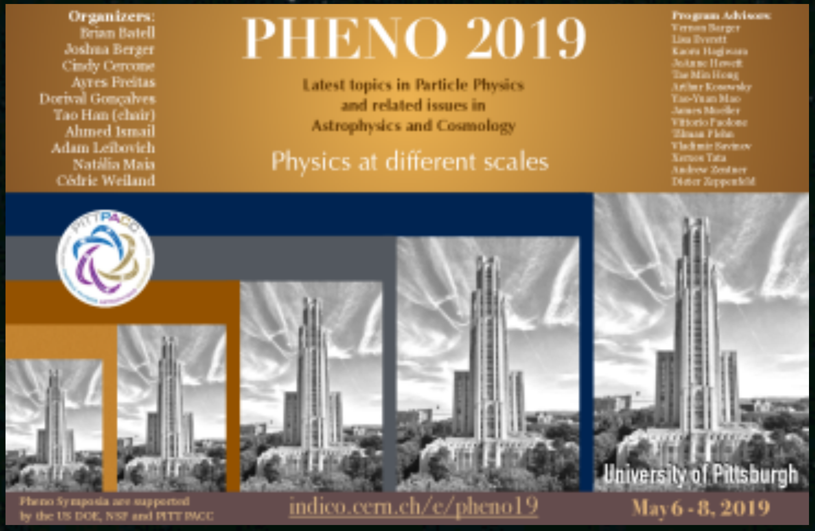
UNIVERSITY OF Southampton
School of Physics and Astronomy

elusives invisiblesPlus
neutrinos, dark matter & dark energy physics

Theories of neutrinos and leptons

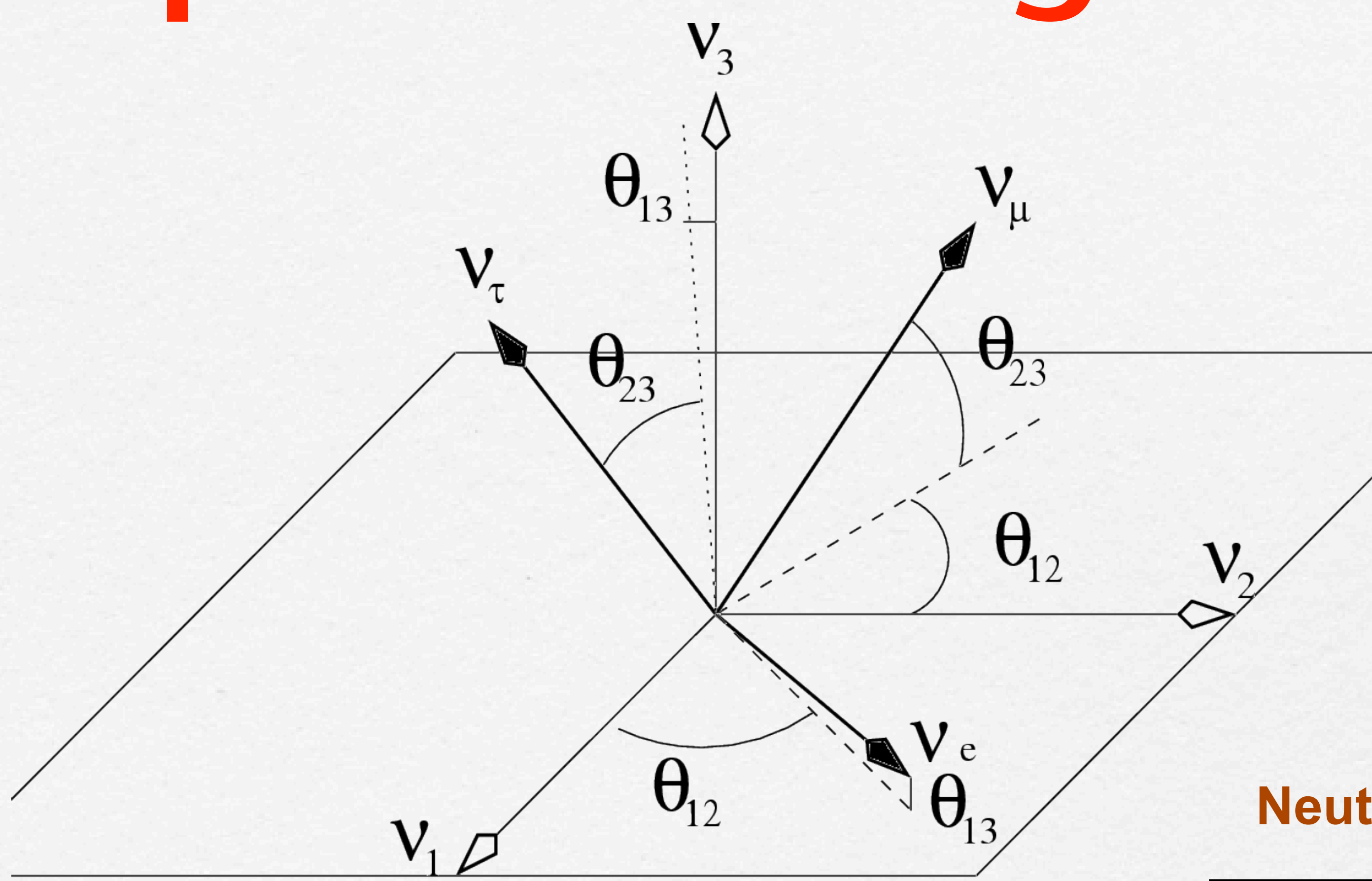
Steve King, 6th May 2019,

Pittsburgh



PMNS Lepton mixing matrix

Pontecorvo
Maki
Nakagawa
Sakata

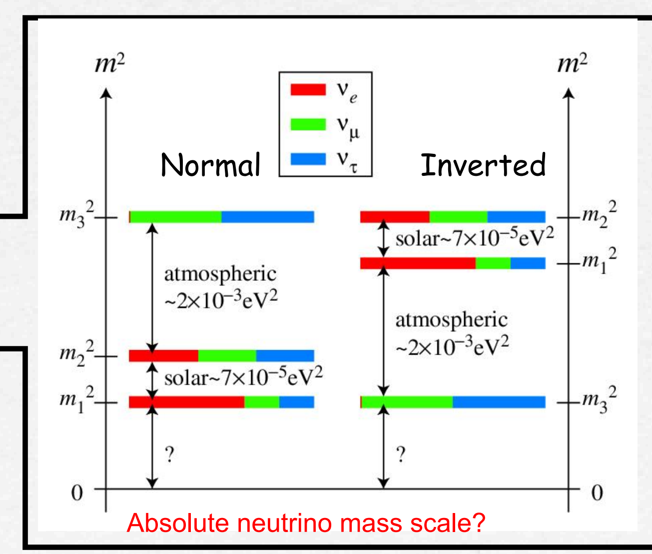


Standard Model states

$$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau \\ \tau^-\end{pmatrix}_L$$

$$\begin{pmatrix}\nu_e \\ \nu_\mu \\ \nu_\tau\end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix} \begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3\end{pmatrix}$$

Neutrino mass states



PMNS Lepton mixing matrix

$$U_{PMNS}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0\\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}}\end{pmatrix}$$

Atmospheric Reactor Solar Majorana

$$=\begin{pmatrix}c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta}\\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}\end{pmatrix}$$

$$\times\ \mathrm{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$$

The 6 parameters measurable in neutrino oscillations:

- ✳ The atmospheric mass squared difference Δm^2_{31}
- ✳ The solar mass squared difference $\Delta m^2_{21} = m^2_2 - m^2_1$
- ✳ The atmospheric angle θ_{23}
- ✳ The solar angle θ_{12}
- ✳ The reactor angle θ_{13}
- ✳ The CP violating phase δ

NuFIT 4.0 (2018)

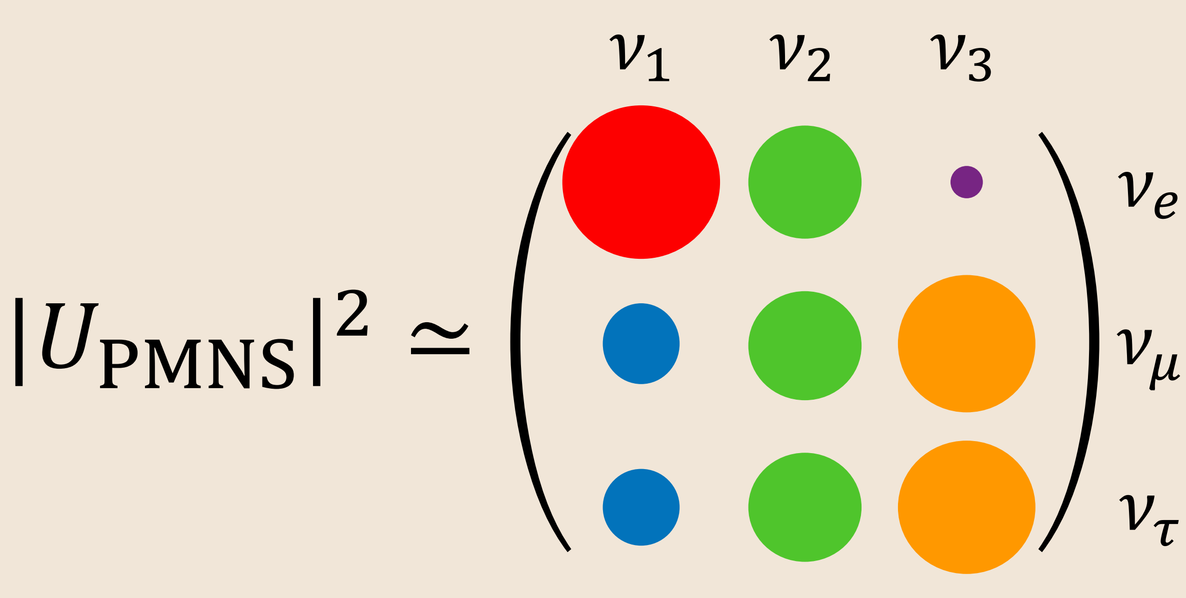
without SK atmospheric data

	Normal Ordering (best fit)	
	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
$\sin^2\theta_{23}$	$0.580^{+0.017}_{-0.021}$	$0.418 \to 0.627$
$\theta_{23}/^\circ$	$49.6^{+1.0}_{-1.2}$	$40.3 \to 52.4$
$\sin^2\theta_{13}$	$0.02241^{+0.00065}_{-0.00065}$	$0.02045 \to 0.02439$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$
$\delta_{\rm CP}/^\circ$	215^{+40}_{-29}	$125 \to 392$
$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.525^{+0.033}_{-0.032}$	$+2.427 \to +2.625$

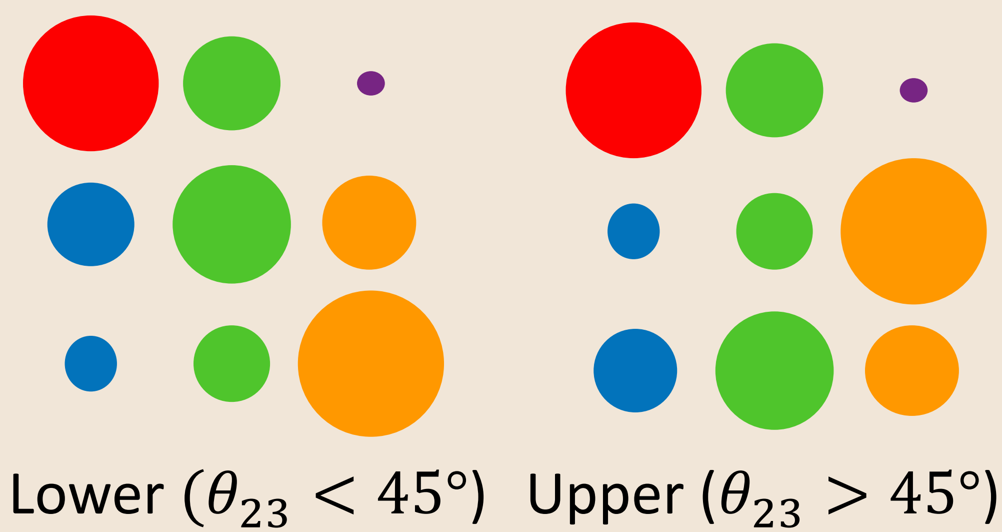
Inverted Ordering ($\Delta\chi^2 = 4.7$)

