On systematic Errors of a "magic Energy" Proton EDM Measurement

Possible a skeleton for the section on systematic effects

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Aim: as thorough as possible summary on possible systematic errors of a proton EDM measurement in an electric ring operated at "magic energy" (not covered: combined magnetic end electric fields, "hybrid ring" with magnetic focusing …).

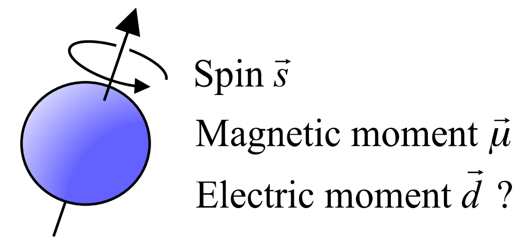
Work in progress!

Content:

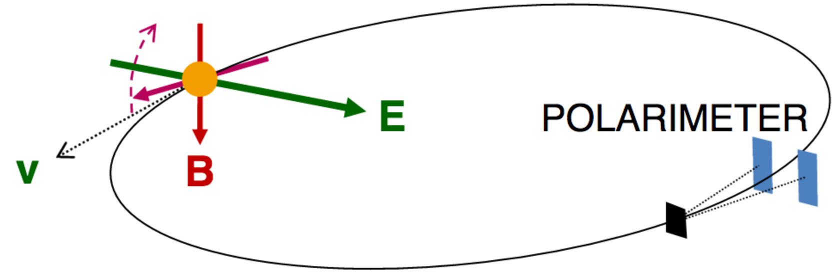
- Recap of the proposal (fully electric ring operated at "magic energy" for "frozen spin")
- Basic equation to describe spin dynamics
- Assumptions on filling patterns, operation and feedback systems
- First order effect(s): average horizontal (radial) magnetic field and gravity
- Second order effects – (some of them are geometric phase effects)
  - Sources for 2nd order systematic effects: electric fields, magnetic fields, (average) horizontal spin …
  - Orbit perturbations in both planes due to misalignments of electric quads
  - Static magnetic field in two planes (e.g. vertical and longitudinal)
  - Magnetic field from cavity and static longitudinal magnetic field
  - Vertical closed orbit from misaligned electric quad and longitudinal vertical magnetic field
  - Vertical magnetic field and horizontal closed orbit from misaligned electric quad

Recap of Proposal

- Aim:
  - Investigate whether charged particles (proton, deuteron ..) have an Electric Dipole Moment (EDM)
  - Cannot be done by applying an electric field to particle at rest due to charge
  - Measurement in synchrotron (using magnetic and/or electric fields for bending and focusing)



- Storage ring combining magnetic and electric field to bend the beam
  - Spin pointing in longitudinal direction (without electric dipole moment) "spin frozen lattice"
  - Beam energy fixes the ratio between magnetic and electric field to keep spin in longitudinal direction
  - An EDM generates a vertical spin
  - Fully electric ring with magnetic shielding to red systematic errors possible for "magic energy protons"

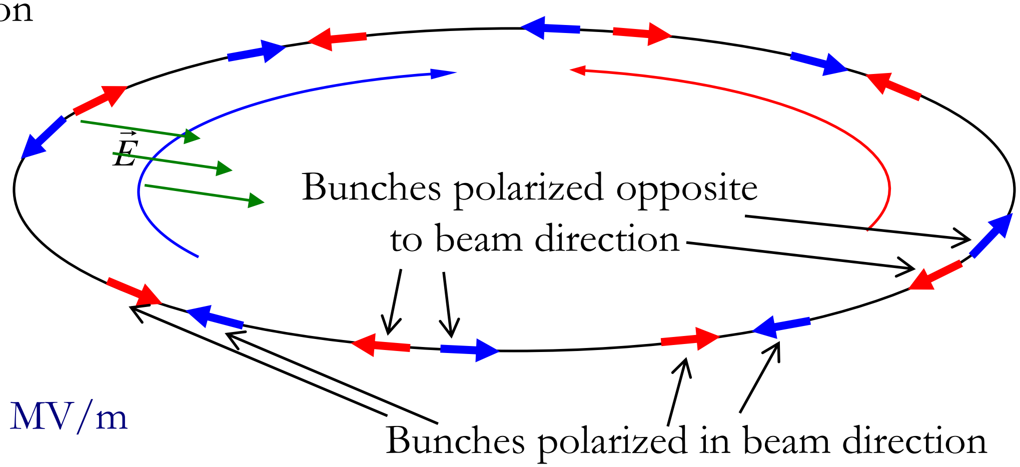


courtesy of E. Stephenson

  - (Other option is a machine with non-frozen spin and an "RF Wien Filter" to generate a vertical spin component, experimental tests in view of this scheme at FZ Jülich)

Recap of Proposal

Arrows indicating bunched with their polarization
CW rotating beam in blue
CCW rotating beam in red



- Circumference of about C=500 m
  - Average radial electric Field of E = 5.27 MV/m
  - Field in bends will be higher
- Small vertical tunes around $Q_V = 0.1$ with large variations to estimate average residual horizontal magnetic field (see later)
- Intra Beam Scattering a potential limitations for some proposals
- Counter-rotating Beams and Polarization in Beam Direction or opposite to it
  - To mitigate systematic effects
  - Some proposals with simultaneously Bunches with polarization in and opposite to beam direction in both Rings (case sketched)
- Proposals for weak focusing (no strong quadrupoles perturbing vertical orbit) and strong focusing (Higher horizontal tune, smaller dispersion, IBS more manageable) rings

Basic Equation for Spin Motion and Proton magic Energy

- Thomas-BMT-Equation to describe change of spin $\vec{S}$ given by (with additional terms for EDM) $$\frac{d\vec{S}}{dt} = \vec{\omega}_s \times \vec{S}$$
- For most Investigations use the change of the spin direction w.r.t. to the direction of motion

$$\Delta\vec{\omega} = \vec{\omega}_s - \vec{\omega}_p = -\frac{e}{m}\left[G\vec{B}_\perp + G\frac{\vec{B}_\parallel}{\gamma} - \left(G - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\frac{\vec{E}}{c} + \frac{\eta}{2}\left(\frac{\vec{E}_\perp}{c} + \frac{1}{\gamma}\frac{\vec{E}_\parallel}{c} + \vec{\beta}\times\vec{B}\right)\right]$$

  - With G = (g-2)/2 describing the magnetic moment (for Protons G = 1.728…)
  - η describes possible EDM (for the sensitivity $d_s = 10^{-29}$ e cm quoted in many publications $\eta_s = 1.9\ 10^{-15}$)
  - Relativistic factors β and γ

- Frozen spin condition for protons in fully electro-static machine
  - Magnetic field B vanishes and η set to zero
  - $\Delta\vec{\omega} = 0$ leads to condition $G - \frac{1}{\gamma_m^2 - 1} = G - \frac{1}{\gamma_m^2\beta_m^2} = 0$
  - Magic momentum $p_m = \frac{\beta_m\gamma_m E_p}{c} = \frac{E_p}{c\sqrt{G}} = 700.74$ MeV/c
  - Strictly fulfilled only for proton executing no betatron or synchrotron oscillations in perfect machine
- (does not work for Deuterons having G < 0)

First order effects

Just a one page recap of earlier discussions, studies, ideas around in the community

- Static radial magnetic field
  - Average field of 9.3 aT mimics the smallest EDM to be detected (10^{-29} e.cm) for a 500 m ring
  - Proposal to measure orbit separation between CW and CCW beam
    - With $Q_V = 0.1$ (realistic?, plus tune modulations in some proposals),
    - Relies on observing orbit separations of 5 pm or less
    - Number and positioning of pick-ups, betatron function variations limit to sensitivities well below aim

  => Likely the dominant contribution to systematic error (quantify?!?)
- Magnet fields from cavity ($h = 100$ and $V_{RF} = 6$ kV)
  - Vertical offset of 0.62 nm generates same vertical spin than smallest EDM to be detected
  - Cancellation of contributions from CW and CCW beam to final EDM value for perfect polarimeter
  - More realistic positioning tolerance of 0.1 mm gives 1.6 10^8 times vertical spin than smallest EDM
    - Impact on final result with limited knowledge of polarimeters (efficiencies)?
    - Feedback to counteract (position of cavity or rather beam in cavity)?
- Gravity
  - About 40 times the spin rotation from smallest EDM to be detected
  - Probably no issue as contributions from CW and CCW on final EDM result cancel

