Qubits as edge state detectors: Illustration using the SSH model

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CAP Congress, SFU, Burnaby, BC
3 June, 2019

Qubits as edge state detectors: Illustration using the SSH model

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NSERC CRSNG
Fonds de recherche Nature et technologies Québec
Université de Montréal
|↑NTR↓Q⟩

Two-level system (TLS)

- most elementary nontrivial quantum system
- many important systems are TLS:
  - spin 1/2
  - photon polarization
  - “two-level atom”
  - quantum dot (empty / full)
  - double well (double quantum dot)
  - KKbar, BBbar systems, etc
  - neutrino oscillations (2 flavours)
- TLS = qubit = building block of
  quantum computers

TLS

An isolated TLS will evolve unitarily:

- pure state remains pure
- oscillates between the two basis states
  (assuming they are coupled)

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- pure state becomes entangled / mixed
- particle can escape to environment

  (point of view of TLS: probability not conserved)

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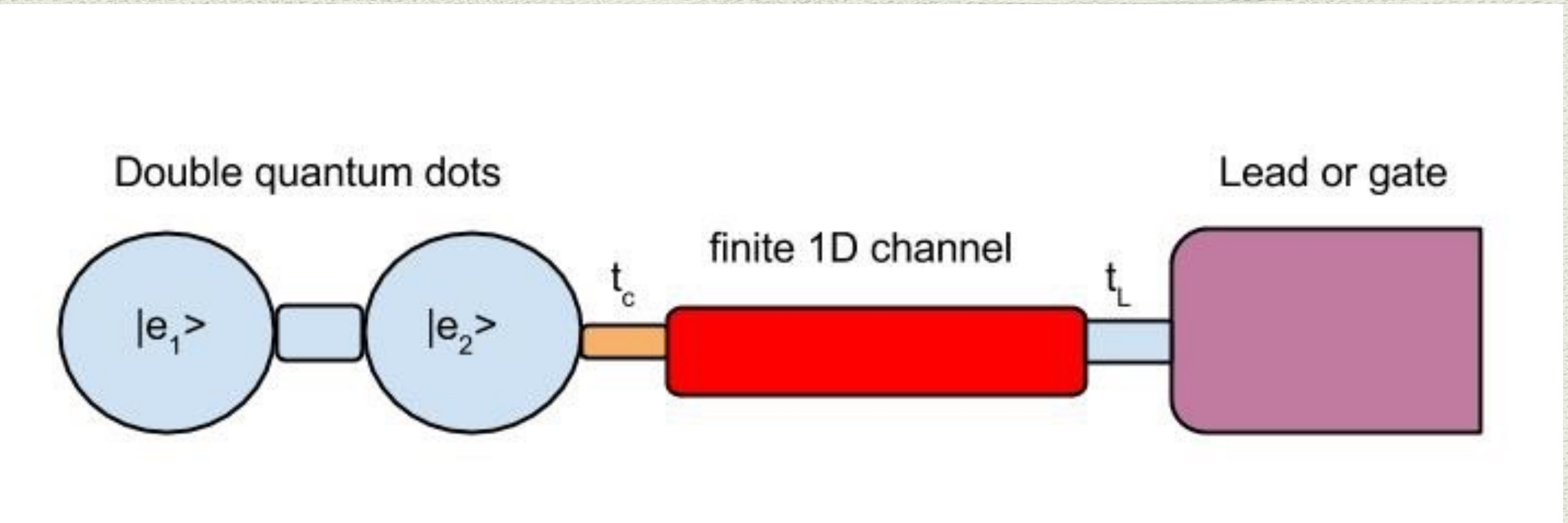
BAD

The system studied:



TLS

SSH chain

Semi-inf. lead

Outline

- TLS in isolation: a rapid review
- Su-Schrieffer-Heeger model
- “Tripartite” system (TLS-SSH-semi∞ chain). Main question addressed: Does the presence of an edge state have a strong effect on decoherence?

Outline

- TLS in isolation: a rapid review
- Su-Schrieffer-Heeger model
- "Tripartite" system (TLS-SSH-semi∞ chain). Main question addressed: Does the presence of an edge state have a strong effect on decoherence? (Answer: yes.)

TLS

Two coupled states, Hamiltonian:

$$H_{DD} = \begin{pmatrix} \epsilon_1 & \tau \\ \tau & \epsilon_2 \end{pmatrix} = \begin{pmatrix} \epsilon_0 + \delta_0/2 & \tau \\ \tau & \epsilon_0 - \delta_0/2 \end{pmatrix}$$

TLS

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Energies, energy splitting:

$$\lambda_{\pm} = \epsilon_0 \pm \delta/2 \quad \text{where} \quad \delta = \sqrt{{\delta_0}^2 + 4\tau^2}$$

TLS

Green's function:

$$G^{DD}(E) = \begin{pmatrix} E - \epsilon_1 & -\tau \\ -\tau & E - \epsilon_2 \end{pmatrix}^{-1}$$

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$$G_{12}^{DD}(t) = -\frac{2\pi i \tau}{\delta} \left(e^{-i\lambda_+ t} - e^{-i\lambda_- t} \right)$$

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(energies:
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What good is it?

(energies:
poles of $G^{DD}(E)$)

TLS

Q: If the system is in the state |1> at t=0,

what's the probability it's in |2> at time t?

TLS

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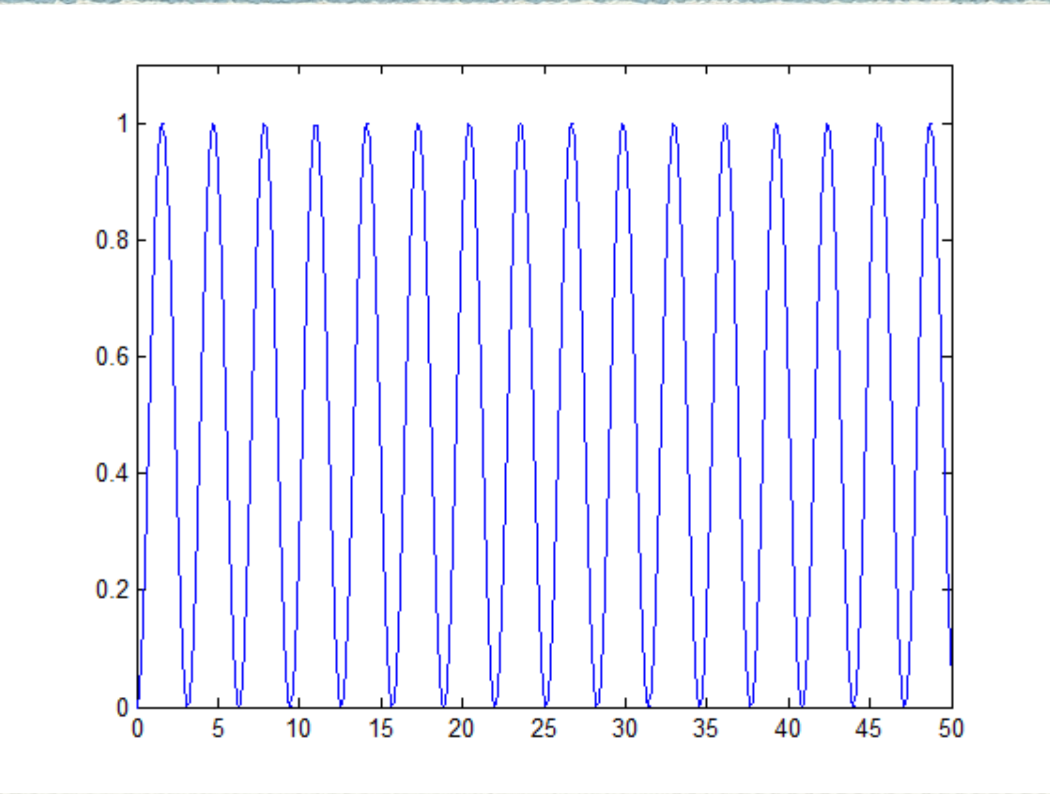
what's the probability it's in |2> at time t?

A:

