$H \to b\bar{b}j$ at Next-To-Next-To-Leading Order Accuracy

Roberto Mondini, University at Buffalo

RM and Ciaran Williams [arXiv:1904.08961]

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Introduction and motivation

A large part of the LHC experimental program consists in measuring the properties of the discovered Higgs boson in relation to the predictions of the Standard Model.

The Higgs boson decays mainly to bottom quarks and therefore precise predictions for $H \to b\bar{b}$ are mandatory to successfully compare theory and experiment (e.g. associated $V(H \to b\bar{b})$ production). Differentially, $H \to b\bar{b}$ is known at NNLO [Anastasiou, Herzog, Lazopoulos (2011); Del Duca, Duhr, Somogyi, Tramontano, Trócsányi (2015); Bernreuther, Cheng, Si (2018)].

Aim of our work:

1. Perform an independent computation of the two-loop amplitudes for the $H \to b\bar{b}g$ process [Ahmed, Mahakhud, Mathews, Rana, Ravindran (2014)].
2. Produce a NNLO Monte Carlo code for $H \to b\bar{b}j$ and establish whether the jet can be integrated out at NNLO (needed for $H \to b\bar{b}$ at N3LO accuracy).

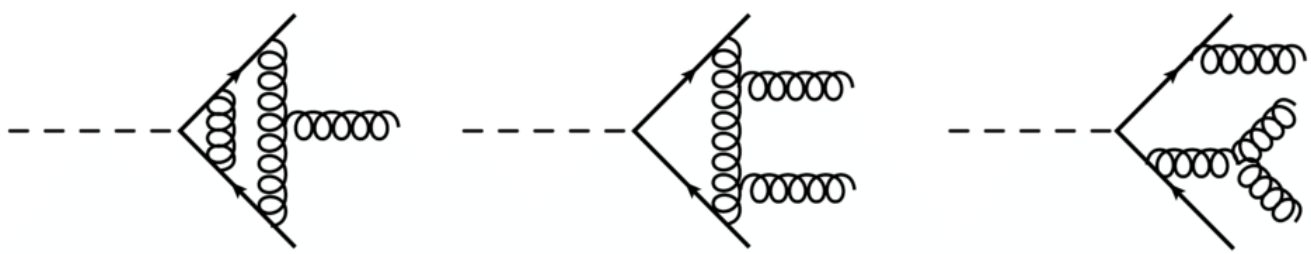
Overview of the calculation

The partial decay width is expanded as

$$\Gamma^{\mathrm{NNLO}}_{H\to b\bar{b}j} = \Gamma^{\mathrm{LO}}_{H\to b\bar{b}j} + \Delta\Gamma^{\mathrm{NLO}}_{H\to b\bar{b}j} + \Delta\Gamma^{\mathrm{NNLO}}_{H\to b\bar{b}j}$$

At NNLO:

- Two-loop amplitudes for $H \to b\bar{b}g$
- One-loop amplitudes for $H \to b\bar{b}gg$ and $H \to b\bar{b}q\bar{q}$ (including identical-quark terms)
- Tree-level amplitudes for $H \to b\bar{b}ggg$ and $H \to b\bar{b}q\bar{q}g$

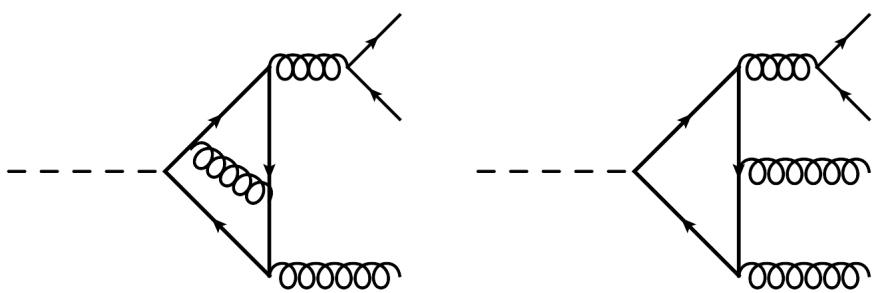


Overview of the calculation

We treat the bottom quark as **massless** in the phase space and kinematics but keep the mass in the bottom Yukawa coupling. In the *full* theory:

