

Improving Conceptual Understanding Through Visual Learning An Example from Quantum Mechanics

Gina Passante, California State University Fullerton



CALIFORNIA STATE UNIVERSITY
FULLERTON



Antje Kohnle
St Andrews University



CALIFORNIA STATE UNIVERSITY FULLERTON



- Part of the California State University (CSU) system
- Physics has BS and MS degrees
- Hispanic-serving institution
 - (42% Hispanic/Latinx, 20% White)
- 40,000 students (57% female) {physics 25% female}
- 47% of Physics BS were 'first generation' (last 10 yrs)

PER @ California State University Fullerton



Faculty

Michael Loverude
Gina Passante

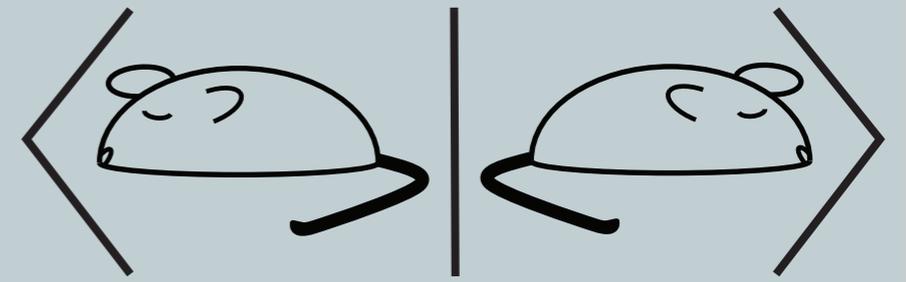
Postdoc

Benjamin Schermerhorn

Masters Students: Mikayla Mays, Anthony Arruda, Zong Yu Wang, Adam Quaal

Undergraduate Students: Anthony Pina, Anderson Fung, Courtney White, Miguel Ramirez, Pachi Her, Kyle Vu, Regan Jones, Thanhnhhan Phan, Sean Young

Research as a base to develop
adaptable curricula bridging instructional
paradigms in quantum mechanics



Homeyra Sadaghiani
Steven Pollock

physport.org/curricula/ACEQM



DUE-1626594

Student learning in Quantum Mechanics

Student thinking about
measurements across the
physics curriculum

Natasha Holmes



DUE-1809178

Visual Learning in
QM: Developing
Simulation-Tutorials

Antje Kohnle
(Shaaron Ainsworth)

Improving Conceptual Understanding Through Visual Learning

“Process of interpreting and constructing visual representations to enhance students’ understanding of the underlying physics concepts and phenomena”

Expert practice and Multiple Representations

The ability to work with and translate between representations...

- Key scientific ability for reasoning and problem-solving

Etkina et al., Phys Rev ST PER 2, 020103 (2006)

- Key difference between novice and expert problem solvers

Dufresne et al., Phys. Teach. 35, 270 (1997)

Kohl & Finkelstein, Phys Rev ST PER 4, 010111 (2008)

- Can lead to emergent insights into a problem, develops conceptual understanding, and facilitates transfer of knowledge across contexts

Ainsworth, Learn. Instr. 16, 183 (2006)

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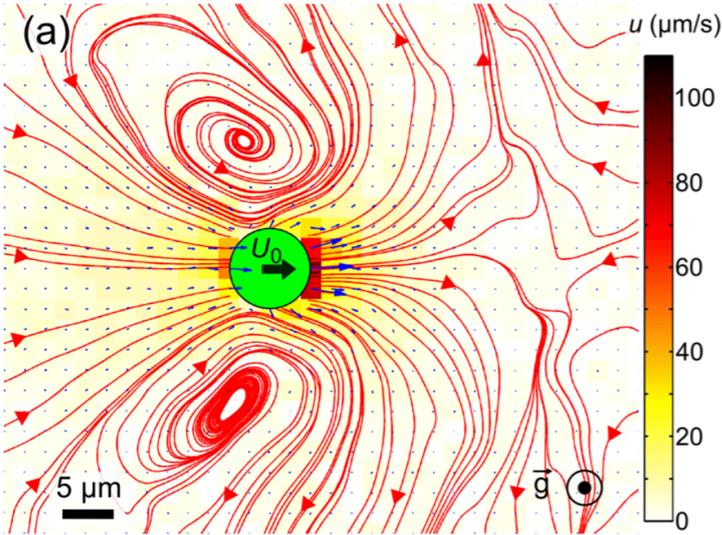
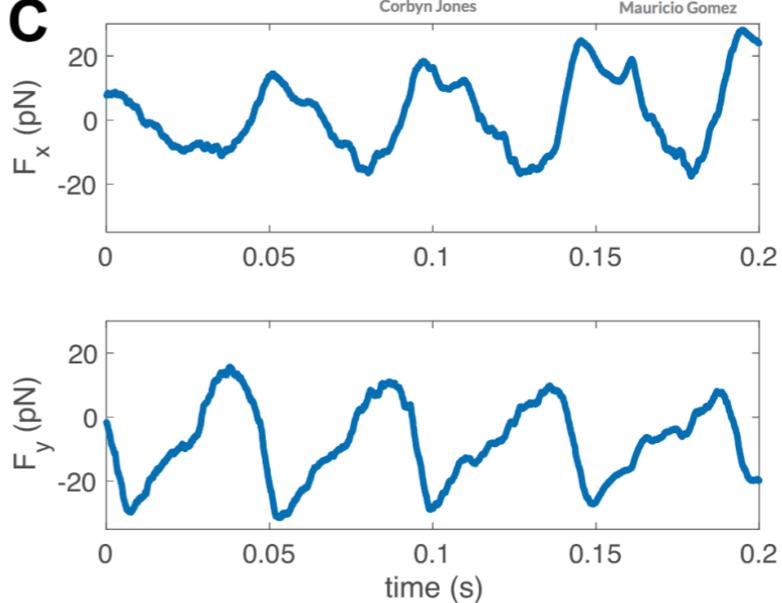
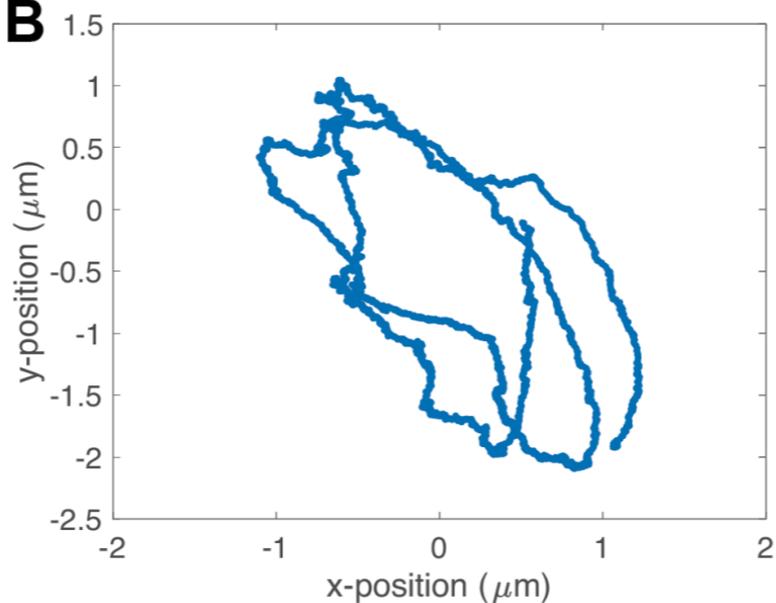
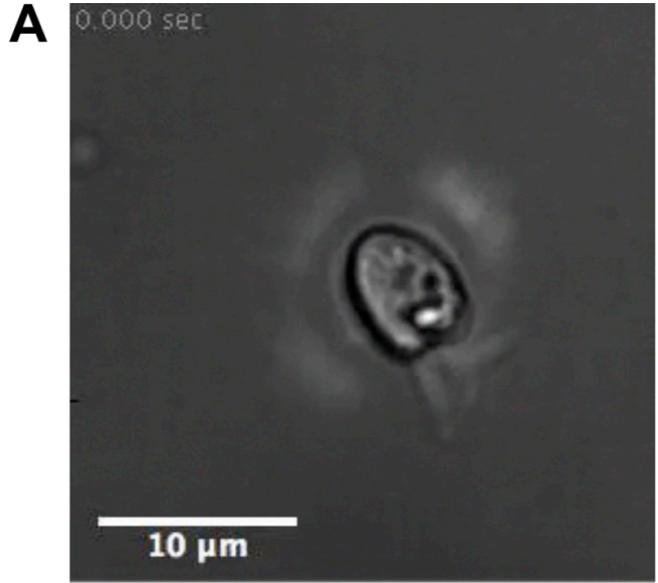
- Can lead to emergent insights into a problem, develops conceptual understanding, and facilitates transfer of knowledge across contexts

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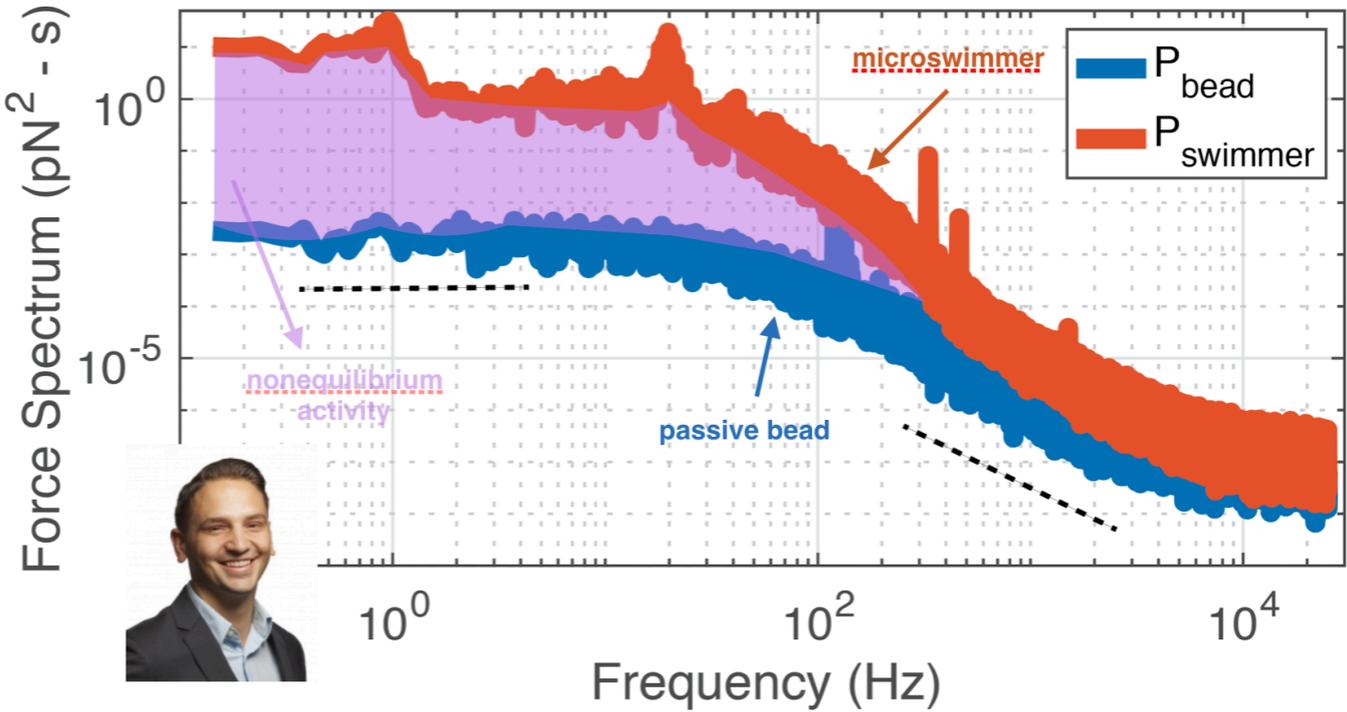
Visual representations are important