$$P_{1\to 2}(t) = |G_{12}^{DD}(t)|^2/4\pi^2$$

TLS

$P_{1\to2}(t)$



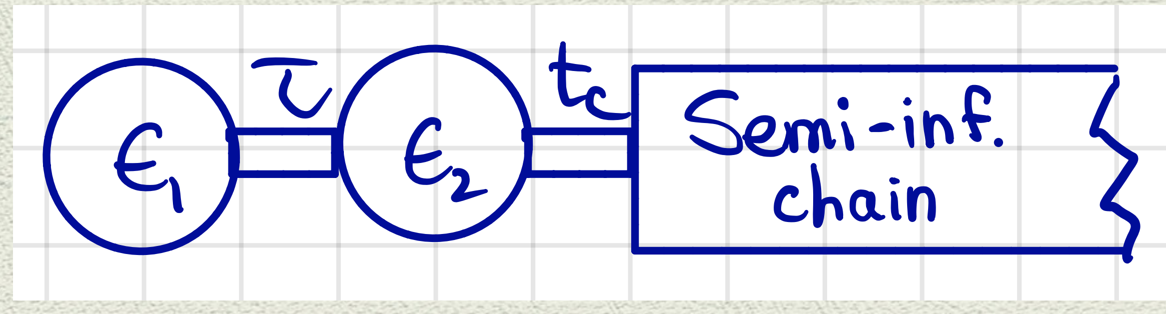
t

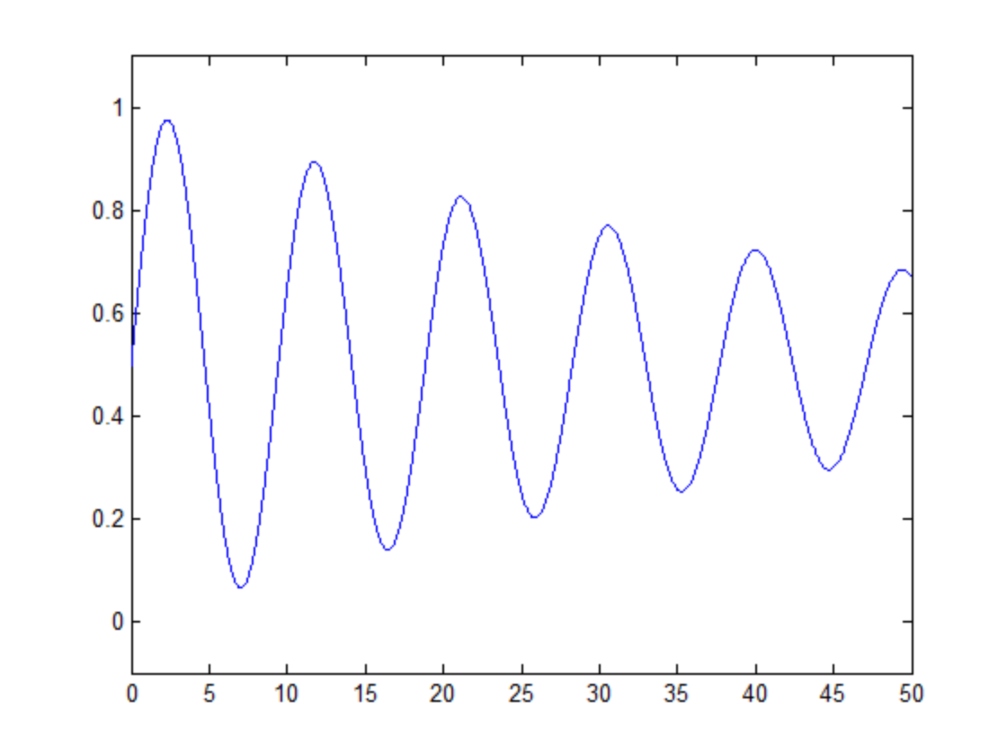
Oscillation |1> —> |2> —> |1> —> …

with frequency = energy splitting δ

TLS

TLS coupled to its environment





Oscillations are damped since particle can escape into its environment

Su-Schrieffer-Heeger Model

Physical context: polymer physics. One-dimensional polymer chains have many interesting and surprisingly exotic features.

Su-Schrieffer-Heeger Model

Physical context: polymer physics.

One-dimensional polymer chains have many interesting and surprisingly exotic features.

Examples:

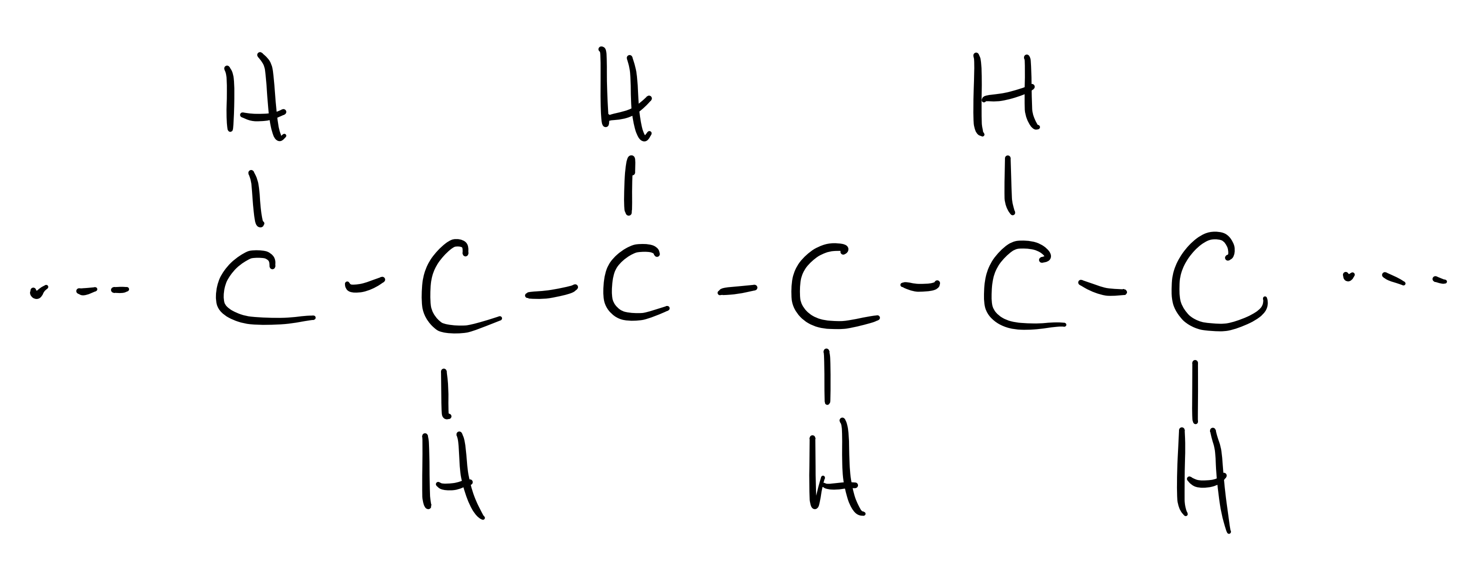
-solitons

-fractional charge

-unusual charge-spin relationships

SSH Model

One of the most elementary, and also interesting and important, polymers is polyacetylene, $(CH)_x$



One free electron per C => conductor.

SSH Model

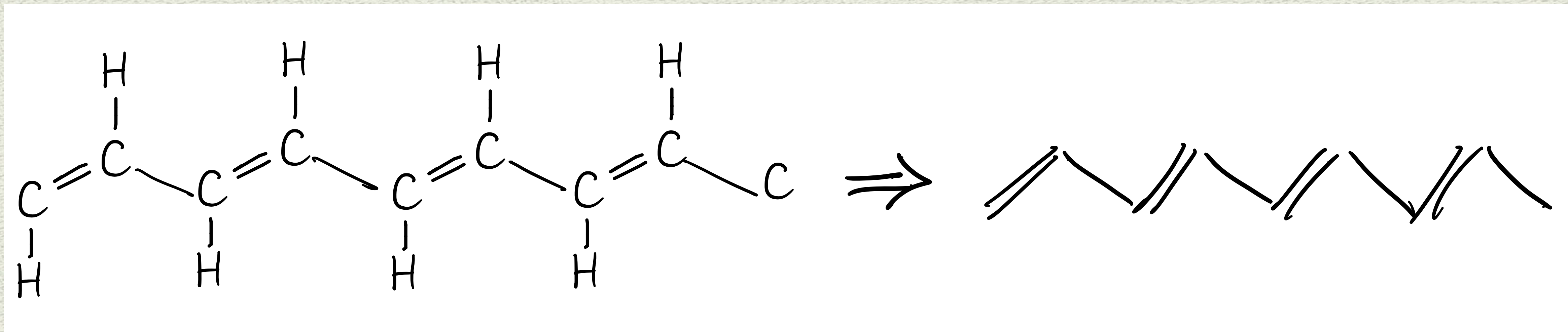
NO!

Polyacetylene is one of the best-known examples of the *Peierls instability,* which states that a one-dimensional chain with equally spaced atoms and one electron per atom is unstable.

SSH Model

Roughly, Peierls found that the electron contribution to the chain energy is reduced by *dimerization,* a process by which bond lengths alternate. So the equal-spacing state is unstable.

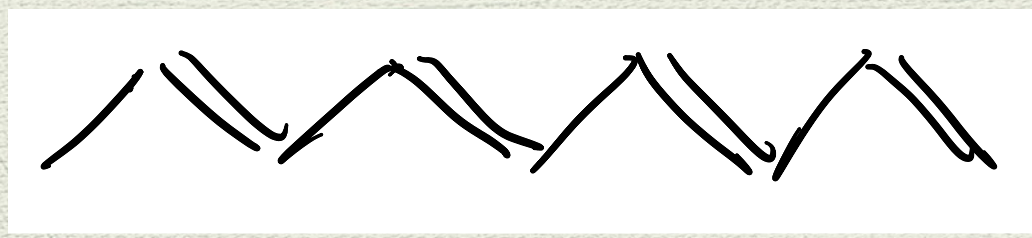
SSH Model



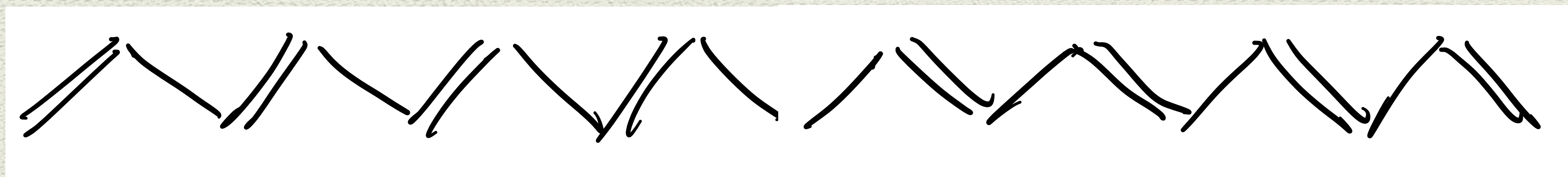
The C-C bonds alternate between single and double bonds, double bonds being stronger and shorter.

SSH Model

Aside: There are two equivalent such states:



A configuration interpolating between the two is the simplest example of a topological soliton:



But that's another story…

SSH Model

The SSH model is a tight-binding 1d chain with alternating hopping parameters:

$$H_{\mathrm{SSH}} = \begin{pmatrix} 0 & t_1 & & & & \\ t_1 & 0 & t_2 & & & \\ & t_2 & 0 & t_1 & & \\ & & t_1 & 0 & \ddots & \\ & & & \ddots & \ddots & t \\ & & & & t & 0 \end{pmatrix}$$

Assume N even for now: $N = 2M;\ t = t_1$

SSH Model

Solutions of the SSH model:

$$H_{\mathrm{SSH}} \left|\psi\right\rangle = E \left|\psi\right\rangle$$

Ansatz reflects translational symmetry:

$$\left|\psi\right\rangle = \sum_{n=0}^{M-1} \big(A \left|2n+1\right\rangle + B \left|2n+2\right\rangle \big) e^{in2k}.$$

SSH Model

Middle equations determine the relationship between E and k and also A/B. In particular:

$$E^2 = t_1^2 + t_2^2 + 2\,t_1 t_2 \cos 2k$$

For any E, there are two solutions with equal and opposite k.