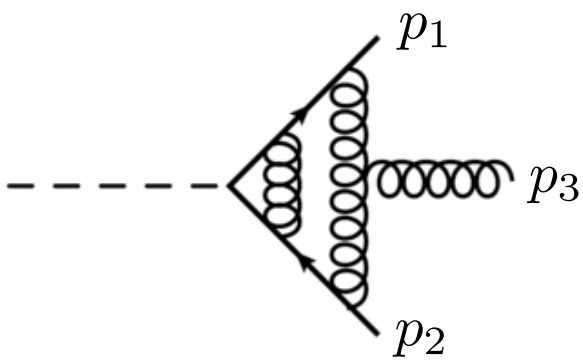
$$\Gamma^{\mathrm{NNLO}}_{H\to b\bar{b}j} \sim \alpha_s y_b^2\, A_b + \alpha_s^2 \left(y_b^2\, B_b + y_t y_b\, B_{tb}\right) + \alpha_s^3 \left(y_b^2\, C_b + y_t^2\, C_t + y_t y_b\, C_{tb}\right) + \mathcal{O}(\alpha_s^4)$$

In the *massless* theory, the mixed terms are zero. We also neglect the (phenomenologically-relevant) top Yukawa contribution.



Two-loop amplitudes for $H \to b\bar{b}g$

- Generate Feynman diagrams:



$$\sim \int \frac{\mathrm{d}^d k_1 \mathrm{d}^d k_2}{(2\pi)^{2d}} \frac{\bar{u}(p_1)\{\gamma^\mu, \not{p}_i, \not{k}_i\} v(p_2)}{D_1\, D_2\, D_3\, D_4\, D_5\, D_6\, D_7}$$

$$D_1 = k_1^2 \quad D_2 = k_2^2 \quad D_3 = (k_1 - k_2)^2 \quad D_4 = (k_2 - p_1)^2$$
$$D_5 = (k_2 - p_1 - p_3)^2 \quad D_6 = (k_2 - p_1 - p_2 - p_3)^2 \quad D_7 = (k_1 - p_1 - p_2 - p_3)^2$$

- Use projectors to rewrite the amplitude in terms of two scalar coefficients:

$$\mathcal{M} = i \left(\frac{\alpha_s}{2\pi}\right)^{\frac{1}{2}} \frac{y_b}{m_H^2}\, \mathbf{T}^a_{ij}\, \epsilon_\mu(p_3)\, [A_1\, T_1^\mu + A_2\, T_2^\mu]$$

$$T_1^\mu = \bar{u}(p_1) \not{p}_3\, \gamma^\mu\, v(p_2) \qquad T_2^\mu = \left[p_1^\mu - \frac{t}{u} p_2^\mu\right] \bar{u}(p_1)\, v(p_2)$$

Two-loop amplitudes for $H \to b\bar{b}g$

- Reduce the scalar integrals in the coefficients A_1 and A_2 to an irreducible set of master integrals (MIs) using LiteRed [Lee (2013)]

- All required two-loop MIs are known in the literature: planar and non-planar topologies [Gehrmann, Remiddi (2001)]

- Results are written in terms of HPLs and 2dHPLs [Gehrmann, Remiddi (2001)] of up to weight 4:

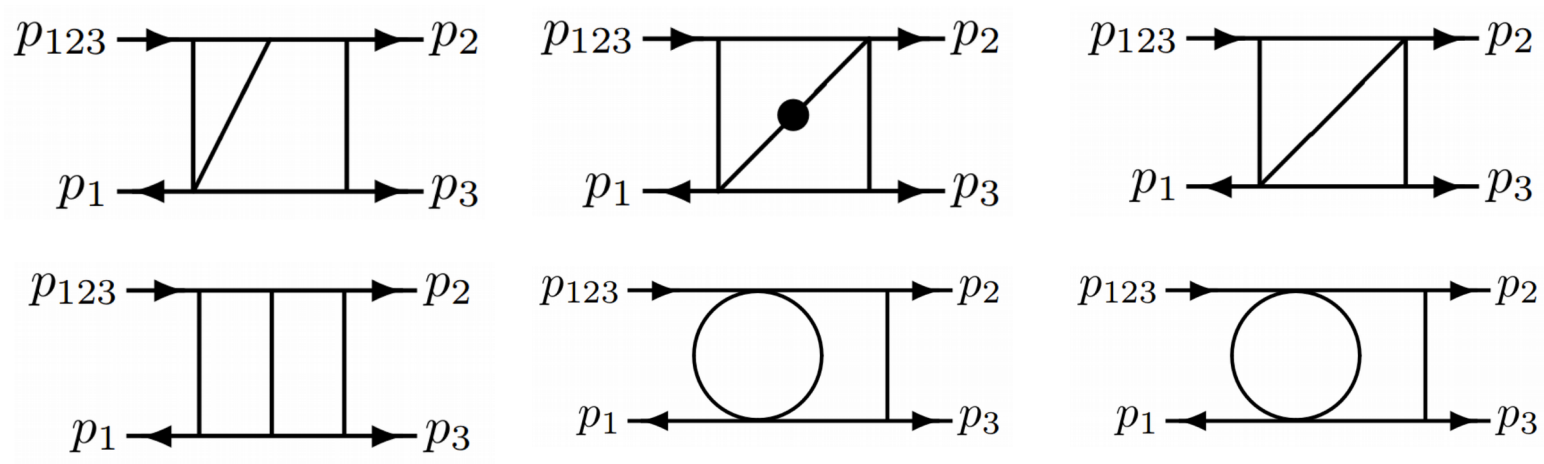
$$H(\vec{m}_z; z) \qquad \vec{m}_z = \{0, 1\}$$
$$G(\vec{m}_y; y) \qquad \vec{m}_y = \{0, 1, z, 1-z\}$$

