Rapidity distributions of $\pi$ mesons produced in $p+p$ and Pb+Pb collisions at CERN SPS energies

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Work done together with
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A. Marcinek, V. Ozvenchuk, N. Davis

1. Introduction;
2. The model;
3. The pion rapidity distribution in $p+p$ collisions;
4. The energy balance in $p+p$ and Pb+Pb collisions;
5. Proton-nucleus reactions;
1) **Introduction**
• This analysis was inspired by our studies of **electromagnetic effects** in heavy ion collisions.

• Charged spectators generate **electromagnetic fields**.

• These modify the trajectories of charged π mesons (π⁺, π⁻).

• We use this effect as a new source of information on the space-time evolution of the system.

→ talk by A. Marcinek
Friday, S3 ;

see also: PRC 87 (2013) 054909,
APB 49 (2018) 711
Faster pions are produced closer to the spectator system (!)
Andrzej Rybicki, 45 Zjazd Fizyków Polskich, Kraków, 13-18 września 2019

\begin{align*}
\sqrt{s_{NN}} &= 17.3 \text{ GeV} \\
Pb+Pb &\text{ peripheral} \\
\text{NA49 preliminary}
\end{align*}

Faster pions are produced closer to the spectator system (!)

\[ x_F = \frac{p_L}{p_L^{\text{beam}}} \text{ (c.m.s.)} \]

\begin{align*}
\text{Note:} \\
1 \text{ fm} &= 10^{-15} \text{ m} \quad :-)
\end{align*}
Faster pions are produced closer to the spectator system (!)

Note: 1 fm = $10^{-15}$ m :-)

$\pi^+ / \pi^-$ vs. $p_T$ for Pb+Pb peripheral collisions at $\sqrt{s_{NN}} = 17.3$ GeV.

$\sqrt{s_{NN}} = 17.3$ GeV Pb+Pb peripheral

$x_F = \frac{p_L}{p_L^{beam}}$ (c.m.s.)

$d_E$ vs. $y/y_{beam}$ for Au+Au, Pb+Pb, Ar+Sc collisions at 7.7-17.3 GeV.

$y = \frac{1}{2} \ln \left( \frac{E+p_L}{E-p_L} \right)$

$1$ fm = $10^{-15}$ m

$1$ fm = $10^{-15}$ m :-)

Note: $1$ fm = $10^{-15}$ m :-)

$v_z = 0$

spectator velocity

Monte Carlo

See also: A. R., A. Szczurek et al., AIP Suppl. 9 (2016) 303
2) The model
Bricks collide 

$1 \times 1 \text{ fm}^2$

... and form fire streaks

with rapidity from $E-p$ conservation

Each fire streak fragments independently into pions

$$\frac{dn}{dy} = A \cdot (E_s^* - m_s) \cdot \exp \left( - \frac{\left[ (y - y_s)^2 + \epsilon^2 \right]^{\frac{n}{2}}}{n \sigma_y^n} \right)$$

available energy  

fire streak rapidity

- Idea by A. Szczurek; see also: R. Hagedorn, CERN-71-12 (1971) W.D. Myers, NPA 296 (1978) 177
Bricks collide...  

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Available energy

Fire streak rapidity

Total fire streak energy

Sum of brick masses

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Each fire streak fragments independently into pions

$$\frac{dn}{dy} = A \cdot (E_s^* - m_s) \cdot \exp\left(-\frac{\left[(y - y_s)^2 + \epsilon^2\right]^{1/2}}{n \sigma_y}\right)$$

available energy

fire streak rapidity

total fire streak energy

sum of brick masses

data points from: NA49, PRC 86 (2012) 054903
3) The pion rapidity distribution in p+p collisions

- From now on: $\sqrt{s_{NN}} = 17.3$ GeV.

Each fire streak fragments independently

Available energy

Fire streak rapidity

Total fire streak energy

Sum of brick masses

$$f(y) = A \cdot (E_s^* - m_s) \cdot \exp\left( -\frac{\left[ (y - y_s)^2 + \epsilon^2 \right]^{\frac{n}{2}}}{n \sigma_y^n} \right)$$

Data points from: NA49, PRC 86 (2012) 054903
Each fire streak fragments independently into pions. For 

\[ \pi^-, \text{ Pb+Pb, } \sqrt{s_{NN}} = 17.3 \text{ GeV} \]

3) The pion rapidity distribution in p+p collisions

- From now on: $\sqrt{s_{NN}} = 17.3$ GeV.