Rotation of a horizontal spin component into vertical plane due to slope of vertical orbit

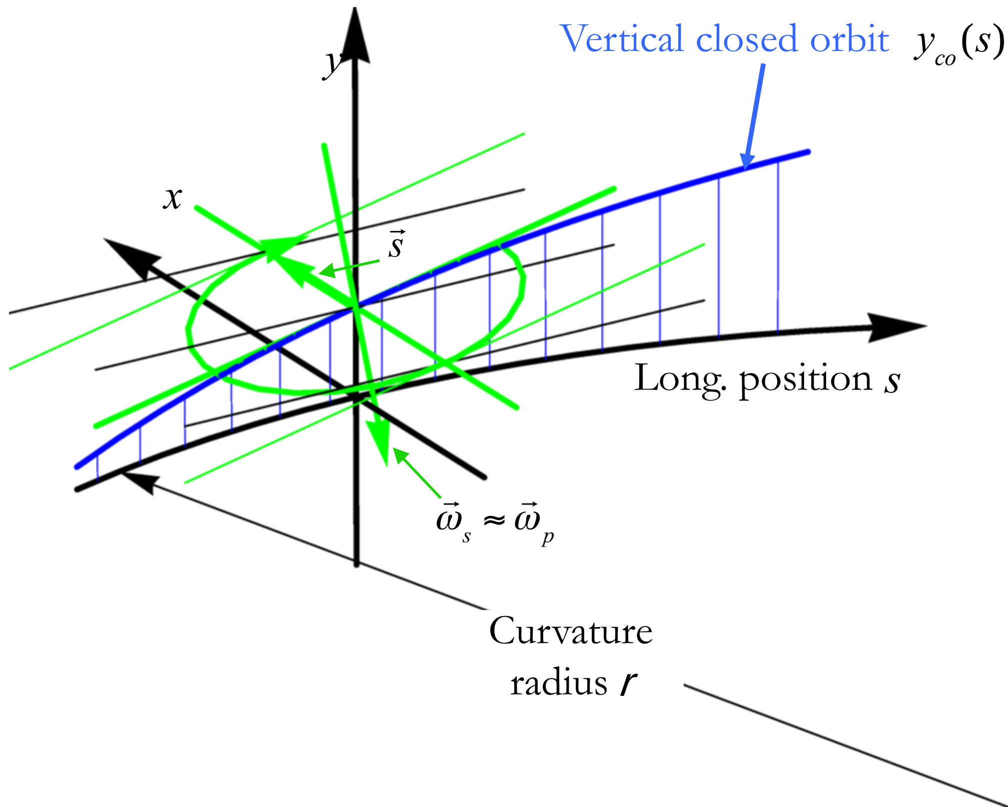
- Close to magic energy $\vec{\omega}_s \approx \vec{\omega}_p$
  - Rotation of spin and of direction of movement described by (almost) the same angular frequency
- Both w_s and w_p with a finite longitudinal component
  - Finite longitudinal component of

    $$w_{s,s} = \frac{bc}{r} y_{co}' \quad \text{with} \quad y_{co}' = \frac{d\, y_{co}}{ds}$$

  - Rotates horizontal ("radial") spin component s_x into vertical

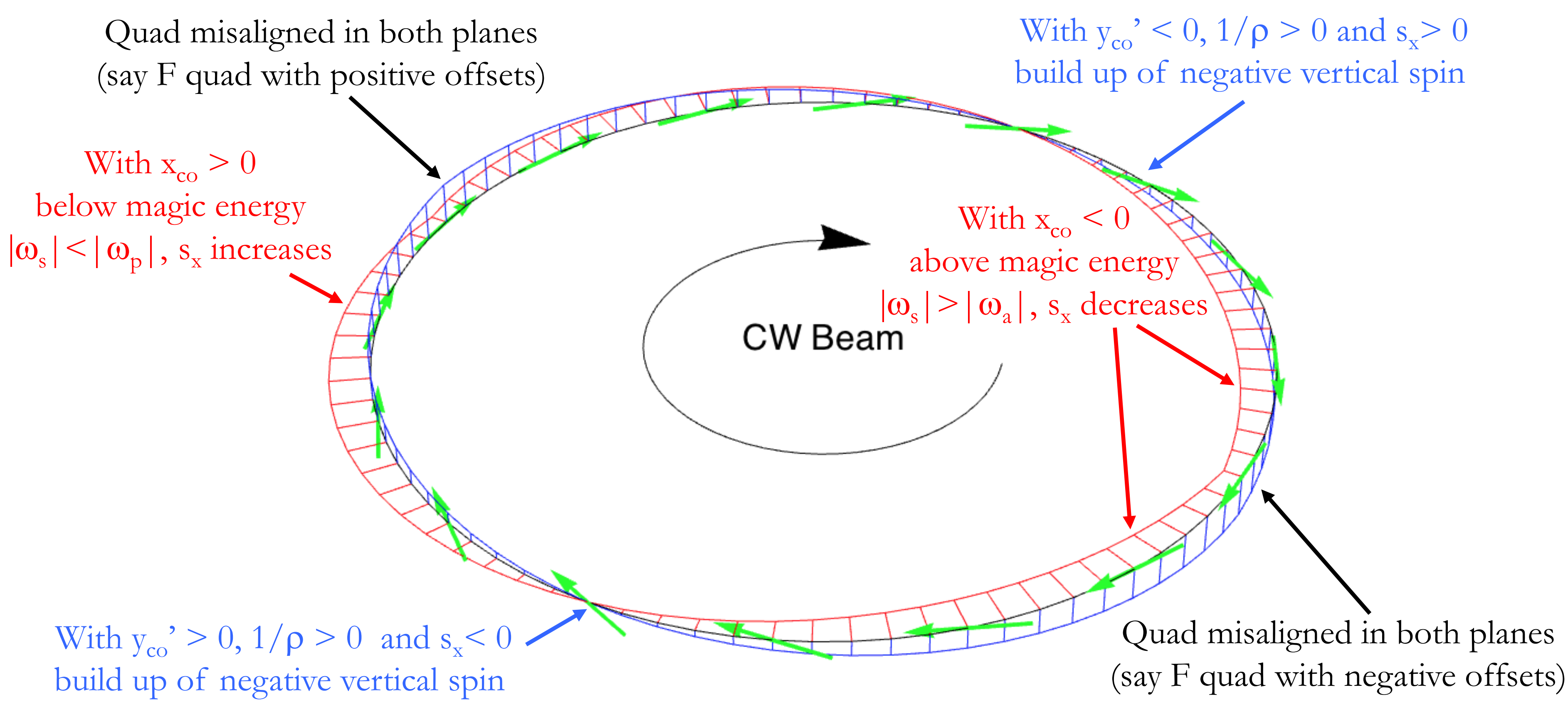
    $$\frac{d\, s_y}{dt} = w_{s,s}\, s_x = \frac{bc\, y_{co}'}{r} s_x$$

    $$s_y' = \frac{d\, s_y}{ds} = \frac{d\, t}{ds} w_{s,s}\, s_x = \frac{y_{co}'}{r} s_x$$



Effect mentioned and formula for vertical spin build-up given by S.Haciomeroglu in a presentation at the EDM meeting on 8th and 9th March 2018 in Jülich (link to workshop program and talks: https://indico.cern.ch/event/712735/)