where we defined

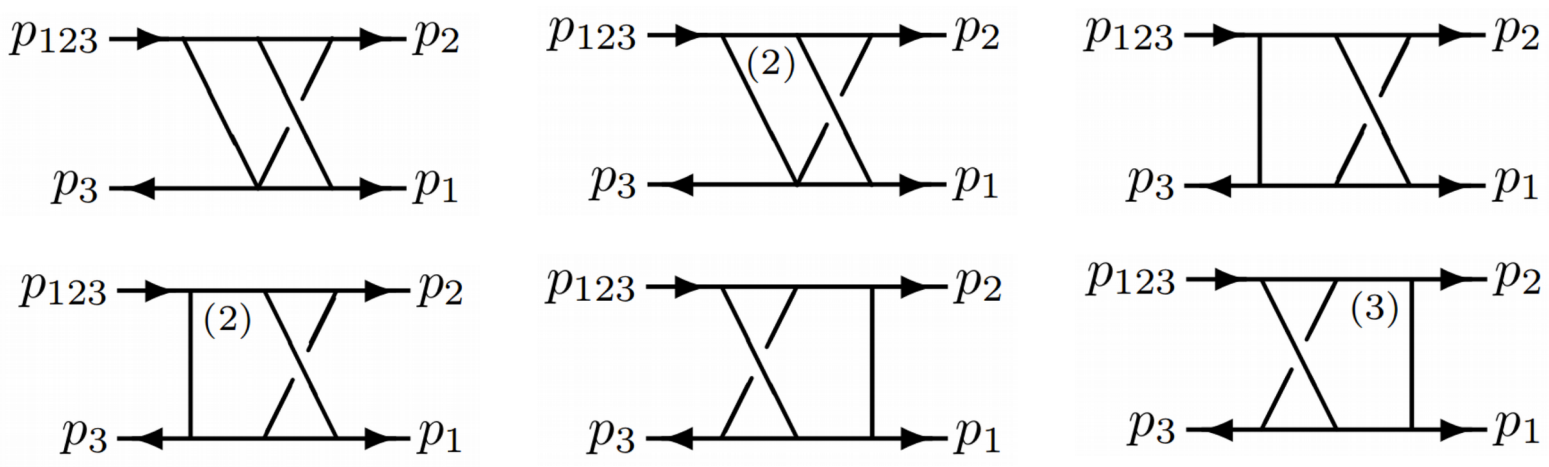
$$y = \frac{t}{m_H^2} \qquad z = \frac{u}{m_H^2} \qquad x = \frac{s}{m_H^2} = 1 - y - z$$
$$0 < y < 1 \qquad 0 < z < 1 \qquad 0 < x < 1$$

Two-loop amplitudes for $H \to b\bar{b}g$

Examples of planar MIs:



Examples of non-planar MIs:



Two-loop amplitudes for $H \to b\bar{b}g$

Checks on our two-loop amplitude

- The IR poles agree with the expected IR structure from Catani's *subtraction operators*:

$$\mathcal{M}^{(2),\mathrm{fin}} = \mathcal{M}^{(2)} - I^{(1)}(\epsilon)\mathcal{M}^{(1)} - I^{(2)}(\epsilon)\mathcal{M}^{(0)}$$

- The finite part of the amplitude agrees with an existing result in the literature [Ahmed, Mahakhud, Mathews, Rana, Ravindran (2014)]

- Additionally, we checked that our two-loop amplitude reproduces the known IR factorization properties of QCD when the external gluon becomes either soft or collinear to one of the quarks

N-jettiness slicing

We have implemented our NNLO calculation into a fully-flexible parton-level Monte Carlo code based on MCFM [Campbell, Ellis, Williams].