Each fire streak fragments independently

$$f(y) = A \cdot (E_s^* - m_s) \cdot \exp \left( - \frac{[(y - y_s)^2 + \epsilon^2]^{n/2}}{n \sigma_y^n} \right)$$

- Available energy
- Fire streak rapidity
- Total fire streak energy
- Sum of brick masses

Summary (1):

- The pion rapidity distribution from one fire streak in Pb+Pb collisions resembles the pion rapidity distribution in p+p reactions;
- There is a difference in absolute normalization: 0.748;
- This difference can be understood by a different energy repartition in p+p and Pb+Pb reactions.

\[
 f(y) = A \cdot \left( E_s^* - m_s \right) \cdot \exp \left( - \frac{\left[ (y - y_s)^2 + \epsilon^2 \right]^{\frac{n}{2}}}{n \sigma_y^n} \right)
\]

Available energy

Fire streak rapidity

Total fire streak energy

Sum of brick masses

\[ E_s^* = \sqrt{s_{NN}} \]
\[ m_s = 2m_N \]
\[ y_s = 0 \]

Data points from: NA49, EPJC 45 (2006) 343
Digression:

isospin
• Pb+Pb collision: 40% protons, 60% neutrons;

• p+p → π−X is not directly comparable to Pb+Pb → π−X!

• isospin symmetry: \( \frac{dn}{dy}(n \rightarrow \pi^-) = \frac{dn}{dy}(p \rightarrow \pi^+) \)

• isospin-averaged \( \pi^- \) distribution:

\[
\frac{dn}{dy}(N + N \rightarrow \pi^- X) = \left( \frac{Z}{A} \right) \cdot \frac{dn}{dy}(p + p \rightarrow \pi^- X) + \left( 1 - \frac{Z}{A} \right) \cdot \frac{dn}{dy}(p + p \rightarrow \pi^+ X)
\]

Once isospin is taken into account, the difference in absolute scaling between p+p and Pb+Pb collisions changes from 0.748 to 0.812.
4) The energy balance in p+p and Pb+Pb collisions

For a more extended description including formulae, numerical values, tables and plots, please see PRC 99 (2019) 024908
**Pb+Pb:** (Fire streak energy) ≈ (baryon energy) + (pion energy) + (kaon energy)

**p+p:** (Fire streak energy) = \( \sqrt{s} \approx (\text{baryon energy}) + (\text{pion energy}) + (\text{kaon energy}) \)
Pb+Pb: (Fire streak energy) \( \approx \) (baryon energy) + (pion energy) + (kaon energy) >

p+p: (Fire streak energy) = \( \sqrt{s} \approx \) (baryon energy) + (pion energy) + (kaon energy)

strangeness enhancement

K. Grebieszkow, NA61/SHINE, CPOD 2018, and references therein

pion production from one fire streak

\( E^*_x = \sqrt{s_{NN}} \)
\( m_s = 2 m_N \)
\( y_s = 0 \)
Pb+Pb: (Fire streak energy) ≈ (baryon energy) + (pion energy) + (kaon energy)

p+p: (Fire streak energy) = \( \sqrt{s} \) ≈ (baryon energy) + (pion energy) + (kaon energy)

**baryon stopping**
**(inelasticity)**

**strangeness enhancement**

---

Protons (also neutrons and $\bar{p}$)

In $p+p$ collisions, the average energies of pions, kaons, and protons can be computed directly from their spectra:

$$
\langle E_i \rangle = \frac{\int_0^1 \int_0^{p_T(\text{max})} E_i(x_F, p_T) \cdot \left( \frac{d^2\sigma}{dx_F dp_T} \right)_i \, dp_T \, dx_F}{\int_0^1 \int_0^{p_T(\text{max})} \left( \frac{d^2\sigma}{dx_F dp_T} \right)_i \, dp_T \, dx_F}
$$

In the nearly-49 GeV $\sqrt{s}$ NA49 collider, $p_L$ = 0.1 GeV/c, $p_T = 0.2$ GeV/c, leading to $x_F = 0.15$. The $p_L$ beam is centered at $x_F = 0$, $N+N = \pi X$, $m_s = 2 m_N$, $y_s = 0$. 