Vertical spin build up due to orbit distortions in planes – quad misalignments

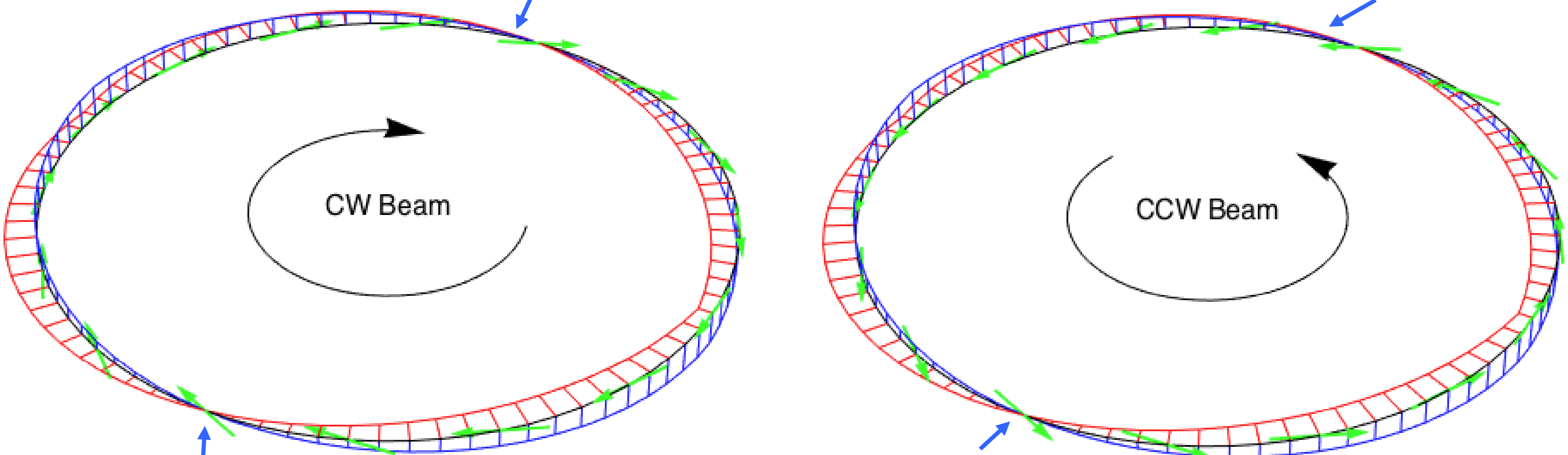


- Distortion of closed orbit by kicks at opposite locations
  - Horizontal orbit distortion generates spin oscillations in horizontal plane (longitudinal spin rotates into horizontal) … strictly speaking with respect to distorted x_{co} and not with respect to reference orbit
  - Longitudinal component of ω_s rotates spin into vertical plane (negative s_y' in example given) … strictly speaking with respect to distorted y_{co} and not with respect to reference orbit

Vertical spin build up due to orbit distortions in both planes – quad misalignments

$y_{co}' < 0$, $1/\rho > 0$ and $s_x > 0$
=> build up of
negative vertical spin

$y_{co}' > 0$, $1/\rho < 0$ and $s_x > 0$
=> build up of
negative vertical spin



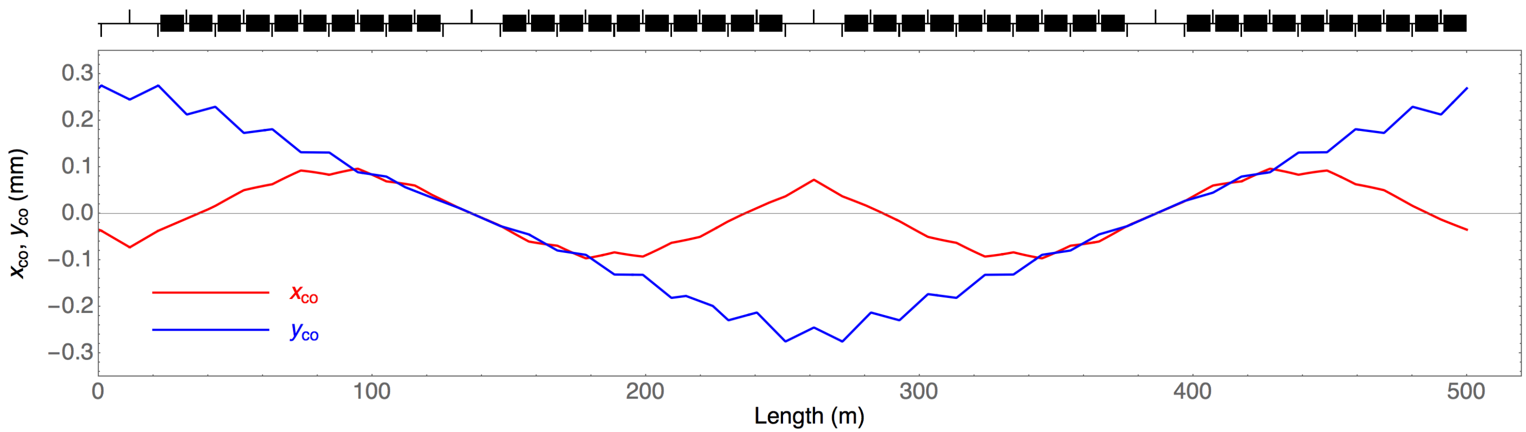
$y_{co}' > 0$, $1/\rho > 0$ and $s_x < 0$
=> build up of
negative vertical spin

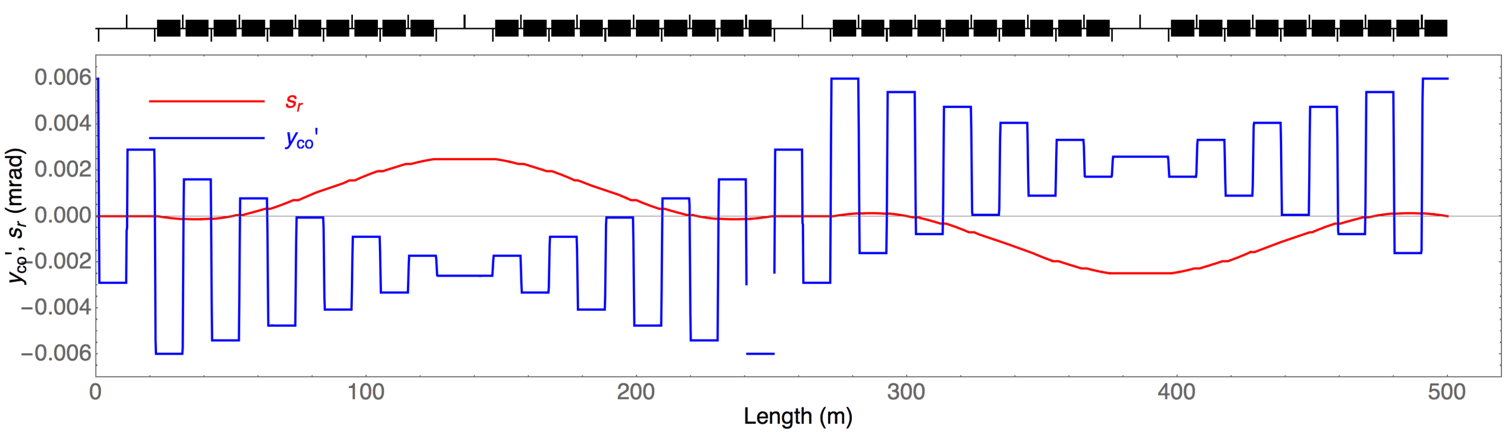
$y_{co}' < 0$, $1/\rho < 0$ and $s_x < 0$
=> build up of
negative vertical spin

- Horizontal and vertical misalignments of electric quads generating horizontal and vertical orbit distortions (details see slide before)
  - Same sign of resulting vertical spin for both the CW and CCW beams (initial polarization parallel to direction of movement)
  - Cancellation of the contributions for final result provided polarimeter efficiencies are precisely known

Vertical spin build up due to orbit distortions in both planes - quad misalignments

- Numerical evaluations using the strong focusing lattice proposal by Lebedev with F quadrupole in first (3rd) straight section offset by +0.1 mm (-0.1 mm) in horizontal and vertical (deflections of ±0.0040 mrad)
  - Horizontal spin $s_x = \int_{s_0}^{s} ds \, \frac{2}{\gamma_m} \frac{x_{co}(s)}{\rho^2(s)}$ and vertical spin build up over one turn $\Delta s_y = \int_0^C ds \, \frac{y_{co}'(s)}{\rho(s)} s_x(s)$





  - Evaluation of integral gives -1.26 10^{-11} rad/turn = -4.5 μrad/s

Vertical spin build up due to orbit distortions in both planes – misalignment of bends

- Simultaneous horizontal offset and tilt of bend generates vertical spin due to
  - Spin rotations in horizontal plane and the vertical orbit distortions – similar, but slightly more complicated than case just considered above) and
  - Direct rotation of longitudinal spin to vertical spin due to vertical electric field and energy offset
- In detail
  - Offset Δx of bend and resulting closed orbit x_{co} generate

$$\Delta\gamma_p = \frac{\Delta E_{kin}}{E_r} = \frac{eE_b\left(\Delta x - x_{co}\right)}{E_r} = \frac{\beta^2\gamma\left(\Delta x - x_{co}\right)}{\rho} \qquad \Delta\left(\beta\gamma\right)_p = \frac{1}{\beta}\Delta\gamma_p$$

$$\left.\frac{\Delta p}{p}\right|_p = \frac{\Delta(\beta\gamma)_p}{\beta\gamma} = \frac{\Delta x - x_{co}}{\rho}$$