We regulate the IR divergences in our calculation by using the **N-jettiness slicing** method [Gaunt, Stahlhofen, Tackmann, Walsh (2015); Boughezal, Focke, Liu, Petriello (2015)]. For a parton-level event we define the 3-jettiness variable [Stewart, Tackmann, Waalewijn (2010)]

$$\tau_3 = \sum_{j=1,m} \min_{i=1,2,3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\}$$

- The index j runs over the m partons in the phase space
- The momenta q_i are the momenta of the three most energetic jets
- $Q_i = 2E_i$ with E_i the energy of the i-th jet.

N-jettiness slicing

We introduce a variable $\tau_3^{\rm cut}$ that separates the phase space into two regions:

- The region $\tau_3 < \tau_3^{\rm cut}$ contains all of the *doubly-unresolved* regions of phase space and here the decay width is approximated using this factorization theorem from SCET [Stewart, Tackmann, Waalewijn]:

$$\Gamma_{H\to b\bar{b}j}\left(\tau_3 < \tau_3^{\rm cut}\right) \approx \int \prod_{i=1}^{3} \mathcal{J}_i \otimes \mathcal{S} \otimes \mathcal{H} + \mathcal{O}(\tau_3^{\rm cut})$$

Jet functions
[Becher, Neubert (2006)]

Soft function
[Campbell, Ellis, RM, Williams (2017)]

Hard function
(finite part of the two-loop amplitudes)

- The region $\tau_3 > \tau_3^{\rm cut}$ contains the *singly-unresolved* and *fully-resolved* regions of phase space and therefore corresponds to the NLO calculation of $H \to b\bar{b}jj$. Can be computed using any NLO code.

Results

In order to compute jet rates and distributions we use the **Durham jet algorithm**. Starting at the parton level, for every pair of partons (i,j):

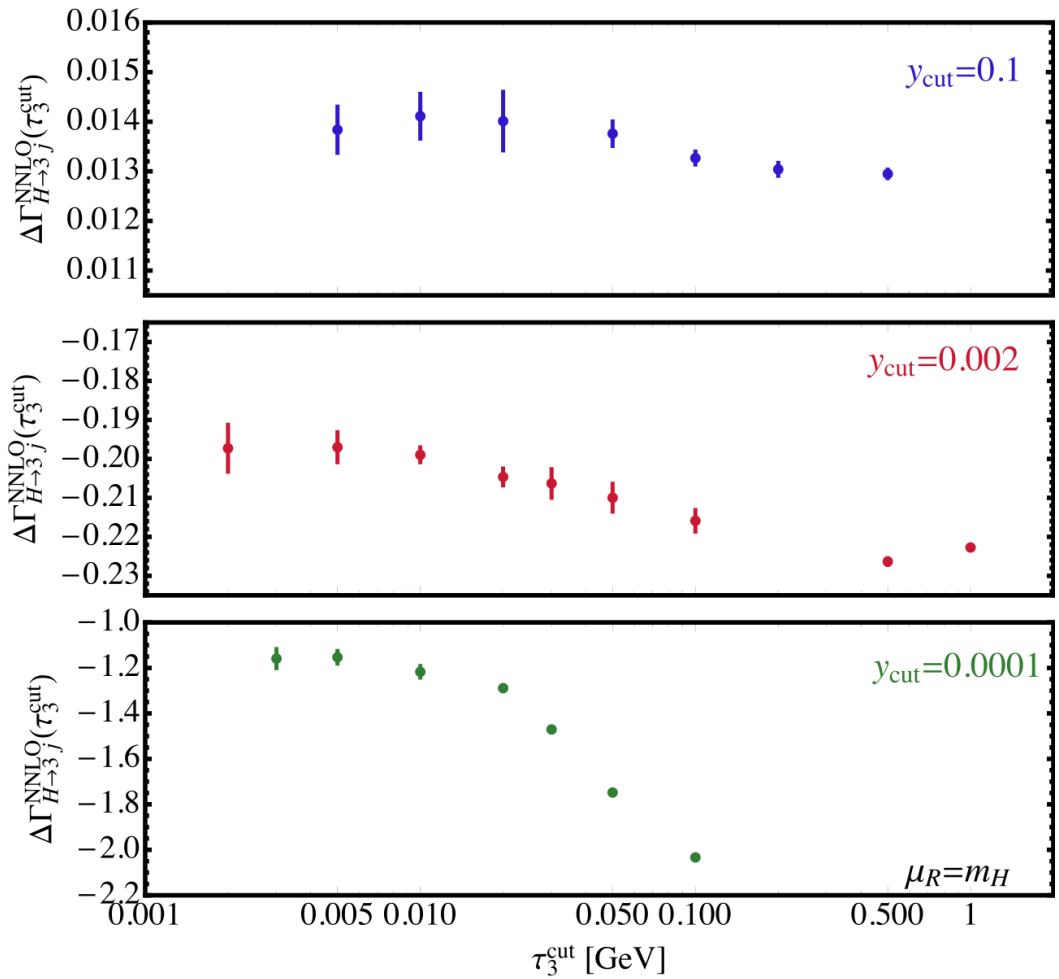
$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{Q^2}$$