Open questions for neutrinos

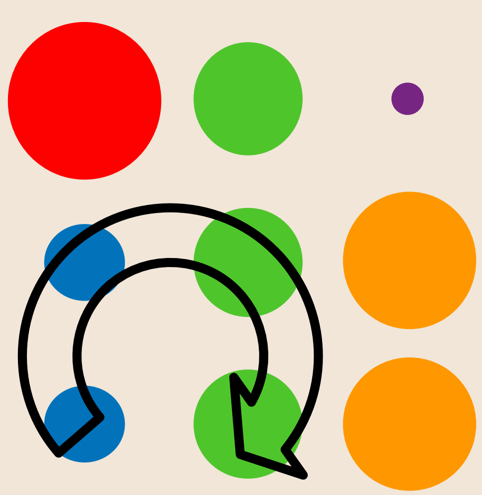
Phill Litchfield



Octant degeneracy



CP Violation

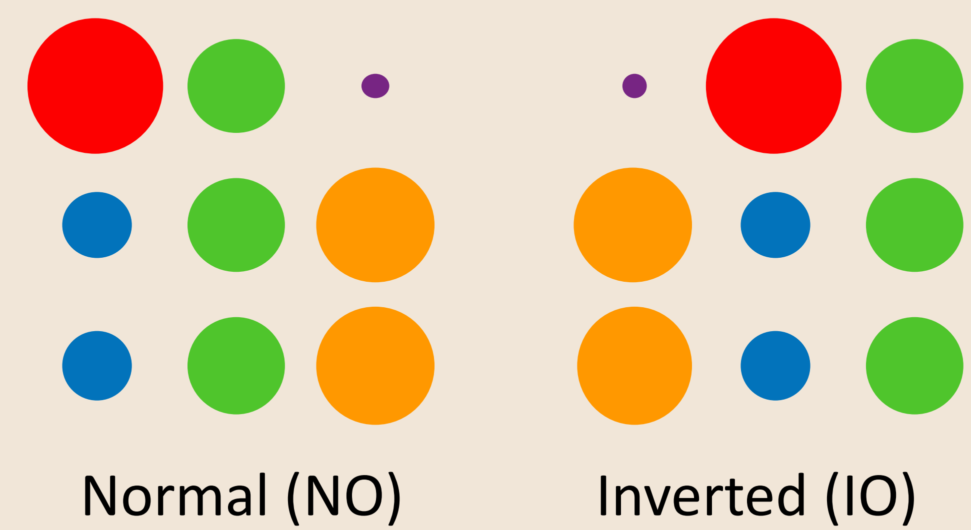


Complex mixing of these 4 elements causes

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

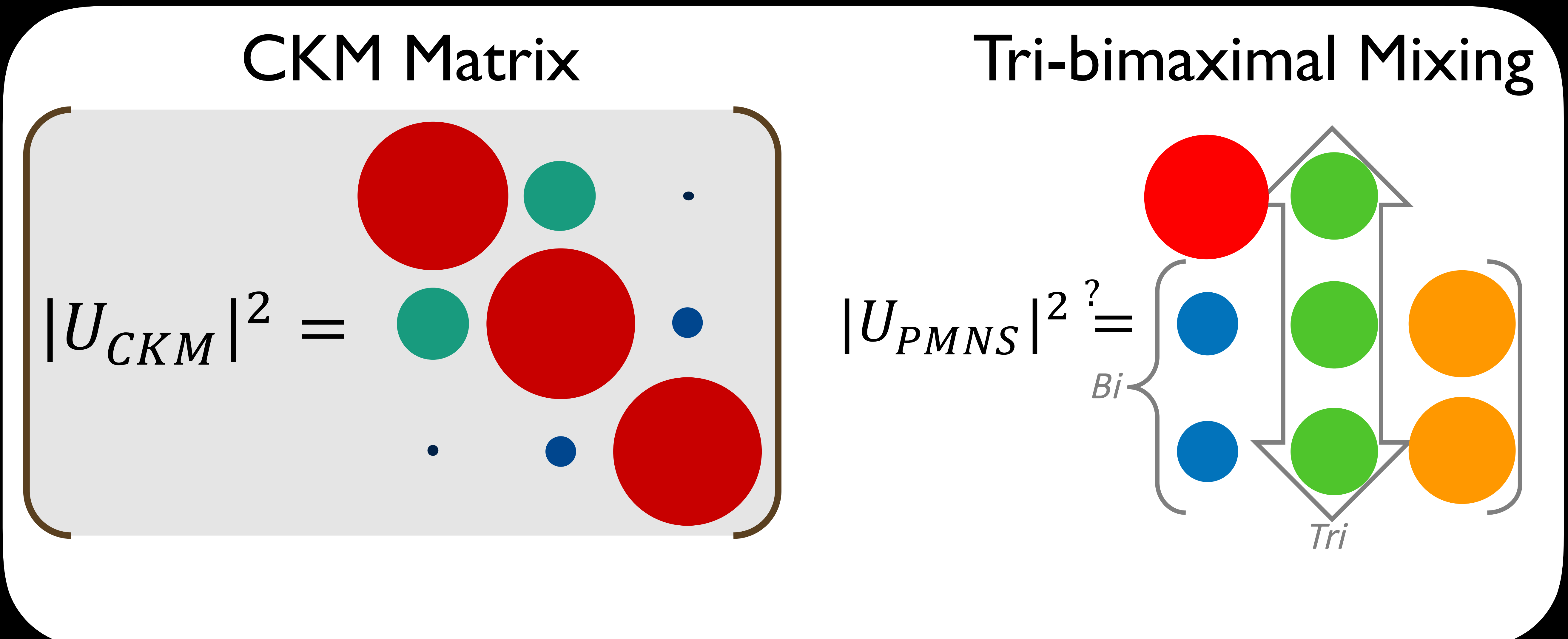
Key parameter: $\boldsymbol{\delta_{CP}}$

Mass Ordering (Hierarchy)



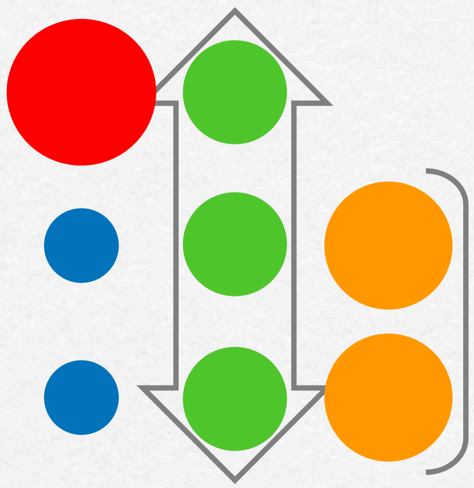
CKM vs Tri-bimaximal Mixing

Phill Litchfield



Tri-Bimaximal Mixing

P.F.Harrison, D.H.Perkins and W.G.Scott, hep-ph/0202074



$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}$$

Allowed at 3 sigma

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$

Allowed at 3 sigma

$$\sin\theta_{13} = 0$$

Excluded at many sigma

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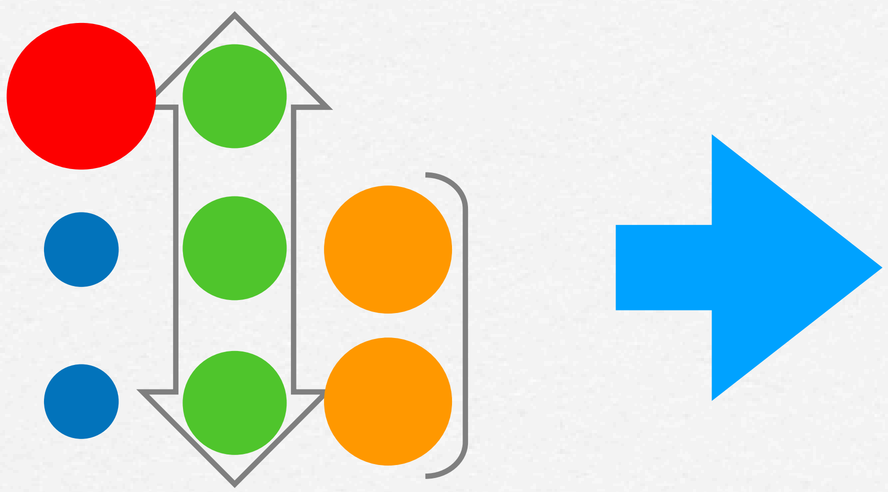

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

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Allowed at 3 sigma

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}$$

Allowed at 3 sigma

$$\sin\theta_{13} = 0$$

Excluded at many sigma

NuFIT 4.0 (2018) Best Fit Preferences:

$$s^2_{12} < \frac{1}{3} \qquad s^2_{23} > \frac{1}{2} \qquad s^2_{13} = 0.02241 \pm 0.00065$$

Charged lepton corrections

Charged lepton rotation

$$U_{\rm PMNS} = \begin{pmatrix} c^e_{12} & s^e_{12}e^{-i\delta^e_{12}} & 0 \\ -s^e_{12}e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-bimaximal

$$= \begin{pmatrix} \cdots & \cdots & \frac{s^e_{12}}{\sqrt{2}}e^{-i\delta^e_{12}} \\ \cdots & \cdots & \frac{c^e_{12}}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Reactor angle generated
- Third row unchanged implies sum rules

Sum rules first derived and studied in: SFK hep-ph/0506297; S.Antusch, SFK hep-ph/0508044;
S.Antusch, P.Huber, S.F.K and T.Schwetz, hep-ph/0702286; S.Antusch, S.F.K., M.Malinsky,0711.4727
More recent detailed phenomenological analyses:
D.Marzocca, S.T.Petcov, A.Romanino and M.C.Sevilla,1302.0423; S.T.Petcov 1405.6006;
P.Ballett, S.F.King, C.Luhn, S.Pascoli and M.A.Schmidt, 1410.7573
I.Girardi, S.T.Petcov and A.V.Titov,1410.8056,1504.00658,1504.02402,1605.04172,…
For asymmetric texture without sum rule see: M.H.Rahat, P.Ramond, B.Xu, 1805.10684

Charged lepton corrections

Charged lepton rotation

$$U_{\rm PMNS} = \begin{pmatrix} c^e_{12} & s^e_{12}e^{-i\delta^e_{12}} & 0 \\ -s^e_{12}e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-bimaximal

$$= \begin{pmatrix} \cdots & \cdots & \frac{s^e_{12}}{\sqrt{2}}e^{-i\delta^e_{12}} \\ \cdots & \cdots & \frac{c^e_{12}}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\rightarrow s_{13} = \frac{s^e_{12}}{\sqrt{2}}$$

Suggests $\theta^e_{12} \approx \theta_C$

$$\rightarrow c_{23}c_{13} = \frac{1}{\sqrt{2}} \rightarrow s^2_{23} < \frac{1}{2}$$

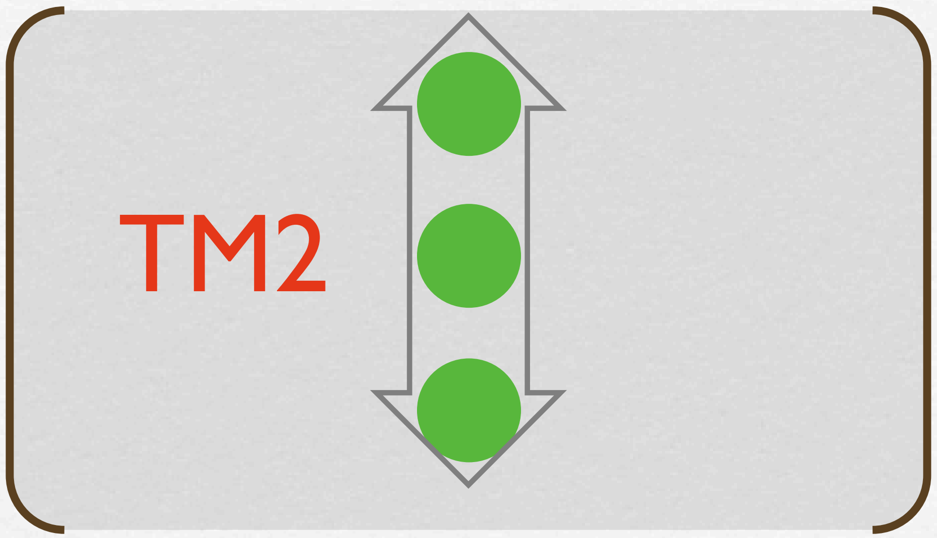
Not best fit

$$\frac{|U_{\tau 1}|}{|U_{\tau 2}|} = \frac{|s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta}|}{|-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}|} = \frac{1}{\sqrt{2}} \rightarrow \cos\delta = \frac{t_{23}s^2_{12} + s^2_{13}c^2_{12}/t_{23} - \frac{1}{3}(t_{23} + s^2_{13}/t_{23})}{\sin 2\theta_{12}s_{13}}$$

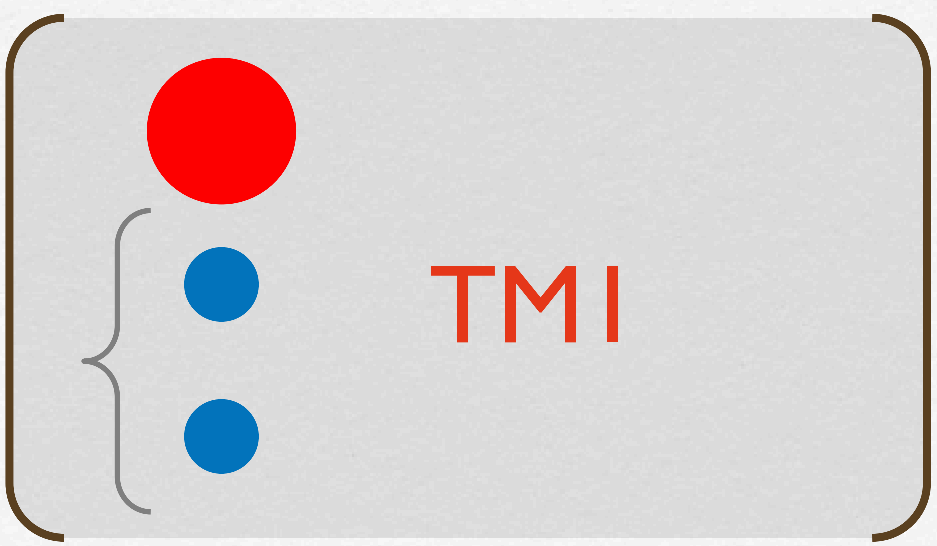
This derivation: P.Ballett, S.F.King, C.Luhn, S.Pascoli and M.A.Schmidt, 1410.7573

