University at Buffalo

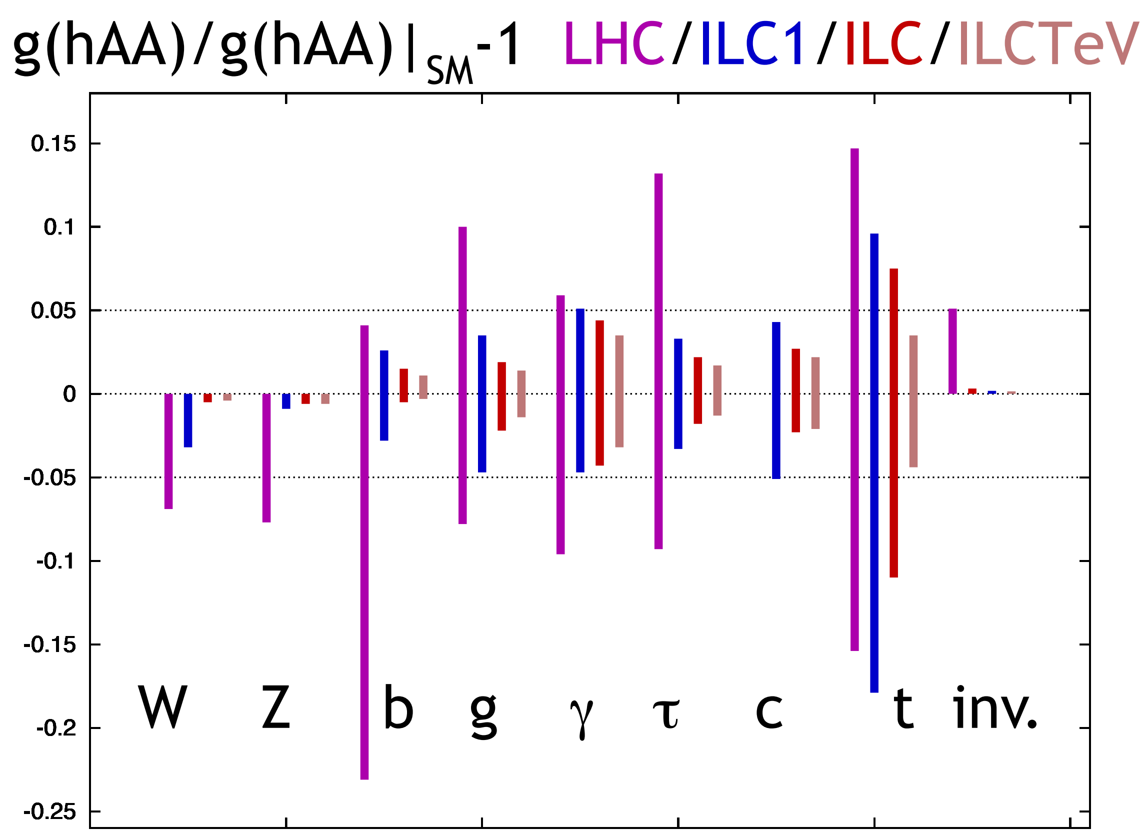
$H \to b\bar{b}$ @ $\mathrm{N}^3\mathrm{LO}$

Ciaran Williams (Buffalo)