Example research slide

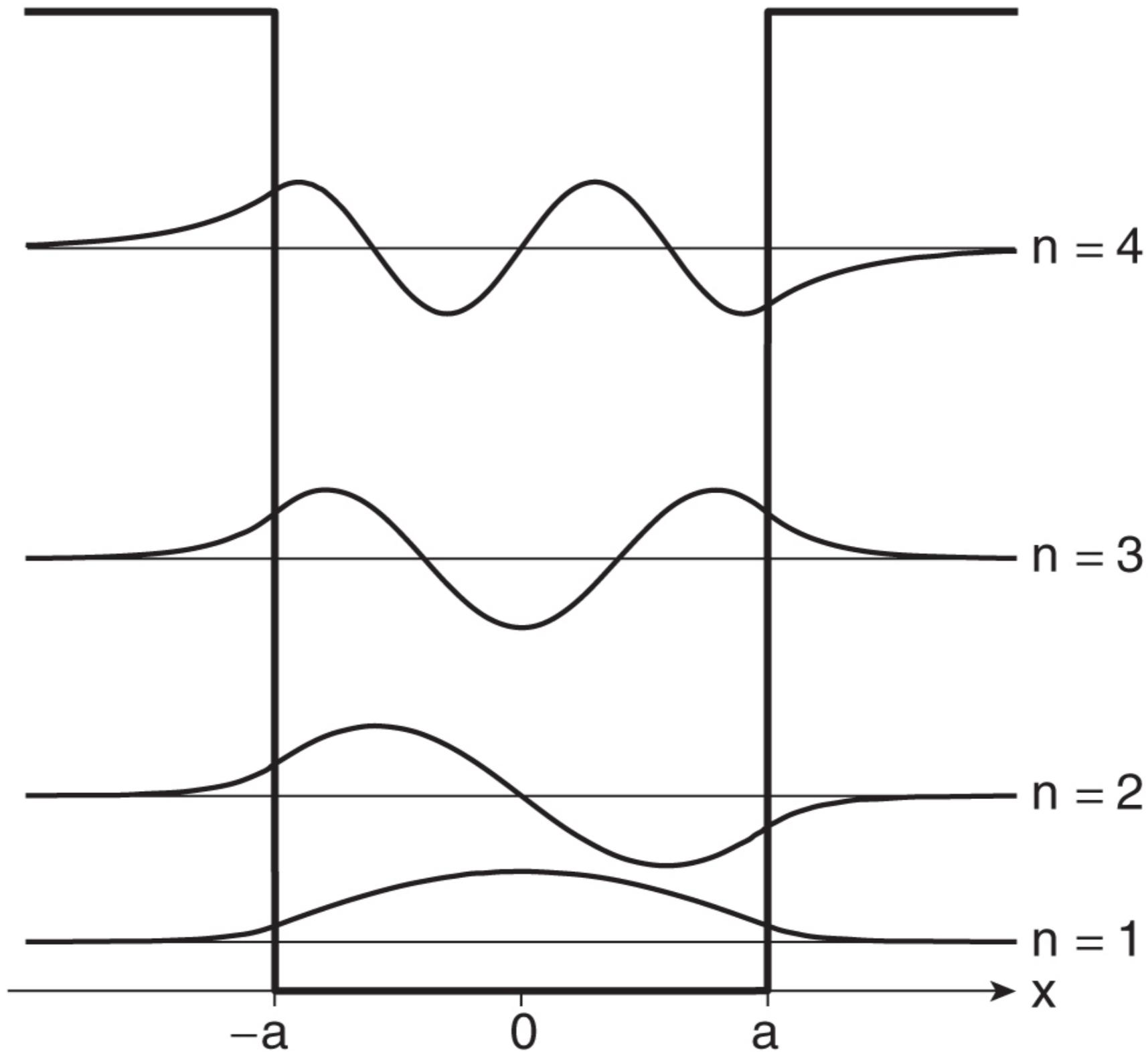
Force spectrum of micro-swimmers

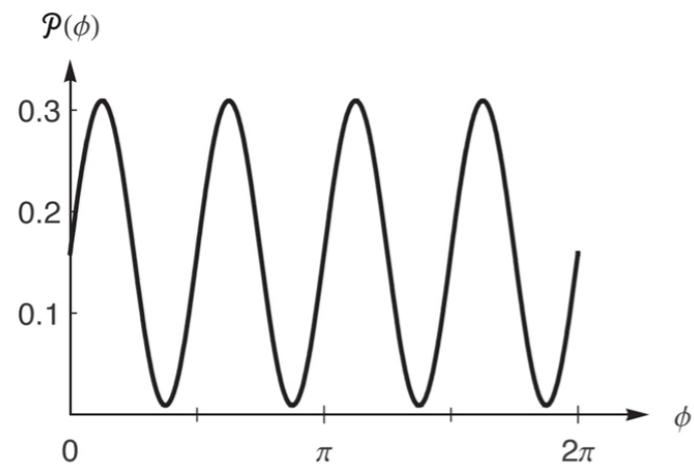


Drescher et al. PRL 2010

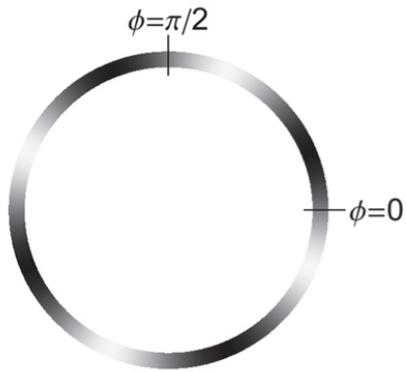


Examples from QM Textbook

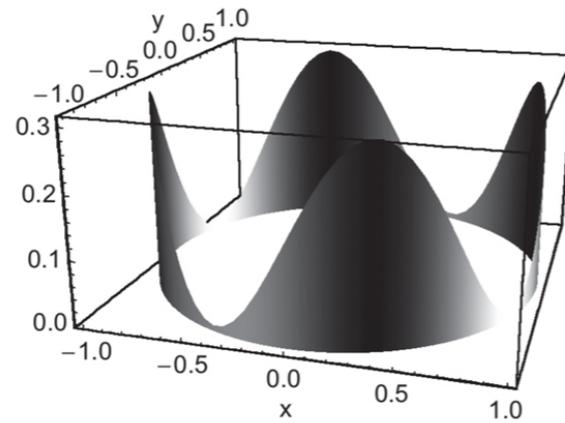




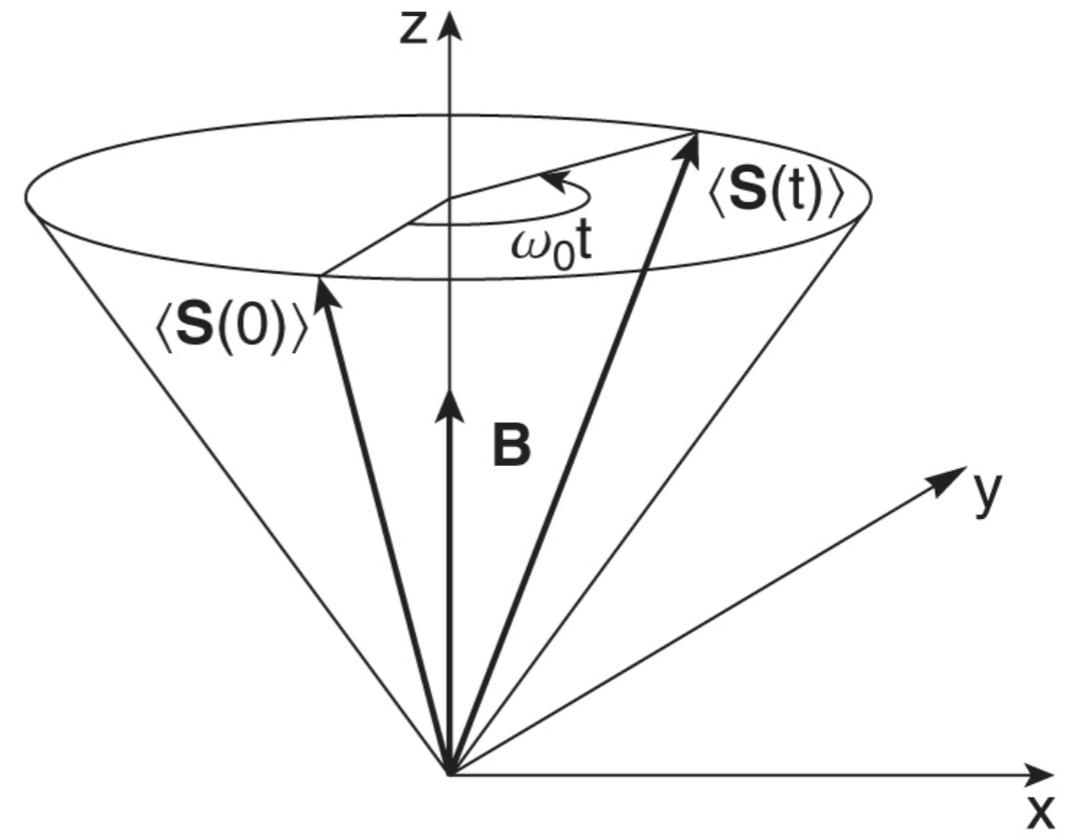
(a)



(b)

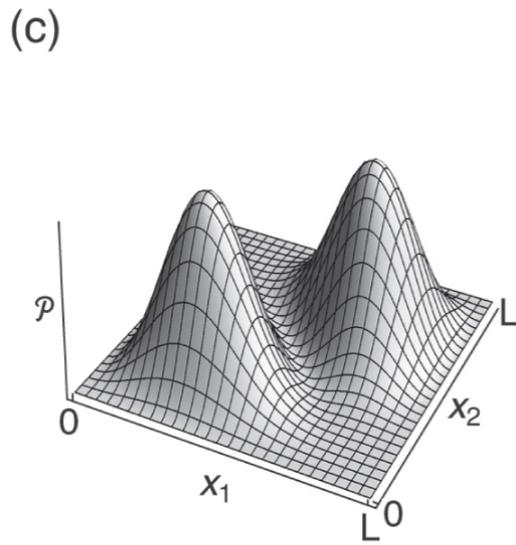


(c)

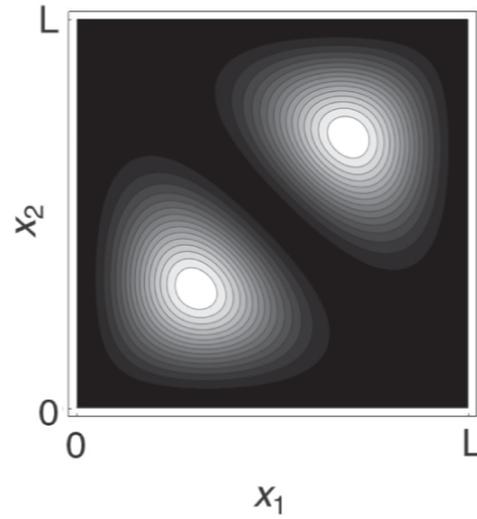


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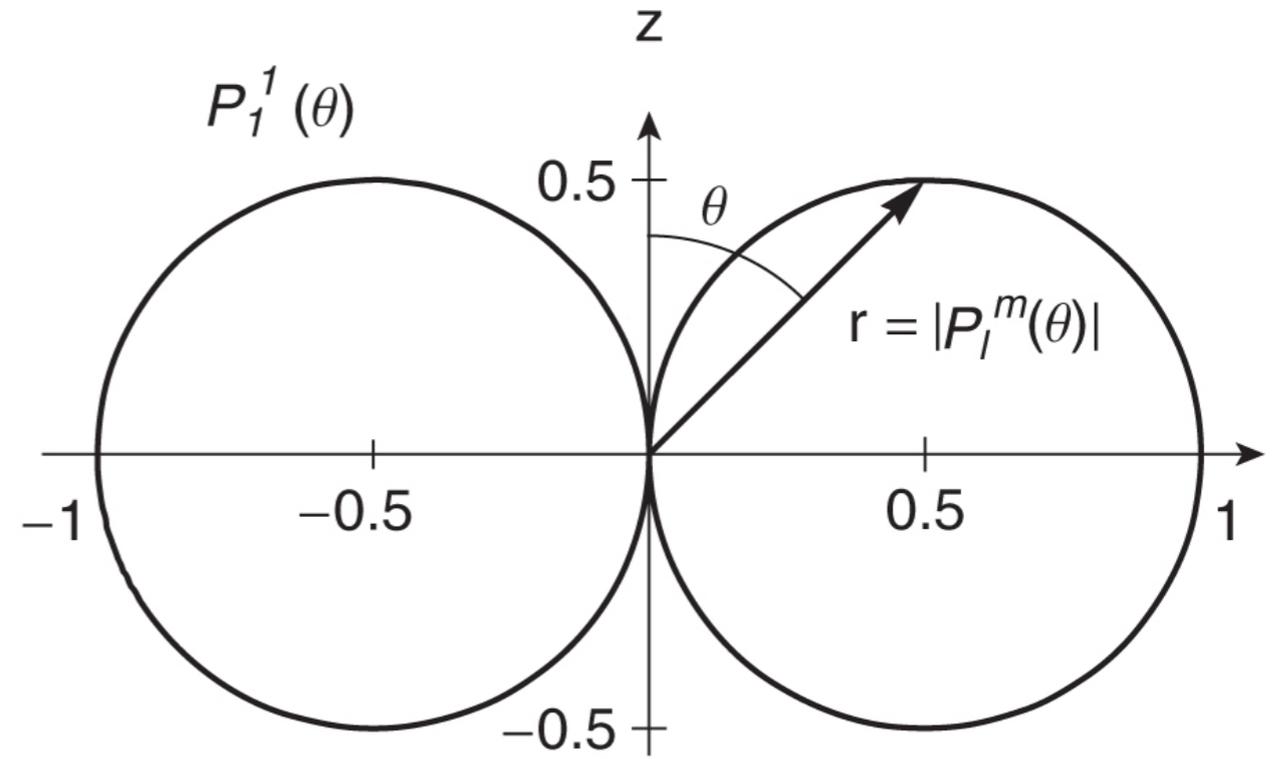
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(c)