If $y_{ij} < y_{cut}$ the pairs are combined into a new object with momentum $p_i + p_j$. The algorithm then repeats until no further clusterings are possible and the remaining objects are classified as jets.

We present results in the Higgs *rest frame*.

Results

We validate our calculation by studying the dependence of the NNLO coefficient on the unphysical parameter $\tau_3^{\rm cut}$ for three clustering options



Asymptotic behavior is established in each region. We set $\tau_3^{\rm cut} = 0.01$ GeV for all subsequent results

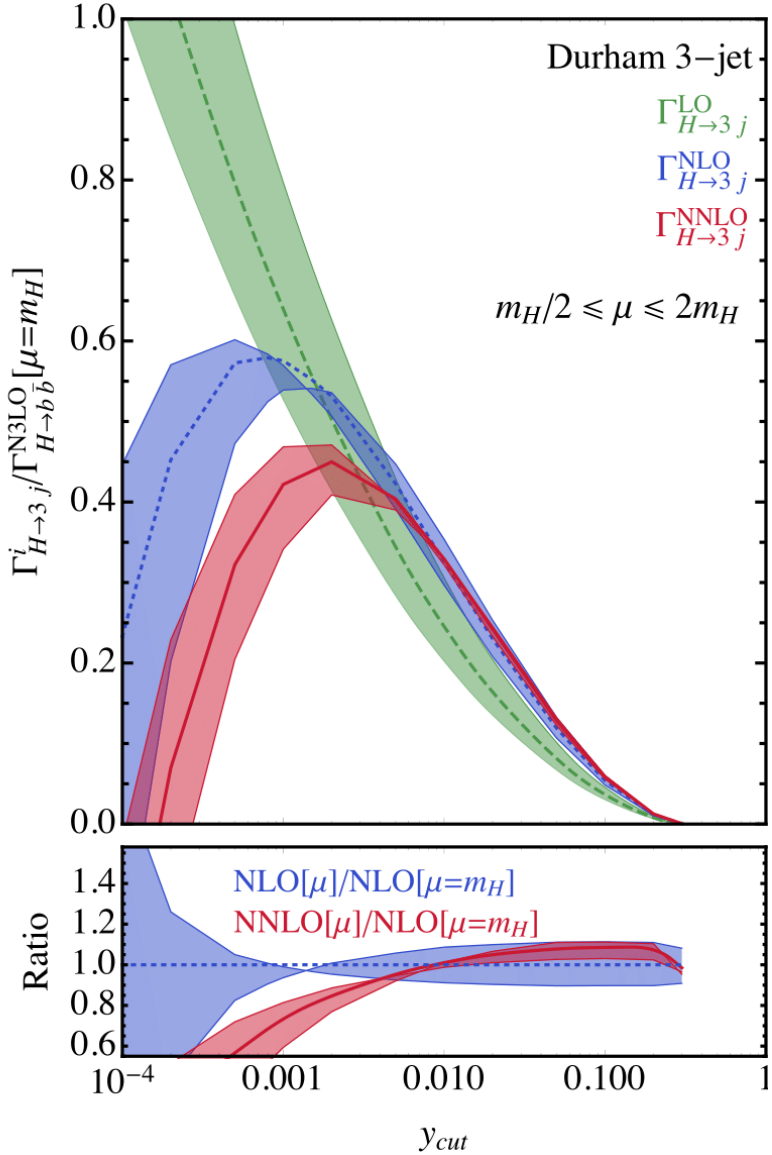
$y_{\rm cut} = 0.0001$ corresponds to imposing a very weak jet cut