SSH Model

Boundary equations determine the ratio of the two solutions, and the energies.

Middle equations:

$$E^2 = t_1^2 + t_2^2 + 2\,t_1 t_2 \cos 2k$$

Boundary equations:

$$\frac{t_1}{t_2} \sin(N+2)k + \sin Nk = 0$$

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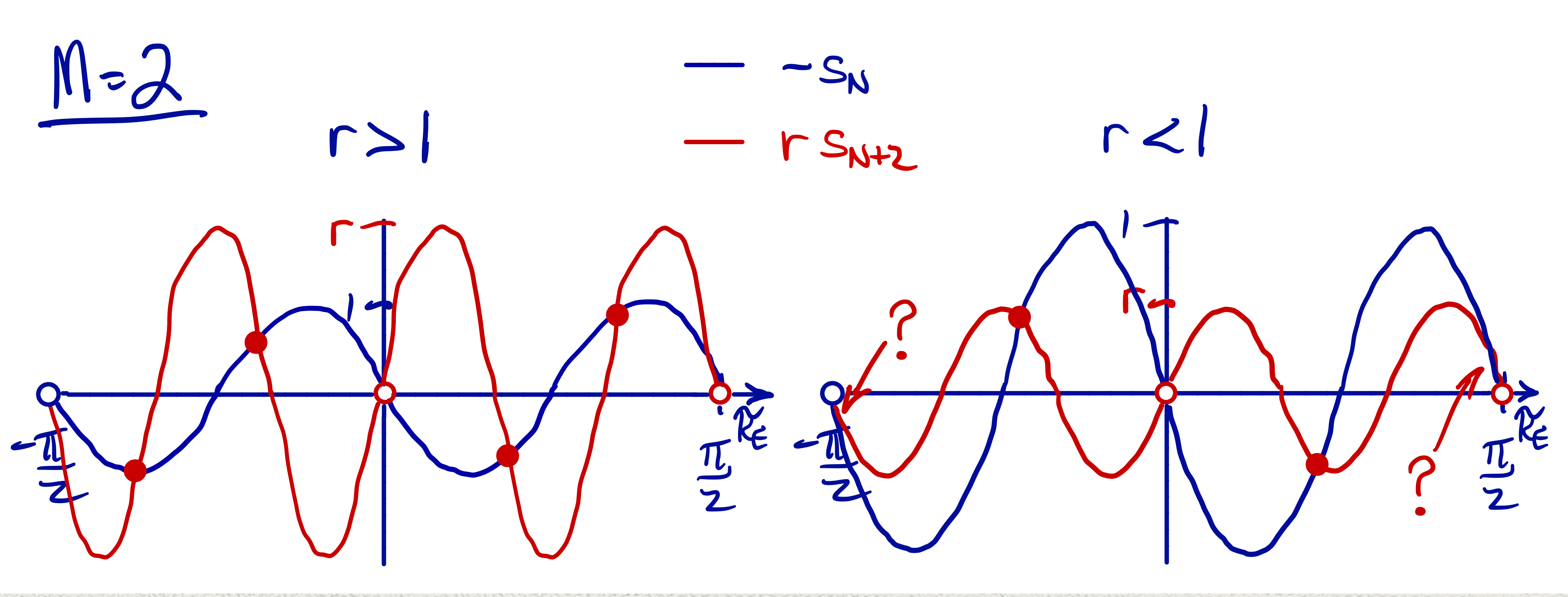
Boundary equations:

$r \equiv t_1/t_2$

$$\frac{t_1}{t_2}\sin(N+2)k + \sin Nk = 0$$

SSH Model

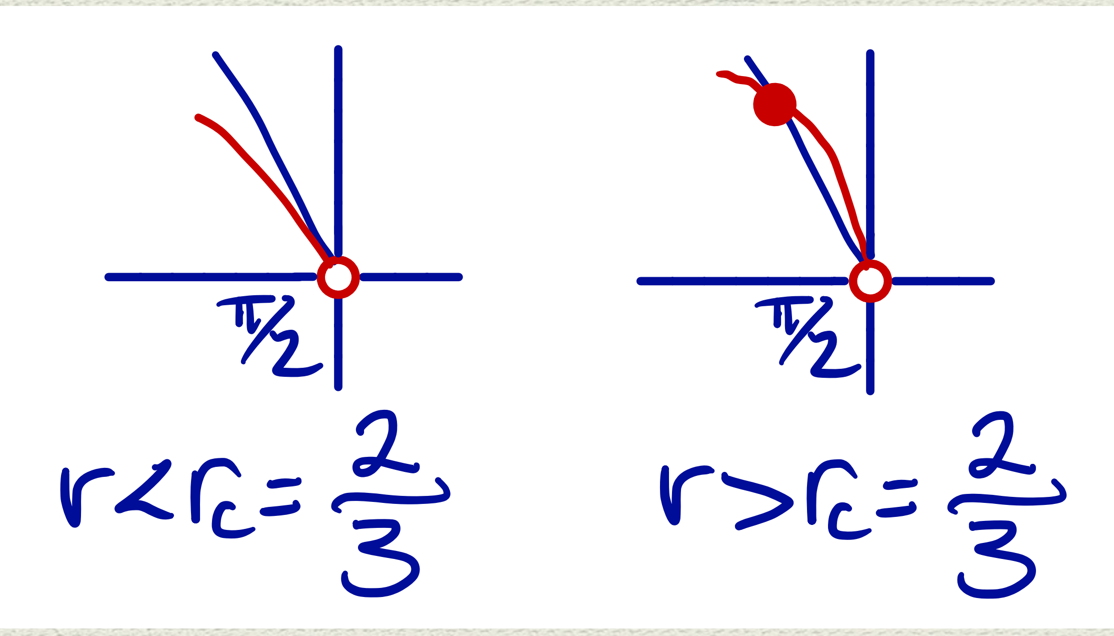
$$r \sin(N+2)k + \sin Nk = 0$$



SSH Model

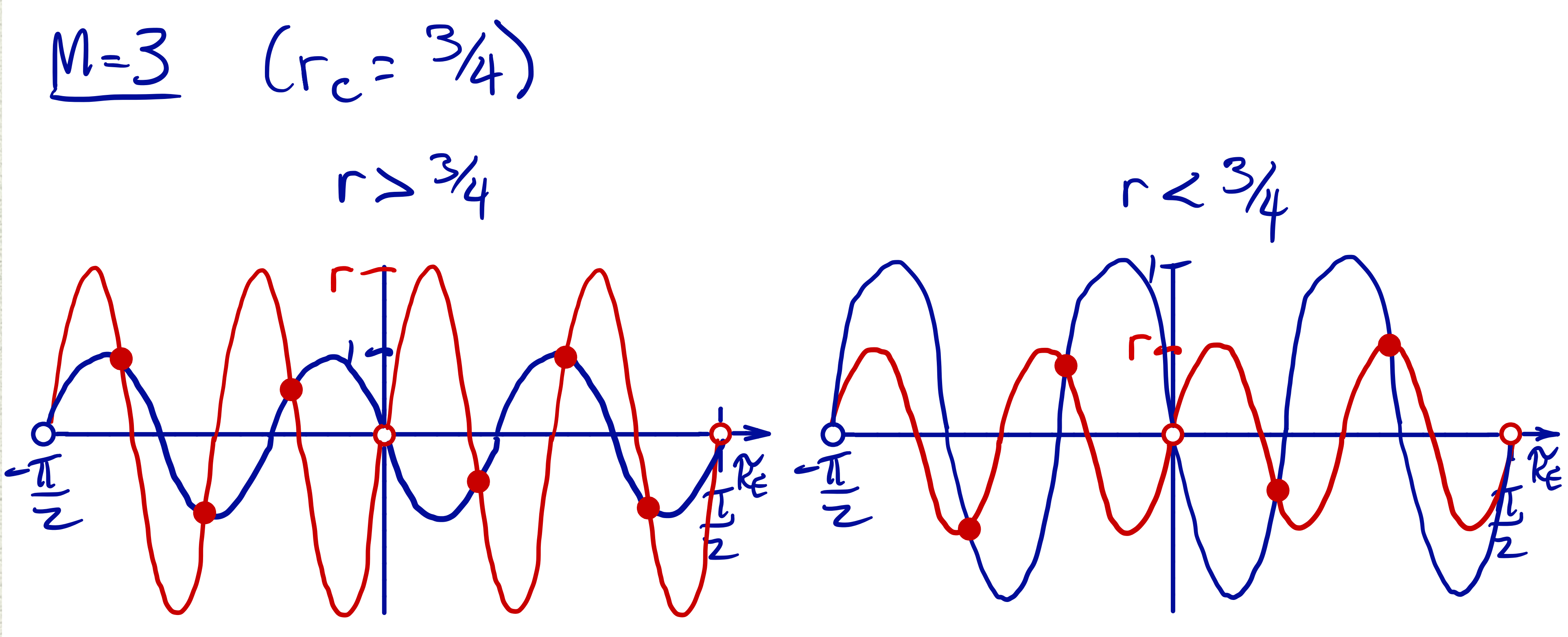
$$r \sin(N+2)k + \sin Nk = 0$$

$$M = 2$$
$$(N = 4)$$



$$r_c = \frac{N}{N+2}$$

SSH Model



6 solutions
(as expected)

4 solutions
(2 too few)