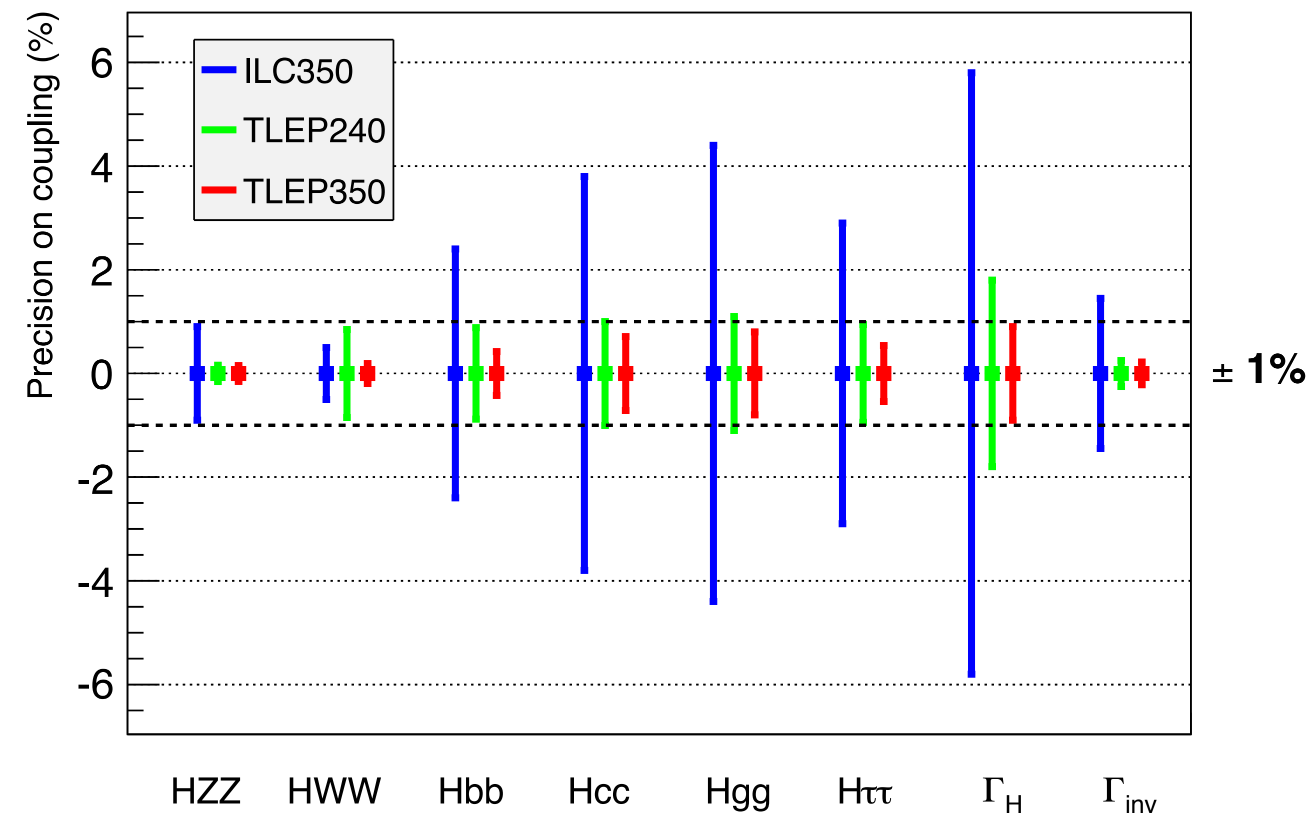
1904.08960 (with R. Mondini and M. Schiavi)
1904.08961 (with R. Mondini)