Tri-maximal Mixing

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798



$$U_{\mathrm{TM2}} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$



$$U_{\mathrm{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

Tri-maximal Mixing

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798

Not best fit

$$U_{\rm TM2} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

$$|U_{e2}| = s_{12}c_{13} = \sqrt{\frac{1}{3}} \rightarrow s_{12}^2 > \frac{1}{3}$$

$$|U_{\mu 2}| = |c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

$$|U_{\tau 2}| = |-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

$$\rightarrow \cos\delta = \frac{2c_{13}\cot 2\theta_{23}\cot 2\theta_{13}}{\sqrt{2-3s_{13}^2}}$$

Tri-maximal Mixing

C.H.Albright and W.Rodejohann, 0812.0436; C.H.Albright, A.Dueck and W.Rodejohann, 1004.2798

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$$U_{\rm TM2} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

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$$|U_{\tau 2}| = |-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{3}}$$

$$\rightarrow \cos\delta = \frac{2c_{13}\cot 2\theta_{23}\cot 2\theta_{13}}{\sqrt{2-3s_{13}^2}}$$

Best fit

$$U_{\rm TM1} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

$$|U_{e1}| = c_{12}c_{13} = \sqrt{\frac{2}{3}} \rightarrow s_{12}^2 < \frac{1}{3}$$

$$|U_{\mu 1}| = |-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta}| = \sqrt{\frac{1}{6}}$$

$$|U_{\tau 1}| = |s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta}| = \sqrt{\frac{1}{6}}$$

$$\rightarrow \cos\delta = -\frac{\cot 2\theta_{23}(1-5s_{13}^2)}{2\sqrt{2}s_{13}\sqrt{1-3s_{13}^2}}$$

Example of TM1 Mixing: The Littlest Seesaw

SFK 1304.6264; 1512.07531

$$\text{Case I}: \quad \mathrm{M}_\nu^{\mathrm{I}} = \omega \mathrm{m_a} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mathrm{m_s} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

$$\omega = e^{i2\pi/3}$$

$$\text{Case II}: \quad \mathrm{M}_\nu^{\mathrm{II}} = \omega^2 \mathrm{m_a} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mathrm{m_s} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix}$$

Two parameters fits all neutrino data with m_1=0

E.g. for m_a/m_s=11 gives Littlest mu-tau seesaw

S.F.K. and C.C.Nishi,1807.00023

$$\text{Case I:} \quad M_\nu = m_s \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9+11\omega & 3+11\omega \\ 1 & 3+11\omega & 1+11\omega \end{pmatrix},$$

$$\text{Case II:} \quad M_\nu = m_s \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+11\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix}.$$

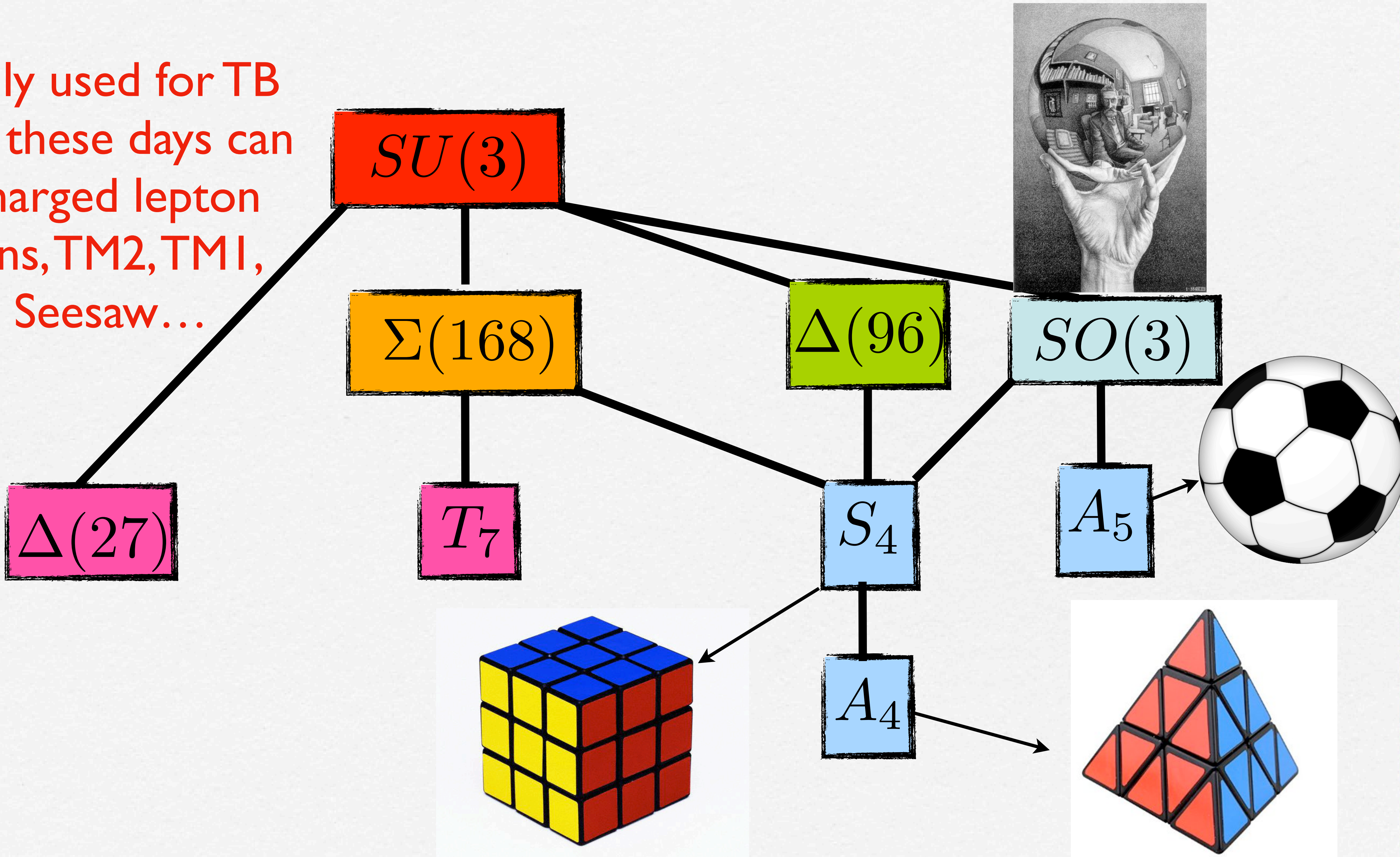
Maximal atmospheric

Maximal CPV

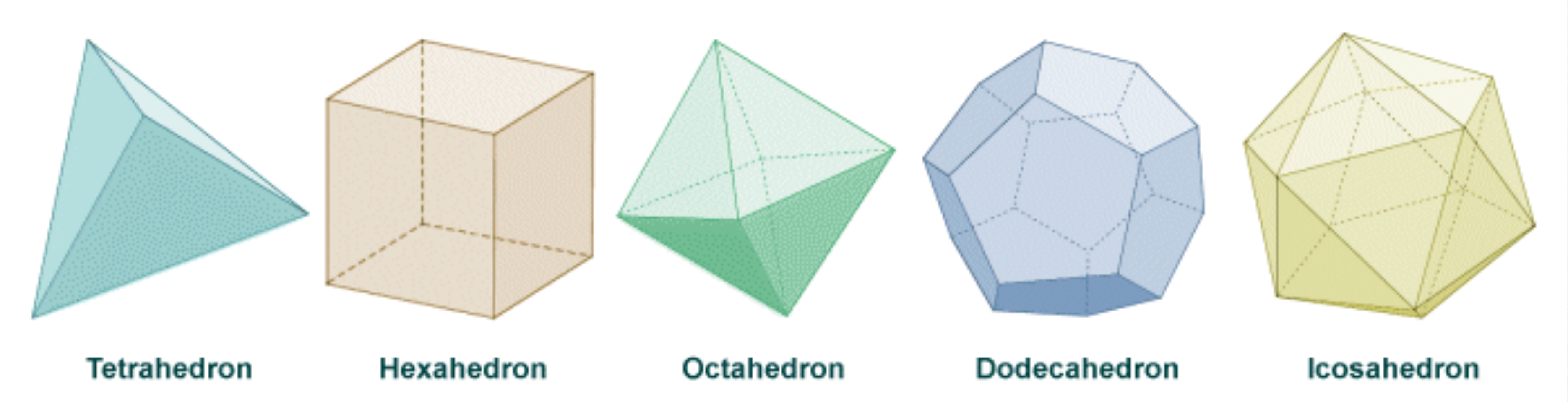
(see later)

Family Symmetry

Traditionally used for TB mixing, but these days can explain charged lepton corrections, TM2, TM1, Littlest Seesaw…



Platonic Solids

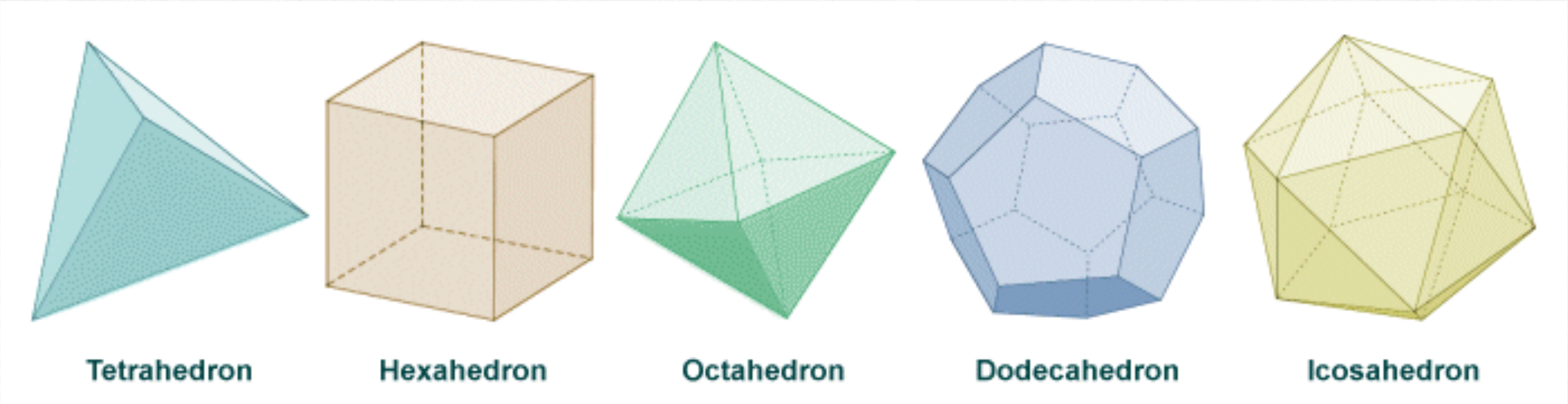


solid	faces	vertices	Plato	Group
tetrahedron	4	4	fire	A_4
octahedron	8	6	air	S_4
icosahedron	20	12	water	A_5
hexahedron	6	8	earth	S_4
dodecahedron	12	20	?	A_5

Plato's fire
A4 can explain
Tri-bimaximal
Mixing

E.Ma and G.Rajasekaran, hep-ph/0106291;
K.S.Babu, E.Ma, J.W.F.Valle, hep-ph/0206292;
G.Altarelli and F.Feruglio, hep-ph/0504165,hep-ph/0512103

Platonic Solids



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hexahedron	6	8	earth	S_4
dodecahedron	12	20	?	A_5

Plato's water A5 can explain Golden Ratio Mixing

A. Datta, F. S. Ling, P. Ramond, hep-ph/0306002;
L. L. Everett and A. J. Stuart, 0812.1057;
F. Feruglio and A. Paris, 1101.0393.