SSH Model

Where are the two missing solutions for

$$r < r_c\,?$$

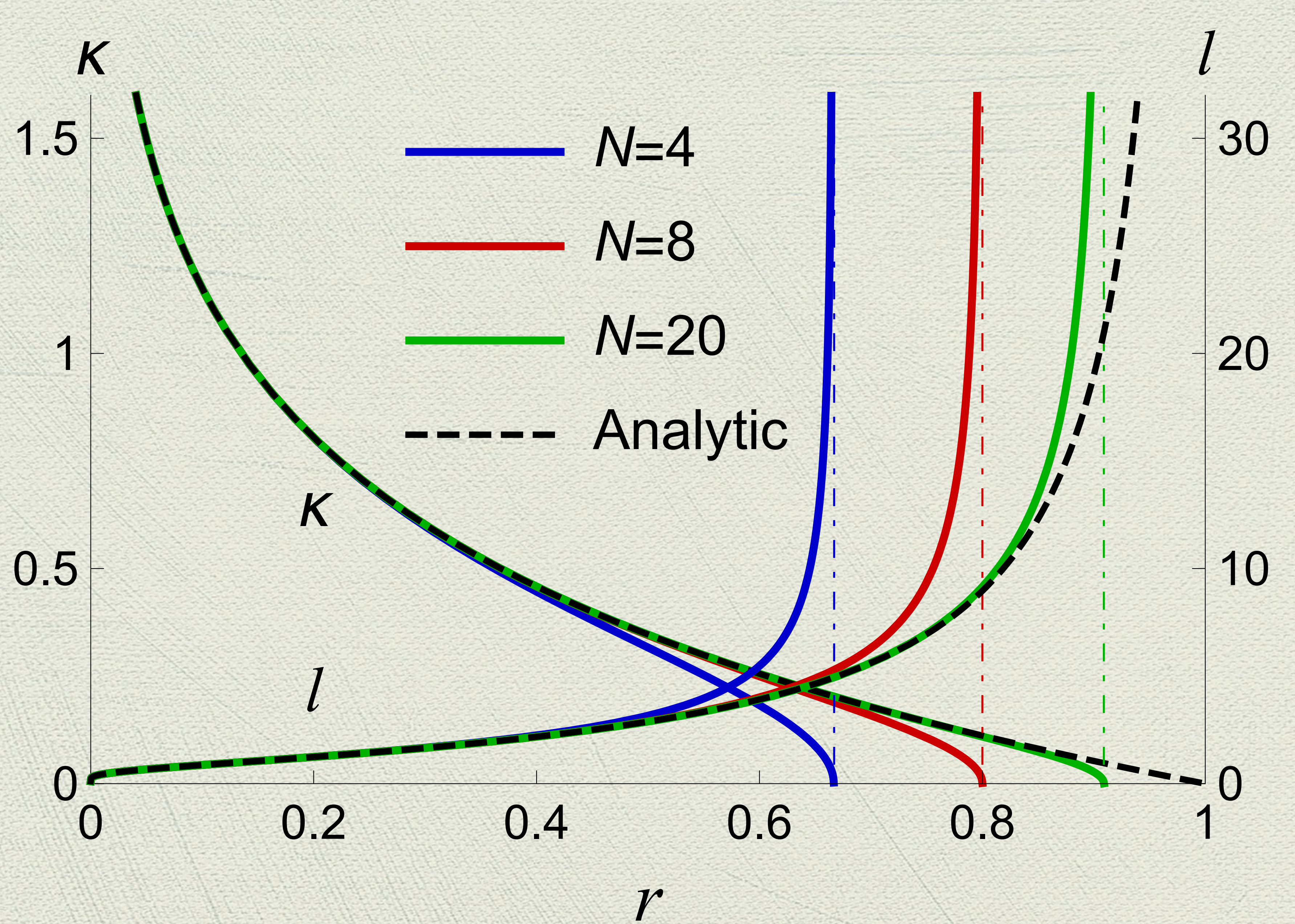
There are two complex solutions: letting

$$k = \frac{\pi}{2} + i\kappa$$

we find
$$\frac{\sinh(N\kappa)}{\sinh((N+2)\kappa)} = r.$$

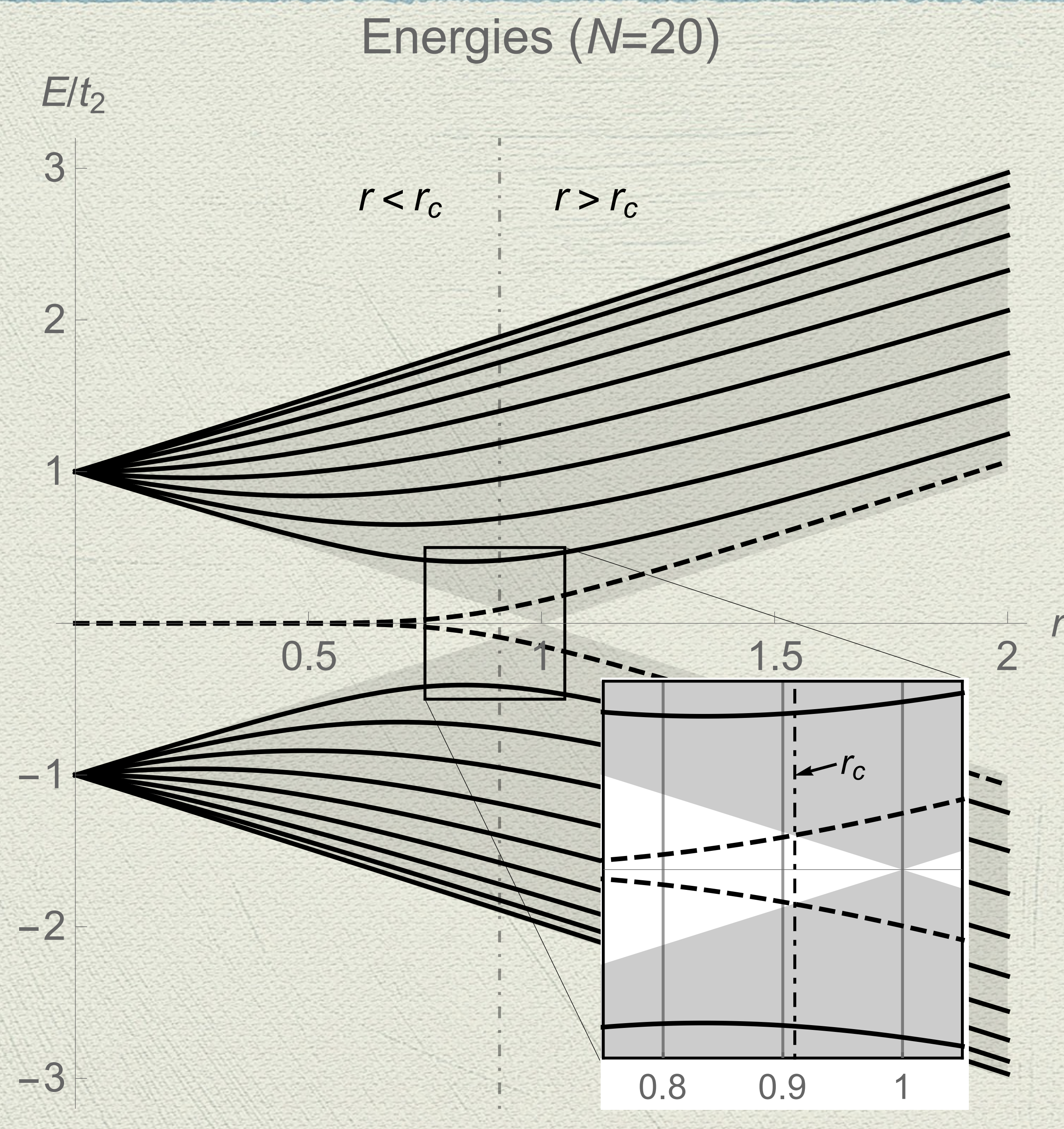
Two equal and opposite solutions for $r < r_c$.