Results

Three-jet rate at LO, NLO, and NNLO as function of y_{cut}

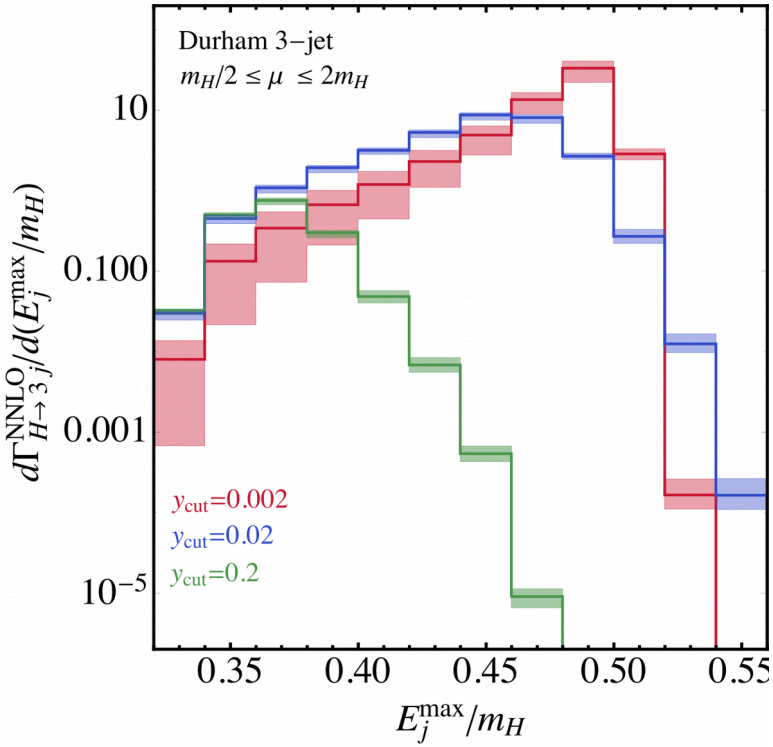
The observed pattern is similar to the results obtained for $e^+e^- \to$ jets [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich (2008); Weinzierl (2008)]

The inclusion of the NNLO corrections reduces the overall scale dependence, especially in the region $y_{cut} > 0.01$



Results

Differential distribution for the maximum energy of the jets in three-jet events



We have also computed distributions for small values of $y_{cut} = 0.0002$. The code can produce stable distributions with small MC uncertainties even in this region.

Summary

- We have presented a calculation of $H \to b\bar{b}j$ at NNLO focusing on the contribution in which the Higgs boson couples directly to massless bottom quarks.

- We have performed an independent calculation of the $H \to b\bar{b}g$ two-loop amplitudes and checked that they reproduce the IR factorization properties of QCD amplitudes.

- We have implemented our results into a Monte Carlo code using the N-jettiness slicing technique to regulate the IR divergences. We have produced differential distributions and jet rates using the Durham jet algorithm in the Higgs rest frame.

- We have established that a stable Monte Carlo code can be constructed even for very small jet cuts. We are therefore able to integrate out the jet at NNLO and use our results in the calculation of $H \to b\bar{b}$ at N3LO **(see Ciaran's talk)**.

Extra slides

Two-loop amplitudes for $H \to b\bar{b}g$

Soft-gluon limit: $p_3 \to 0$ which means $y,z \to 0$ simultaneously

$$2\,\mathrm{Re}\left(\mathcal{M}^{(2)}_{H\to b\bar{b}g}\mathcal{M}^{(0)*}_{H\to b\bar{b}g}\right) \to 2\,\mathrm{Re}\Big(S^{(0)}(y,z)\mathcal{M}^{(2)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}} + S^{(1)}(y,z)\mathcal{M}^{(1)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}} + S^{(2)}(y,z)\mathcal{M}^{(0)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}}\Big)$$

$y = z = 10^{-10}$

Coefficient	Known limit	Our result
ϵ^{-4}	81.7702729678	81.7702729678
ϵ^{-3}	3818.49680411	3818.49680413
ϵ^{-2}	130763.8079162	130763.8079168
ϵ^{-1}	$3.26338843478 \cdot 10^6$	$3.26338843480 \cdot 10^6$
ϵ^{0}	$6.52342650778 \cdot 10^7$	$6.52342650793 \cdot 10^7$