We're grateful for funding from
the National Science Foundation

Motivation



TLEP 1308.6176



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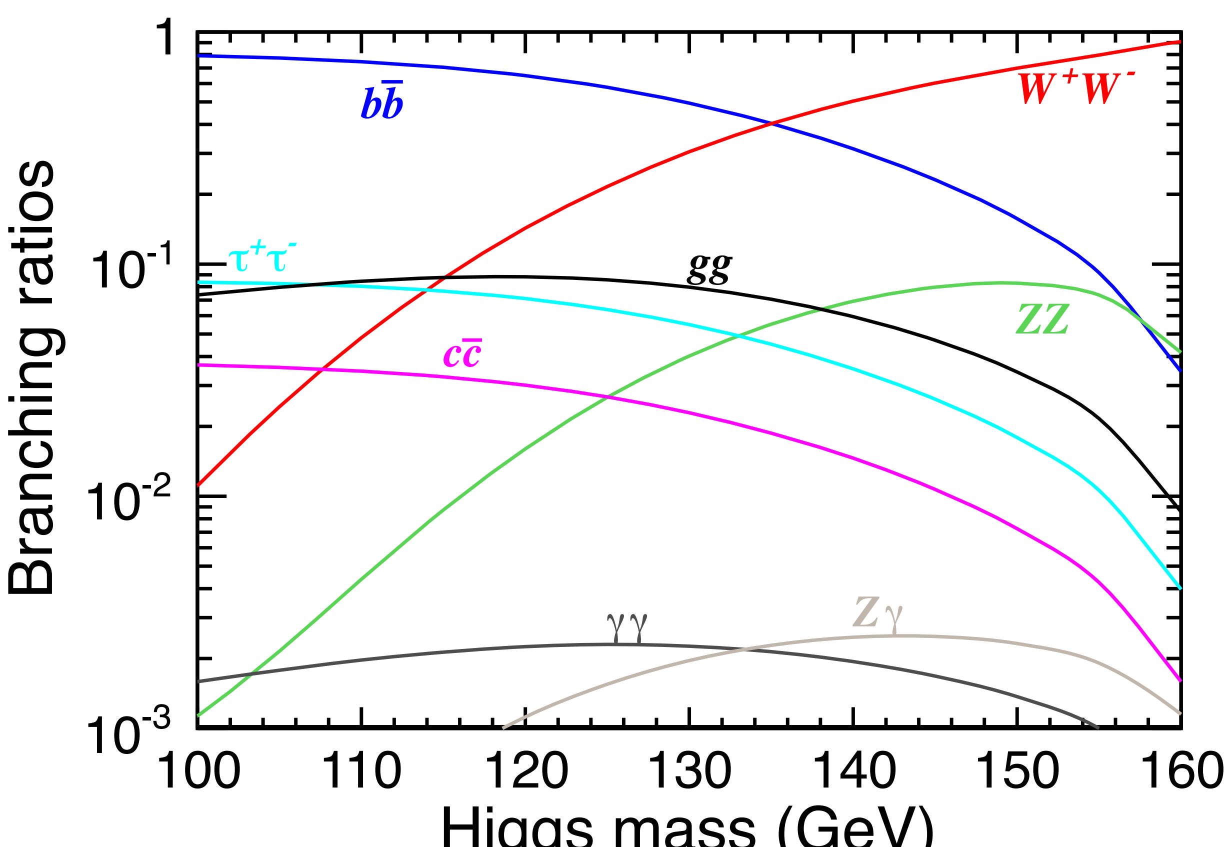
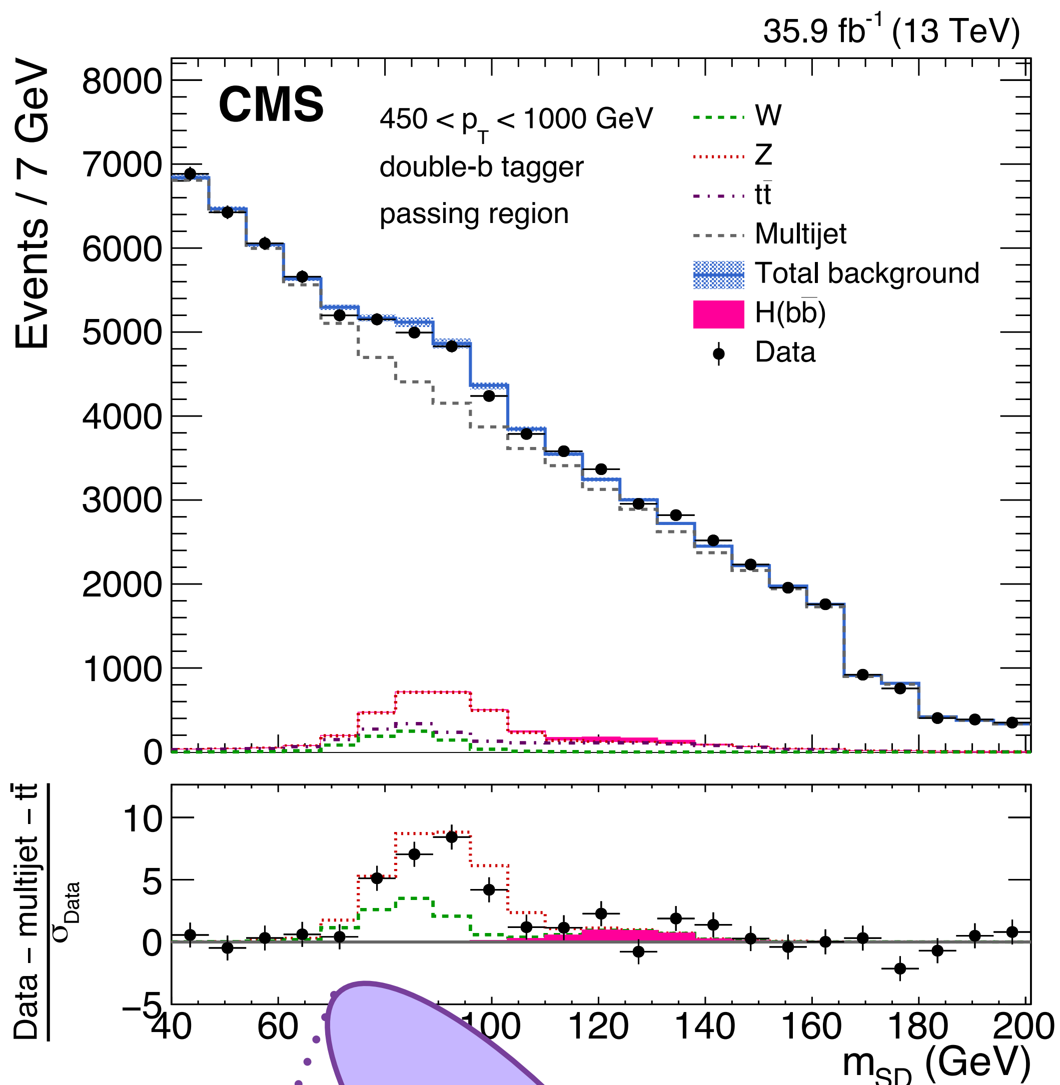