SSH Model



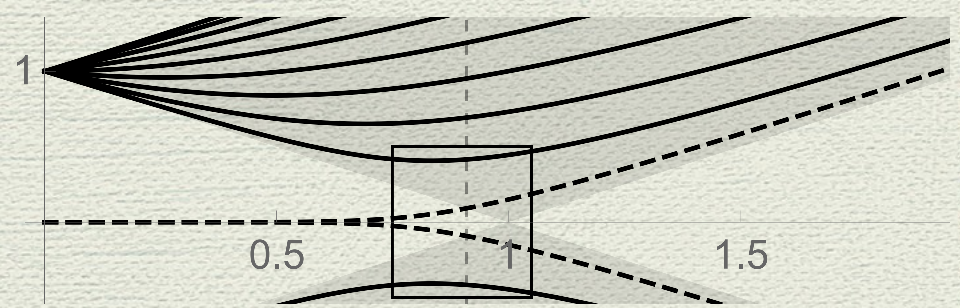
SSH Model

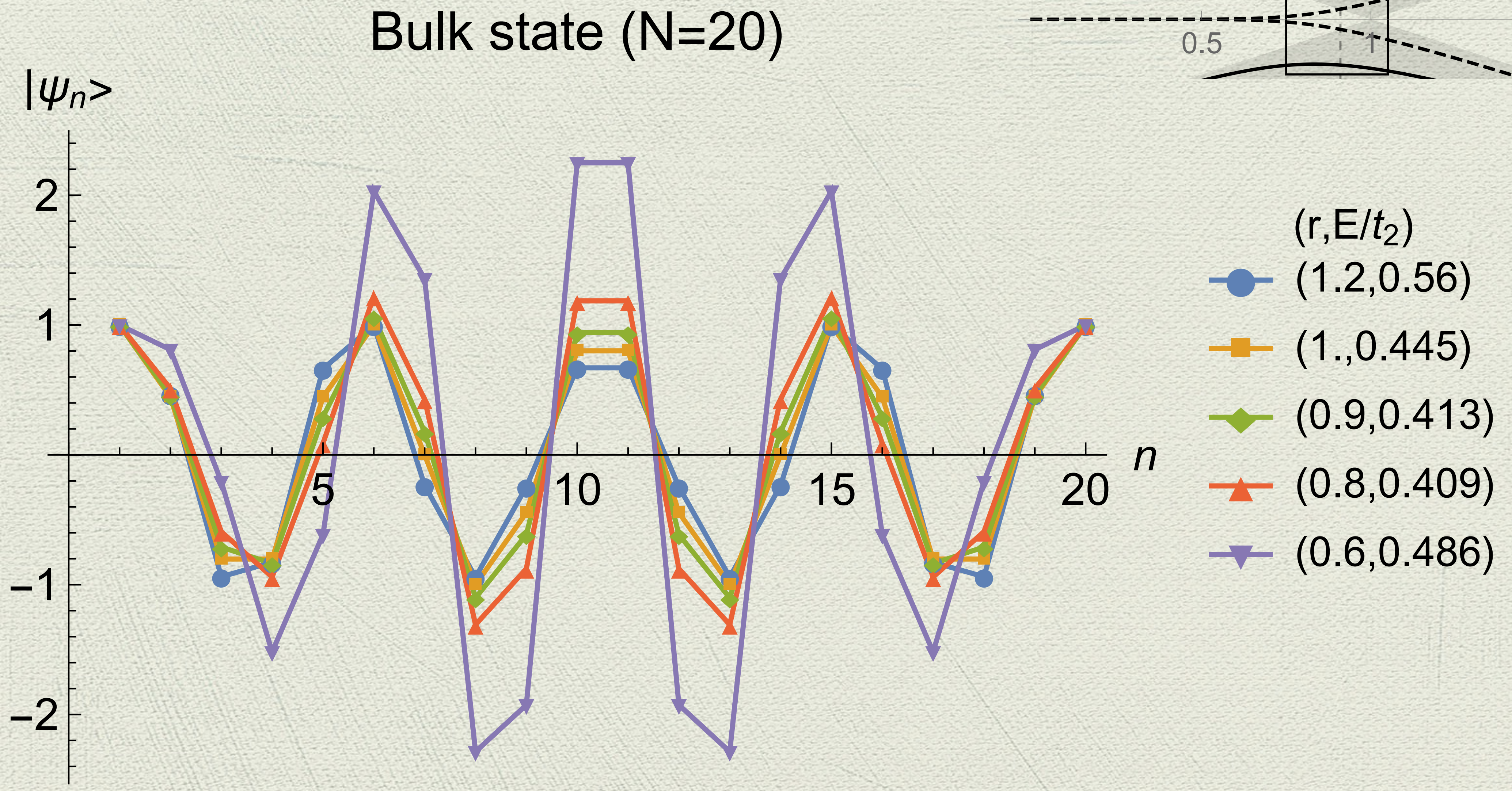
Energy spectrum (N=20):