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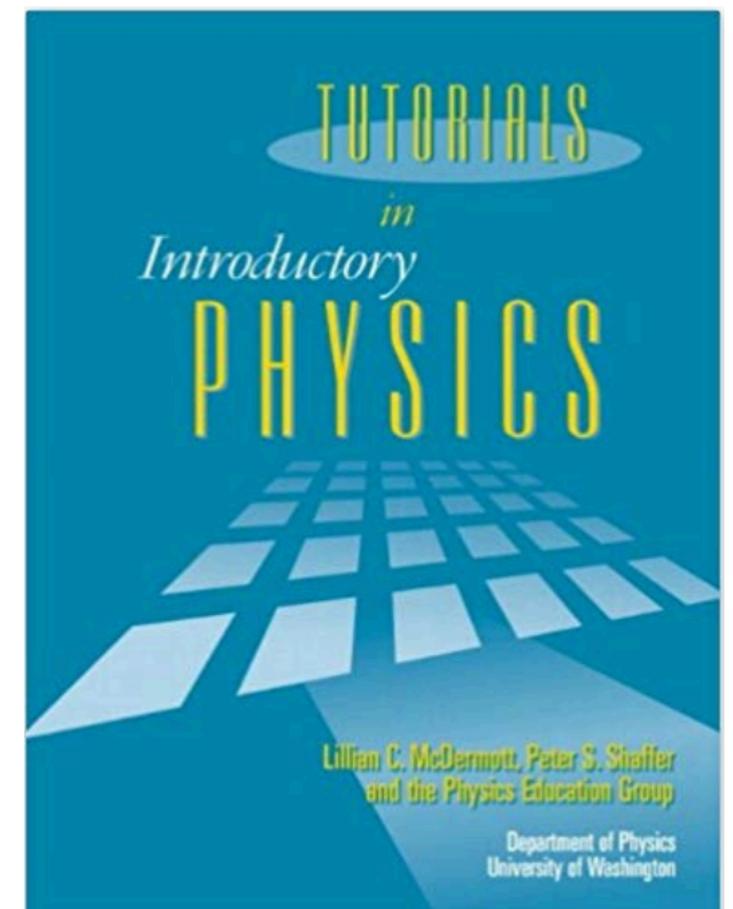
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Our goal: Develop materials that help develop student proficiency with multiple representations by combining the affordances of **interactive simulations** and **tutorials***

 The **Q**uantum Mechanics **V**isualisation Project



www.st-andrews.ac.uk/physics/quvis/



A specific example: Time evolution of quantum states

A specific example: Time evolution of quantum states

Time evolution comes from the Schrödinger equation

$$|\Psi(t)\rangle = \sum_i c_i e^{-iE_i t/\hbar} |\psi_i\rangle$$

For example

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{-iE_1 t/\hbar} |\psi_1\rangle + e^{-iE_2 t/\hbar} |\psi_2\rangle)$$

A specific example: Time evolution of quantum states

Time evolution comes from the Schrödinger equation

$$|\Psi(t)\rangle = \sum_i c_i e^{-iE_i t/\hbar} |\psi_i\rangle$$

Common difficulties:

- One phase
- Ignore imaginary
- Belief that stationary states don't evolve in time

C. Singh, Am. J. Phys. 69, 885 (2001)

H. R. Sadaghiani and S. J. Pollock, Phys. Rev. ST Phys. Educ. Res. 11, 010110 (2015)

P. J. Emigh, G. Passante, and P. S. Shaffer, Phys. Rev. ST Phys. Educ. Res. 11, 020112 (2015)

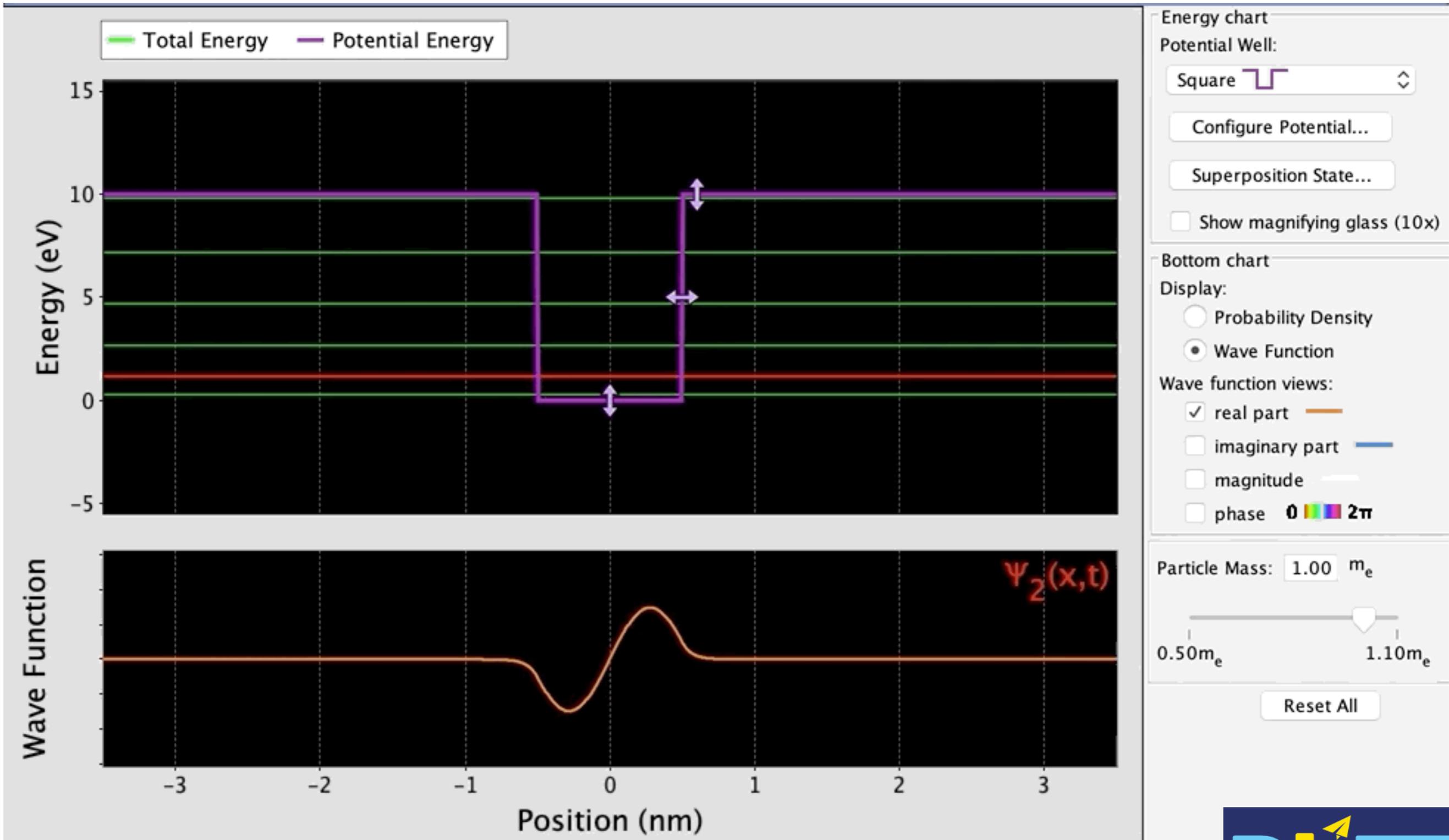
G. Zhu and C. Singh, Am. J. Phys. 80, 252 (2012)

Example of a common visualization of the time evolution of the ground state:

Describe the time evolution of the first excited state of the infinite square well:



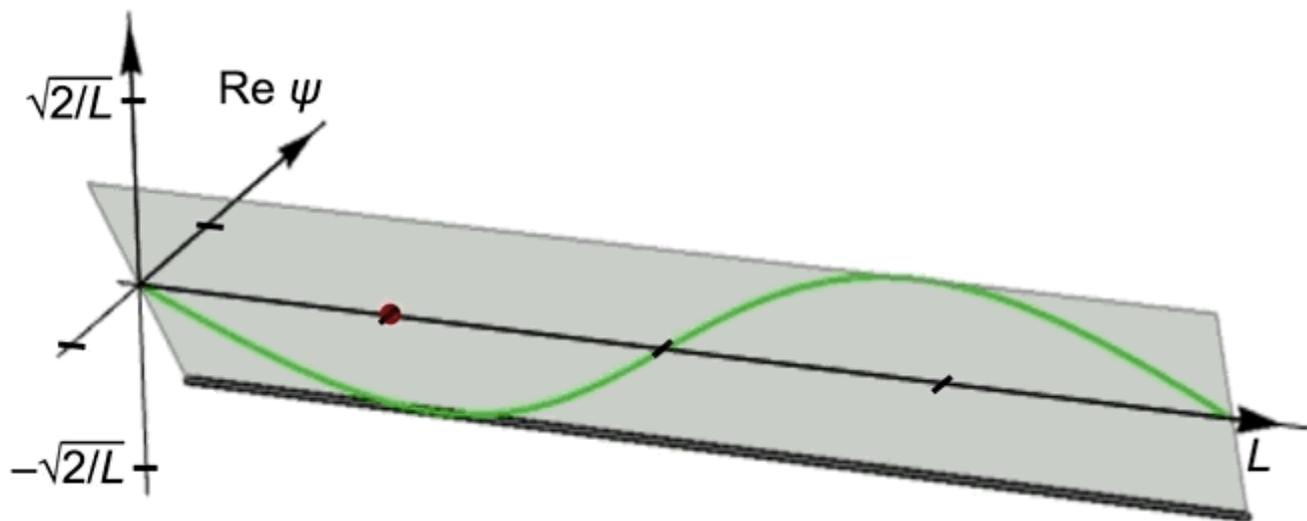
Example of a common visualization of the time evolution of the ground state:



<https://phet.colorado.edu/en/simulation/legacy/bound-states>

Time-development of infinite well quantum states

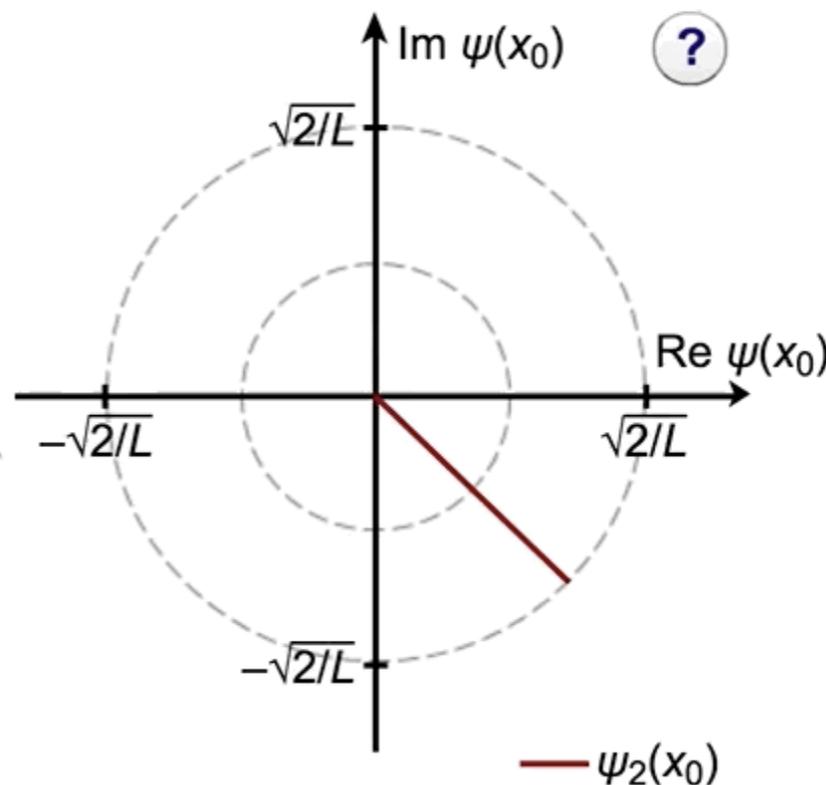
Wave function:
Im ψ



$$\psi_2(x)e^{-i(22\pi+0.27\pi)}$$

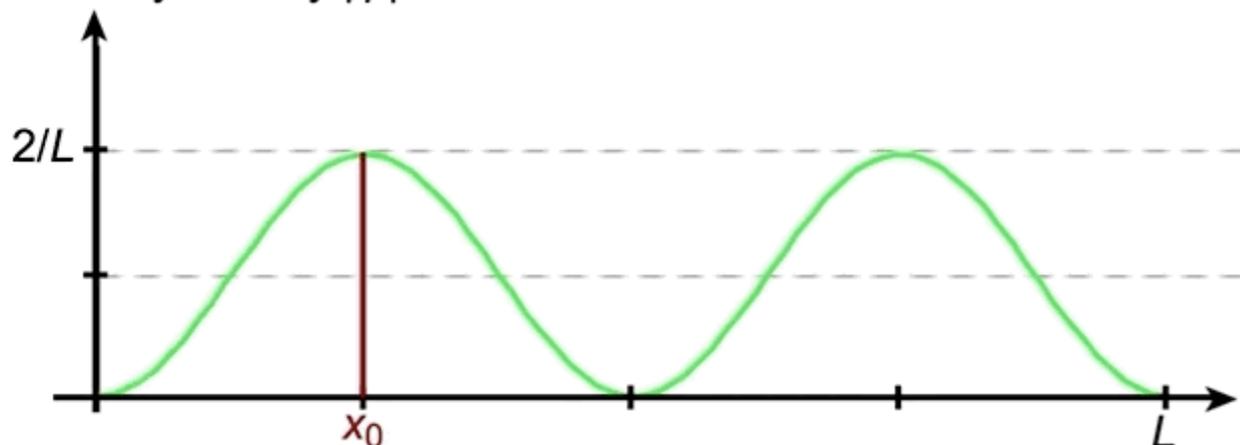
Time $t = (2 + 0.78) h/E_1$

Complex plane at point $x_0 = 0.25L$



The time-dependence of the one-dimensional infinite square well energy eigenstate $\psi_n(x,t) = \psi_n(x) e^{-iE_n t/\hbar}$ corresponds to a rotation of $\psi_n(x)$ in the complex plane, with angular frequency $\omega_n = E_n/\hbar$ where $E_n = n^2 E_1$. Use the "?" buttons for more information. Then try the Challenges!

