.

A sum-rules study of the strangeonium hybrid content of the *Y(2175)* gives…

- a relative hadronic coupling *r* consistent with zero
- a heavy 2.9 GeV resonance in agreement with [Govaerts NPB262 (1985)]
- **a preference for a *[ss][$\overline{ss}$]* or *$\Lambda\overline{\Lambda}$* interpretation.**