S.F.K., C.Luhn, 1301.1340

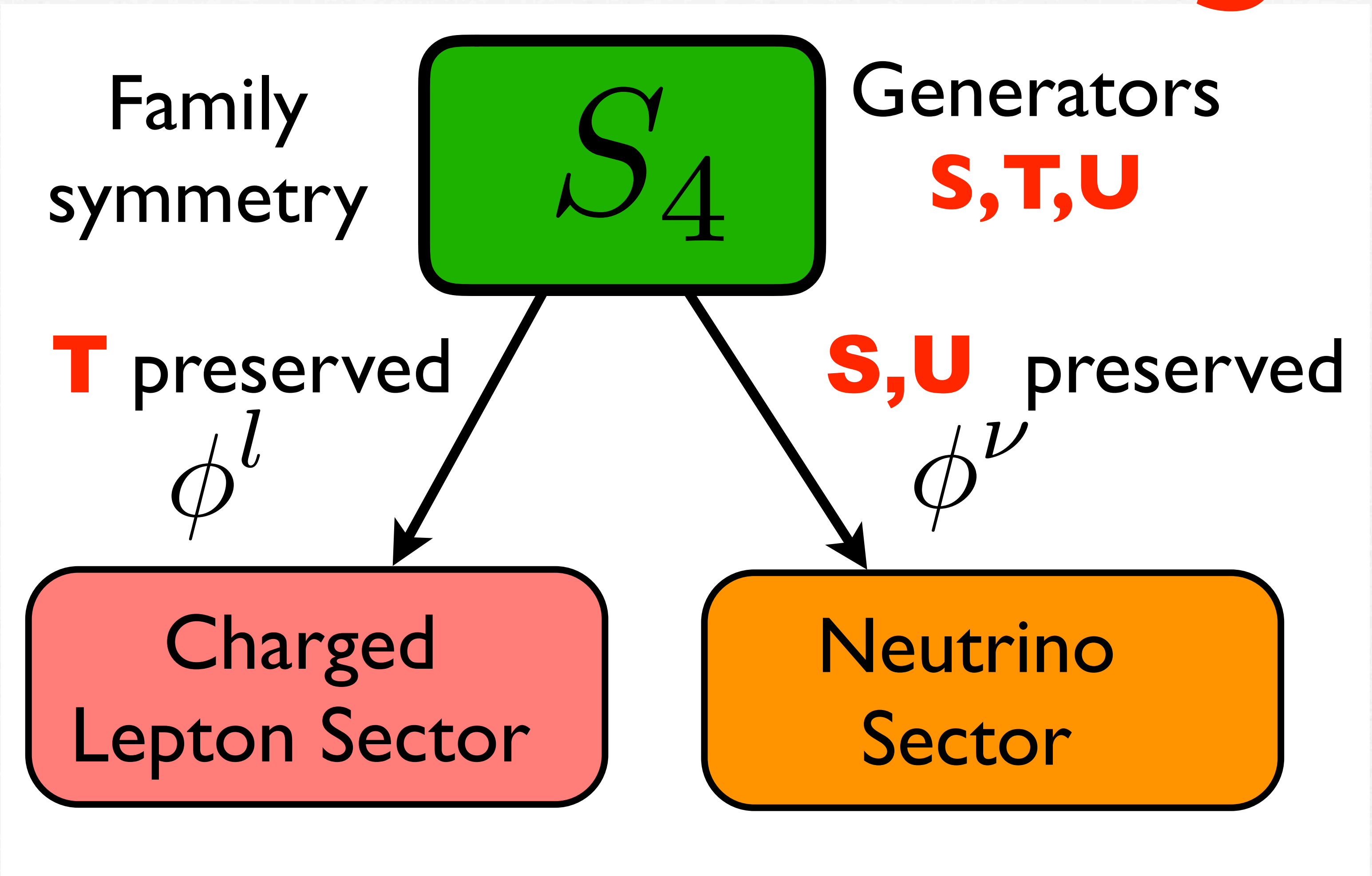
A_4 and S_4 Group Theory

S_4	A_4	S	T	U
$\mathbf{1}, \mathbf{1}'$	$\mathbf{1}$	1	1	± 1
$\mathbf{2}$	$\begin{pmatrix} \mathbf{1}'' \\ \mathbf{1}' \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\mathbf{3}, \mathbf{3}'$	$\mathbf{3}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$	$\mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Diagonalised by TB matrix