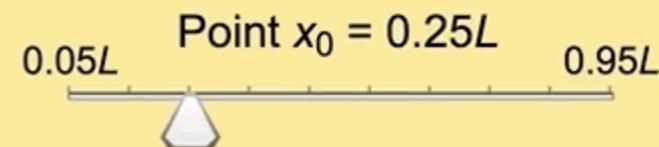
Probability density $|\psi|^2$



$|\psi(x_0)|^2 = 1.00 (2/L)$ (red line)

Main Controls

Show $\psi(x,t)$



Show probability density $|\psi(x,t)|^2$ graph

ψ_1 ψ_2 $1/\sqrt{2} (\psi_1 + \psi_2)$ Quantum state

Reset to $t = 0$

Stop

Time step back

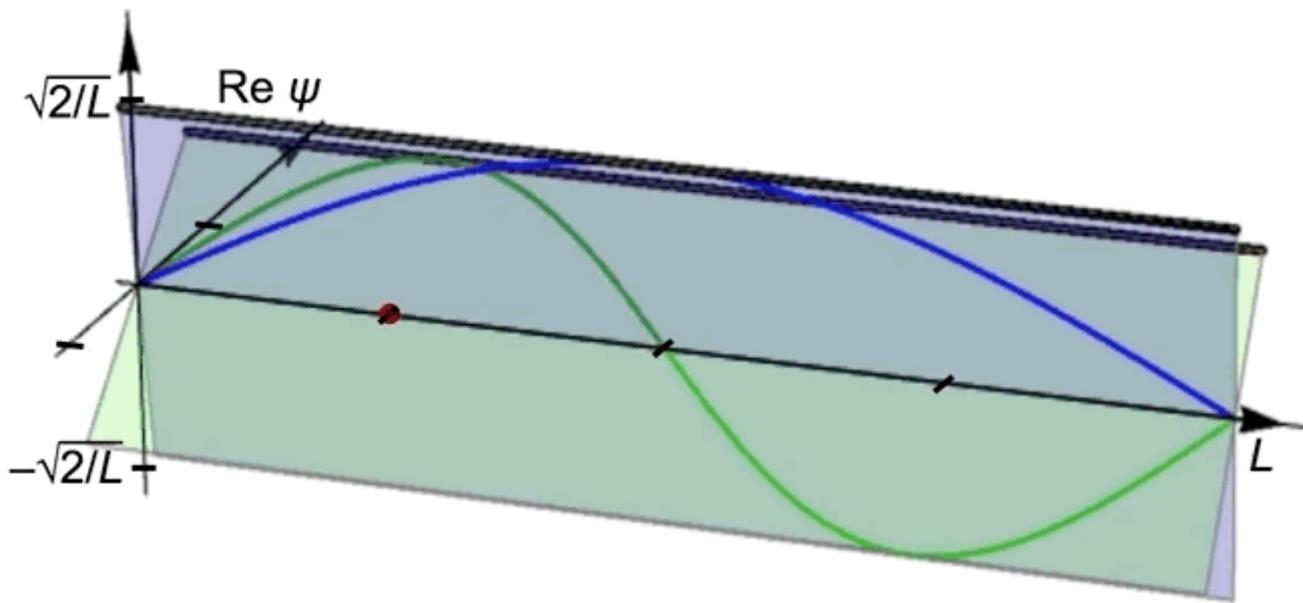


Time step forward

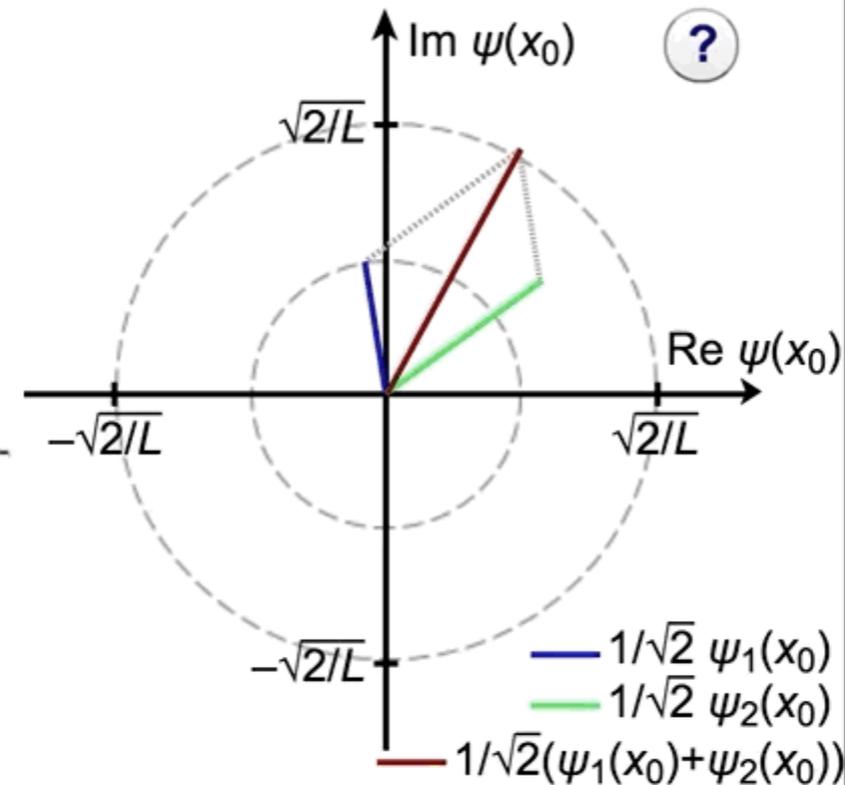
Time-development of infinite well quantum states



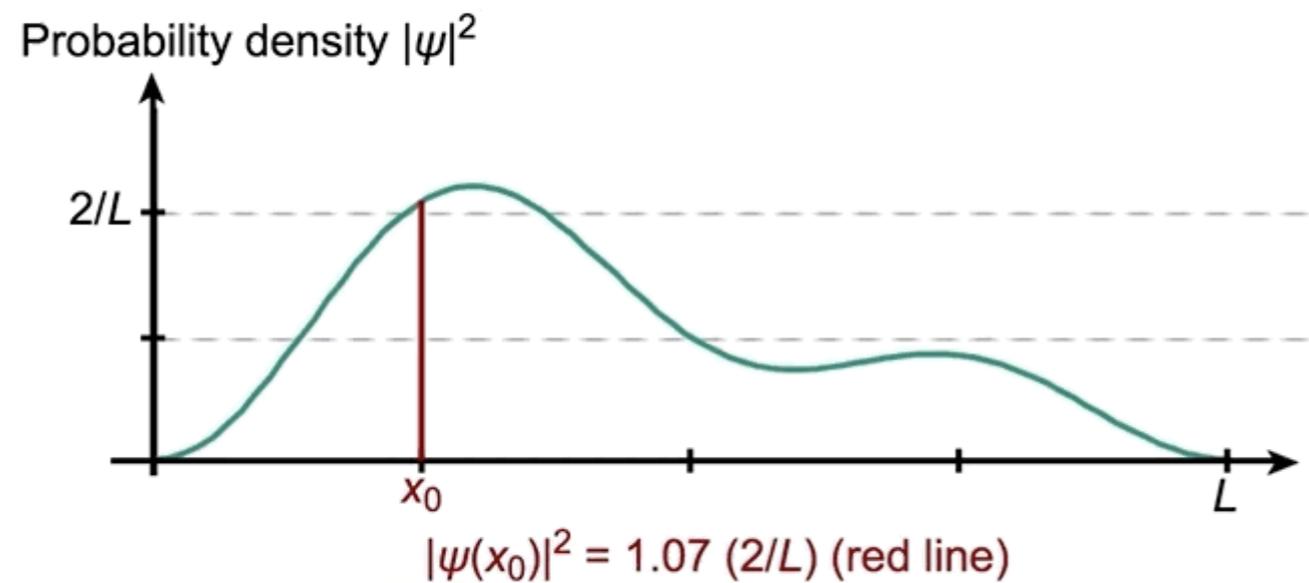
Wave function: $1/\sqrt{2}(\psi_1(x)e^{-i(4\pi+1.45\pi)} + \psi_2(x)e^{-i(20\pi+1.80\pi)})$
 Time $t = (2 + 0.73) h/E_1$



Complex plane at point $x_0 = 0.25L$

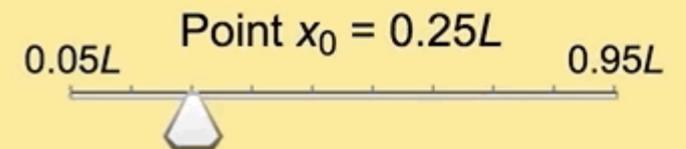


The time-dependence of the one-dimensional infinite square well energy eigenstate $\psi_n(x,t) = \psi_n(x) e^{-iE_n t/\hbar}$ corresponds to a rotation of $\psi_n(x)$ in the complex plane, with angular frequency $\omega_n = E_n/\hbar$ where $E_n = n^2 E_1$. Use the "?" buttons for more information. Then try the Challenges!



Main Controls

Show $\psi(x,t)$



Show probability density $|\psi(x,t)|^2$ graph

ψ_1 ψ_2 $1/\sqrt{2}(\psi_1 + \psi_2)$ Quantum state

Reset to $t = 0$

Stop

Time step back

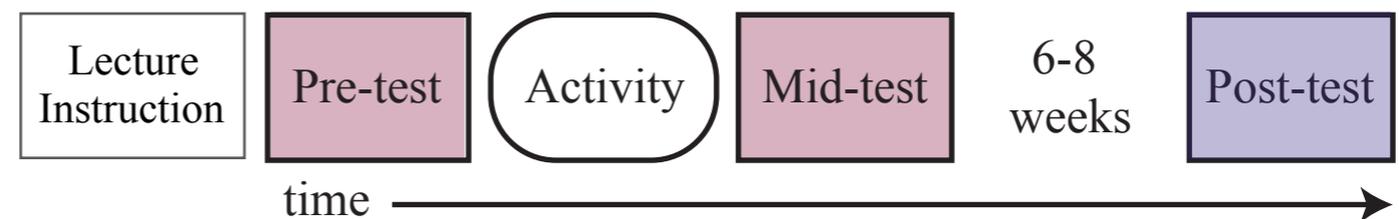


Time step forward

Research Questions:

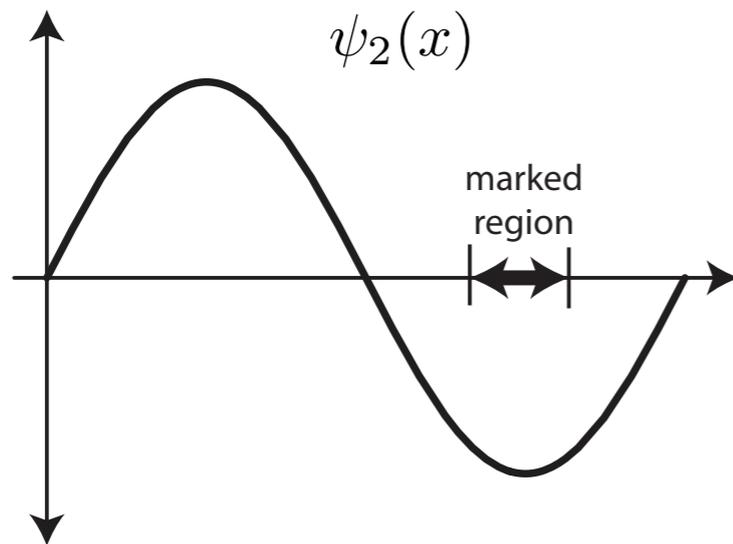
1. Does visual reasoning improve student performance?
2. Does the simulation-tutorial develop a visual understanding?