  - Using general equation for spin rotation (neglecting EDM and magnetic fields)

$$\Delta\vec{\omega} = \vec{\omega}_s - \vec{\omega}_p = -\frac{e}{m}\left[G\vec{B}_\perp + G\frac{\vec{B}_\parallel}{\gamma} - \left(G - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\frac{\vec{E}}{c}\right]$$

  - and $E_y = -E_b\alpha$ with α the bend tilt angle and

$$\left(G - \frac{1}{(\beta\gamma)^2}\right) \approx \frac{2}{(\beta\gamma)^3}\Delta(\beta\gamma) = \frac{2}{(\beta\gamma)^2}\frac{\Delta p}{p} = 2G\frac{\Delta x - x_{co}}{\rho}$$

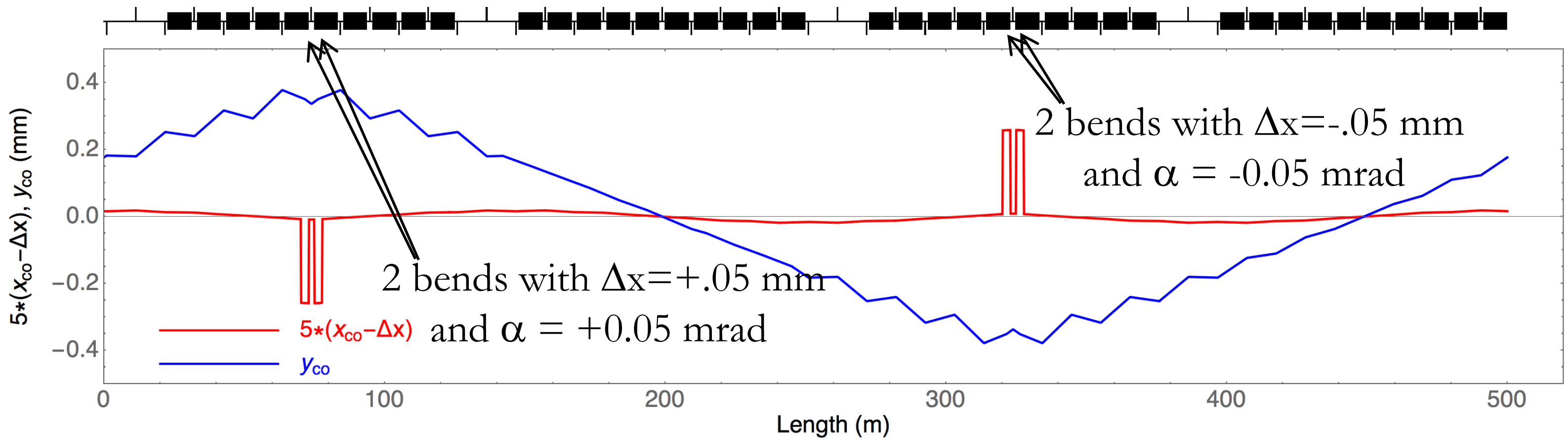
  - obtain spin precession rates (w.r.t. to direction of movement)

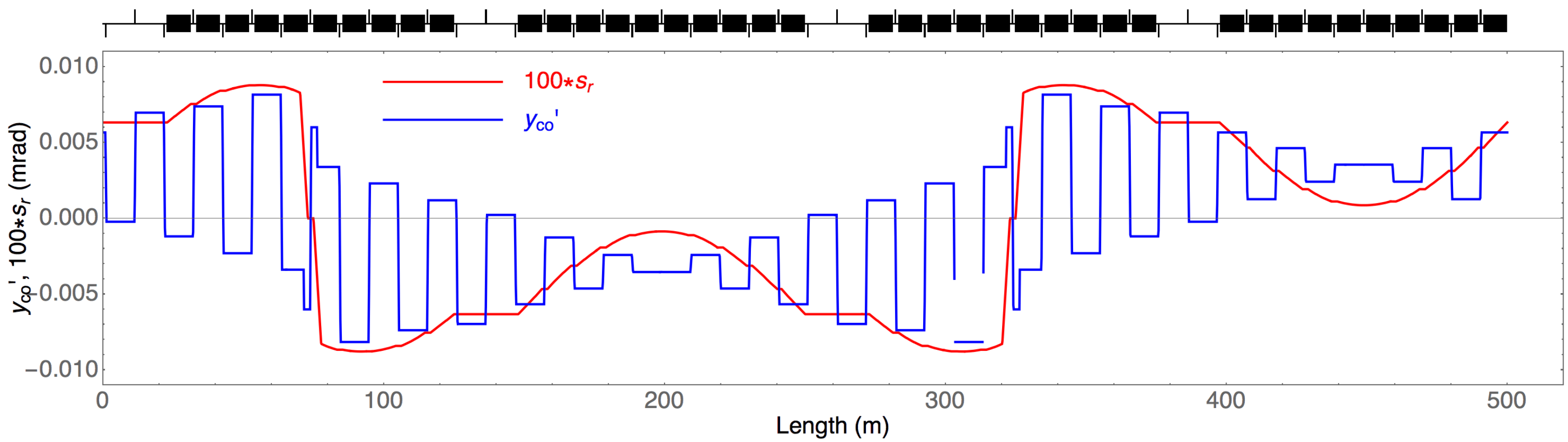
$$\frac{ds_y}{dt} = -\Delta\omega_x = -\frac{2eG}{m}\frac{\Delta x - x_{co}}{\rho c}\left(\vec{\beta}\times\vec{E}\right)_x = -\frac{2eG}{m}\frac{\Delta x - x_{co}}{\rho c}\beta E_b\alpha = \frac{2eG}{m}\frac{x_{co} - \Delta x}{\rho c}\frac{\beta^3\gamma E_r}{e\rho}\alpha = \frac{2\beta c}{\gamma}\frac{x_{co} - \Delta x}{\rho^2}\alpha$$

$$\frac{ds_x}{dt} = \Delta\omega_s = \frac{2eG}{m}\frac{\Delta x - x_{co}}{\rho c}\left(\vec{\beta}\times\vec{E}\right)_s = -\frac{2eG}{m}\frac{\Delta x - x_{co}}{\rho c}\beta E_b = \frac{2eG}{m}\frac{\left(x_{co} - \Delta x\right)}{\rho c}\frac{\beta^3\gamma E_r}{e\rho}\alpha = \frac{2\beta c}{\gamma}\frac{x_{co} - \Delta x}{\rho^2}\alpha$$

Vertical spin build up due to orbit distortions in both planes – misalignment of bends