Two-loop amplitudes for $H \to b\bar{b}g$

Collinear limit: t→0 which means y→0 while z is fixed

$$2\,\mathrm{Re}\left(\mathcal{M}^{(2)}_{H\to b\bar{b}g}\mathcal{M}^{(0)*}_{H\to b\bar{b}g}\right) \to 2\,\mathrm{Re}\Big(C^{(0)}(y,z)\mathcal{M}^{(2)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}} + C^{(1)}(y,z)\mathcal{M}^{(1)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}} + C^{(2)}(y,z)\mathcal{M}^{(0)}_{H\to b\bar{b}}\mathcal{M}^{(0)*}_{H\to b\bar{b}}\Big)$$

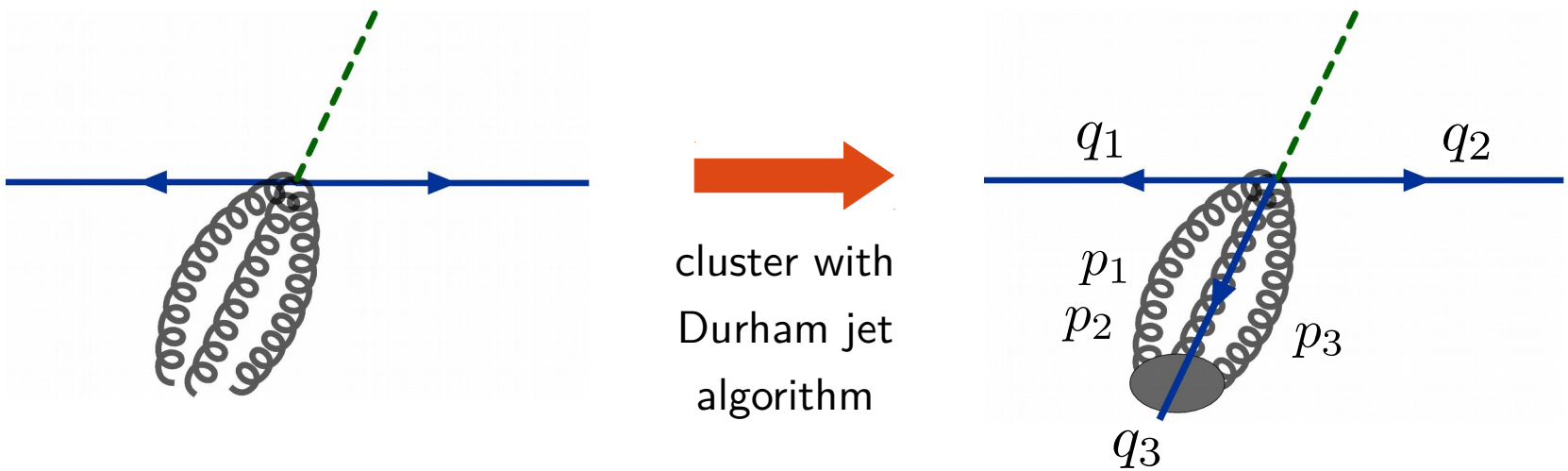
$y = 10^{-12}$

$z = 0.23$

Coefficient	Known limit	Our result
ϵ^{-4}	283.156234427	283.156234427
ϵ^{-3}	8122.55721506	8122.55721505
ϵ^{-2}	170379.942318	170379.942317
ϵ^{-1}	$2.584146 \cdot 10^6$	$2.584189 \cdot 10^6$
ϵ^{0}	$3.09852 \cdot 10^7$	$3.09870 \cdot 10^7$