Research Methods:

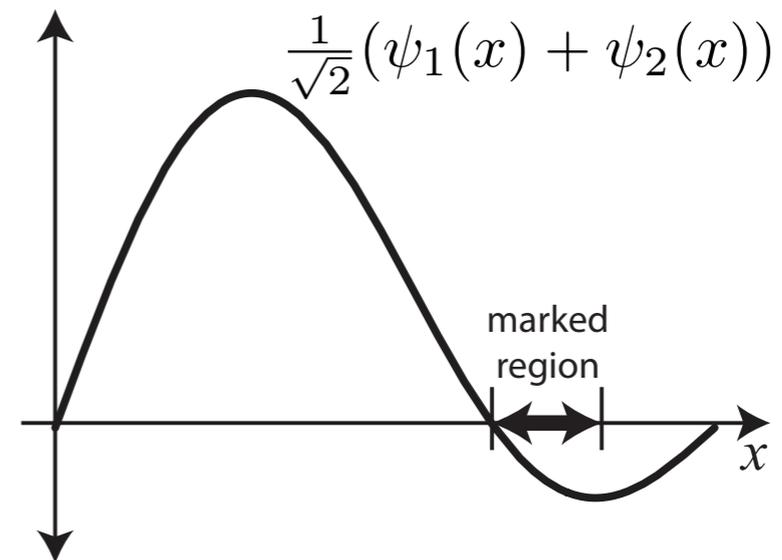


Pre- and Mid-test:

Describe graphically or in words (not just with formulas) how the full wave function evolves with time.



First excited state



Superposition state

Data analysis

Math:

Real

e^{-t} or similar

Complex

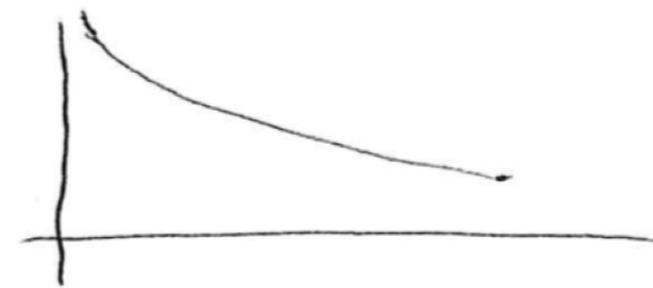
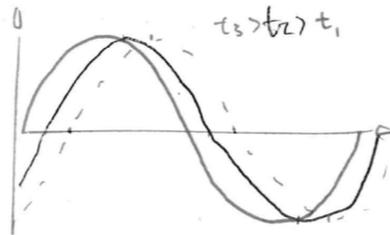
$$\Psi(x,t) = \frac{1}{\sqrt{2}} \left(\Psi_1(x) e^{-\frac{iE_1 t}{\hbar}} + \Psi_2(x) e^{-\frac{iE_2 t}{\hbar}} \right)$$

$\Rightarrow \Psi(x,t) \rightarrow 0$ as $t \rightarrow \infty$ as before

Visual:

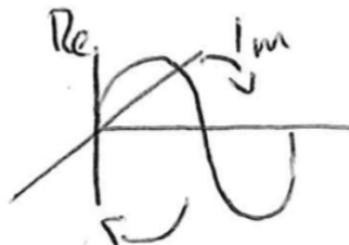
“Classical”

“Wavefunction moves left or right as t increases.”



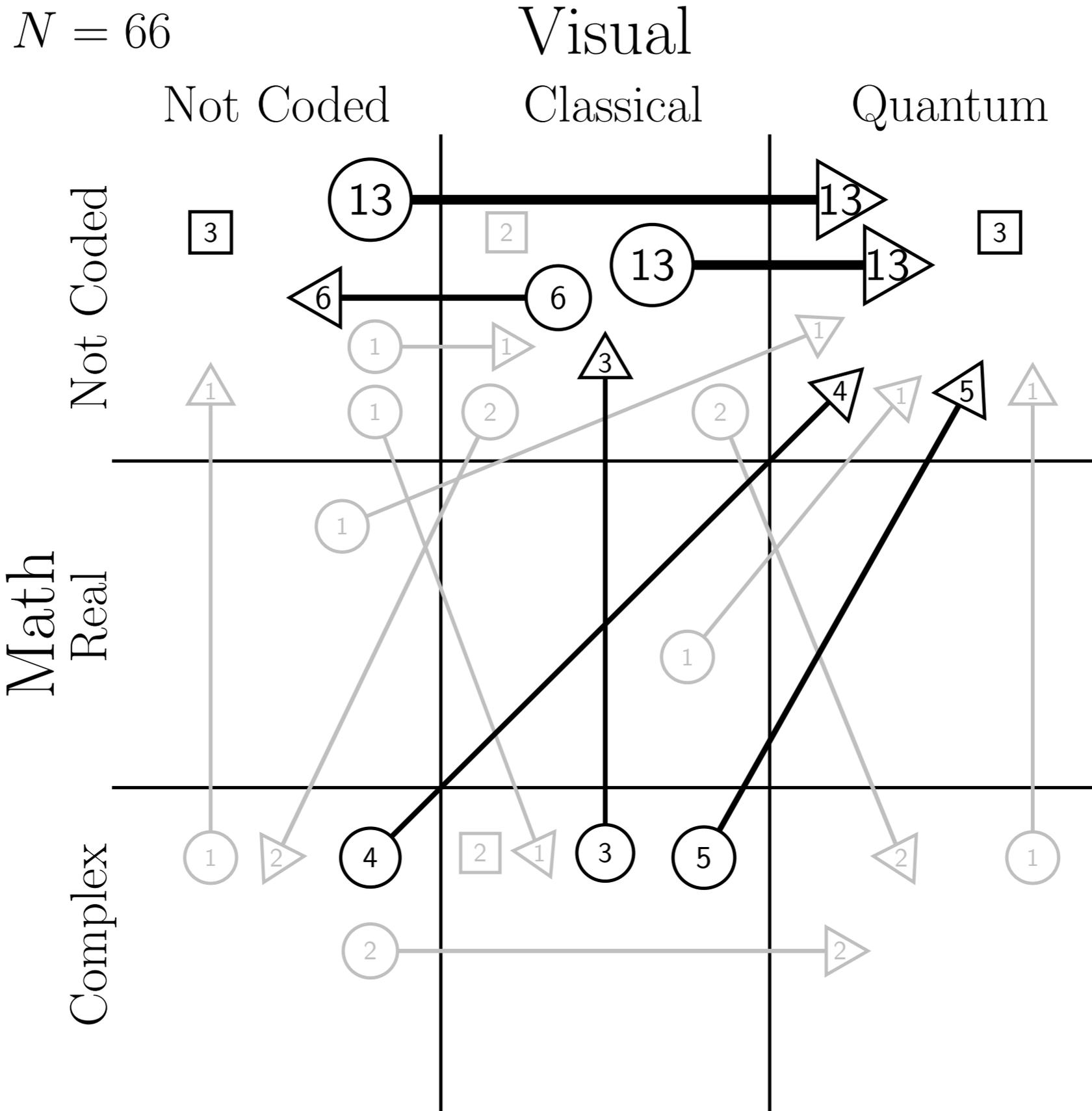
“Quantum”

“ $\psi_2(x, t)$ rotates clockwise in the complex plane”



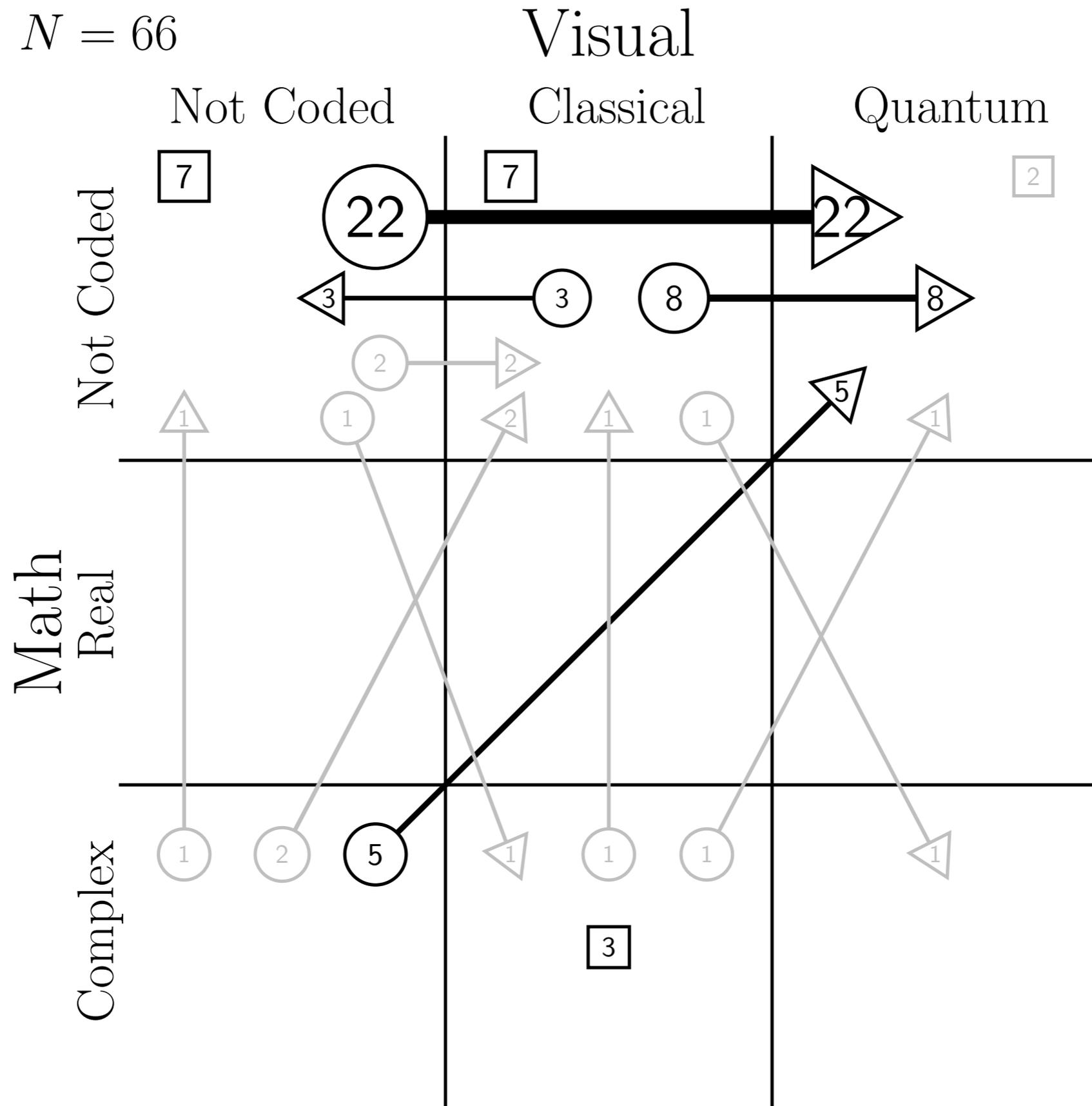
(Responses were also coded for correctness)

Results: Pre \rightarrow Mid (First excited state)



Results: Pre \rightarrow Mid (Superposition state)

$N = 66$

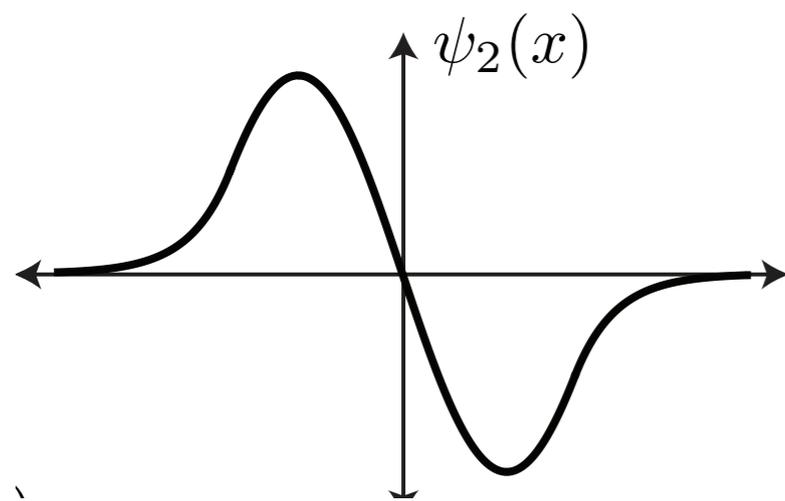
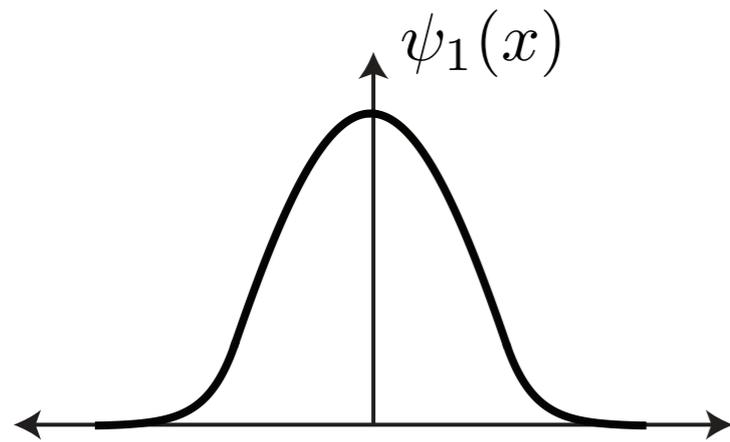


Research Questions:

1. Does visual reasoning improve student performance?
2. Does the simulation-tutorial develop a visual understanding?

After the activity, students are more likely to use a quantum visual explanation for how the wave function evolves with time.