SSH Model

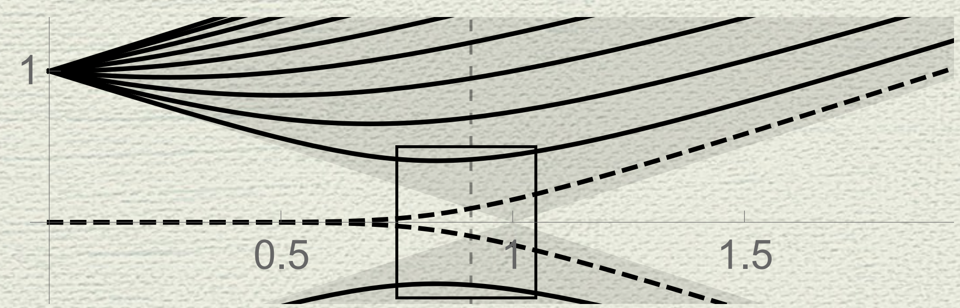
Bulk state

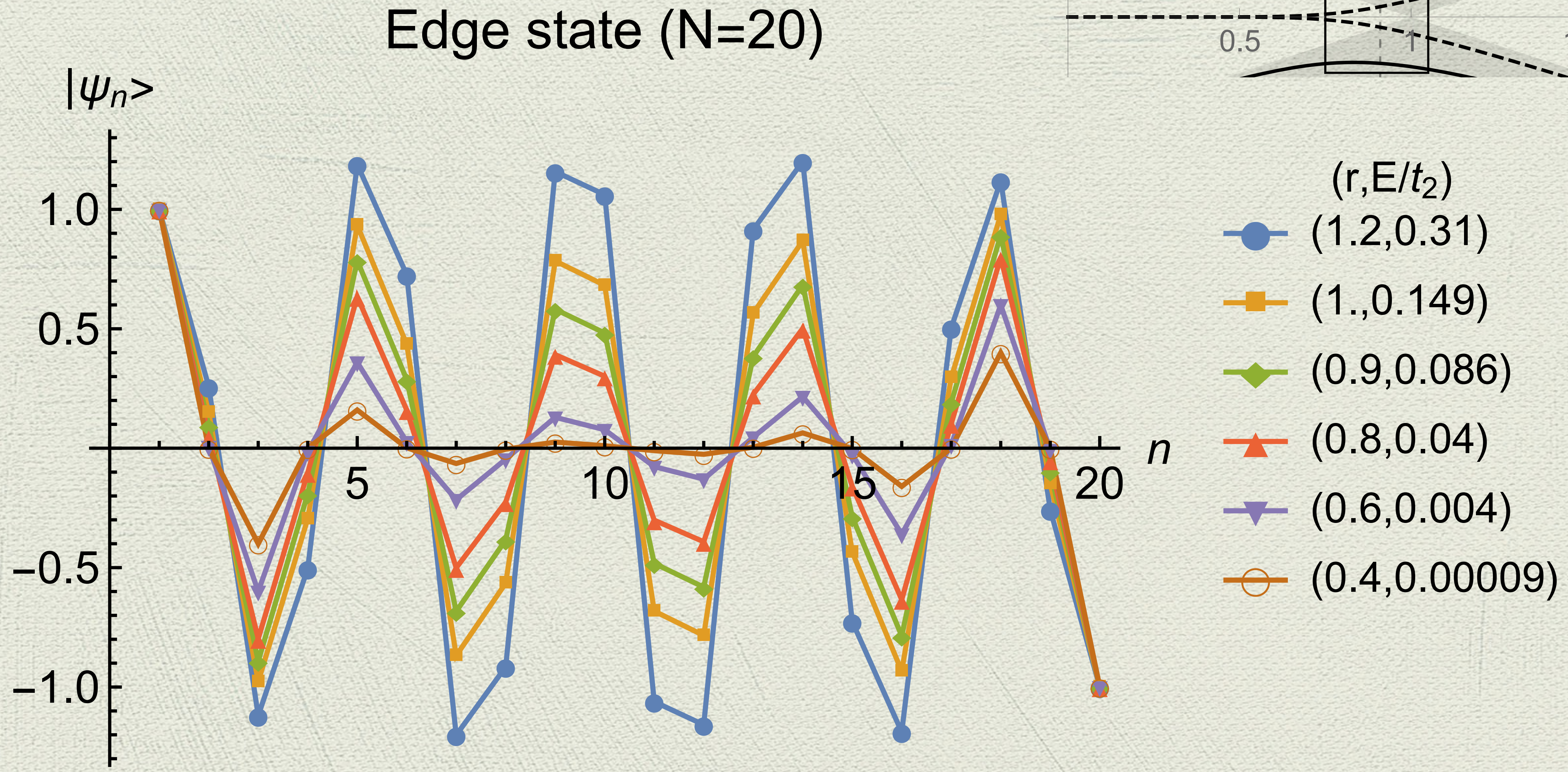




SSH Model

Edge state

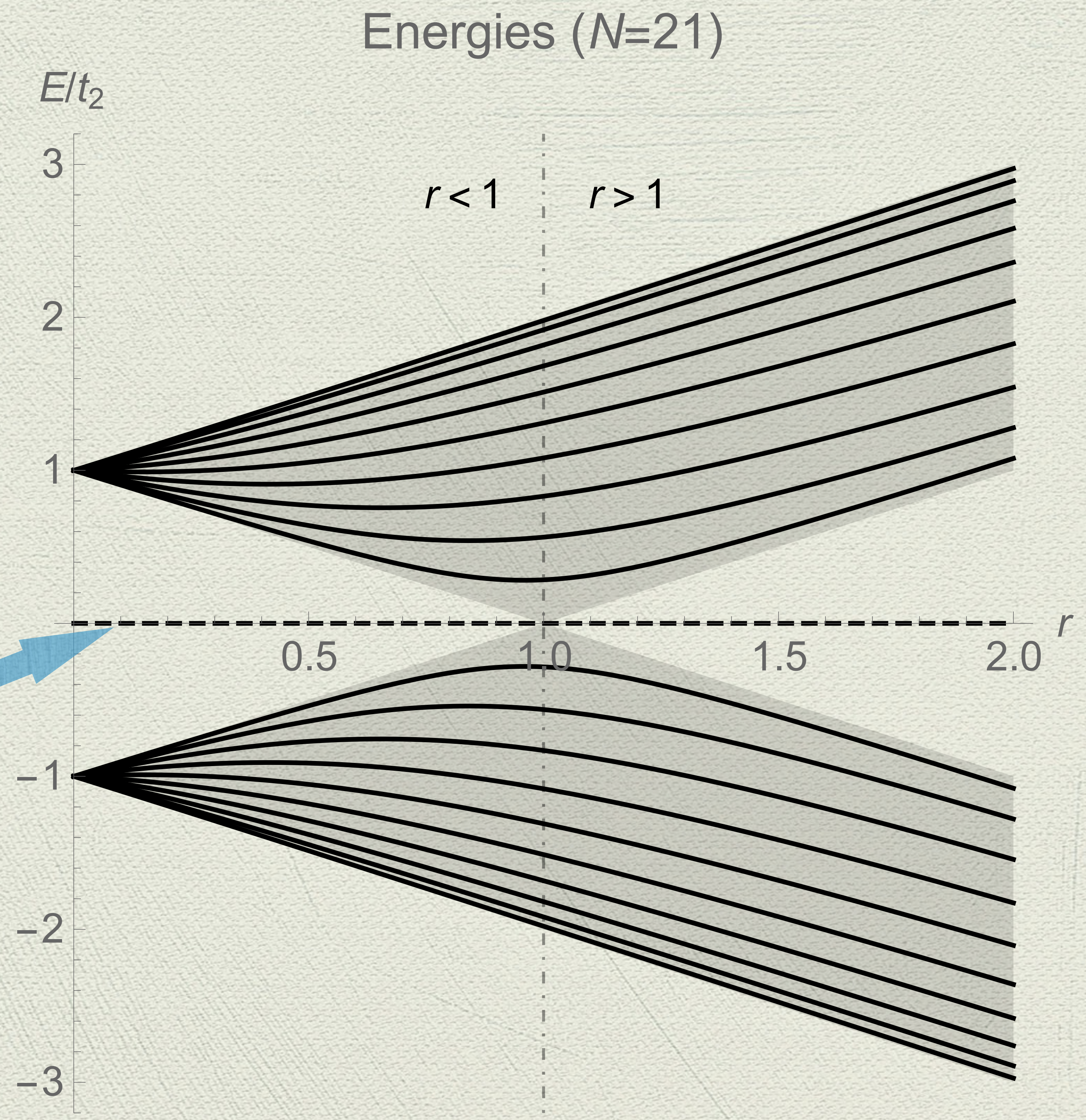




SSH Model

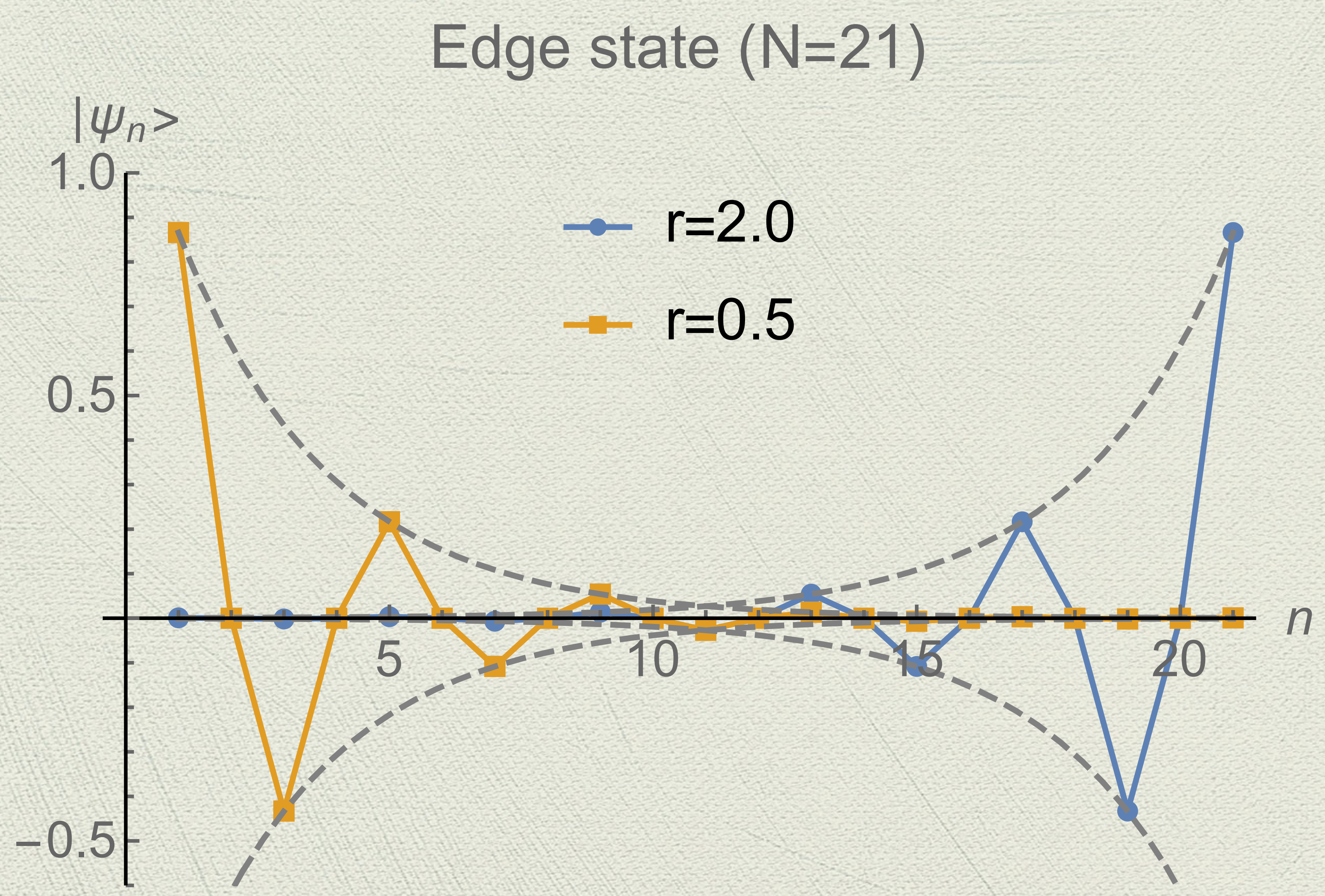
Energy spectrum (N=21):

Exact zero-mode



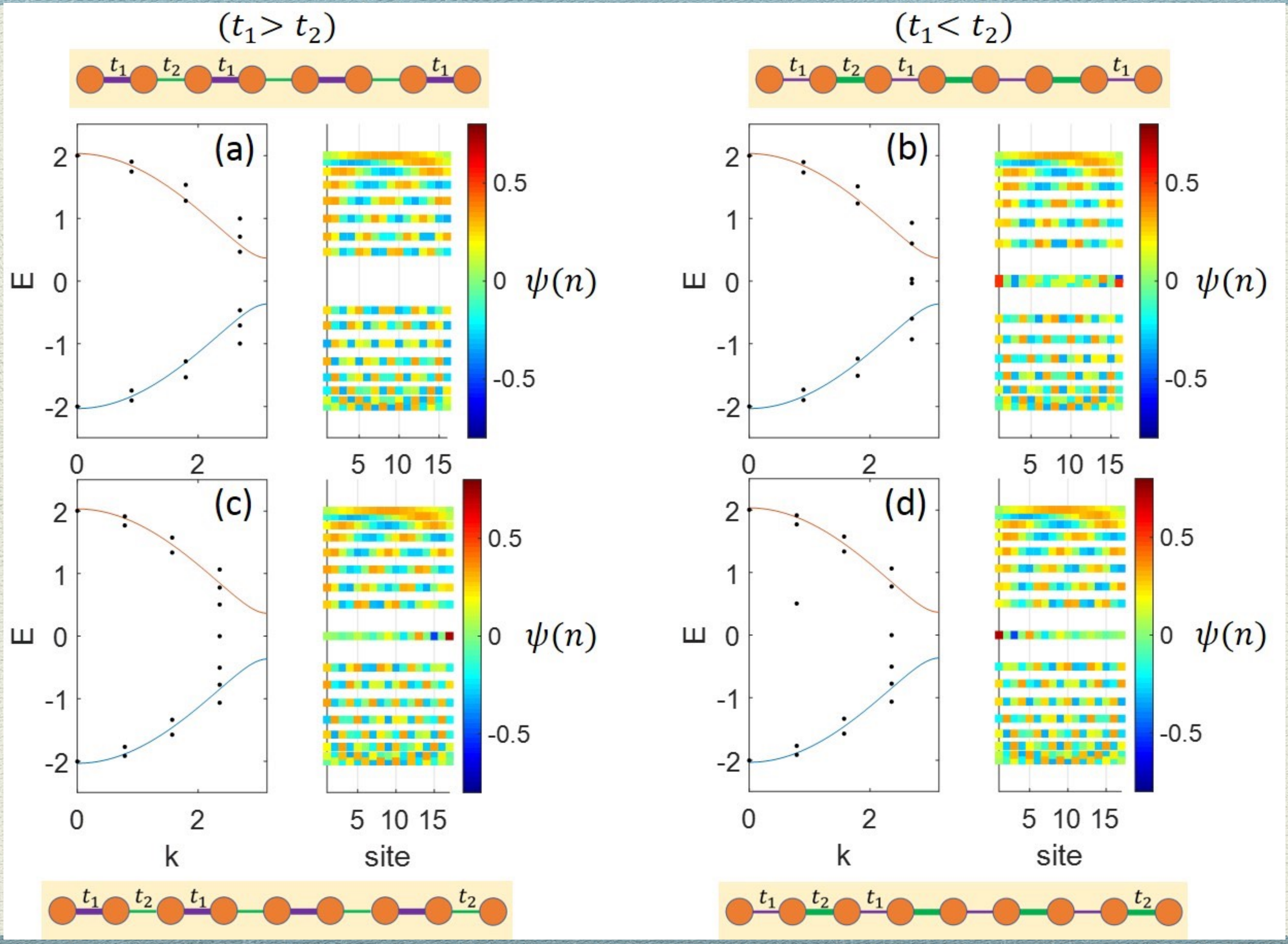
SSH Model

Zero-mode edge state



SSH Model

Summary



SSH Model

How to detect an edge state?

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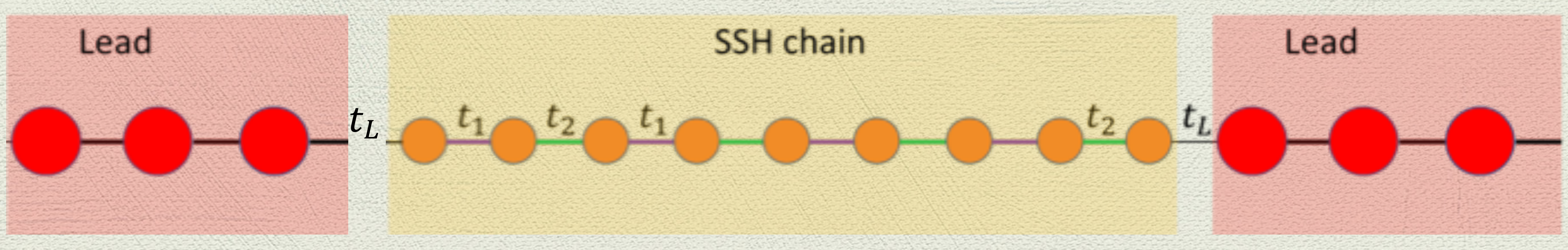

SSH Model

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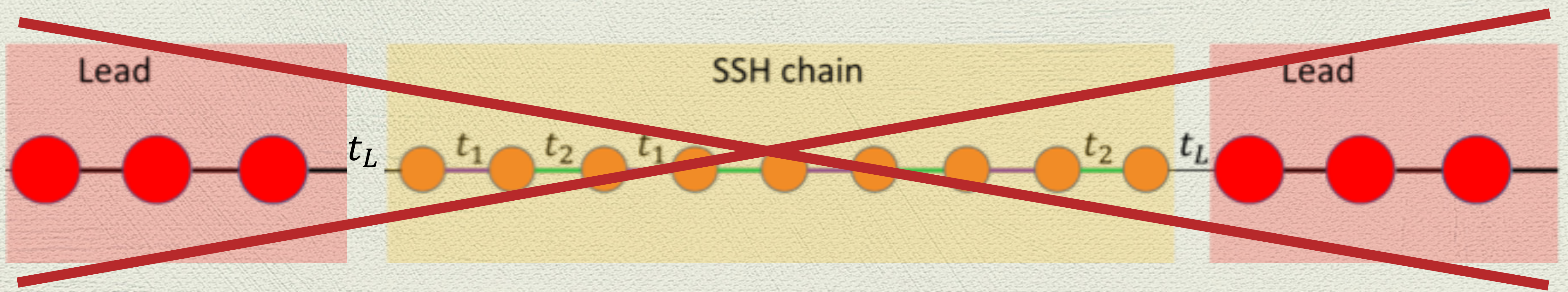

No. Measuring resistance will not tell us if we have a L edge state, R edge state or both.