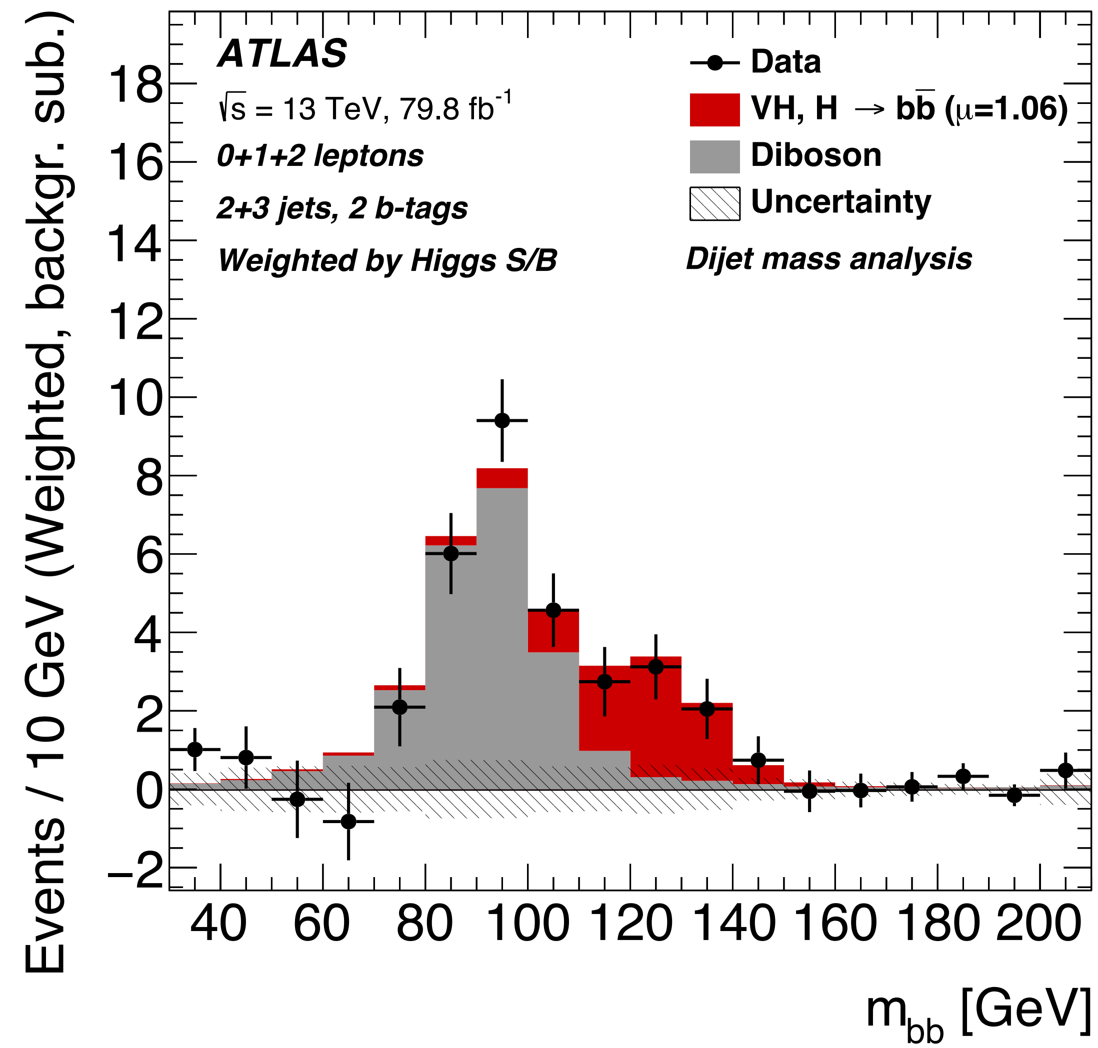
H=>bb is the largest BR for the Higgs.