Posttest:

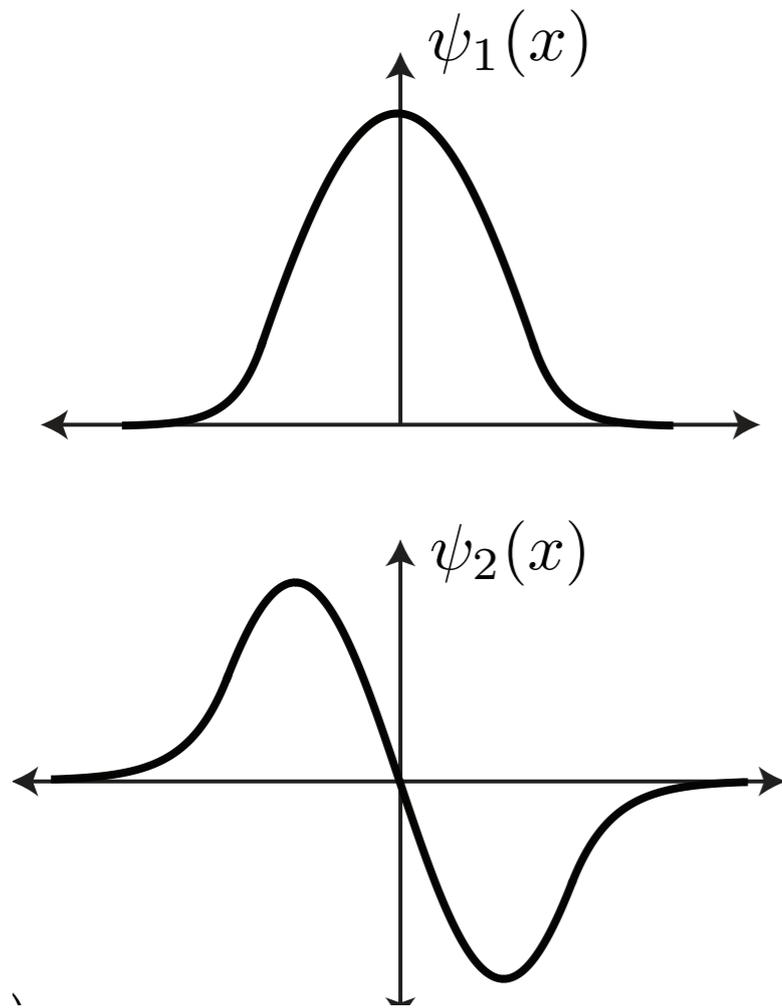


State is initially given by:

$$\psi(x, t = 0) = \frac{1}{\sqrt{2}}(\psi_1(x) + i\psi_2(x))$$

- (a) Sketch the probability density for this state at time $t = 0$. Show your reasoning.
- (b) Does the probability density for this state depend on time? Explain your reasoning.

Results: Posttest



State is initially given by:

$$\psi(x, t = 0) = \frac{1}{\sqrt{2}}(\psi_1(x) + i\psi_2(x))$$

(b) Does the probability density for this state depend on time? Explain your reasoning.

	Math	Visual
# responses	43 (50%)	30 (35%)
A Correct	72%	100%
A + Reasoning	54%	73%

Total N = 86

Research Questions:

1. Does visual reasoning improve student performance?

Yes! On an exam question, all students that used a visual explanation got the answer correct (and 73% provided correct reasoning).

2. Does the simulation-tutorial develop a visual understanding?

After the activity, students are more likely to use a quantum visual explanation for how the wave function evolves with time.

The activity works.

Why?

Sketching as a Learning Tool

- Highly successful strategy for learning text

Ainsworth et al., Science 333, 1096 (2011)

- Correlation between sketch accuracy/completeness and posttest performance

Van Meter & Garner, Educ. Psychol. Rev., 17, 295 (2005)

- Sketching supports extraction of conceptually relevant information

Mason et al., Contemp. Educ. Psychol. 38, 211 (2013)

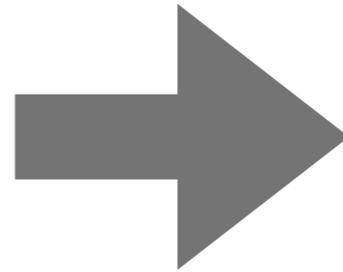
BUT Ploetzner & Fillisch, Learn. Instr. 47 (2017)



Shaaron Ainsworth
University of Nottingham
(Learning Sciences)

Simulation-tutorials have a two-part structure:

Phase 1: Without simulation support

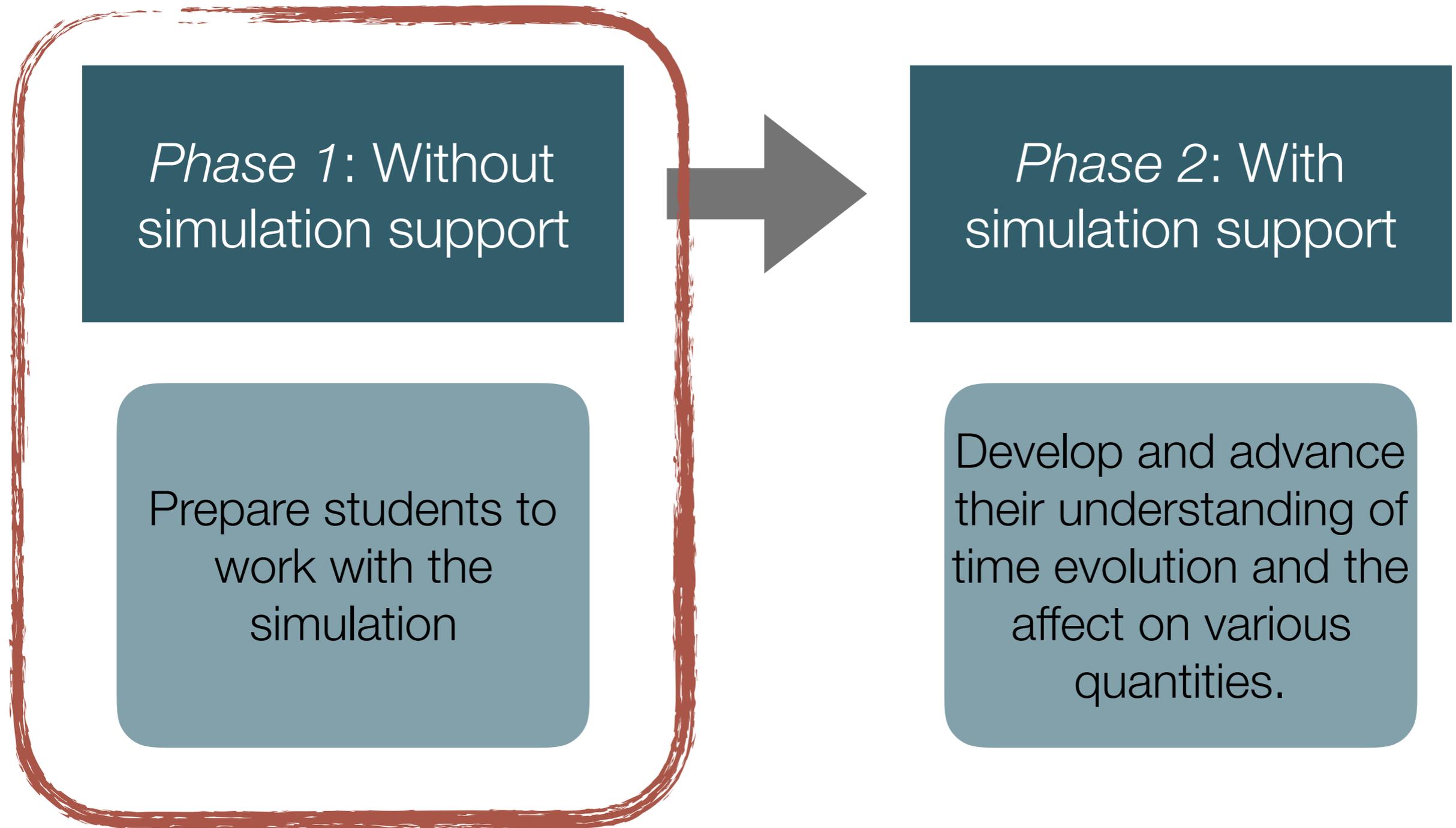


Phase 2: With simulation support

Prepare students to work with the simulation

Develop and advance their understanding of time evolution and the affect on various quantities.

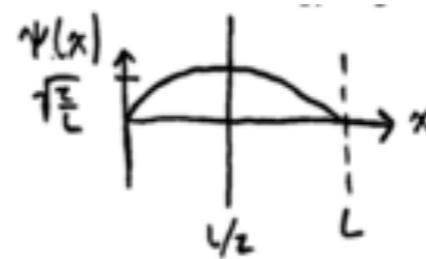
Simulation-tutorials have a two-part structure:



Abbreviated Question Text

- A** Sketch the ground state energy eigenfunction at $t = 0$.

Example Sketches



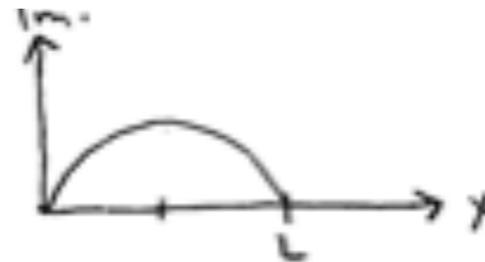
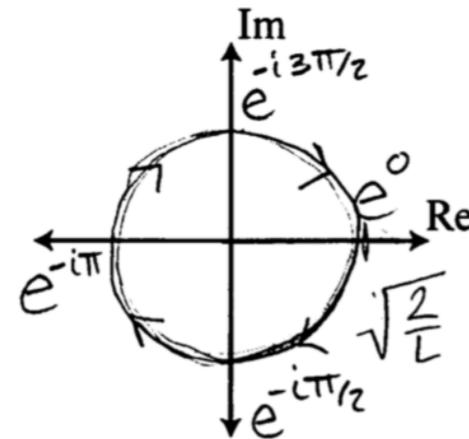
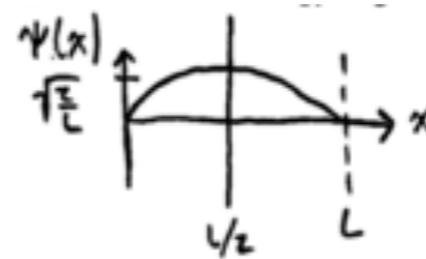
Role of Sketching

Sketch to
activate prior
knowledge

Abbreviated Question Text

- A** Sketch the ground state energy eigenfunction at $t = 0$.
- B** Plot the time evolution of $\psi_1(x)$ at $x = L/2$. Describe this evolution in words.
- C** Sketch $\psi_1(x, t)$ at time $E_1 t / \hbar = 3\pi/2$

Example Sketches



Role of Sketching

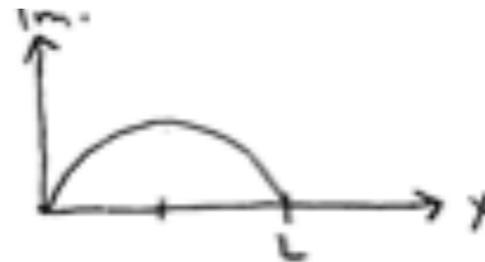
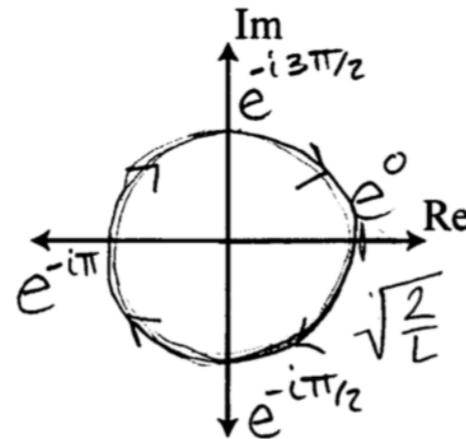
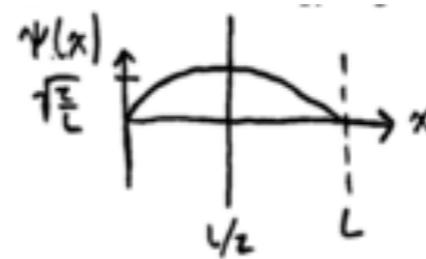
Sketch to activate prior knowledge

Sketch to transform

Abbreviated Question Text

- A** Sketch the ground state energy eigenfunction at $t = 0$.
- B** Plot the time evolution of $\psi_1(x)$ at $x = L/2$. Describe this evolution in words.
- C** Sketch $\psi_1(x, t)$ at time $E_1 t / \hbar = 3\pi/2$
- D** How would you plot the time evolution for the entire function using a 3D representation?