$E^*_s = \sqrt{S_{NN}} \cdot 0.812 \cdot f(y)$

Calculation of energy balance (simplified):

\[
\langle E_i \rangle = \frac{\int_0^1 \int_0^{p_T^{\text{max}}} E_i(x_F, p_T) \cdot \left(\frac{d^2\sigma}{dx_F dp_T}\right)_i \, dp_T \, dx_F}{\int_0^1 \int_0^{p_T^{\text{max}}} \left(\frac{d^2\sigma}{dx_F dp_T}\right)_i \, dp_T \, dx_F}
\]

\[E_* = \sqrt{s_{NN}}\]

\[m_s = 2m_N\]

\[y_s = 0\]

\[K = \frac{2 \cdot E_{inel}}{\sqrt{s} - 2m_p}\]

\[K = 0.547\]

The relation between (baryon energy), (pion energy) and (kaon energy) in \(\text{Pb+Pb}\) collisions is calculated on the basis of:

→ baryon inelasticity in \(\text{Pb+Pb}\), \(K \approx 0.78\);

→ the change in \(<K>/<\pi>\) ratios between \(\text{p+p}\) and \(\text{Pb+Pb}\) (~2).

In this way we get (per unit of total collision energy):

\[
\frac{\text{Energy spent on pions in } \text{p+p}}{\text{Energy spent on pions in } \text{Pb+Pb}} = 0.781
\]
The pion rapidity distribution from one fire streak in Pb+Pb collisions reproduces the pion rapidity spectrum in p+p collisions …

… with a difference in absolute normalization which comes from the different energy repartition in the two reactions.

Thus, a correspondence exists between $dn/dy$ spectra in p+p and Pb+Pb collisions.

The pion rapidity distribution from one fire streak in Pb+Pb collisions reproduces the pion rapidity spectrum in p+p collisions …

… with a difference in absolute normalization which comes from the different energy repartition in the two reactions.

Thus, a correspondence exists between $dn/dy$ spectra in p+p and Pb+Pb collisions.
**π⁻, Pb+Pb, \(\sqrt{s_{NN}} = 8.8\) GeV**

![Graph for Pb+Pb at 8.8 GeV](image1)

- \(b = 2.47\) fm
- \(b = 4.35\) fm

**π⁻, N+N, \(\sqrt{s_{NN}} = 8.8\) GeV**

![Graph for N+N at 8.8 GeV](image2)

- Pion production from one fire streak
- \(E^* = \sqrt{s_{NN}}\)
- \(m_s = 2m_N\)
- \(y_s = 0\)
- 2-4%

**Valid for**

\[8.8\ \text{GeV} \leq \sqrt{s_{NN}} \leq 17.3\ \text{GeV}\]

(or better)

**π⁻, Pb+Pb, \(\sqrt{s_{NN}} = 17.3\) GeV**

![Graph for Pb+Pb at 17.3 GeV](image3)

- \(b = 8.41\) fm
- \(b = 6.64\) fm

**π⁻, N+N, \(\sqrt{s_{NN}} = 17.3\) GeV**

![Graph for N+N at 17.3 GeV](image4)

- Pion production from one fire streak
- \(E^*_s = \sqrt{s_{NN}}\)
- \(m_s = 2m_N\)
- \(y_s = 0\)
- 4%

**\( \pi^- \), Pb+Pb, \( \sqrt{s_{NN}} = 8.8 \) GeV**

- **B = 2.47 fm**

**\( \pi^- \), N+N, \( \sqrt{s_{NN}} = 8.8 \) GeV**

- pion production from one fire streak
- \( E^* = \sqrt{s_{NN}} \)
- \( m_s = 2m_N \)
- \( y_s = 0 \)

\( C_0 \)

- **\( \pi^- \), Pb+Pb, \( \sqrt{s_{NN}} = 17.3 \) GeV**

- **B = 6.64 fm**

**\( \pi^- \), N+N, \( \sqrt{s_{NN}} = 17.3 \) GeV**

- **Reinvented by A. Szczurek**
- **Local energy-momentum conservation**
- **R. Hagedorn, CERN-71-12 (1971)**
- **W.D. Myers, Nucl. Phys. A 296 (1978) 177**


*Andrzej Rybicki, 45 Zjazd Fizyków Polskich, Kraków, 13-18 września 2019*
$\pi^-$, Pb+Pb, $\sqrt{s_{NN}} = 8.8$ GeV

$\pi^-$, Pb+Pb, $\sqrt{s_{NN}} = 17.3$ GeV

Local energy-momentum conservation

R. Hagedorn, CERN-71-12 (1971)


(Re)invented by A. Szczurek.