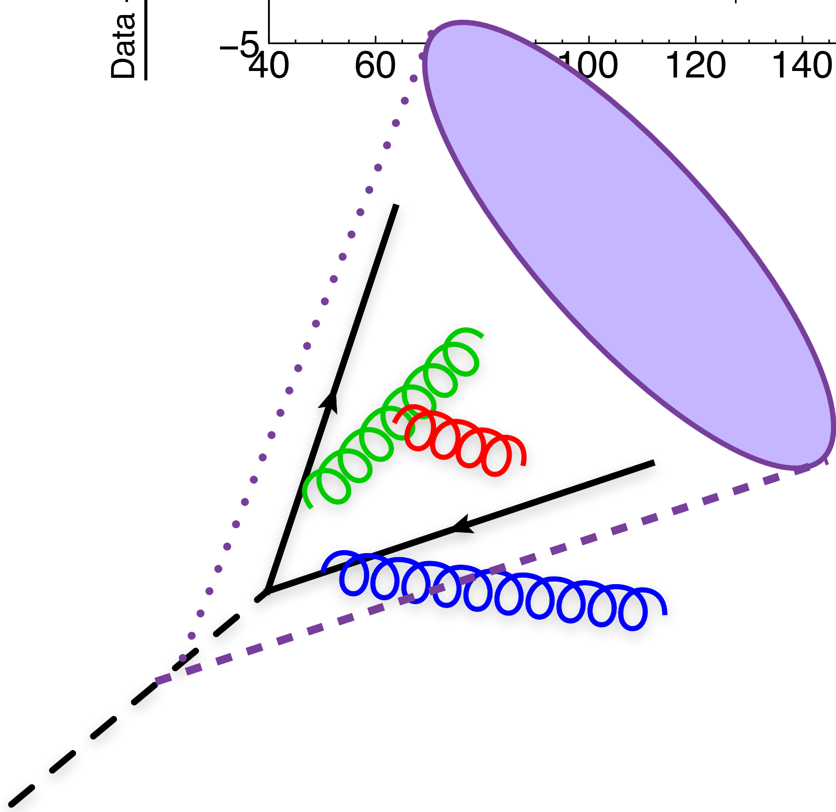
It effects every (on-shell) Higgs measurement through the width.