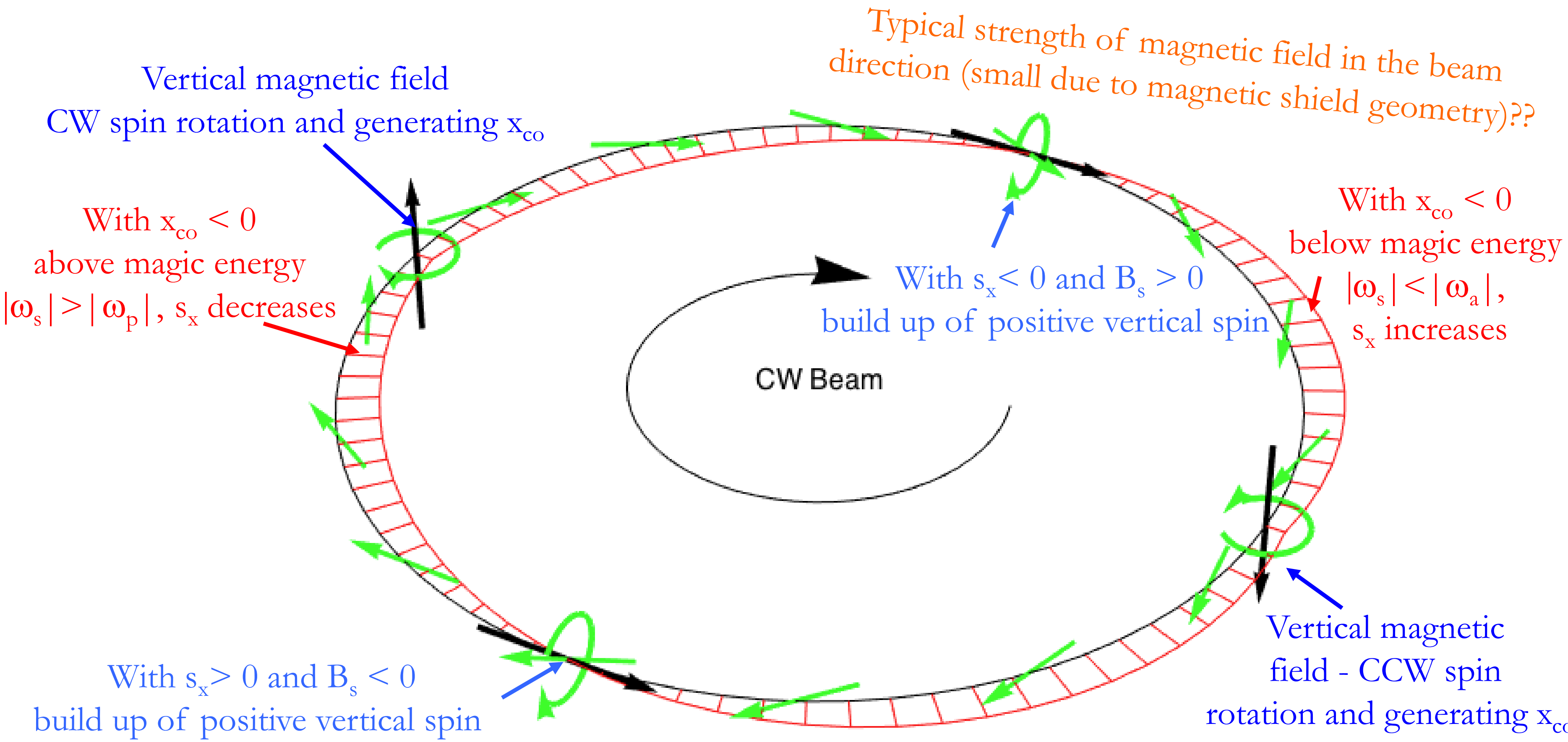
- Finally obtain (with Δs_y vertical spin per turn) $$s_x = \int_{s_0}^{s} d\hat{s}\, \frac{2}{g_m} \frac{x_{co}(\hat{s}) - \Delta x(\hat{s})}{r^2(\hat{s})} \qquad \Delta s_y = \int_0^C ds\, \frac{2}{g_m} \frac{x_{co}(s) - \Delta x}{r^2(s)} \alpha(s) + \int_0^C ds\, \frac{y_{co}'(s)}{r(s)} s_x(s)$$
- Simultaneous horizontal offset by Δx=0.05 mm and tilt α = 0.05 mrad of two pairs of bends





- Evaluation of integral gives (0.8 - 586) 10^{-12} rad/turn = (0.30 - 210) μrad/s

Vertical spin from static magnetic fields
- vertical and longitudinal field 90° out of phase



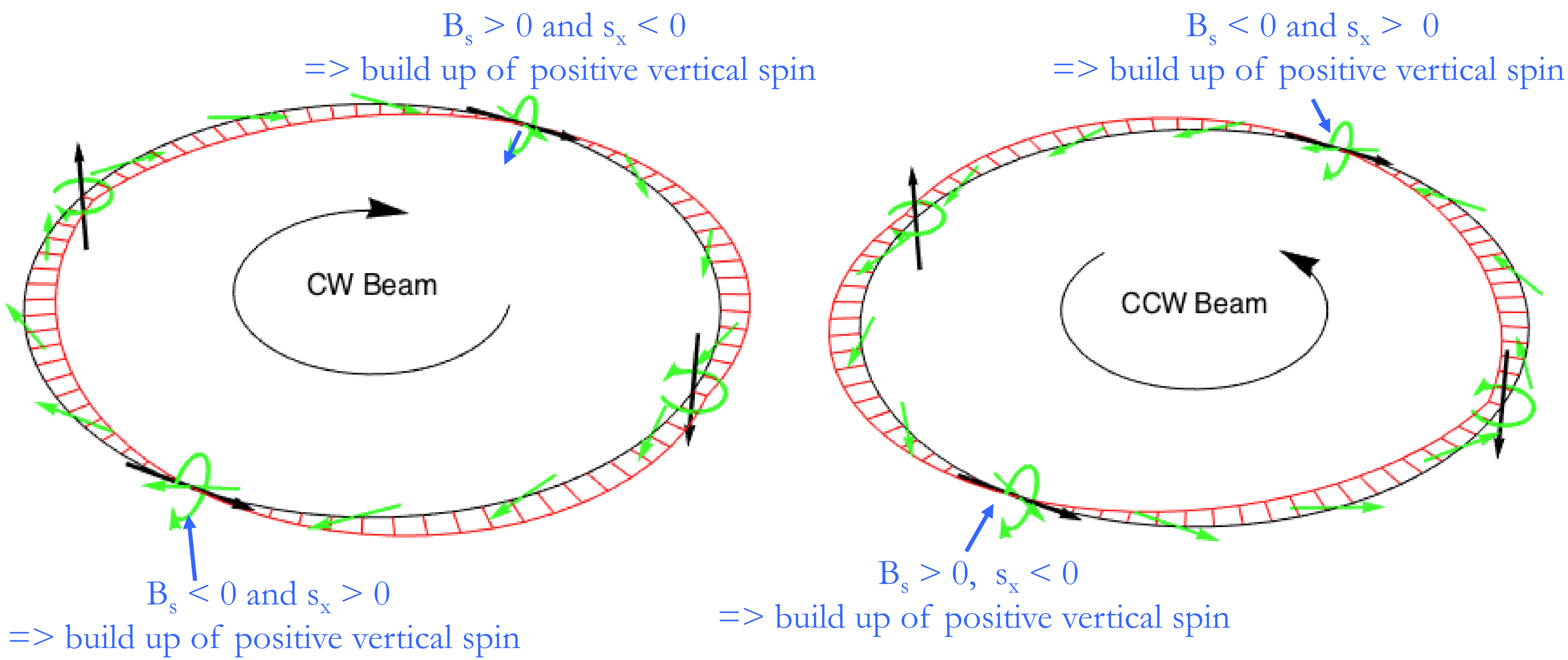
- Vertical magnetic field B_y generates spin rotation in horizontal plane directly and via horizontal closed orbit (two locations with opposite polarity of B_y such that spin rotation over turn cancels in first order)

$$s_x(s) = \int_{s_0}^{s} d\hat{s}\left[\frac{2}{g_m}\frac{x_{co}(\hat{s})}{r^2(\hat{s})} - \frac{e}{m}G B_y(\hat{s})\right]$$

- Longitudinal magnet rotates spin into vertical plane - vertical spin over one turn given by

$$Ds_y = -\int_0^C ds\,\frac{e}{m g_m}\,G B_s(s)\, s_x(s)$$

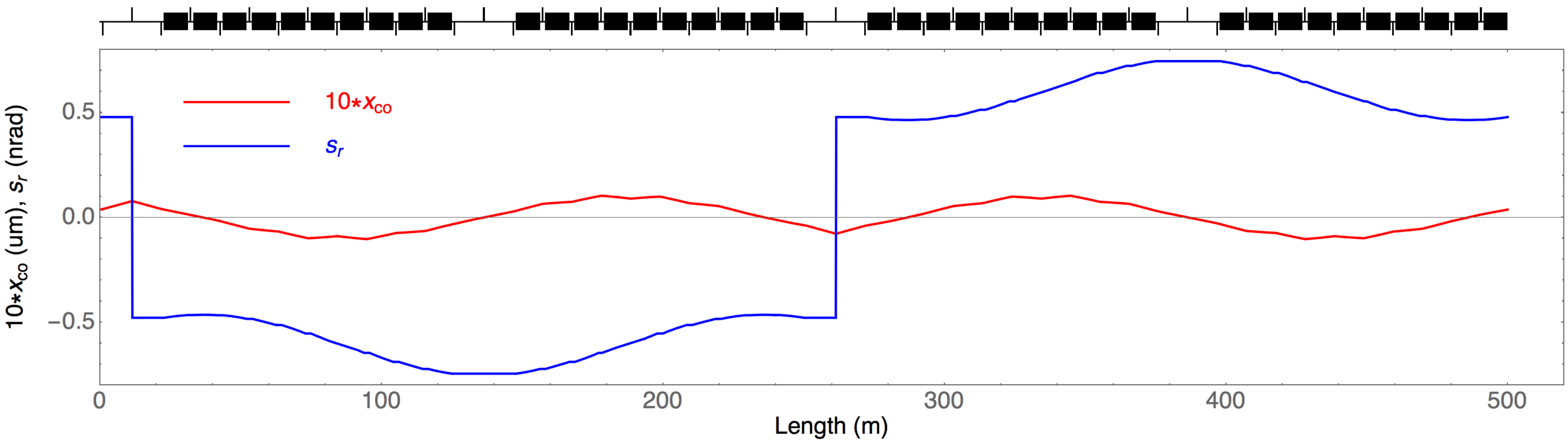
Vertical spin from static magnetic fields
- vertical and longitudinal field 90° out of phase