SSH Model

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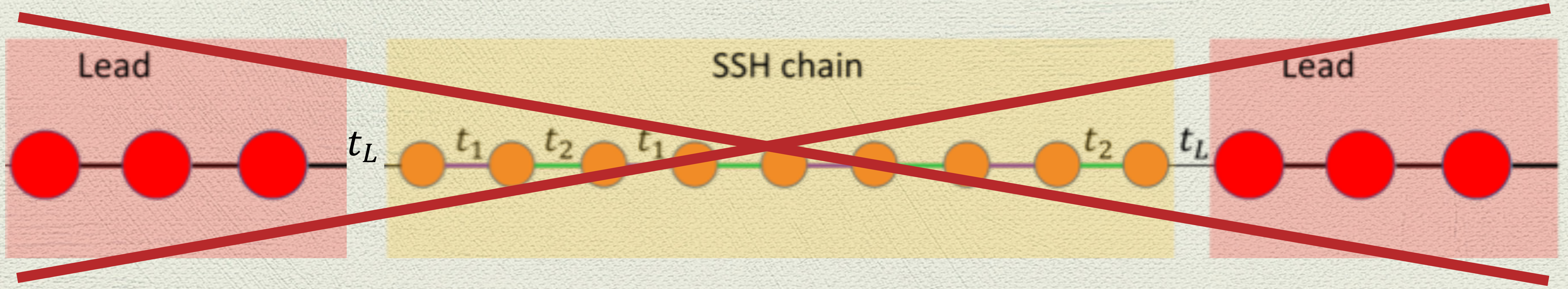

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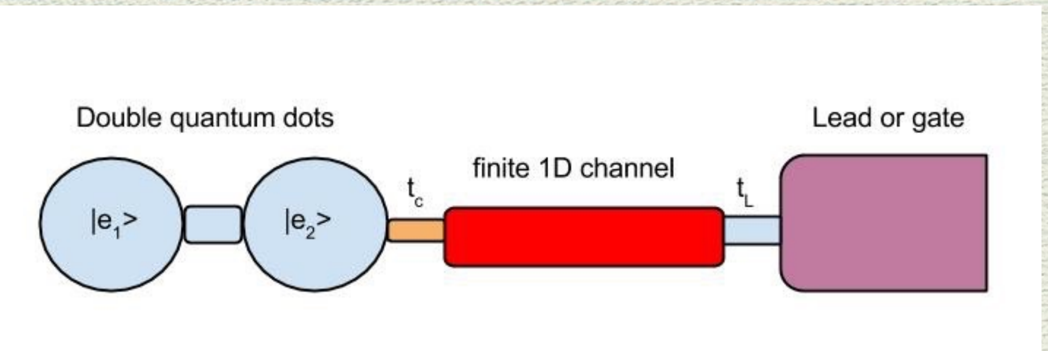
Our proposal:

Tripartite system

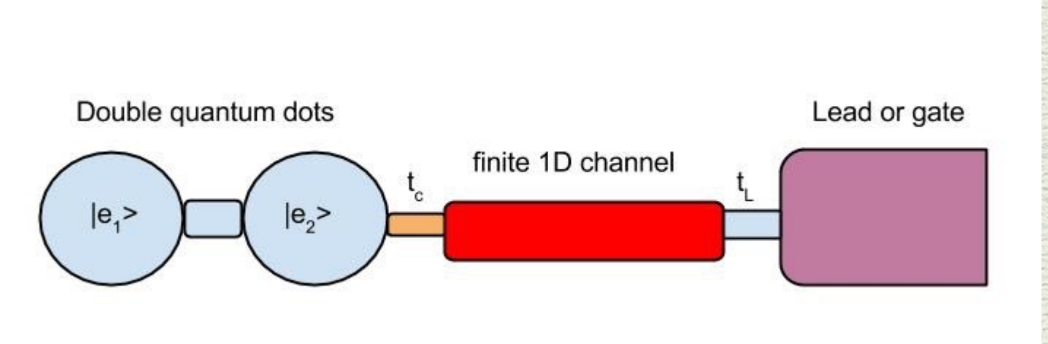


$$(V_N)_{21} = t_c \qquad W_{N1} = t_L$$

$$H = \left(\begin{array}{c|c|c} H_{\mathrm{DD}} & V_N & 0 \\ \hline V_N^\dagger & H_{\mathrm{SSH}} & W \\ \hline 0 & W^\dagger & H_\infty \end{array}\right)$$

$$t_1 = t_2 = 1$$

Tripartite system



The effect of the channel + lead on the TLS can be incorporated into an effective TLS Hamiltonian.

$$H \to \begin{pmatrix} \epsilon_1 & \tau \\ \tau & \epsilon_2 + \Sigma_{N\infty}(E) \end{pmatrix}$$

Tripartite system

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The added term $\Sigma_{N\infty}(E)$ is proportional to the surface Green's function of the channel+lead, $G^S_{N\infty}(E)$.

Tripartite system

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The added term $\Sigma_{N\infty}(E)$ is proportional to the surface Green's function of the channel+lead, $G^S_{N\infty}(E)$.
The rate of escape can then be calculated in the same way as for the isolated TLS.

Tripartite system

Some nasty math (1/2): Surface Green's fcn

$$G^{\mathrm{S}}_{N\infty} = \begin{cases} \dfrac{Et_2\tilde{s}_N - \Sigma'_\infty(t_1\tilde{s}_{N-2}+t_2\tilde{s}_N)}{t_2^2(t_1\tilde{s}_{N+2}+t_2\tilde{s}_N)-Et_2\Sigma'_\infty\tilde{s}_N}, & (N \text{ even}) \\ \dfrac{t_1(t_1\tilde{s}_{N-1}+t_2\tilde{s}_{N+1})-E\Sigma'_\infty s_{N-1}}{t_1t_2E\tilde{s}_{N+1}-t_2\Sigma'_\infty(t_1\tilde{s}_{N+1}+t_2\tilde{s}_{N-1})}, & (N \text{ odd}) \end{cases}$$

$$E = 2\cos(k) = \pm\sqrt{t_1^2 + t_2^2 + 2t_1t_2\cos(2\tilde{k})};\ \Sigma'_\infty = e^{ik};\ \tilde{s}_N = \sin(\tilde{k}N)$$

Tripartite system

Some nasty math (2/2): Decoherence time

$$(\tau_\phi)^{-1} \approx \min\left(-\frac{1}{2}\Im\left\{\Sigma_{N\infty}(\lambda_\pm) \pm \delta_{N\infty}(\lambda_\pm)\right\}\right)$$

$$\Sigma_{N\infty} = t_c^2 G_{N\infty}^S \quad \delta_{N\infty} = \sqrt{(\epsilon_1 - \epsilon_2 - \Sigma_{N\infty})^2 + 4\tau^2}$$

Tripartite system

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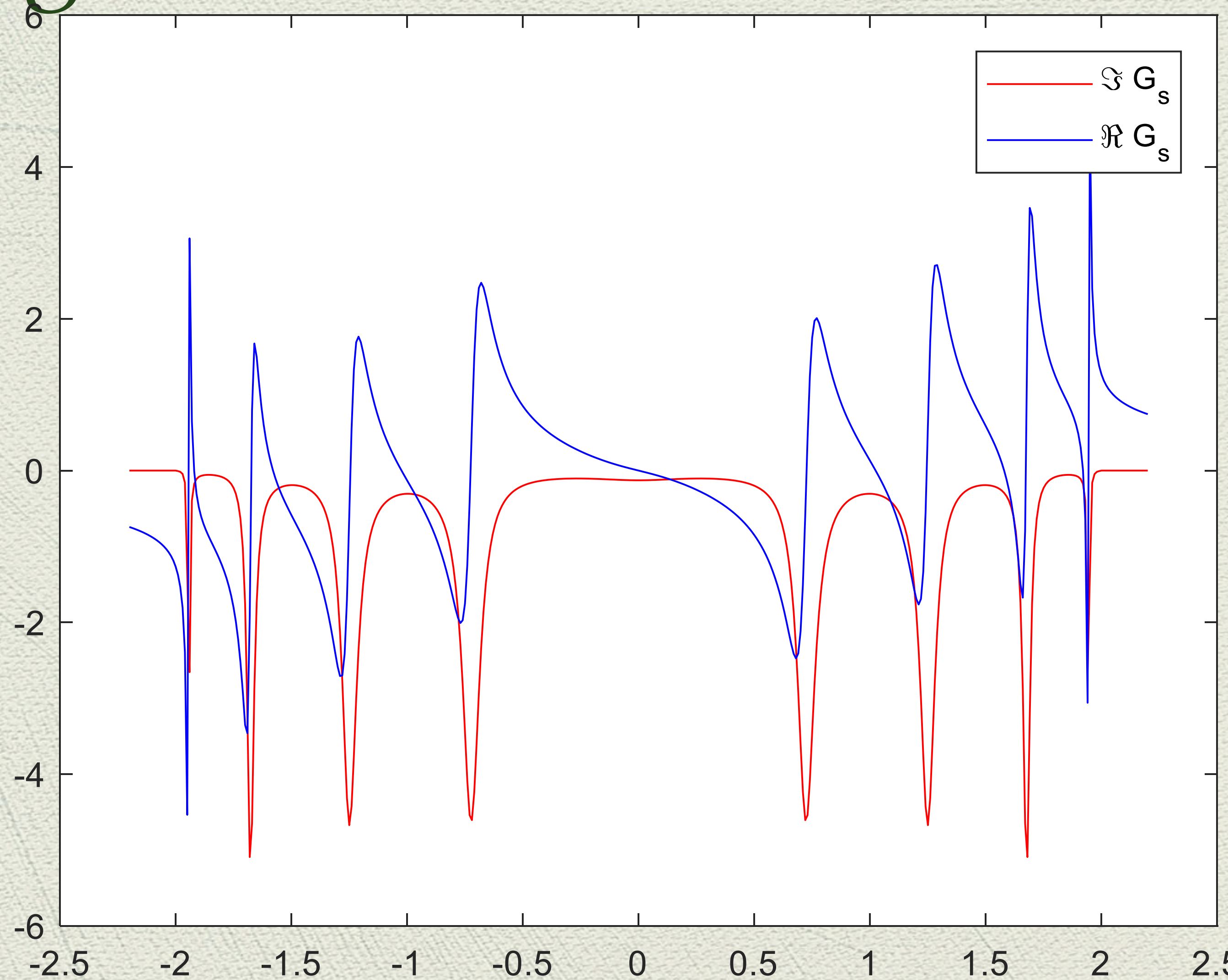
Bottom line: $(\tau_\phi)^{-1} \propto G_{N\infty}^S$

