What Is the Condition for Conservation of Momentum?

- A question in teaching introductory physics

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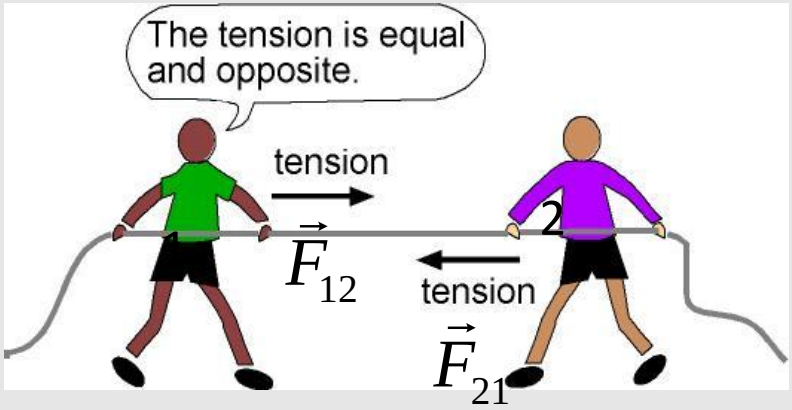
From Newton's Laws



Internal forces: $\vec{F}_{12}$ **and** $\vec{F}_{21}$.

External forces: $m_1\vec{g},\ m_2\vec{g},\ \vec{F}_{N1}$ *and* $\vec{F}_{N1}$

Newton's laws: $m_1\vec{g}+\vec{F}_{N1}+\vec{F}_{12}=\dfrac{\Delta\vec{p}_1}{\Delta t},\quad m_2\vec{g}+\vec{F}_{N2}+\vec{F}_{21}=\dfrac{\Delta\vec{p}_2}{\Delta t}$

$$m_1\vec{g}+\vec{F}_{N1}+m_2\vec{g}+\vec{F}_{N2}+\vec{F}_{12}+\vec{F}_{21}=\frac{\Delta\vec{p}_1}{\Delta t}+\frac{\Delta\vec{p}_2}{\Delta t}$$

$$\sum\vec{F}_{ext}+\sum\vec{F}_{int}=\frac{\Delta\vec{P}}{\Delta t}$$

$$\vec{F}_{21}=-\vec{F}_{12},\quad \sum\vec{F}_{int}=0$$

Consequence of the 3rd law: Internal forces cancel out.

$$\sum\vec{F}_{ext}=\frac{\Delta\vec{P}}{\Delta t}\qquad \text{When } \sum\vec{F}_{ext}=0,\quad \frac{d\vec{P}}{dt}=0,\ \ i.e.,\ \ \vec{P}=\text{constant}$$

"Theorem" of conservation of momentum:

When the net external force is zero, the linear momentum of a system is constant.

What is the condition for conservation of momentum?

A. When (if and only if) the net external force is zero, the total momentum of a system is constant.

B. The total momentum of an **isolated system** is constant.

Definitions of an isolated system

A. When the net external force acting on the system is zero.

B. When there is no exchange of momentum with other objects outside the system.

How do you know if there is no exchange in momentum?

- If and only if the net external force is zero.

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Using the Concept of Isolated System

- When the term "isolated system" is properly defined

Pros

- Introducing the concept of conserved quantities.

Cons

- The term isolated system is defined differently elsewhere.
- May cause confusion
  – Can we apply the theorem of conservation of momentum to a system that is not isolated?