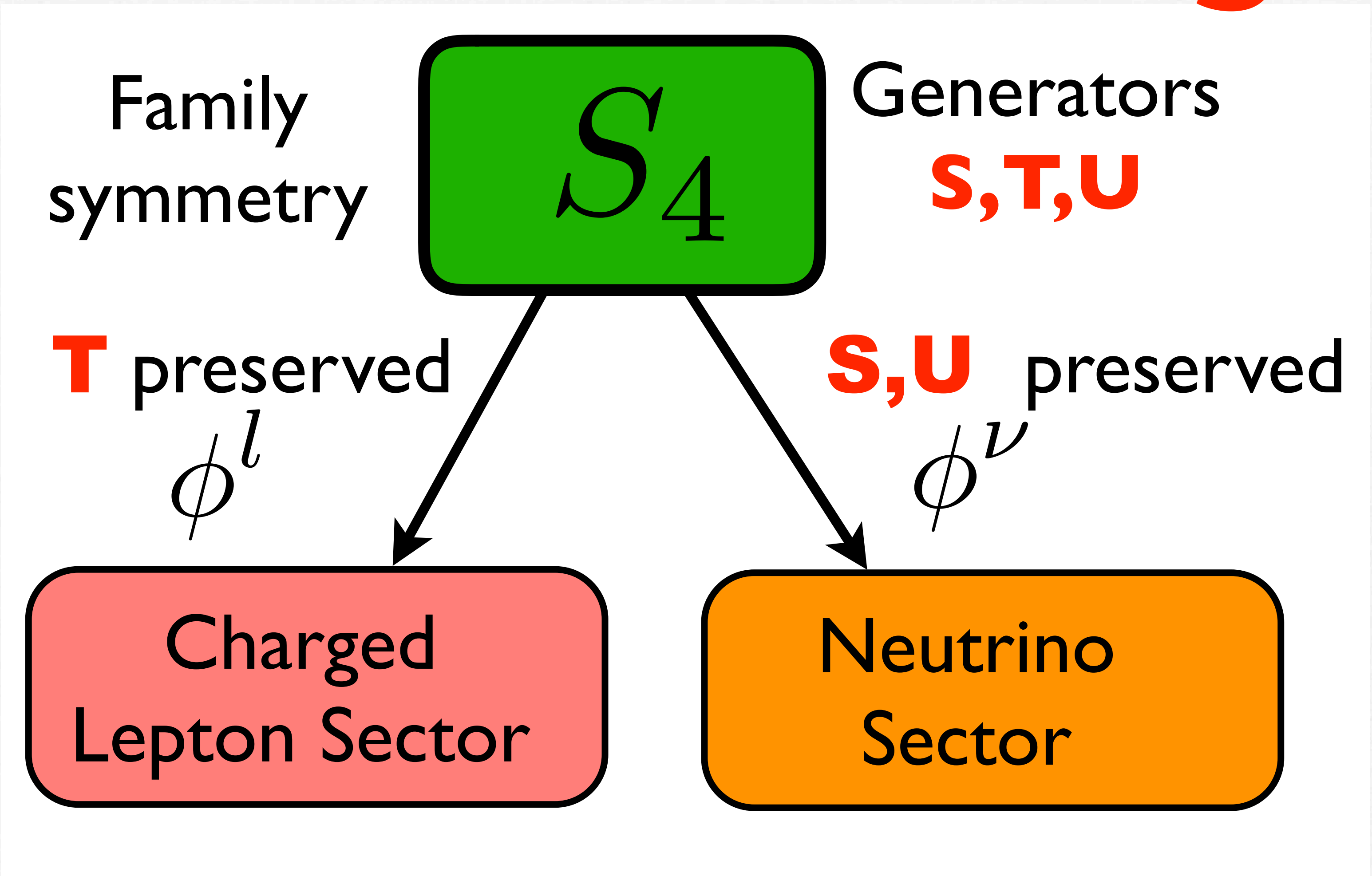
Tri-bimaximal mixing from S_4

S.F.K., C.Luhn, 1301.1340



Tri-bimaximal mixing from S_4

S.F.K., C.Luhn,
1301.1340

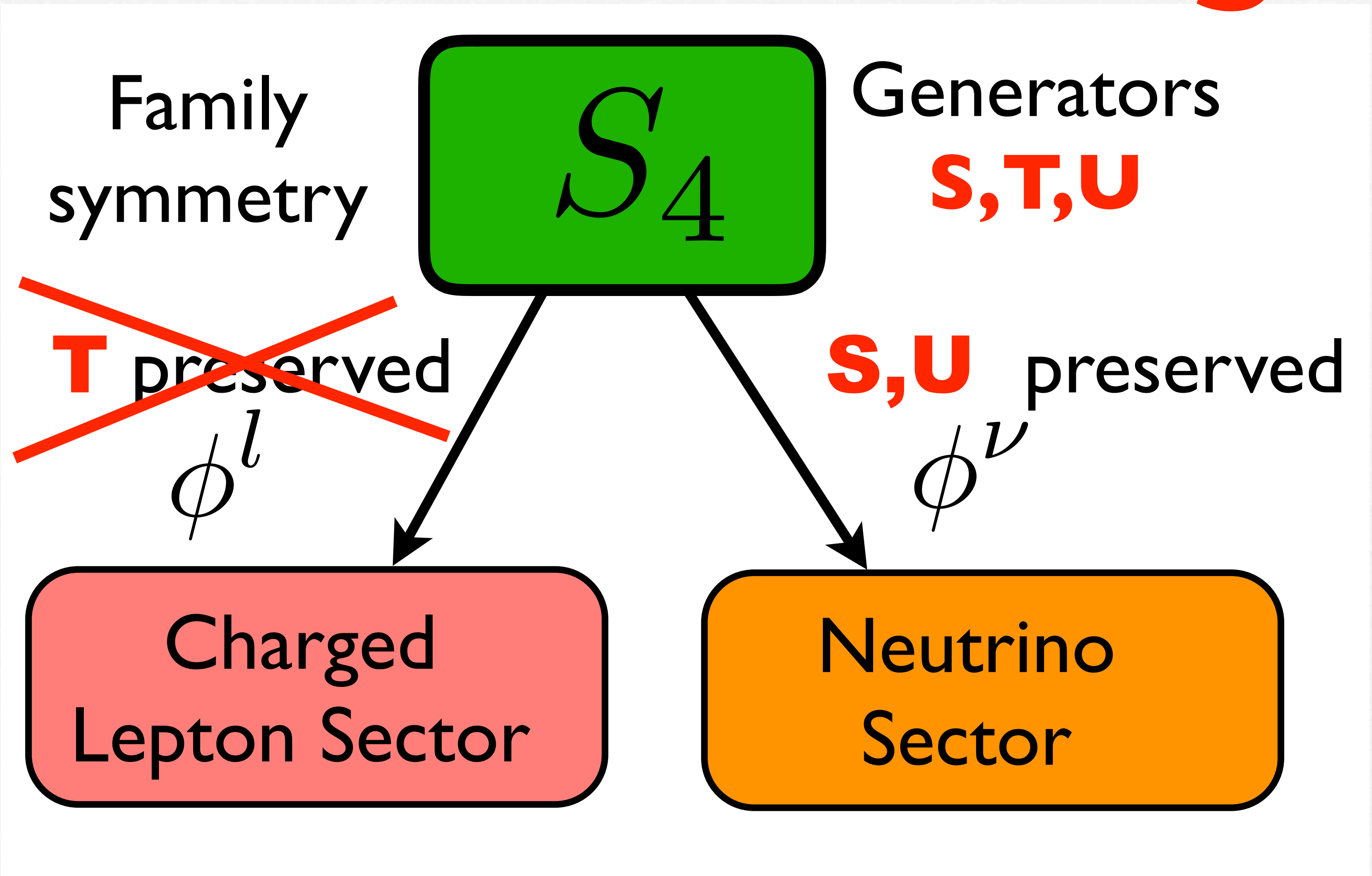


$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

TB mixing
excluded
so need to
break S,T,U

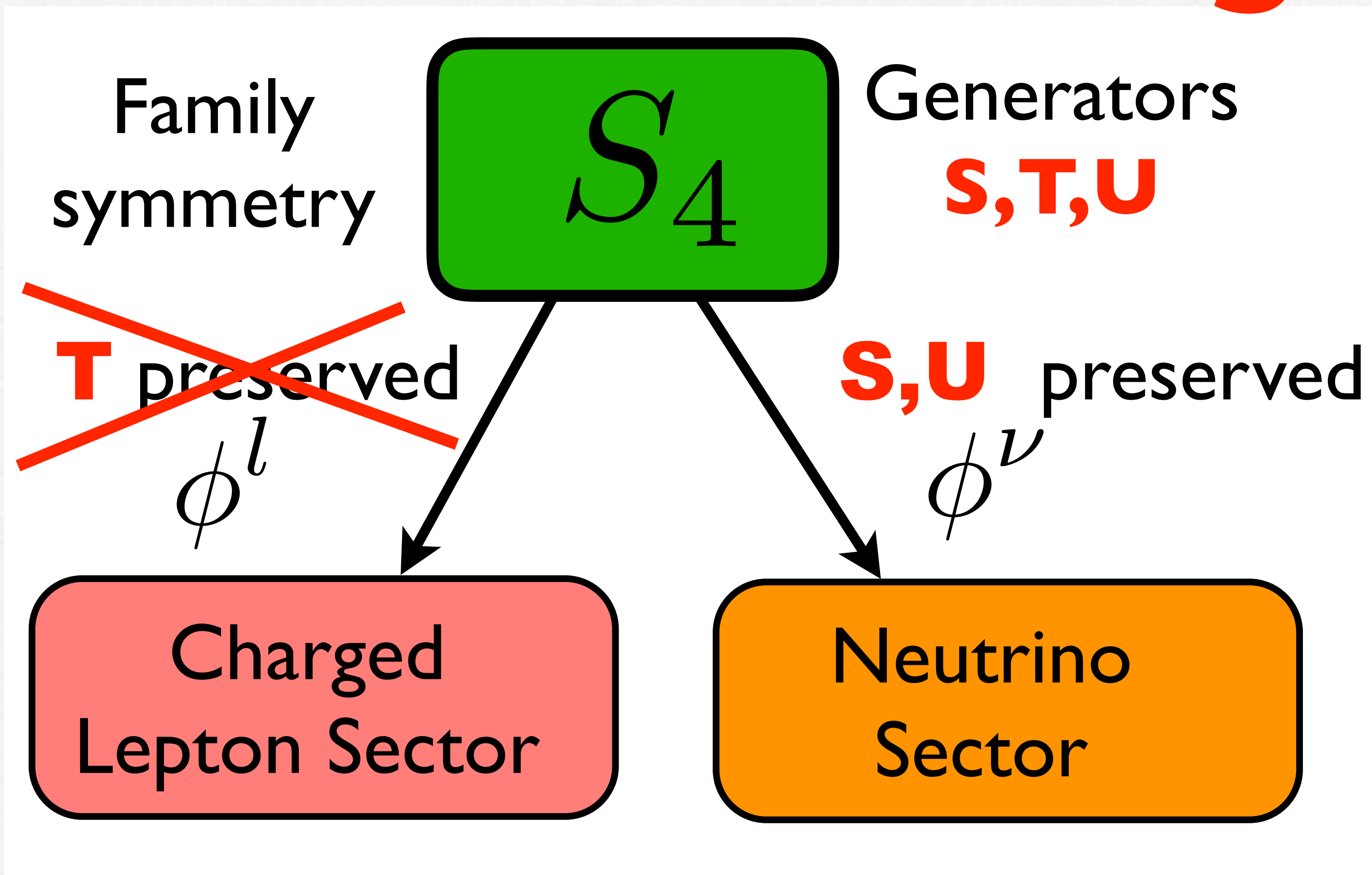
Tri-bimaximal mixing from S_4

break T



Tri-bimaximal mixing from S_4

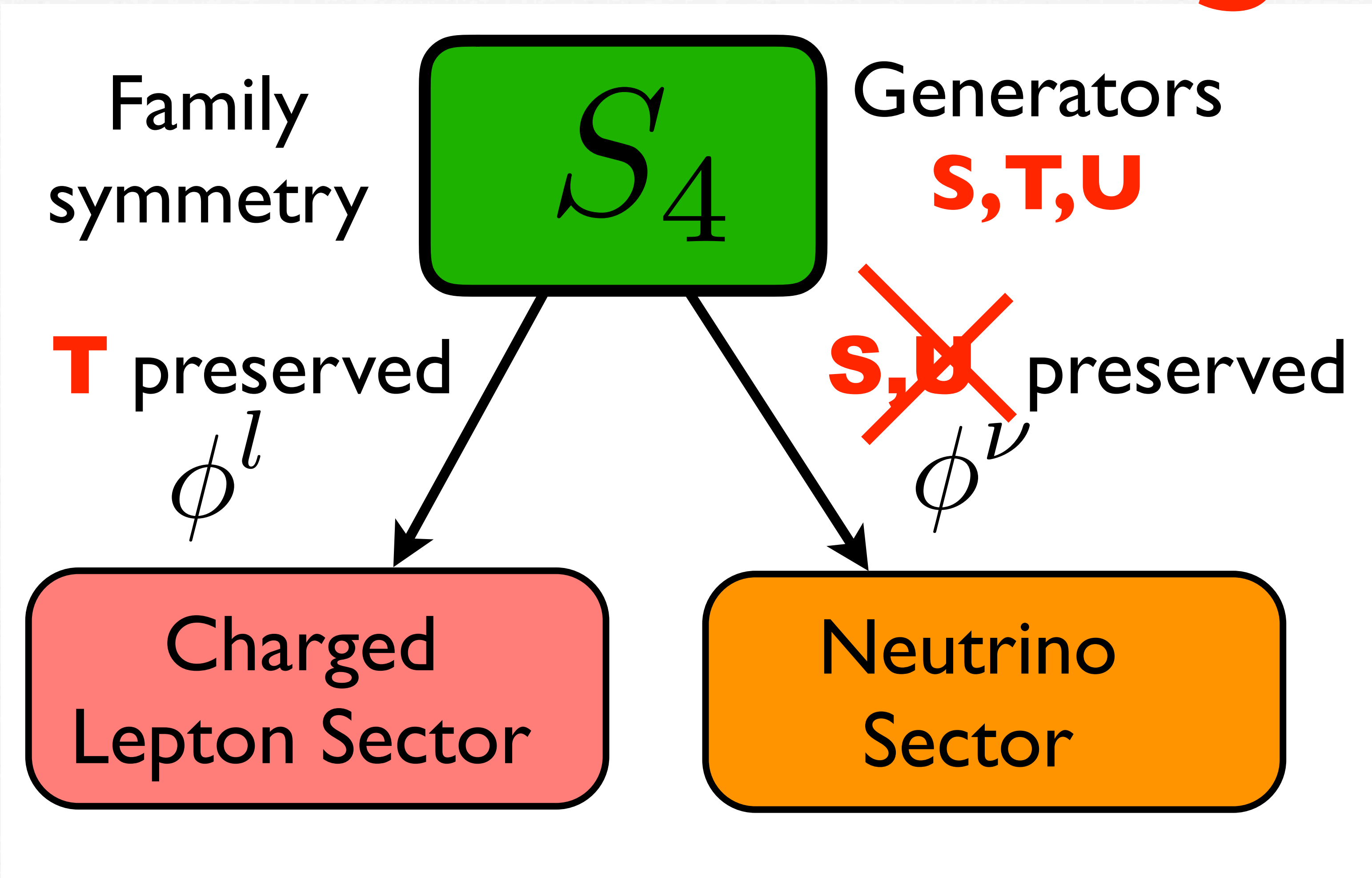
break T



$$U_{\mathrm{PMNS}} = \begin{pmatrix} c^e_{12} & s^e_{12} e^{-i\delta^e_{12}} & 0 \\ -s^e_{12} e^{i\delta^e_{12}} & c^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Charged lepton rotation

Tri-bimaximal mixing from S_4

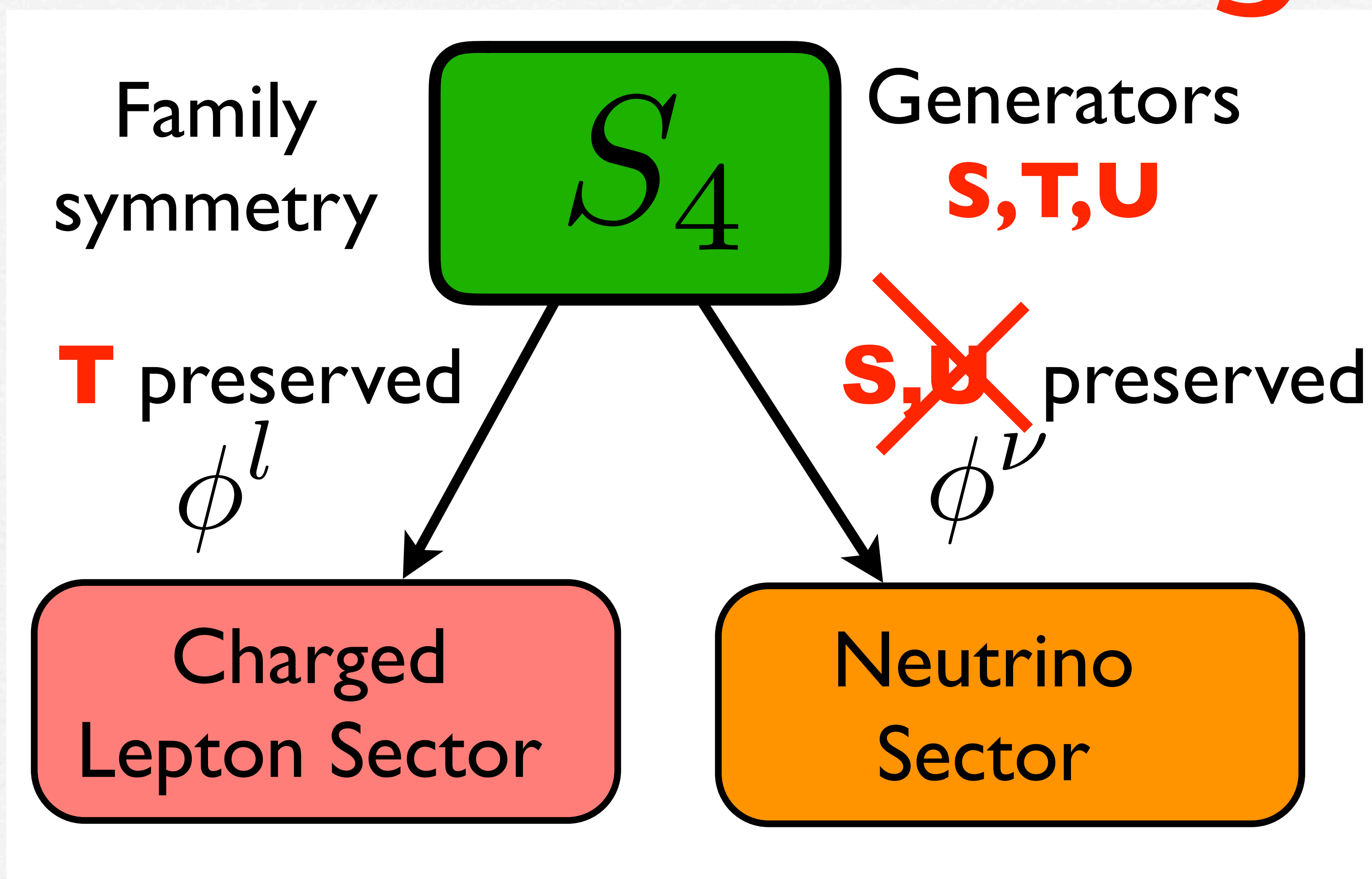


S.F.K., C.Luhn, 1301.1340

Y.Shimizu, M.Tanimoto, A.Watanabe, 1105.2929; S.F.K.,C.Luhn,1107.5332

break U

Tri-bimaximal mixing from S_4



S.F.K., C.Luhn, 1301.1340

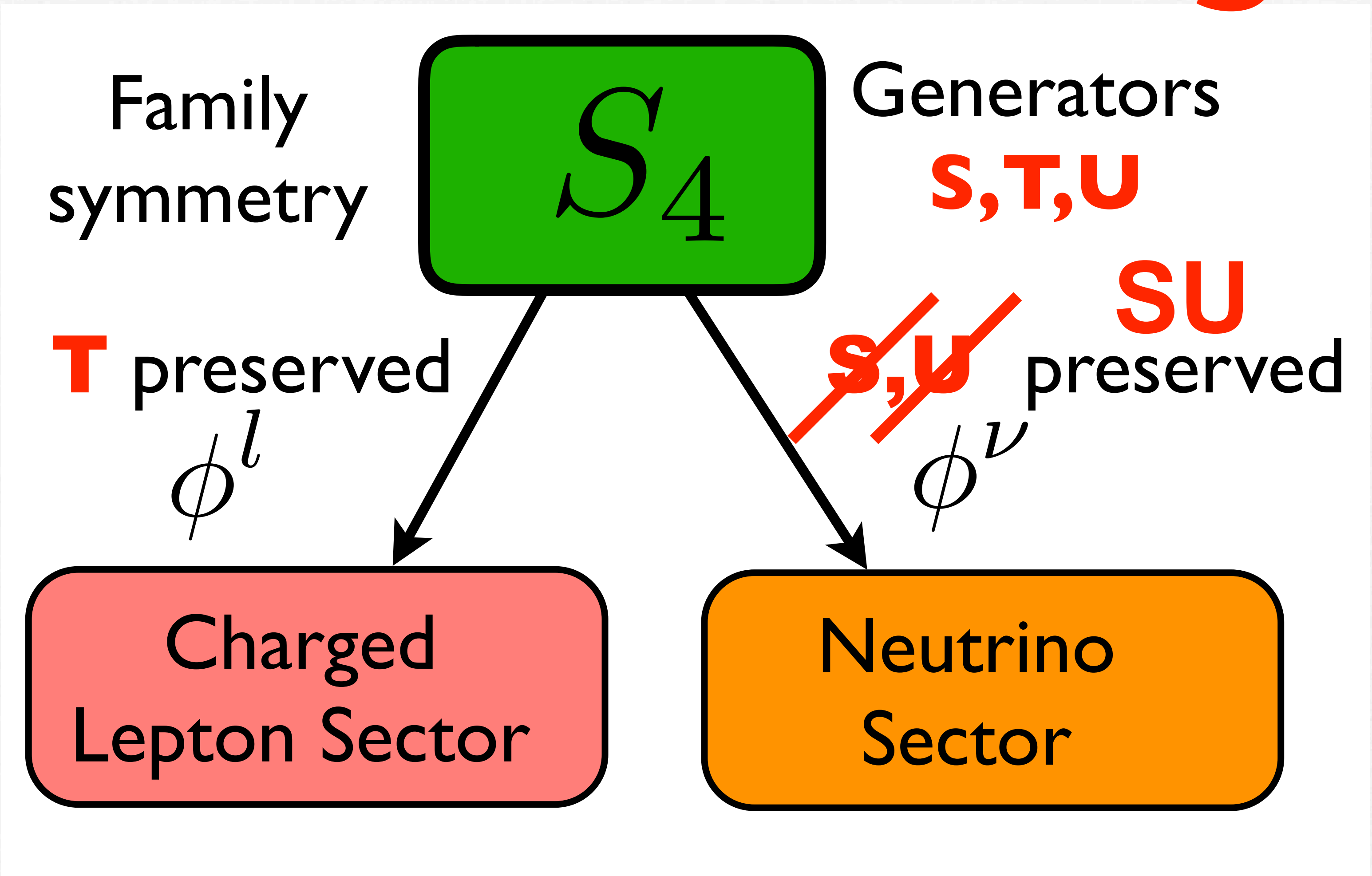
Y.Shimizu, M.Tanimoto, A.Watanabe, 1105.2929; S.F.K.,C.Luhn,1107.5332