N-jettiness slicing

$$H \to 3j \text{ at NNLO}$$



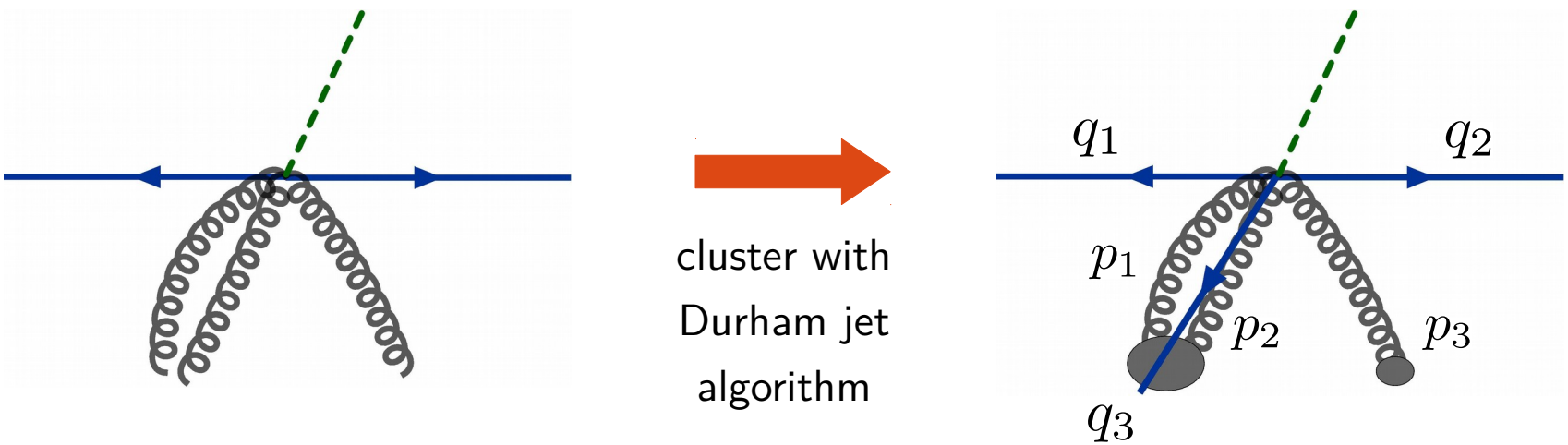
$$\tau_3 = \sum_{j=1,m} \min_{i=1,2,3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\} \approx 0$$

Doubly-unresolved region

All radiation is either soft or collinear

N-jettiness slicing

$$H \to 3j \text{ at NNLO}$$



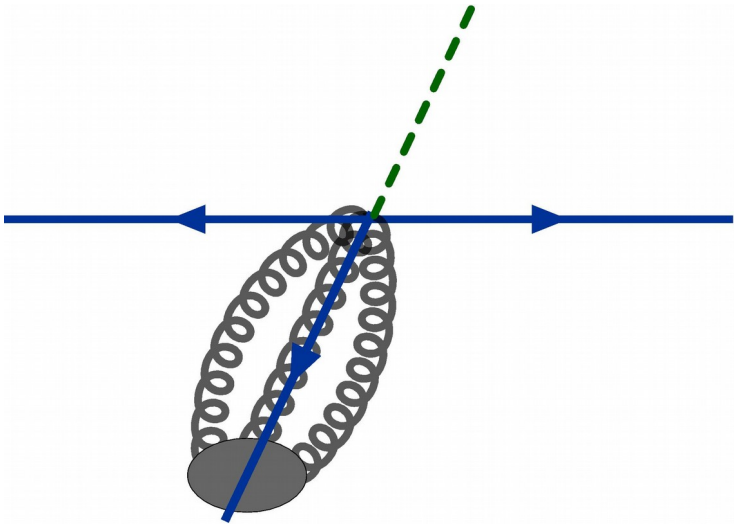
$$\tau_3 = \sum_{j=1,m} \min_{i=1,2,3} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\} > 0$$

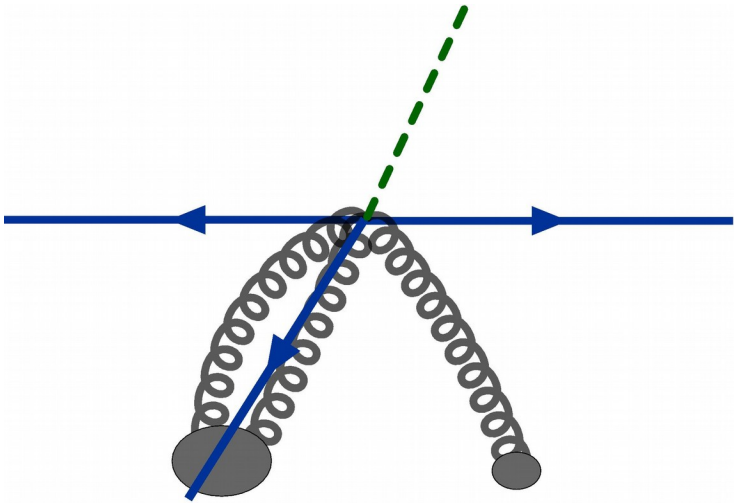
Singly-unresolved region

At least one parton is resolved

N-jettiness slicing

$$H \to 3j \text{ at NNLO}$$





Doubly-unresolved region $\tau_3 \approx 0$

All radiation is either soft or collinear

Introduce τ_3^{cut} and use factorization theorem for region $\tau_3 < \tau_3^{\text{cut}}$

Singly-unresolved region $\tau_3 > 0$

At least one parton is resolved

NLO calculation of $H \to 4j$ in the region $\tau_3 > \tau_3^{\text{cut}}$