Experimental data: NA61/SHINE, EPJC 74 (2014) 2794, EPJC 77 (2017) 671
5) Proton-nucleus reactions
$\pi^+ + \pi^-$, $\sqrt{s_{NN}} = 17.3$ GeV

Energy balance:
change in energy spent on pions by +13% w.r.t. p+p, most probably can be explained by baryon stopping [in progress].


\[ \pi^+ + \pi^-, \sqrt{s_{NN}} = 17.3 \text{ GeV} \]

Energy balance: change in energy spent on pions by **+13 %** w.r.t. p+p, most probably can be explained by baryon stopping [in progress].
6) Summary:

- Our study started from EM effects on final state particles...
- ... and resulted in a model of the longitudinal evolution of the system, which
  - explains the centrality dependence of pion yields and rapidity distributions in Pb+Pb collisions;
  - links p+p and Pb+Pb reactions;
  - is valid in some extended range of collision energy ($\sqrt{s_{NN}} = 9 - 17$ GeV, for now).
Our study started from EM effects on final state particles...

... and resulted in a model of the longitudinal evolution of the system, which

→ explains the centrality dependence of pion yields and rapidity distributions in Pb+Pb collisions;
→ links p+p and Pb+Pb reactions;
→ is valid in some extended range of collision energy ($\sqrt{s_{NN}} = 9 - 17$ GeV, for now).

Note: the experimental data used in this talk come from the NA49 and NA61/SHINE experiments at the CERN SPS, with participation of 2 and 9 Polish groups, respectively.

Thank you.
Acknowledgments.

This work was supported by the National Science Centre, Poland (grant no. 2014/14/E/ST2/00018).
Extra slides
More on
Proton-nucleus collisions

\[ \pi^+ + \pi^-, \sqrt{s}_{NN} = 17.3 \text{ GeV} \]

Energy balance: change in energy spent on pions by +13 % w.r.t. p+p, most probably can be explained by baryon stopping [in progress].
More on energy dependence
Bricks collide . . .

1 x 1 fm\(^2\)

. . . and form fire streaks

with rapidity from \(E-p\) conservation

Each fire streak fragments independently into pions

\[
\frac{dn}{dy} = A \cdot (E_s^* - m_s) \cdot \exp \left( - \frac{\left((y - y_s)^2 + \epsilon^2\right)^{\frac{n}{2}}}{n\sigma_y^n} \right)
\]

available energy

fire streak rapidity

total fire streak energy

sum of brick masses

 Peripheral (\(b=9.72\) fm)

\(\sqrt{s_{NN}} = 17.3\) GeV

Pb+Pb

Peripheral (\(b=9.72\) fm)

Central (\(b=2.55\) fm)

\[
\sqrt{s_{NN}} = 17.3\text{ GeV}
\]

Pb+Pb

\(\pi^-\)

Peripheral

Central

data points from: NA49, PRC 86 (2012) 054903
Bricks collide...

$1 \times 1 \text{ fm}^2$

...and form fire streaks

with rapidity from $E-p$ conservation

Each fire streak fragments independently into pions

$\pi^-$, Pb+Pb, $\sqrt{s_{NN}} = 17.3 \text{ GeV}$

$d n / dy = A \cdot \left( E_s^* - m_s \right) \cdot \exp \left( - \frac{\left[ (y - y_s)^2 + \epsilon^2 \right]^{n/2}}{n \sigma_y^n} \right)$

available energy

fire streak rapidity

total fire streak energy

sum of brick masses

data points from: NA49, PRC 86 (2012) 054903
Bricks collide . . .

$1 \times 1 \text{ fm}^2$

. . . and form fire streaks

with rapidity from $E$-$p$ conservation

Each fire streak fragments independently into pions

$\pi^-$, Pb+Pb, $\sqrt{s}_{NN} = 8.8$ GeV

$dn/dy = A \cdot (E_s^* - m_s) \cdot \exp \left( \frac{-[(y - y_s)^2 + \epsilon^2]^{\frac{n}{2}}}{n\sigma_y^n} \right)$

total fire streak energy

$\sum$ of brick masses

available energy

fire streak rapidity

Peripheral

Central

data points from: NA49, PRC 86 (2012) 054903
Our simple model explains the full centrality dependence of the pion $dn/dy$ spectrum. It is valid in some extended range of collision energy ($\sqrt{s_{NN}} = 9 - 17$ GeV at the least).