break U

$$U_{\mathrm{TM2}} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

TM2 as A4 with just S and T

Tri-bimaximal mixing from S_4

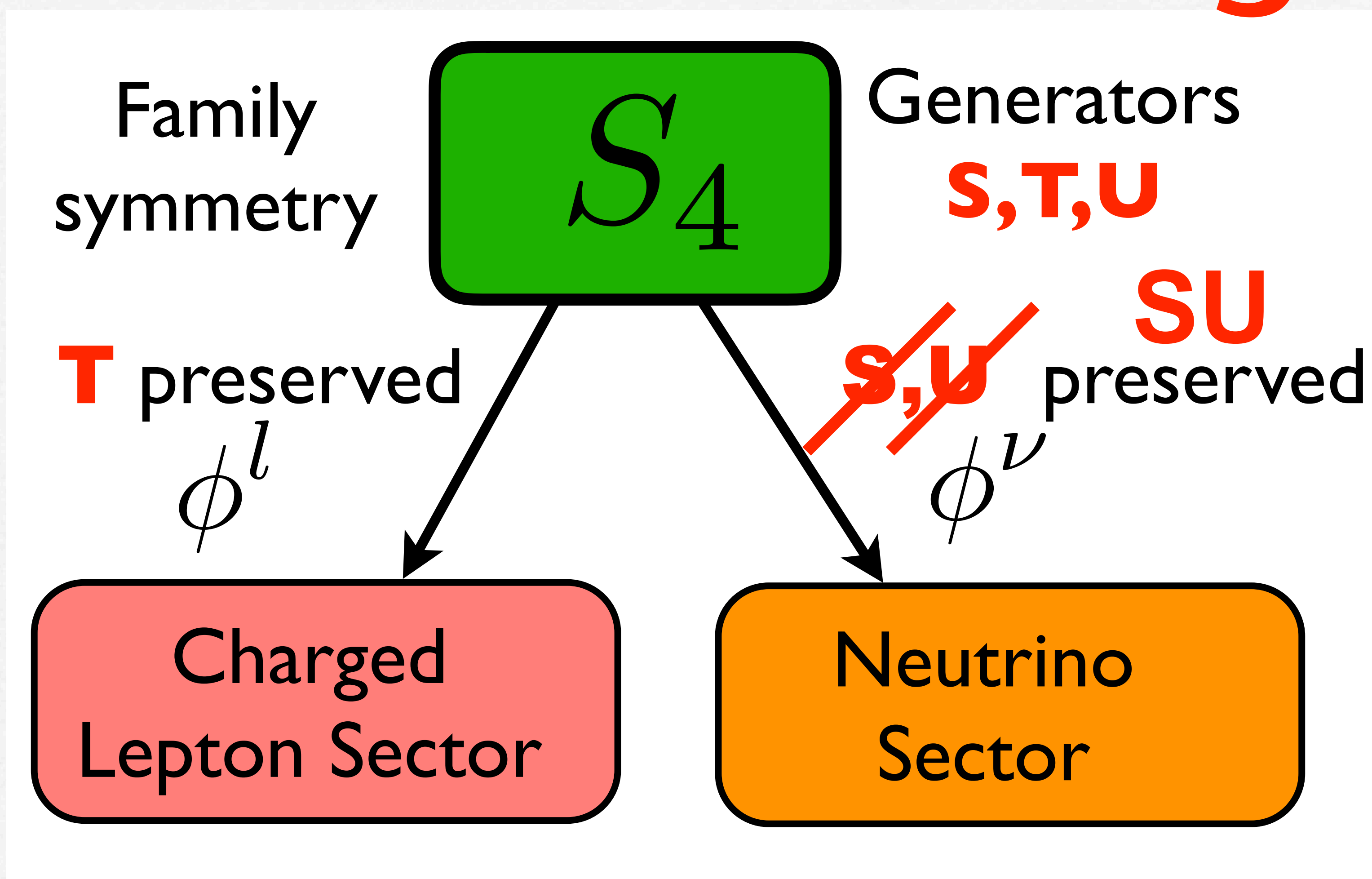


S.F.K., C.Luhn, 1301.1340

break S,U separately preserve SU

Tri-bimaximal mixing from S_4

S.F.K., C.Luhn, 1301.1340



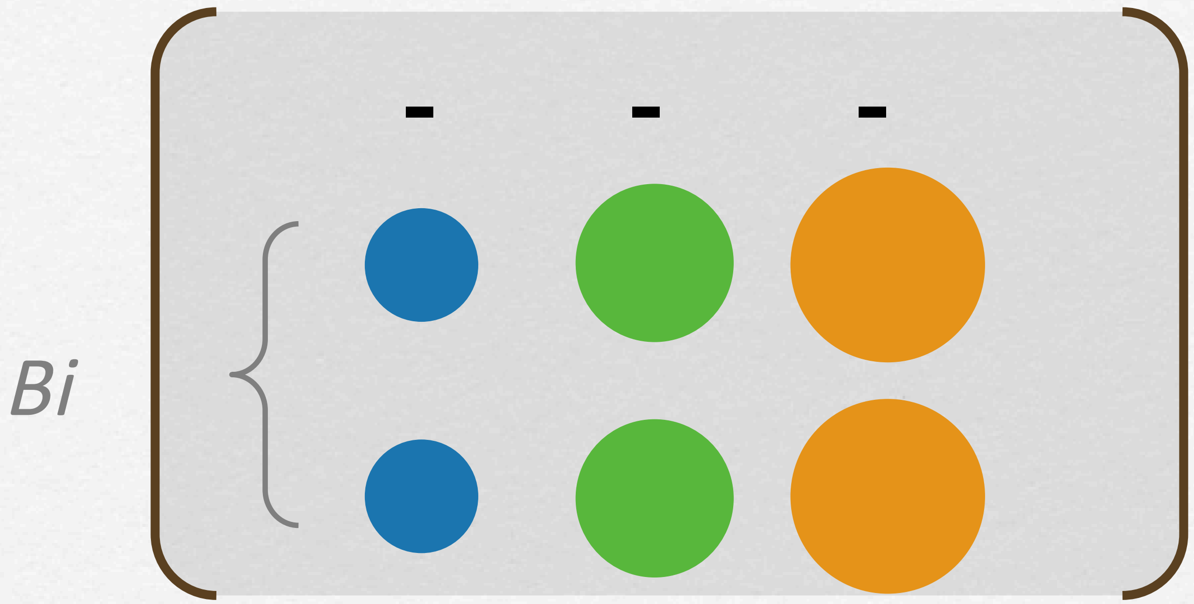
break S,U separately preserve SU

$$U_{\mathrm{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

TM1 with SU and T

D.Hernandez and A.Y.Smirnov 1204.0445,1212.2149,1304.7738; C.Luhn, 1306.2358
S.F.K.,C.Luhn,1607.05276

Mu-Tau Symmetry $\nu_\mu \leftrightarrow \nu_\tau^*$



Basic Idea:

→ Two rows have

→ equal magnitudes

Z.z.Xing and S.Zhou, 0804.3512

$$\rightarrow \quad \theta_{13} \neq 0\,, \quad \theta_{23} = 45^\circ, \quad \delta_{\mathrm{CP}} = \pm 90^\circ$$

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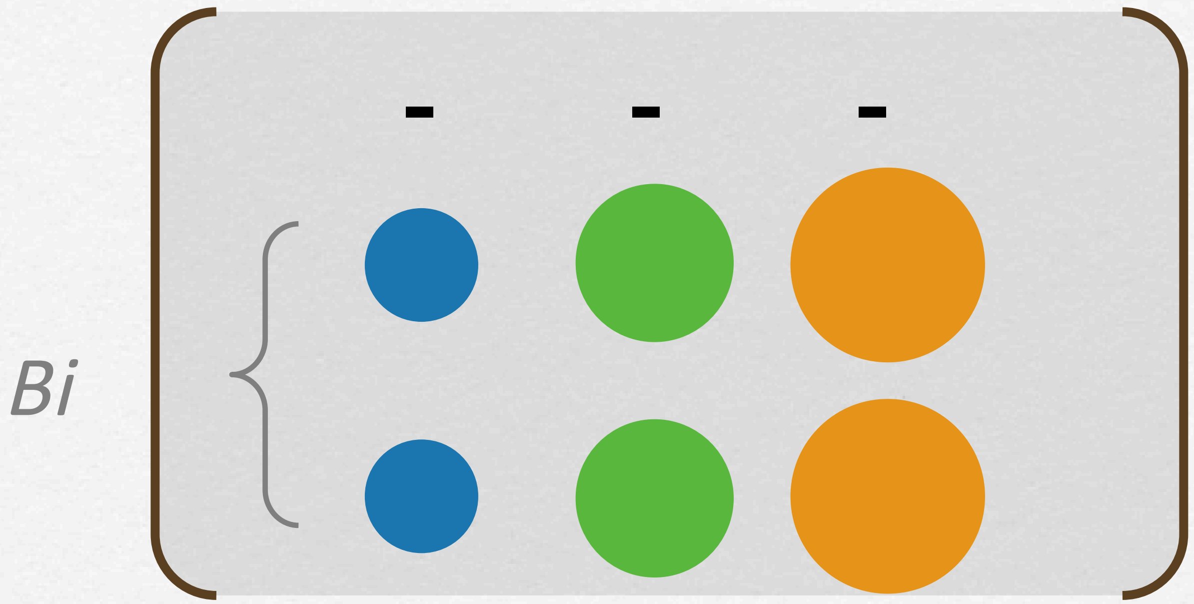

Basic Idea:

→ Two rows have
→ equal magnitudes

Z.z.Xing and S.Zhou, 0804.3512

→ $\theta_{13} \neq 0\,, \quad \theta_{23} = 45^\circ, \quad \delta_{\text{CP}} = \pm 90^\circ$

$$V_0 = \begin{pmatrix} |V_{e1}| & |V_{e2}| & |V_{e3}| \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\mu 1}^* & V_{\mu 2}^* & V_{\mu 3}^* \end{pmatrix}$$

Generalisation of:

Mu-tau reflection symmetry

P.F.Harrison and W.G.Scott, hep-ph/0210197

Mu-Tau Symmetry $\nu_\mu \leftrightarrow \nu_\tau^*$

Mu-tau reflection symmetric Majorana mass matrix:

$$H_\nu = M_\nu^\dagger M_\nu = \begin{pmatrix} A & D & D^* \\ D^* & B & C^* \\ D & C & B \end{pmatrix}$$

P.F.Harrison and W.G.Scott, hep-ph/0210197

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P.F.Harrison and W.G.Scott, hep-ph/0210197

Can arise from:

$$M_\nu = \begin{pmatrix} a & d & d^* \\ d & c & b \\ d^* & b & c^* \end{pmatrix}$$

W.Grimus and L.Lavoura,hep-ph/0305309

More general examples:

H.J.He, W.Rodejohann and X.J.Xu, 1507.03541

A.S.Joshipura and K.M.Patel,1507.01235

Mu-Tau Symmetry $\nu_\mu \leftrightarrow \nu_\tau^*$

Littlest mu-tau seesaw

S.F.K. and C.C.Nishi,1807.00023

$$M_\nu = m_s \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix}$$

$$\omega = e^{i2\pi/3}$$

unequal

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$$H_\nu = M_\nu^\dagger M_\nu = 11\,|m_s|^2 \begin{pmatrix} 1 & -1-2i\sqrt{3} & 1-2i\sqrt{3} \\ -1+2i\sqrt{3} & 19 & 17+4i\sqrt{3} \\ 1+2i\sqrt{3} & 17-4i\sqrt{3} & 19 \end{pmatrix}$$

equal

Mu-Tau Symmetry $\nu_\mu \leftrightarrow \nu_\tau^*$

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S.F.K. and C.C.Nishi,1807.00023

$$M_\nu = m_s \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1+11\omega^2 & 3+\omega^2 \\ 3 & 3+11\omega^2 & 9+11\omega^2 \end{pmatrix}$$