Tripartite system

Surface Green's function in two cases:

No left edge state:



Tripartite system

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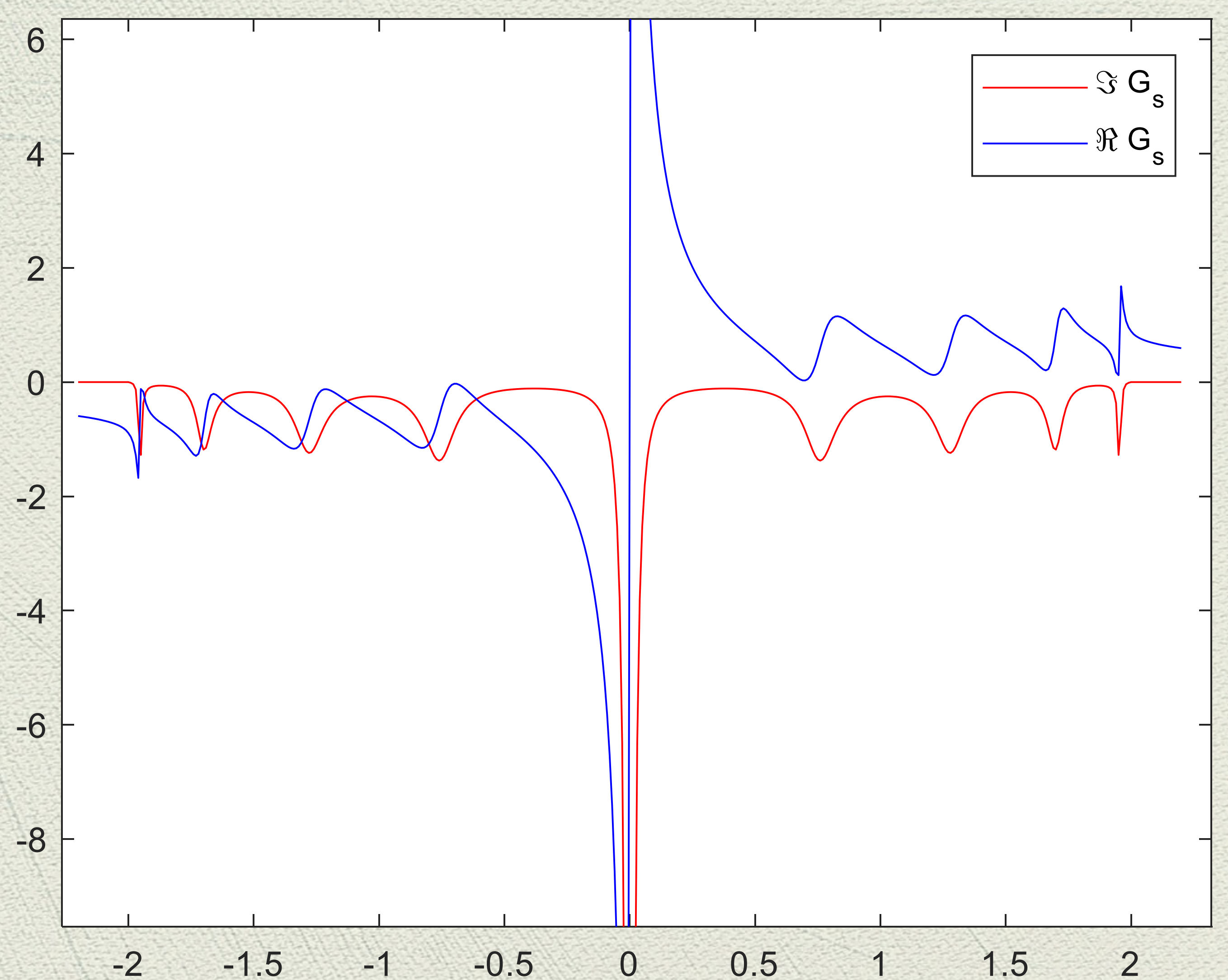
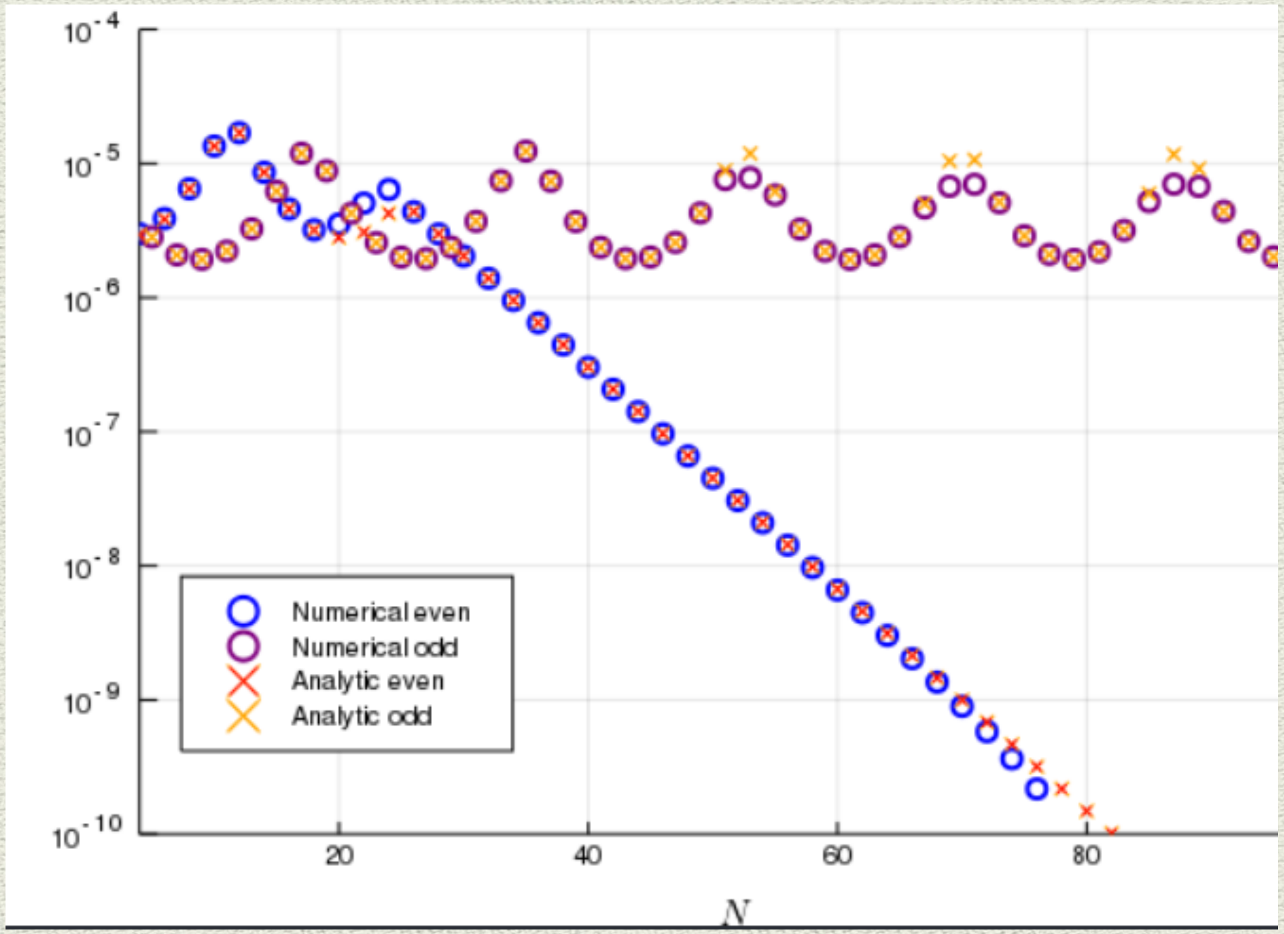


Tripartite system

Comparison of decoherence times:



Edge state: decoherence remains high as N ↗

No edge state: deco. drops off as N ↗

Tripartite system

Conclusion:

Measurement of the decoherence rate of a TLS attached to a channel, itself attached to an infinitely long chain (or as it's known in the real world: a VERY long chain!), can be used to probe the existence or not of an edge state on the TLS edge of the channel.

Tripartite system

What I have described is a work in progress (keep your eyes out for a preprint this summer). Some further ideas:

-effect of soliton?

-connecting TLS to an interior state of chain (soliton detector?!)

Thank you

Coming to a preprint near you