Example Sketches



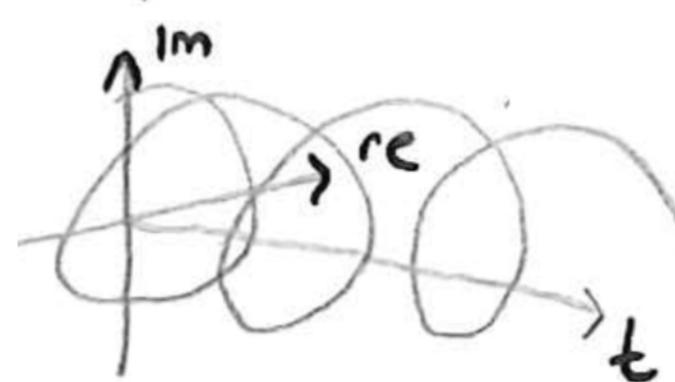
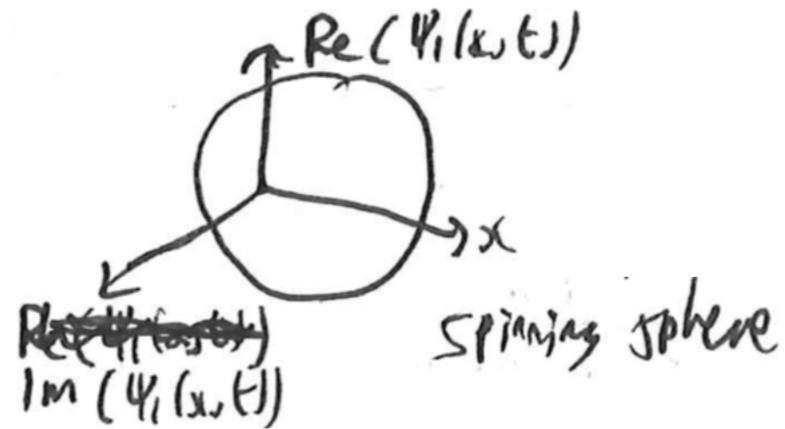
Role of Sketching

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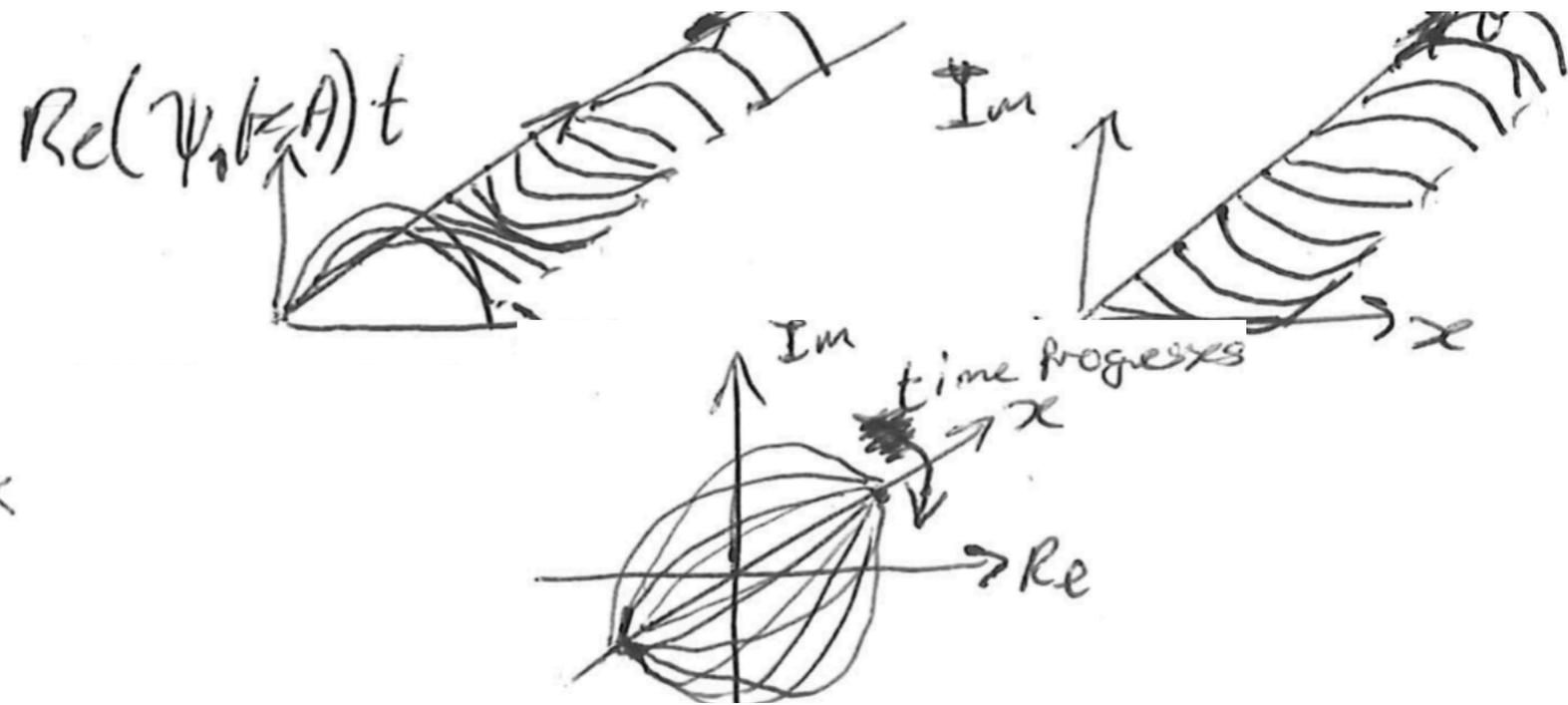
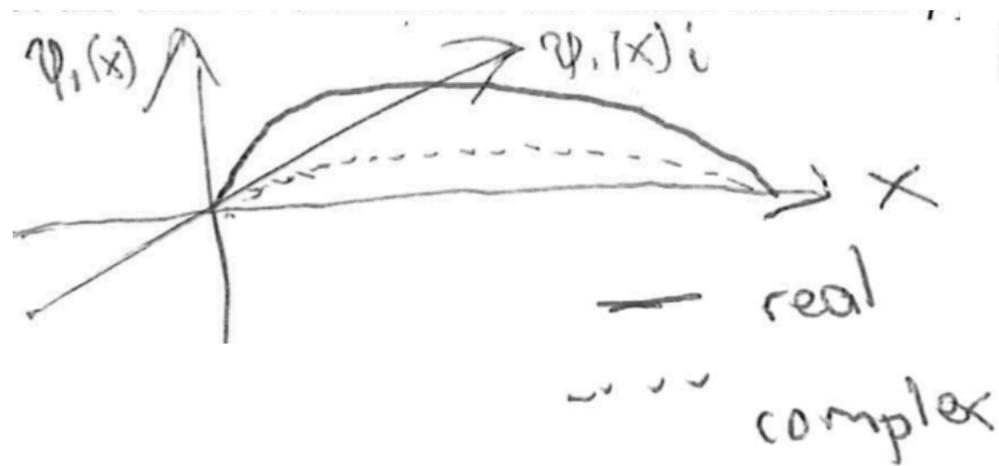
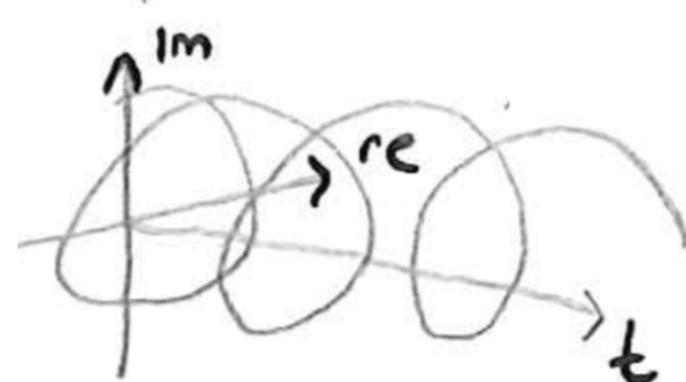
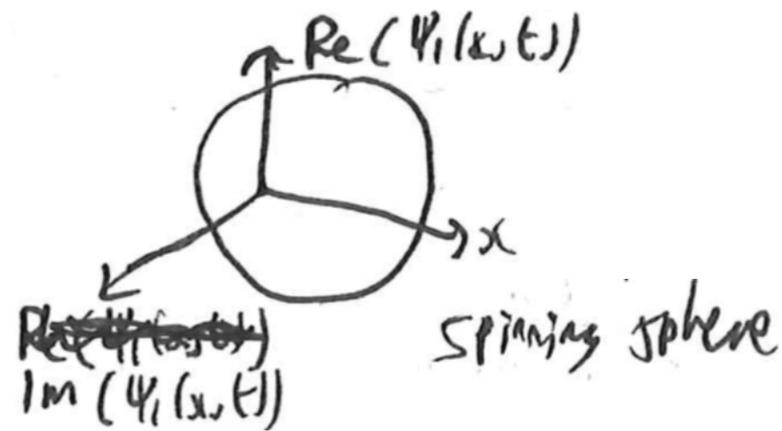
Sketch to transform