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equal

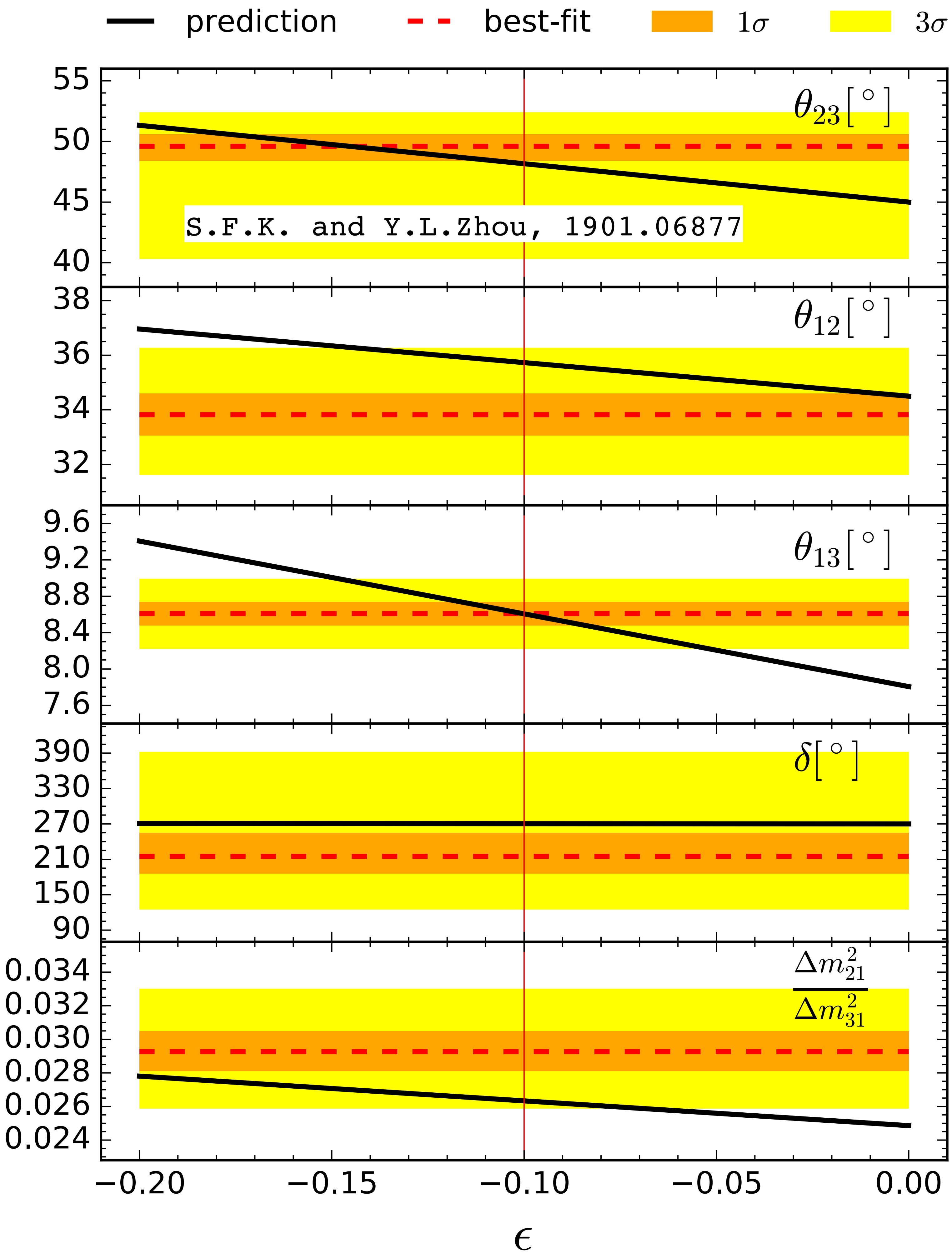
$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_+}{\sqrt{6}} & \frac{c_-}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} - i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} + i\frac{c_+}{2} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} + i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} - i\frac{c_+}{2} \end{pmatrix}$$

TM1

Mu-tau reflection symmetry

$$c_\pm = \sqrt{1 \pm \frac{11}{3\sqrt{17}}}$$

Littlest mu-tau seesaw



$$m_1 = 0$$

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_+}{\sqrt{6}} & \frac{c_-}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} - i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} + i\frac{c_+}{2} \\ \frac{1}{\sqrt{6}} & -\frac{c_+}{\sqrt{6}} + i\frac{c_-}{2} & -\frac{c_-}{\sqrt{6}} - i\frac{c_+}{2} \end{pmatrix}$$

$$c_\pm = \sqrt{1 \pm \frac{11}{3\sqrt{17}}}$$

Renormalisation Group Corrections

$$\theta_{13} \approx 7.807^\circ - 8.000^\circ\epsilon\,,$$

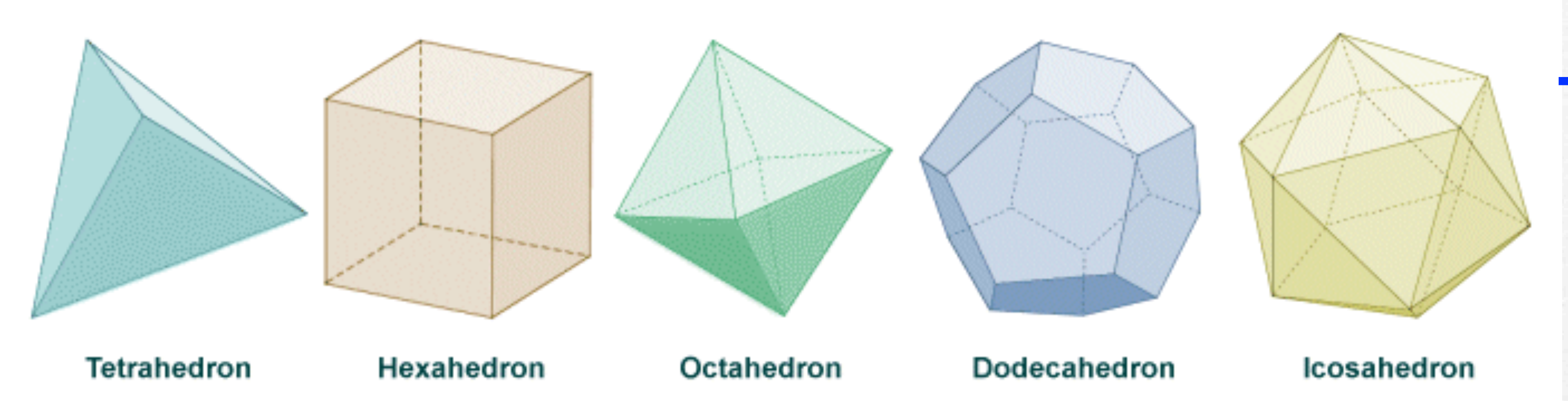
$$\theta_{12} \approx 34.50^\circ - 12.30^\circ\epsilon\,,$$

$$\theta_{23} \approx 45.00^\circ - 31.64^\circ\epsilon\,,$$

$$\delta \approx 270.00^\circ + 3.23^\circ\epsilon\,,$$

$$\Delta m^2_{21}/\Delta m^2_{31} \approx 0.0247 - 0.0147\epsilon$$

Origin of Plato's symmetry?



solid	faces	vertices	Plato	Group
tetrahedron	4	4	fire	A_4
octahedron	8	6	air	S_4
icosahedron	20	12	water	A_5
hexahedron	6	8	earth	S_4
dodecahedron	12	20	?	A_5

Two possibilities:

1. From gauge group e.g. SU(3) or SO(3)
2. From extra dimensions e.g. string theory

Origin of Plato's symmetry?

Possibility 1:

Y.Koide,0705.2275; T.Banks and N.Seiberg,1011.5120;
Y.L.Wu,1203.2382; A.Merle and R.Zwicky,1110.4891;
B.L.Rachlin and T.W.Kephart,1702.08073; C. Luhn, 1101.2417;
S.F.K. and Ye-Ling Zhou, 1809.10292

Break SO(3) using large Higgs reps

irrep	$\underline{1}$	$\underline{3}$	$\underline{5}$	$\underline{7}$
subgroups	$SO(3)$	$SO(2)$	$Z_2 \times Z_2$	$\mathbf{1}$
		$SO(3)$	$SO(2)$	A_4
			$SO(3)$	Z_3
				D_4
				$SO(2)$
				$SO(3)$

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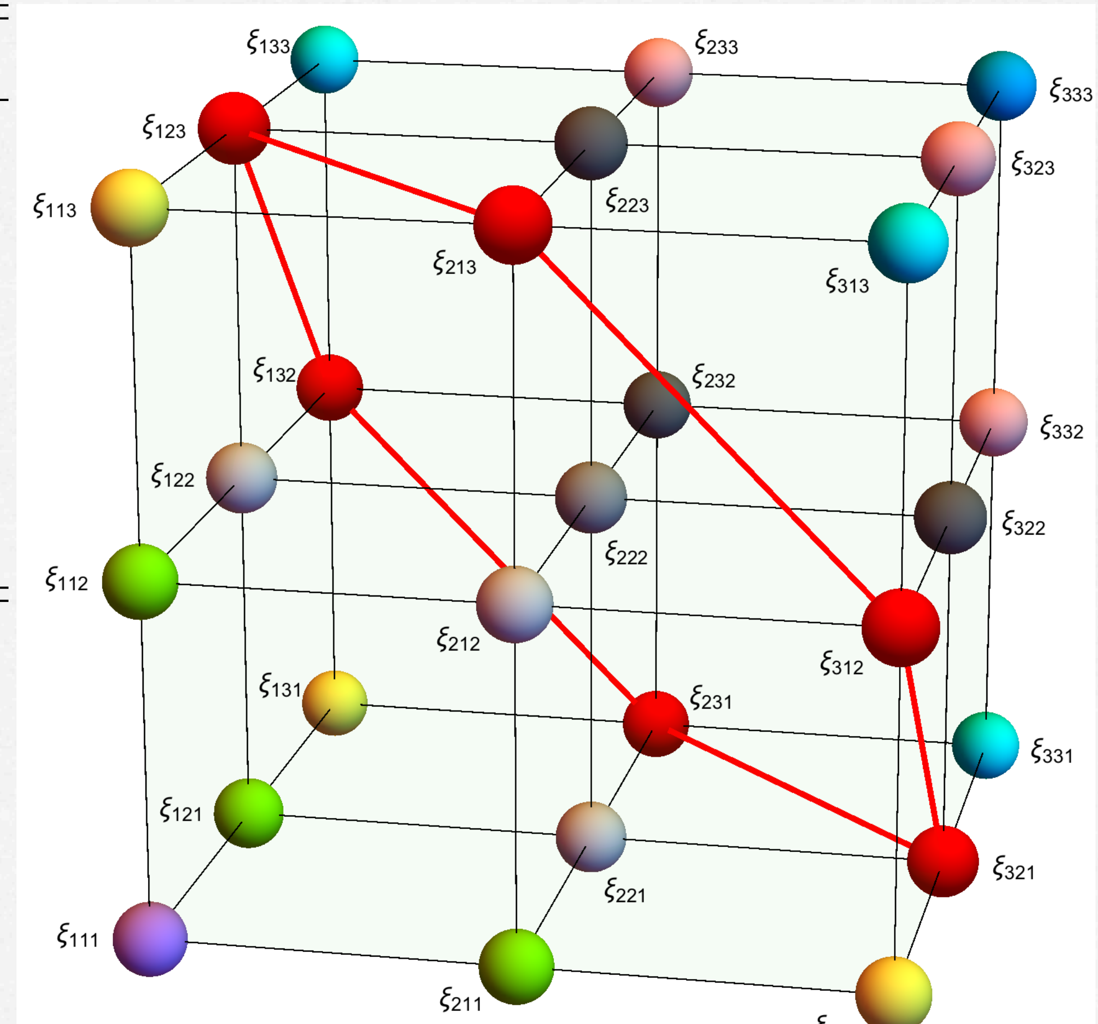
S.F.K. and Ye-Ling Zhou, 1809.10292

Break SO(3) using large Higgs reps E.g. 7-plet

irrep	$\underline{1}$	$\underline{3}$	$\underline{5}$	$\underline{7}$
subgroups	$SO(3)$	$SO(2)$	$Z_2 \times Z_2$	$\mathbf{1}$
		$SO(3)$	$SO(2)$	A_4
			$SO(3)$	Z_3
				D_4
				$SO(2)$
				$SO(3)$



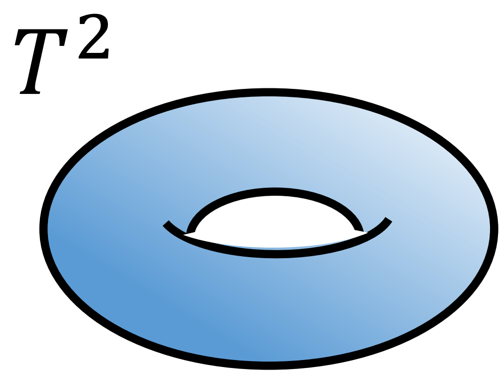
A4 preserving direction of 7-plet VEV

$$\langle \xi_{123} \rangle \equiv \frac{v_\xi}{\sqrt{6}}, \quad \langle \xi_{111} \rangle = \langle \xi_{112} \rangle = \langle \xi_{113} \rangle = \langle \xi_{133} \rangle = \langle \xi_{233} \rangle = \langle \xi_{333} \rangle = 0$$