Will be measured, to sub percentage at a future Lepton Collider

Motivation



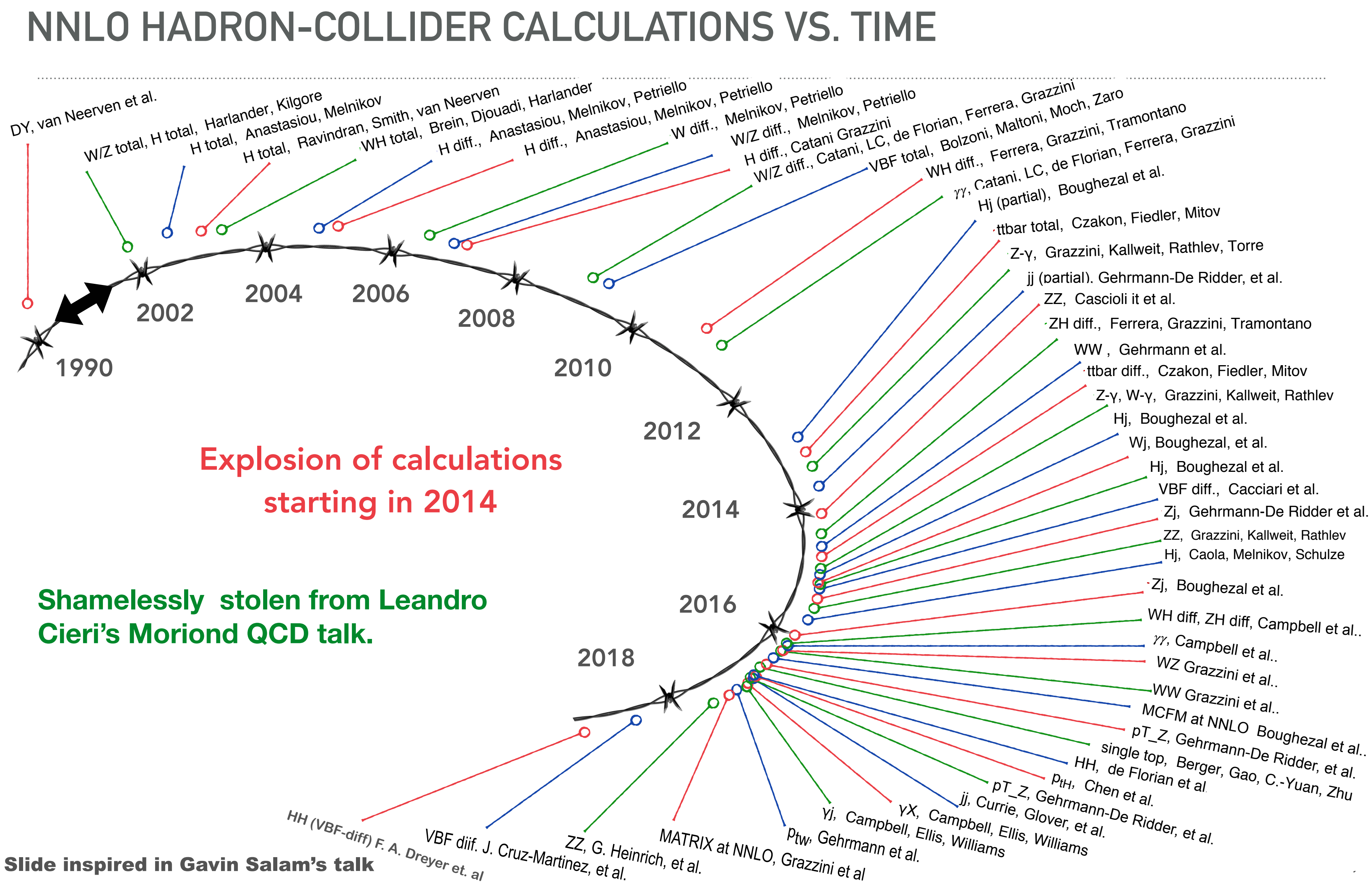




Traditional and boosted searches at the LHC also need precise QCD predictions.