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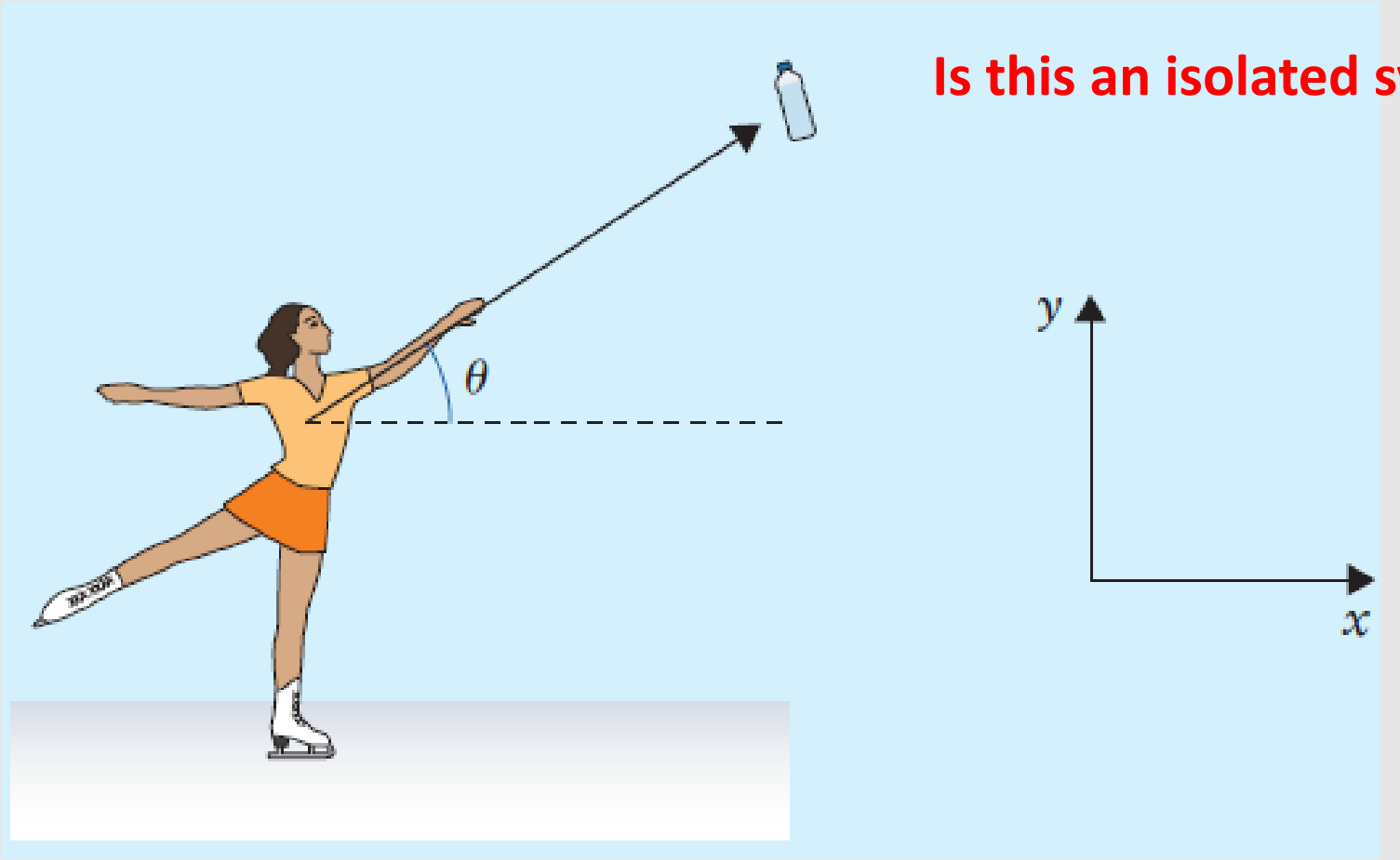
Example. An ice skater of mass 45.0 kg initially standing on ice tosses a 1.20-kg water bottle at a speed of 14.0 m/s and an angle of 30.0° above the ground. The skater recoils on the ice. If the friction is ignored, what would be the recoil velocity of the skater?



Is this an isolated system?

The total momentum of the system is not conserved, but the x-component of the momentum of the system is.

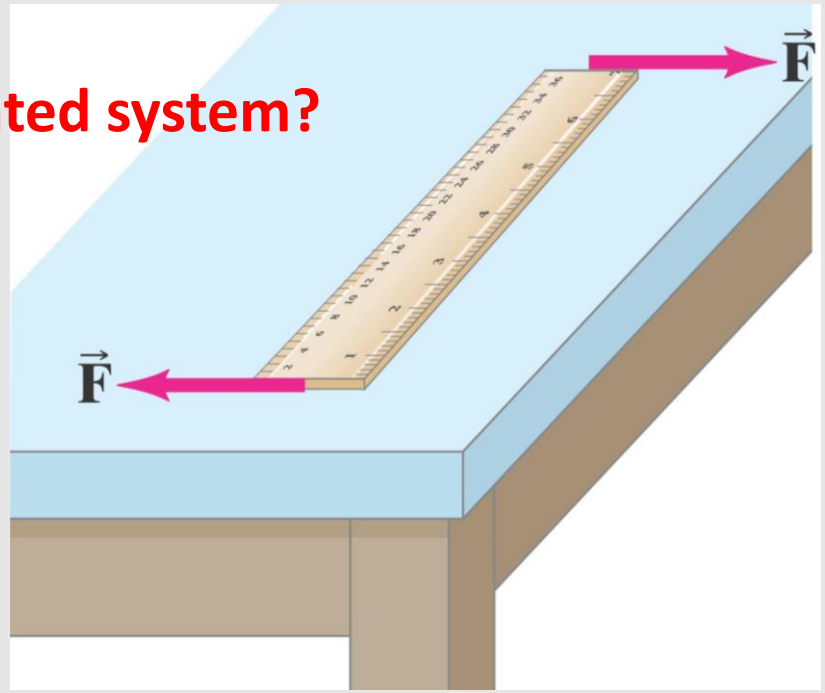
Using the Concept of Isolated System

- When the term "isolated system" is defined differently

Eric Mazur: No **interaction** with external objects

But interaction includes forces and torques. Then, the condition for conservation of linear momentum and angular momentum is the same: when it is an isolated system.