- Vertical B_y and longitudinal B_s magnetic fields:
  - Horizontal spin due to B_y (direct B_y and via horizontal orbit) rotated into vertical direction by B_s
  - Same sign of resulting vertical spin for both the CW and CCW beams (initial polarization parallel to direction of movement)
  - Cancellation of the contributions for final result provided polarimeter efficiencies are precisely known

Vertical spin from static magnetic fields - vertical and longitudinal field 90° out of phase

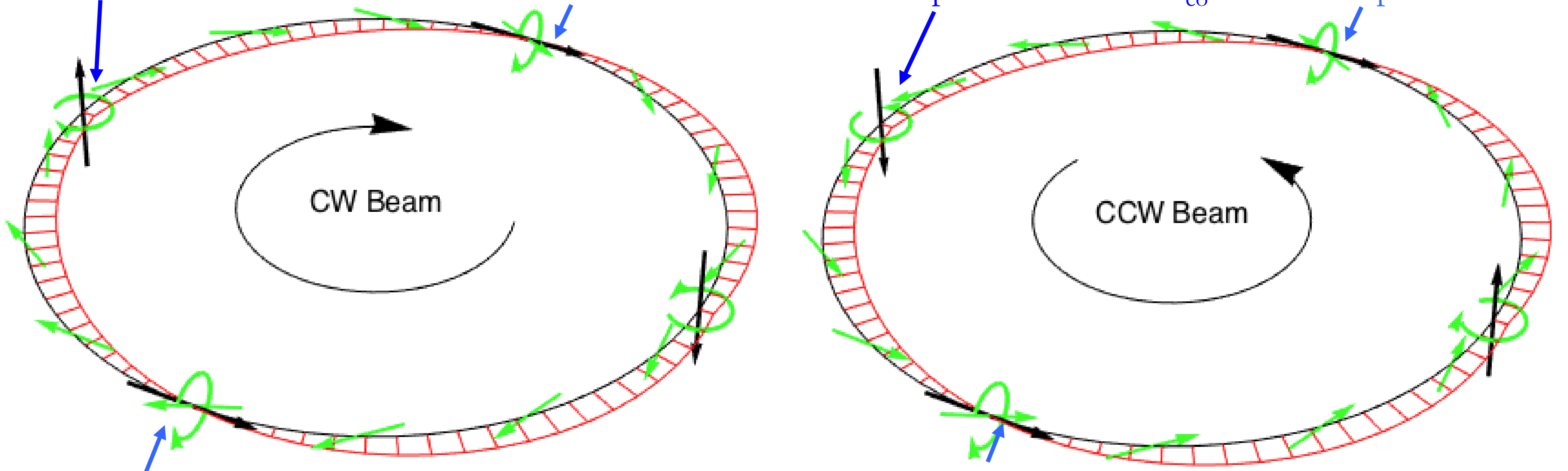
- Integrated vertical magnetic field of $B_y dl$ = 1 nTm and $B_y dl$ = -1 nTm at center of first straight section and opposite gives deflection and $\Delta x' = -0.43 \cdot 10^{-9}$ spin rotation $\Delta \sigma_r = -0.96$ nrad (and additional spin rotation due to closed orbit)



- Integrated longitudinal magnetic field of $(B_s dl)_1$ = 1 nTm and $(B_s dl)_2$ = -1 nTm at center of 2nd and 4th straight section with $\sigma_{r1} = -0.75 \times 10^{-10}$ and $\sigma_{r2} = 0.75 \times 10^{-10}$ gives

  for one turn $\Delta \sigma_y = -\frac{eG}{m \gamma_m \beta_m c}\left(\sigma_{r1}(Bdl)_1 + \sigma_{r2}(Bdl)\right) = 1.14 \times 10^{-18}$ turn^{-1} or 4.1×10^{-13} s^{-1}
  - Question on formula used based on $\Delta \omega$ and whether this quantity is appropriate in all cases
  - Orders of magnitude less than the smallest EDM to be detected (somewhat optimistic assumptions) … cancellation between CW and CCW beam for final result, effect not an issue
- Similarly horizontal and longitudinal magnetic fields generate spin rotation in horizontal plane
- No second order effects with horizontal and vertical magnetic field components?

Vertical spin from vertical magnetic field from cavity and longitudinal magnetic field



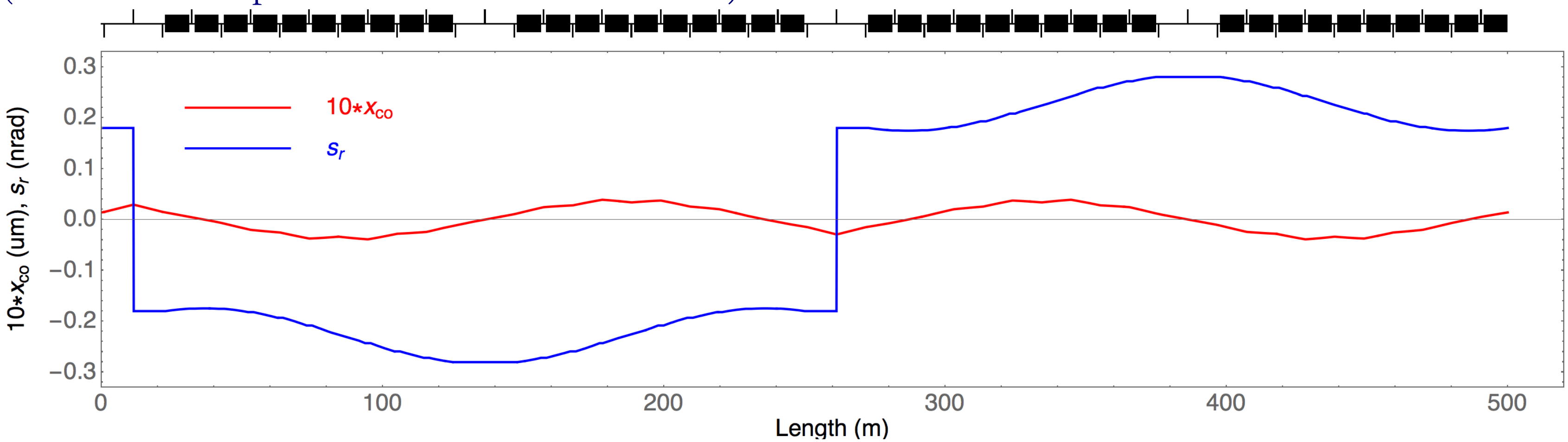
- Case similar to static vertical and longitudinal B – but polarity of vertical field B_y opposite for CW and CCW
- Integrated vertical field with V_{RF}=6 kV, h=100 and horizontal offset r = -0.1 mm

$$B_y dl = -\frac{V_{RF} h p}{c^2 T_{rev}} r = 0.75\ \text{nT}$$

  - Assume two cavities (sharing the vert. magnet field) for treatment consistent with "geometric phase"
  - Sign of B_y different for CW and CCW beam (phase stability!)
  - Effect mimics EDM – no cancellation between CW and CCW beam!