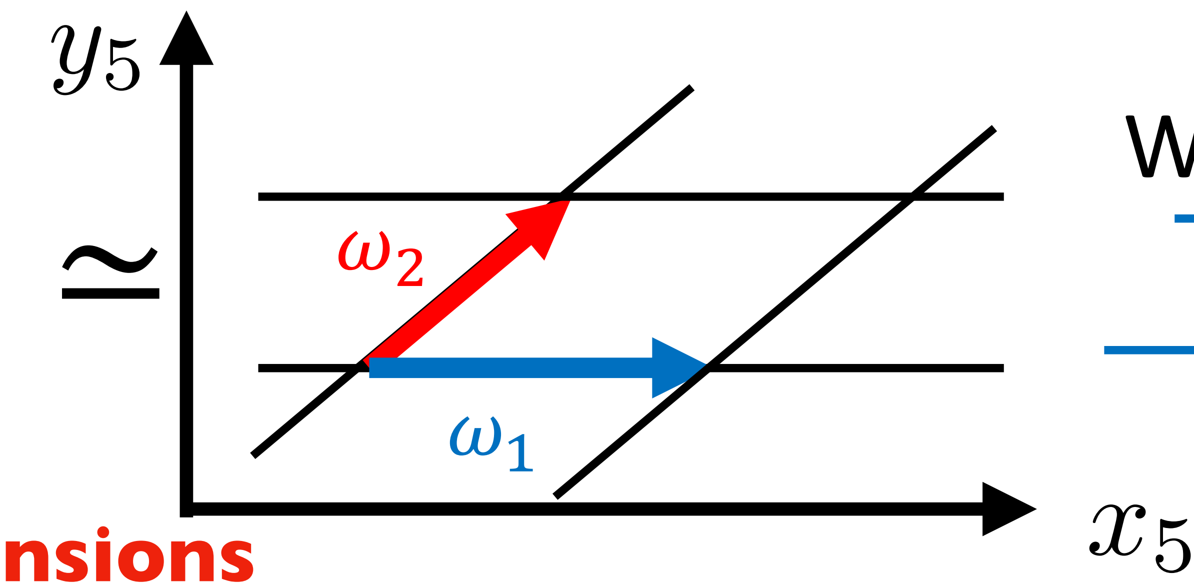
Origin of Plato's symmetry?

Possibility 2: Extra dimensions (string theory)

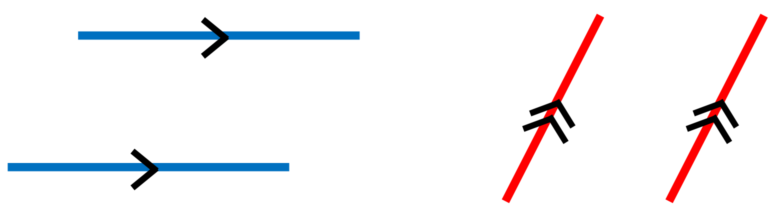
The structure of a torus $T^2 \simeq$ The structure of a lattice on $\mathbb{C}$-plane



$\simeq$



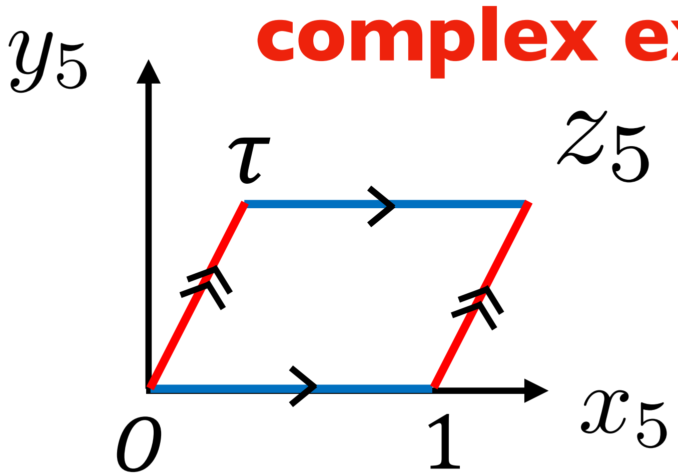
With identification



two extra dimensions compactified on torus

complex extra dimension

$$z_5 = x_5 + iy_5$$



Without loss of generality,

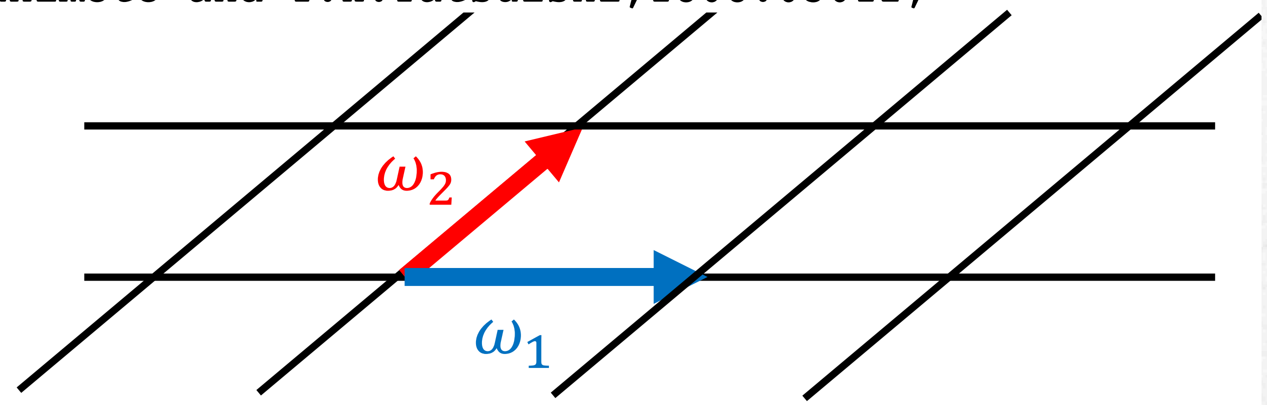
$$(\omega_1, \omega_2) \to \left(1, \frac{\omega_2}{\omega_1}\right) \equiv (1, \tau)$$

modulus

T.H.Tatsuishi

G.Altarelli and F.Feruglio, hep-ph/0512103
R.de Adelhart Toorop, F.Feruglio and C.Hagedorn, 1112.1340
F.Feruglio, 1706.08749; J.C.Criado and F.Feruglio, 1807.01125; J.T.Penedo and S.T.Petcov 1806.11040;
P.P.Novichkov, J.T.Penedo, S.T.Petcov and A.V.Titov, 1811.04933, 1812.02158;
T.Kobayashi, K.Tanaka and T.H.Tatsuishi,1803.10391; F.de Anda,S.F.K.,E.Perdomo,1812.05620
T.Kobayashi, N.Omoto, Y.Shimizu, K.Takagi, M.Tanimoto and T.H.Tatsuishi,1808.03012;
G.J.Ding, S.F.King and X.G.Liu, 1903.12588

There are two independent lattice invariant transformations.

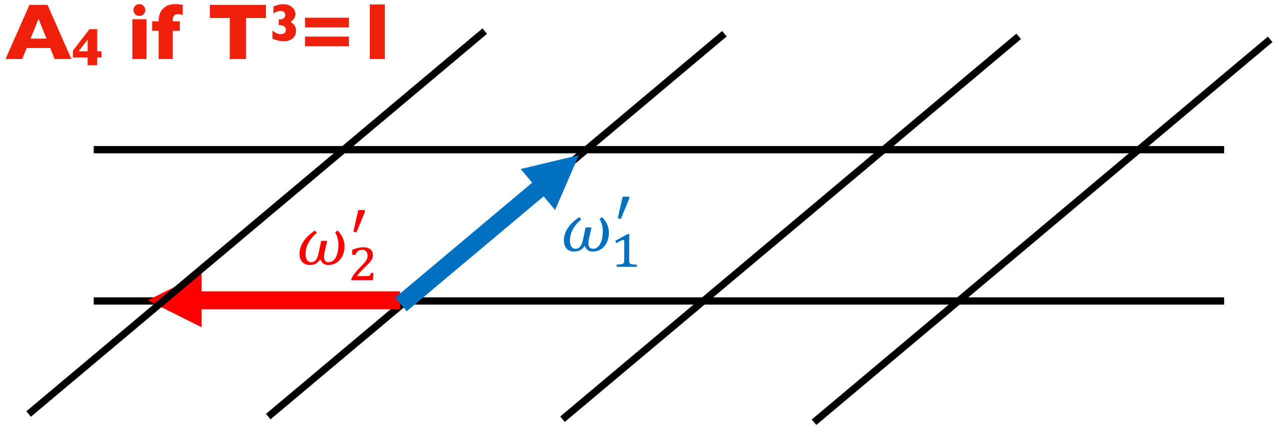


Modular Symmetry:

E.g. S, T are generators of A_4 if $T^3=1$

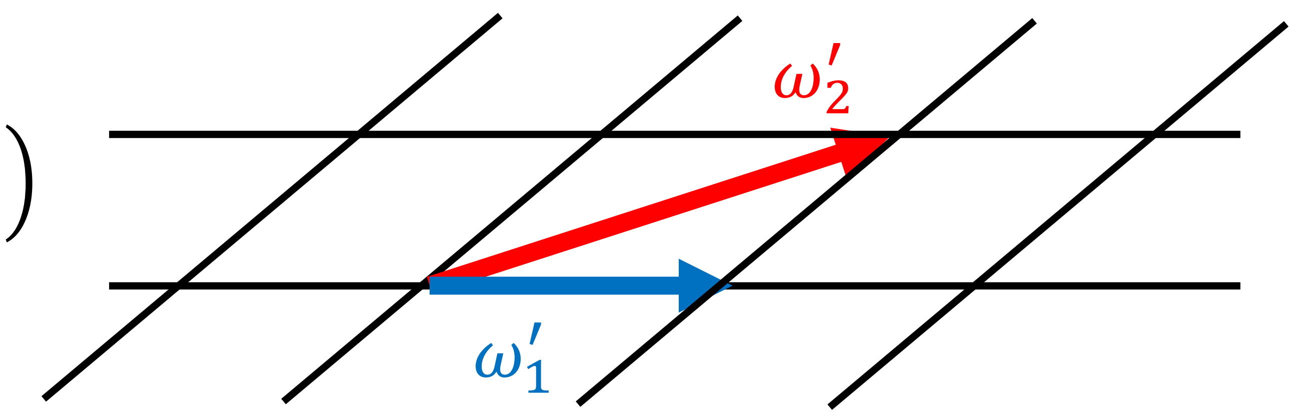
S-transformation

$$\begin{pmatrix}\omega_1' \\ \omega_2'\end{pmatrix} = \begin{pmatrix}0 & 1 \\ -1 & 0\end{pmatrix}\begin{pmatrix}\omega_1 \\ \omega_2\end{pmatrix} = \begin{pmatrix}\omega_2 \\ -\omega_1\end{pmatrix}$$



T-transformation

$$\begin{pmatrix}\omega_1' \\ \omega_2'\end{pmatrix} = \begin{pmatrix}1 & 0 \\ 1 & 1\end{pmatrix}\begin{pmatrix}\omega_1 \\ \omega_2\end{pmatrix} = \begin{pmatrix}\omega_1 \\ \omega_2+\omega_1\end{pmatrix}$$



Yukawa couplings involving twisted states whose modular weights do not add up to zero are modular forms

F.Feruglio, 1706.08749

E.g. Weight 2 Y acts as A_4 triplet:

$$\begin{aligned} Y_1(\tau) &= 1 + 12q + 36q^2 + 12q^3 + \dots \\ Y_2(\tau) &= -6q^{1/3}(1 + 7q + 8q^2 + \dots) \\ Y_3(\tau) &= -18q^{2/3}(1 + 2q + 5q^2 + \dots) \end{aligned}$$

$$q \equiv e^{i2\pi\tau}$$

Weinberg operator $\frac{1}{\Lambda}(H_u H_u \; LL \; Y) \longrightarrow m_\nu = \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix} \frac{v_u^2}{\Lambda}$

A_4: 3 3 3

Neutrino mass matrix depends on the complex modulus

E.g. Weight 6 $\tau = \omega$ gives mu-tau $Y = (-1, 2\omega, 2\omega^2)$

F.de Anda,S.F.K.,E.Perdomo,1812.05620

Conclusions

- Most parameters well measured in oscillation experiments…but…CP phase, octant, ordering? Also: Dirac or Majorana? Absolute masses?
- TB mixing explained by S_4…excluded by reactor angle…but…S_4 violations allow: charged lepton corrections, or TM1,TM2, with testable sum rules
- Mu-tau symmetry predicts $\theta_{23} = 45^\circ, \delta = -90^\circ$ Littlest mu-tau seesaw…one parameter…wow!
- Origin of Plato's symmetry: SO(3) or extra dims?