Sketch to invent

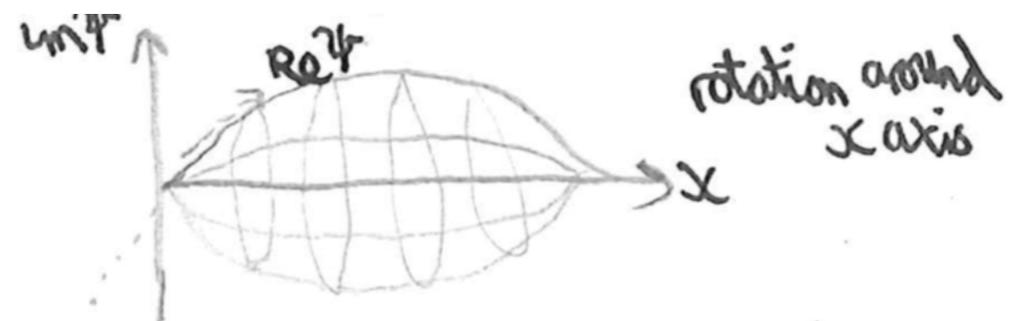
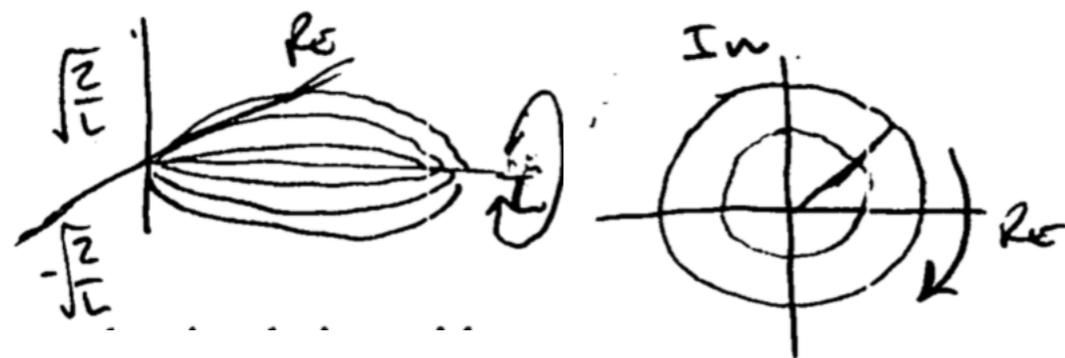
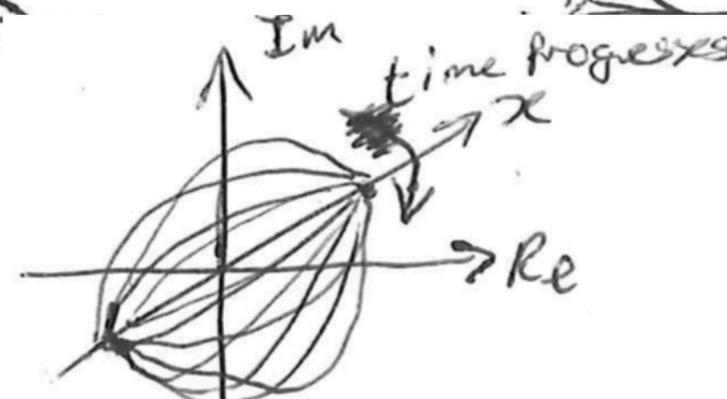
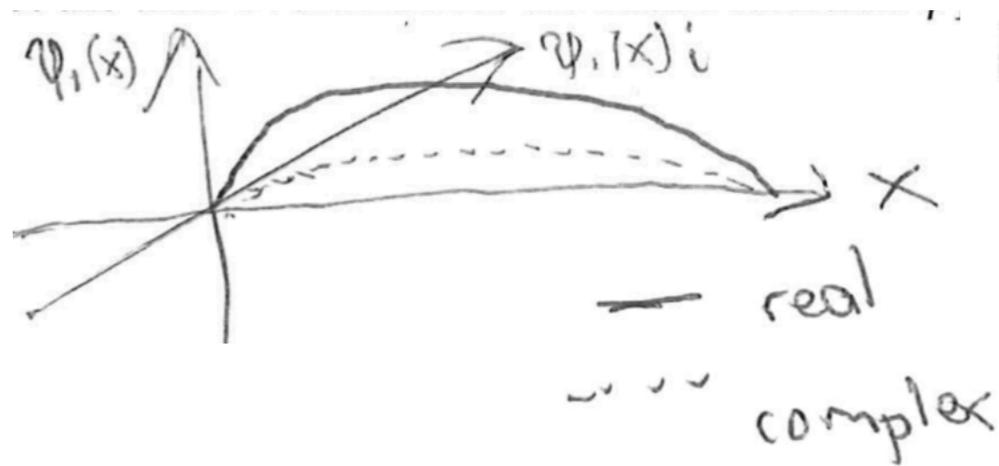
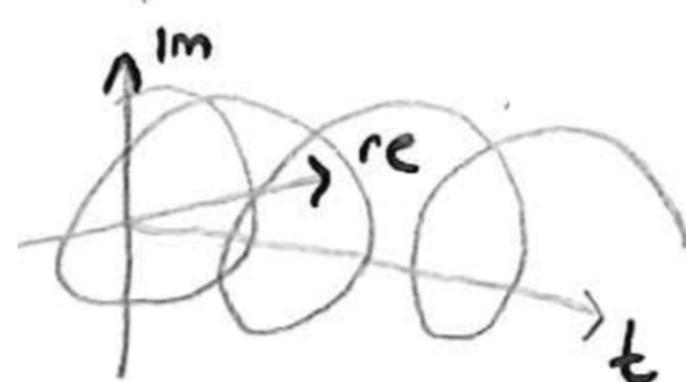
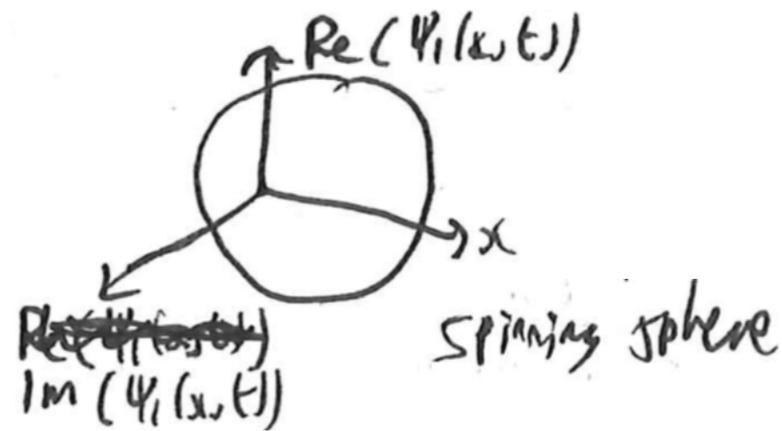
D How would you plot the time evolution for the entire function using a 3D representation?



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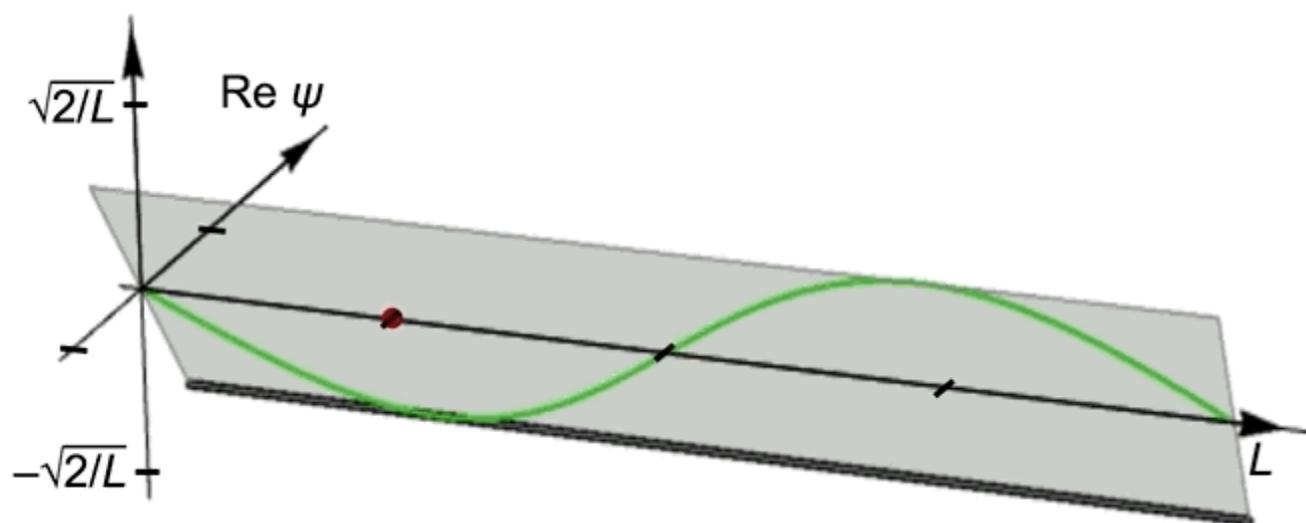


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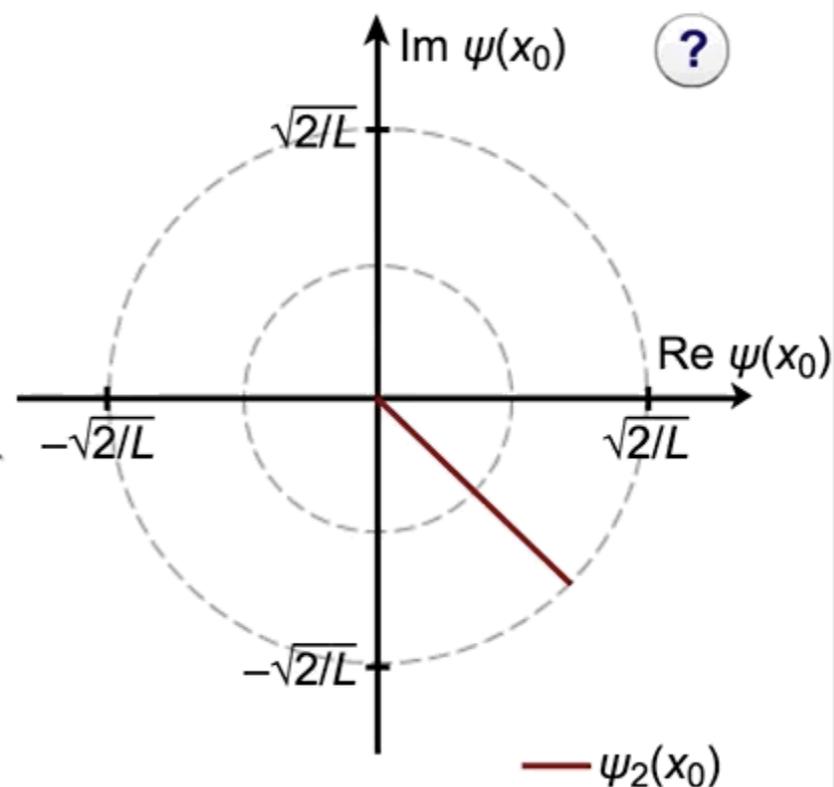
Time-development of infinite well quantum states

Wave function:
Im ψ



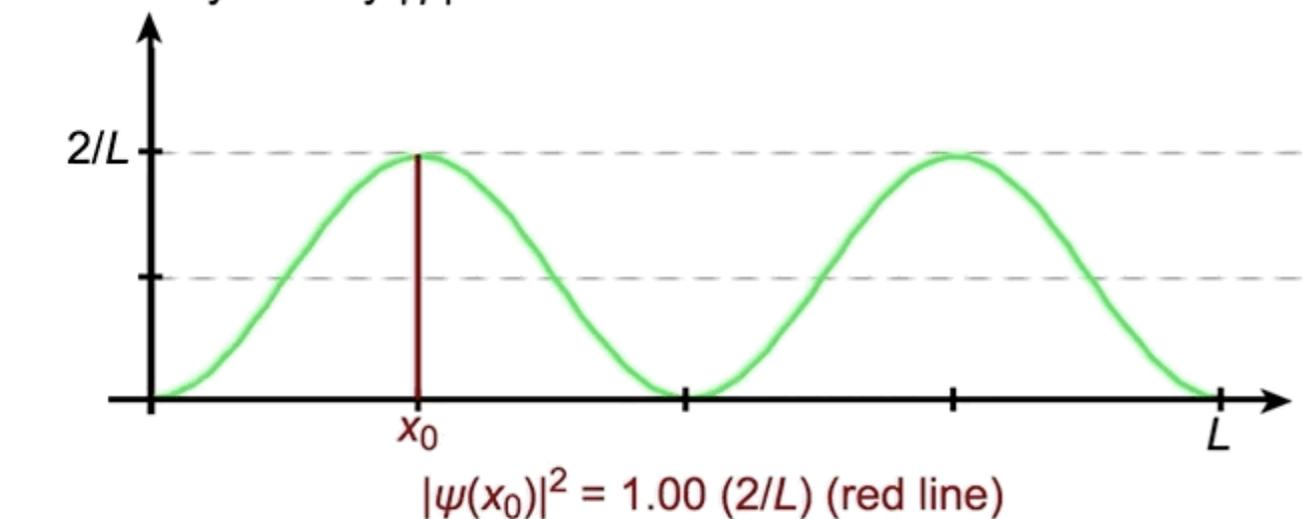
$\psi_2(x)e^{-i(22\pi+0.27\pi)}$
Time $t = (2 + 0.78) h/E_1$

Complex plane at point $x_0 = 0.25L$



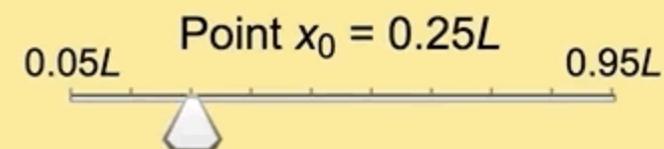
The time-dependence of the one-dimensional infinite square well energy eigenstate $\psi_n(x,t) = \psi_n(x) e^{-iE_n t/\hbar}$ corresponds to a rotation of $\psi_n(x)$ in the complex plane, with angular frequency $\omega_n = E_n/\hbar$ where $E_n = n^2 E_1$. Use the "?" buttons for more information. Then try the Challenges!

Probability density $|\psi|^2$



Main Controls

Show $\psi(x,t)$



Show probability density $|\psi(x,t)|^2$ graph

ψ_1 ψ_2 $1/\sqrt{2} (\psi_1 + \psi_2)$ Quantum state

Reset to
 $t = 0$

Stop

Time step
back



Time step
forward

Thank you

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Some useful links:

Curricula: physport.org/curricula/ACEQM

QuVis Sims: www.st-andrews.ac.uk/physics/quvis/

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