Vertical spin from vertical magnetic field from cavity and longitudinal magnetic field

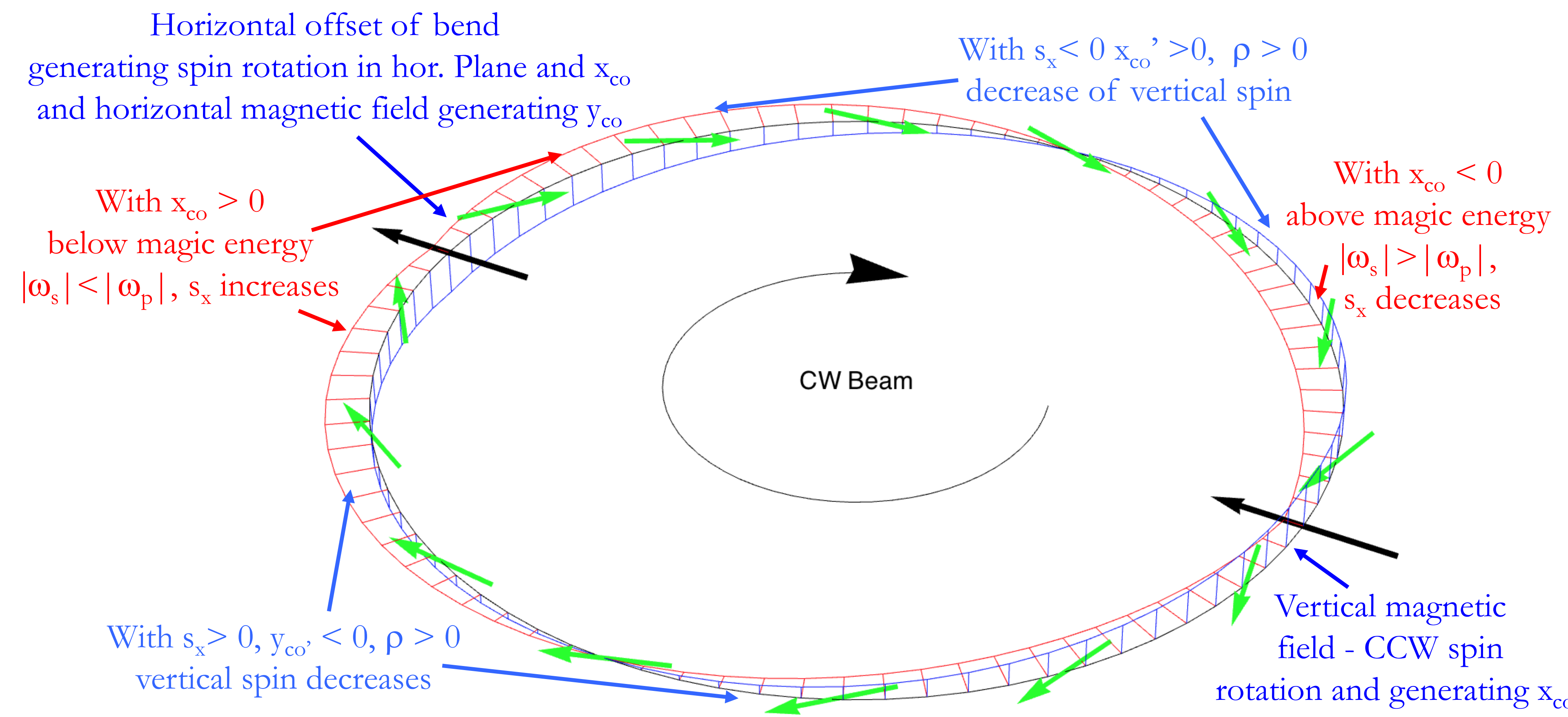
- Integrated vertical magnetic field of $B_y$dl = 1 nTm and $B_y$dl = -1 nTm at center of first straight section and opposite gives deflection and $\Delta x' = -0.16$ nrad spin rotation $\Delta s_r = -0.36$ nrad (and additional spin rotation due to closed orbit)



- Integrated longitudinal magnetic field of $B_s$dl = 1 nTm and $B_s$dl = -1 nTm at center of 2nd and 4th straight section with $\Delta s_r = -0.75 \cdot 10^{-10}$ gives $\Delta s_y = 0.46 \times 10^{-18}\ \text{turn}^{-1} = 1.5 \times 10^{-13}\ \text{s}^{-1}$
- Orders of magnitude less than smallest EDM to be detected .. no issue even without cancellation of contributions from CW and CCW beam on final result

- Similarly horizontal and longitudinal magnetic fields generate spin rotation in horizontal plane looking like operation slightly different from magic energy
- No second order effects with horizontal and vertical magnetic field components?

Vertical spin from horizontal offset of bend and vertical orbit from horizontal magnetic field



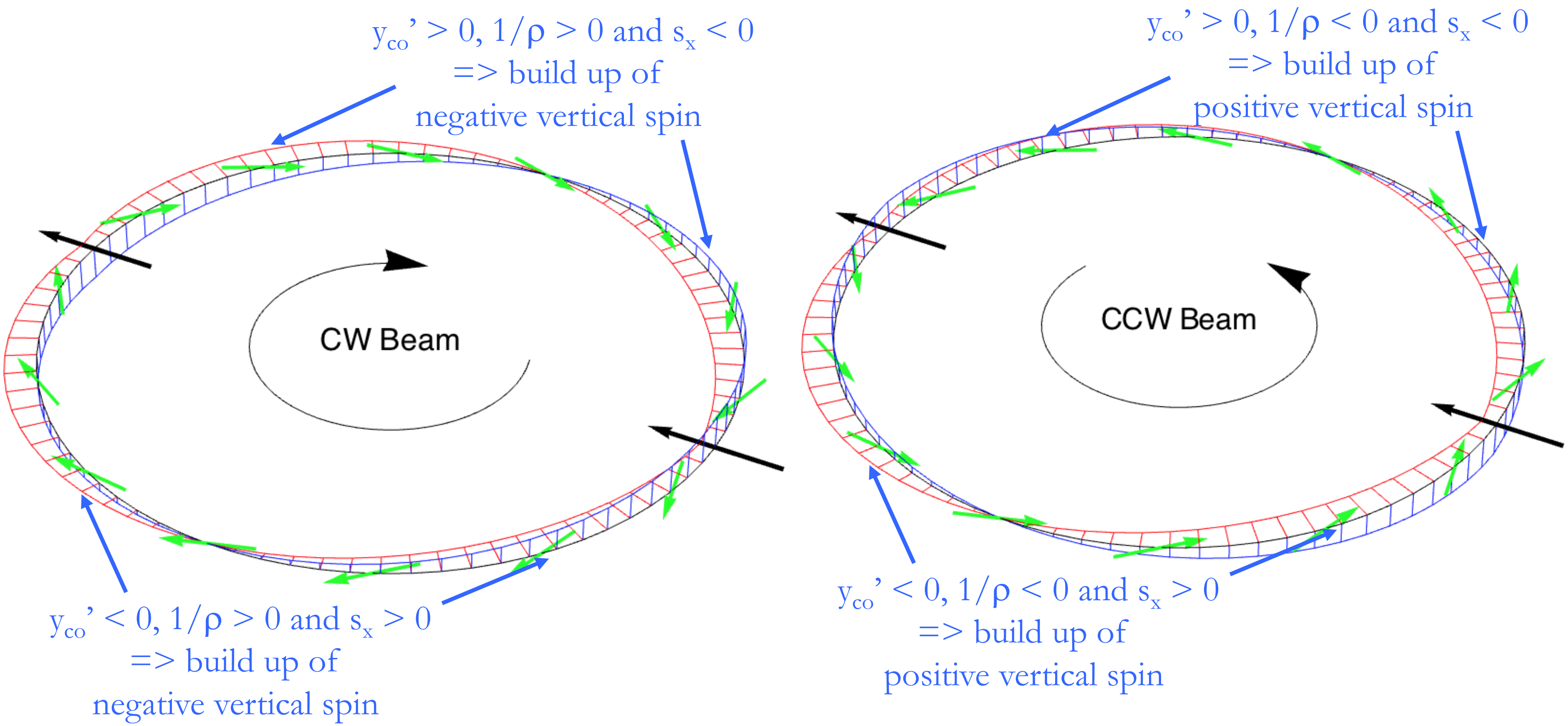
- Rotations of the spin in the horizontal plane due horizontal offset of bend
- Slope of vertical orbit (excited by horizontal magnetic field) generates vertical spin over one turn

$$s_x = \int_{s_0}^{s} d\hat{s}\, \frac{2}{g_m} \frac{x_{co}(\hat{s}) - \Delta x(\hat{s})}{r^2(\hat{s})}$$

$$\Delta s_y = \int_0^C ds\, \frac{y_{co}'(s)}{r(s)}\, s_x(s)$$

Probably (?) most pessimistic case of vertical spin due to orbit distortions by electric fields in one plane and magnetic field in the other

Vertical spin from horizontal offset of bend and vertical orbit from horizontal magnetic field