Each fire streak fragments independently into pions with rapidity from $E$-$p$ conservation:

\[
\frac{dn}{dy} = A \cdot (E_s^* - m_s) \cdot \exp \left( -\frac{\left[ (y - y_s)^2 + \epsilon^2 \right]^{\frac{\eta}{2}}}{n \sigma_y^n} \right)
\]

available energy

<table>
<thead>
<tr>
<th>Model</th>
<th>$b$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C0</td>
<td>4.35</td>
</tr>
<tr>
<td>C1</td>
<td>6.32</td>
</tr>
<tr>
<td>C2</td>
<td>8.03</td>
</tr>
<tr>
<td>C3</td>
<td>8.41</td>
</tr>
<tr>
<td>C4</td>
<td>9.46</td>
</tr>
</tbody>
</table>

total fire streak energy

sum of brick masses

data points from: NA49, PRC 86 (2012) 054903
The energy balance in p+p reactions at SPS energies
(PRCE 99 (2019) 024908, Appendix A)

\[ \sqrt{s} \approx \text{(baryon energy)} + \text{(pion energy)} + \text{(kaon energy)} \]

Taking the values for K, (pion energy) and (kaon energy) from the Tables, we get

\[ \sqrt{s} \approx 16.629 \text{ GeV} \rightarrow \text{agreement within 3.7%} \]

Including pair production of protons, antiprotons, neutrons and antineutrons:

\[ \langle E_p \rangle = 1451 \text{ MeV} \]

\[ E(pp \rightarrow \bar{p}) = 0.0386 \cdot 1451 = 56 \text{ MeV} \]

Following the considerations made in [17], we multiply the above by 1.66 in order to obtain the average energy spent on antineutron production. Finally we multiply the

\[ E(pp \rightarrow \text{non-strange, pair-produced } B \text{ and } \bar{B}) = 2 \cdot (1 + 1.66) \cdot 56 = 298 \text{ MeV} \]

We obtain:

\[ \sqrt{s} \approx 16.927 \text{ GeV} \rightarrow \text{agreement within 2%} \]
The energy dependence of the fire streak fragmentation function
(PRC 99 (2019) 024908, Appendix B)

FIG. 7:

Comparison of single fire-streak fragmentation functions used for the description of $\pi^-$ rapidity distributions in Pb+Pb collisions at $\sqrt{s_{NN}} = 8.8$ GeV (solid) and at $\sqrt{s_{NN}} = 17.3$ GeV (dotted). The two presented functions are given by Eq. (2.1) with $(E_s^* - m_s) \equiv 1$ GeV. The numerical values of the function parameters are given in the text.
Each fire streak fragments independently into pions:

Finally, we get (per unit of total collision energy):

\[
\frac{\text{Energy spent on pions in } p+p}{\text{Energy spent on pions in } Pb+Pb} = 0.781 \quad 4 \%
\]
References cited in the Tables

Bricks collide …

1 x 1 fm²

… and form fire streaks

Peripheral (b=9.72 fm)

Central (b=2.55 fm)

R. Hagedorn, CERN-71-12
W.D. Myers, Nucl. Phys. A
(Re)invented by A. Szczurek

Characteristics of fire streaks


$\sqrt{s_{NN}} = 17.3$ GeV

Pb+Pb

$\sqrt{s_{NN}} = 17.3$ GeV

Pb+Pb

$\sqrt{s_{NN}} = 17.3$ GeV

Pb+Pb

$\sqrt{s_{NN}} = 17.3$ GeV

Pb+Pb
More on EM effects

Note: the remaining points on the bottom plot are deduced from EM effects on azimuthal anisotropies:

A. R.,
B42 (2011) 867

\[ y = y_{\text{beam}} = \frac{1}{2} \ln \left( \frac{E + p_L}{E - p_L} \right) \]
The simplest model (first attempt for Pb+Pb) reproduces overall systematic effect, but hardly fits to individual points.

Addition of spectator expansion (motivated by Ar+Sc) already visibly improves the situation for Pb+Pb.

Building EM simulation on top of the fire streaks model (motivated by observations from EM effects) makes it follow most of the points.

There is still room for improvement for $x_F \approx 0$. 