Is this an isolated system?



Linear momentum is conserved, but angular momentum is not.

Using the Concept of Isolated System

- When the term “isolated system” is defined differently

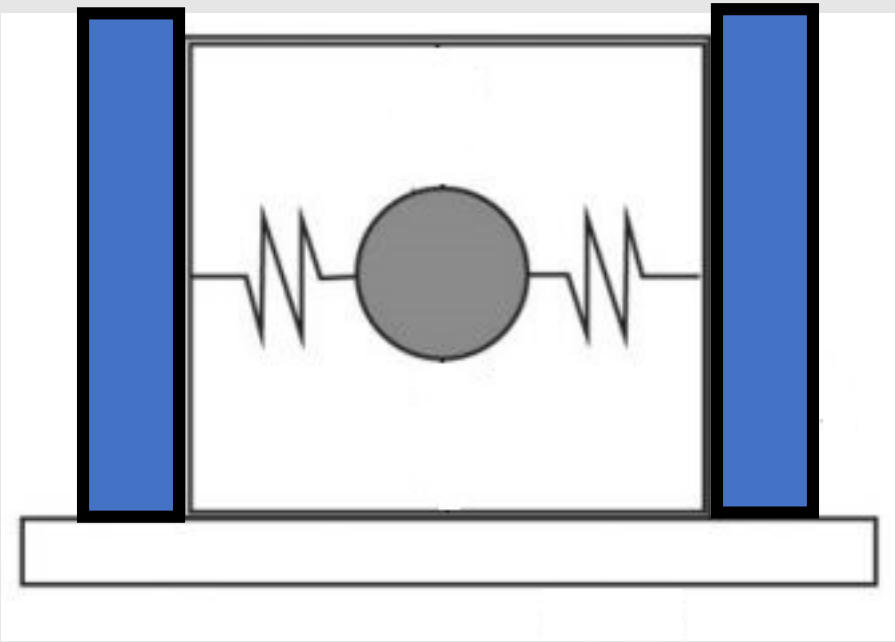
Wikipedia: In physical science, an **isolated system** is either of the following:

1. A physical system so far removed from other systems that it does not interact with them.
2. A thermodynamic system enclosed by rigid immovable walls through which neither mass nor energy can pass.

[Spring + mass + box]
Insulated fixed

Is this an isolated system?

It’s momentum is not conserved.



Do Our Students Get Confused?

Phys100 at SFU

Textbook: OpenStax College Physics

An isolated system is defined to be one for which the net external force is zero.

Instructor: The condition for conservation of momentum is that the net external force is zero, without using the concept of isolated system.

A Quizz at the end of the semester

For an **isolated system**,

A. both the energy and momentum are conserved.
B. the energy is conserved, but the momentum doesn't have to be conserved.
C. the momentum is conserved, but the energy doesn't have to be conserved.
D. neither the energy nor the momentum has to be conserved.

Spring 2017

Attempts: 228 out of 229

For an isolated system,

both the energy and momentum are conserved.	131 respondents	57 %	
the energy is conserved, but the momentum doesn't have to be conserved.	32 respondents	14 %	
the momentum is conserved, but the energy doesn't have to be conserved.	61 respondents	**27 %**	
neither the energy nor the momentum has to be conserved.	4 respondents	2 %	
No Answer	1 respondents	0 %	

Spring 2018

Attempts: 166 out of 167

For an isolated system,

both the energy and momentum are conserved.	71 respondents	43 %	
the energy is conserved, but the momentum doesn't have to be conserved.	11 respondents	7 %	
the momentum is conserved, but the energy doesn't have to be conserved.	79 respondents	**47 %**	
neither the energy nor the momentum has to be conserved.	5 respondents	3 %	
No Answer	1 respondents	1 %	

Spring 2019

Attempts: 154 out of 154

For an isolated system,

both the energy and momentum are conserved.	60 respondents	39 %	
the energy is conserved, but the momentum doesn't have to be conserved.	11 respondents	7 %	
the momentum is conserved, but the energy doesn't have to be conserved.	80 respondents	**52 %**	
neither the energy nor the momentum has to be conserved.	3 respondents	2 %	

Conclusion

We need to be careful when using the term “isolated system”.

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