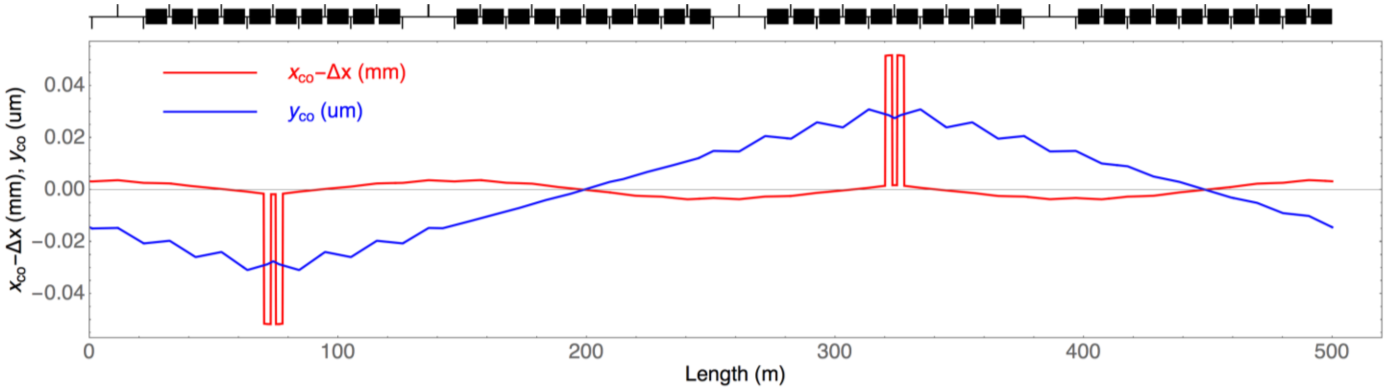


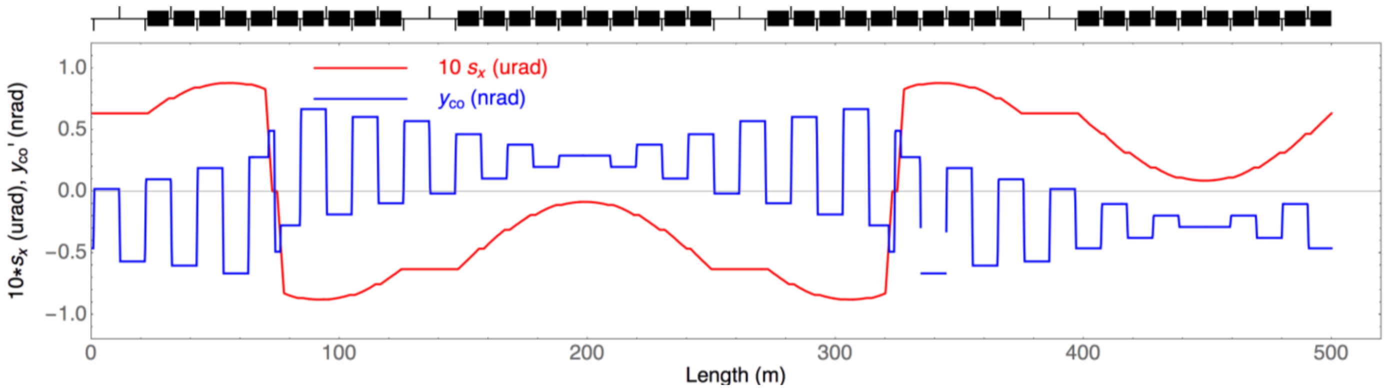
- Same horizontal orbit for both CW and CCW beam and thus opposite orientation of horizontal spin
- Vertical orbit opposite for CW and CCW beam (magnetic field generates orbit separation)
- In this example: vertical spin decreases for CW beam and increases for CCW beam

=> Effects mimics EDM (no cancellation of contributions from CW and CCW beam in final EDM value!!)

Vertical spin from horizontal offset of bend and vertical orbit from horizontal magnetic field

- For two pairs of bends with a horizontal offset of $\Delta x = 0.05$ mm (0.16 urad deflection) and a vertical integrated field of 1 nTm (deflection 0.43 nrad)



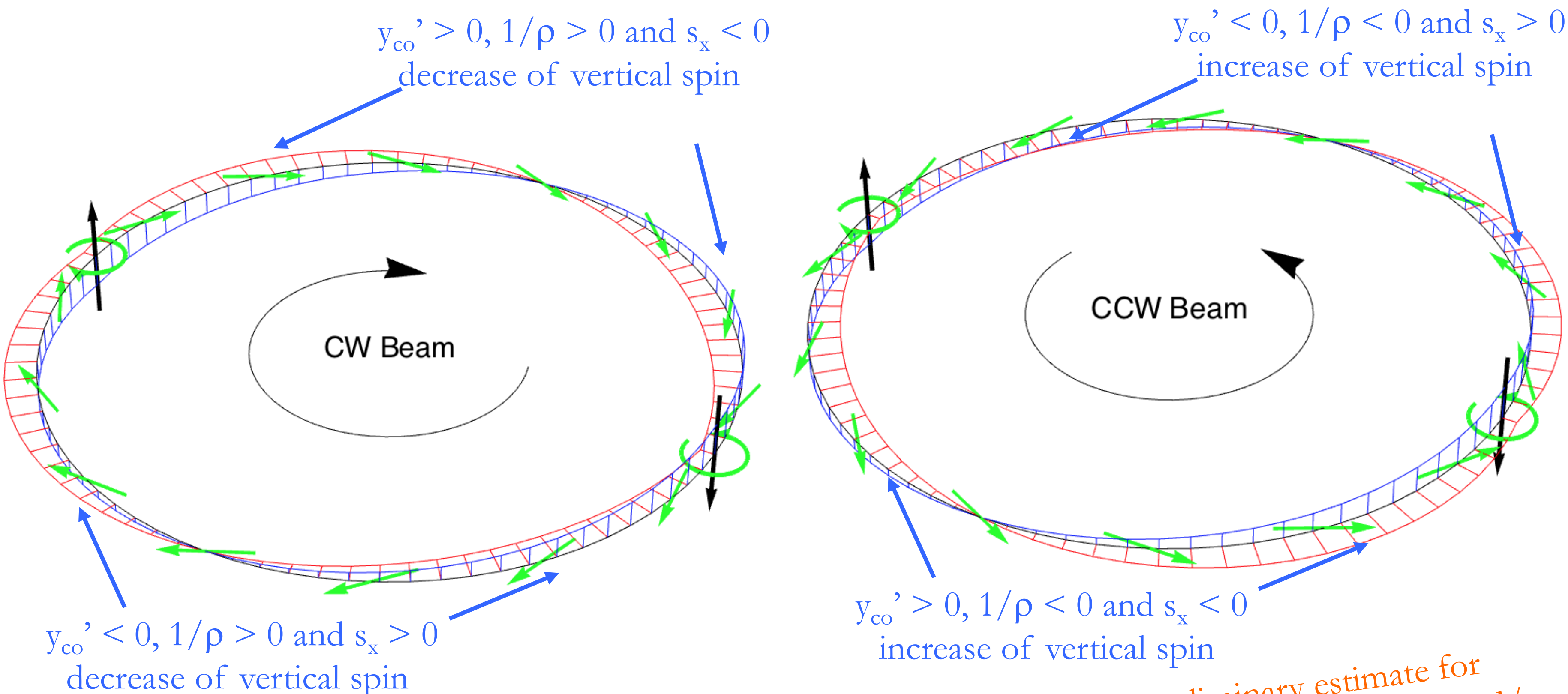


- Result $\Delta s_y = -6.8 \cdot 10^{-17}\ \mathrm{turn}^{-1} = -0.024$ nrad/s
- Almost two orders of magnitude smaller then EDM signal for 1E-29 e cm, but with somewhat optimistic assumptions
- Significantly higher magnetic fields for “hybrid ring”?

Not yet considered 2^{nd} order effects

- Vertical magnetic field generating spin rotations in horizontal plane and vertical orbit due to electric fields (no cancellation of contributions from CW and CCW beam)
- Average horizontal spin
  - Imperfections of polarimeter (argument for bunches polarized parallel to direction of movement and opposite?)
  - Position of polarimeter (can be at extremum of spin oscillations around ring put to zero by feedback)
  - In combination with other imperfection
- Betatron oscillations (together with other effects)

Vertical magnetic field and vertical orbit distortions from quad misalignment



Preliminary estimate for 1 nTm and 0.1 mm: 3.1 nrad/s

- Vertical magnetic field (and resulting horizontal orbit) generate spin rotations in horizontal plane
- Slope of vertical orbit in bends rotates horizontal spin into vertical
- In this example: vertical spin decreases for CW beam and increases for CCW beam

=> Effects mimics EDM (no cancellation of contributions from CW and CCW beam in final EDM value!!)