Motivation

Its good to have an indoor project



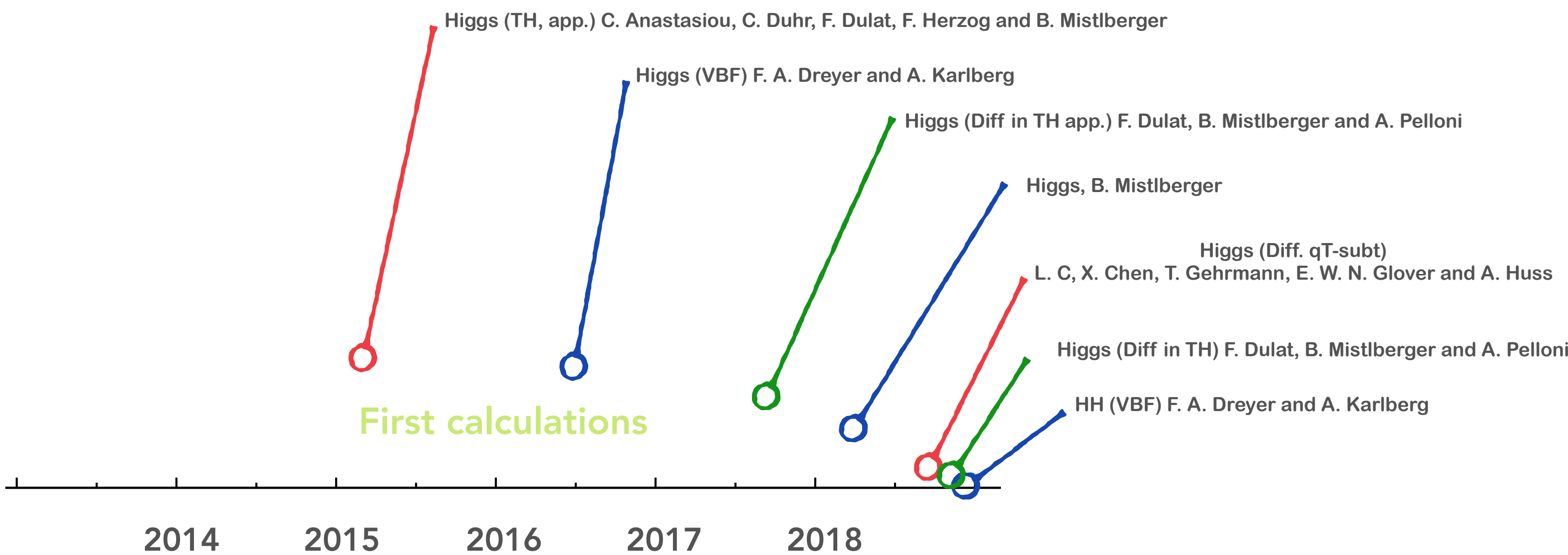
Progress in QCD

NNLO 2->2 for LHC now standard.

N3LO

N^3LO HADRON-COLLIDER CALCULATIONS VS. TIME

Shamelessly stolen from Leandro Cieri's Moriond QCD talk.



N3LO is still something we are learning to do!

Handful of different calculations for cross sections.

Higgs production known differentially.

N3LO

Coefficient of N3LO correction to the width is

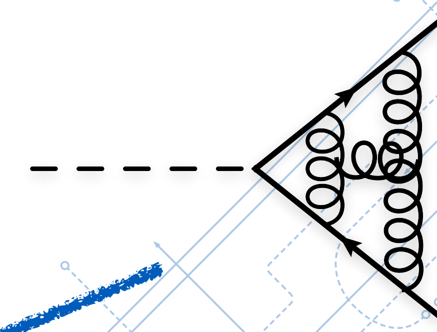
$$\frac{d\,\Delta\Gamma^{\rm N3LO}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = \int d\Gamma^{VVV}_{H\to b\bar{b}} F_2^m(\Phi_2) d\Phi_2$$

$$+ \int d\Gamma^{RVV}_{H\to b\bar{b}} F_3^m(\Phi_3) d\Phi_3$$

$$+ \int d\Gamma^{RRV}_{H\to b\bar{b}} F_4^m(\Phi_4) d\Phi_4$$

$$+ \int d\Gamma^{RRR}_{H\to b\bar{b}} F_5^m(\Phi_5) d\Phi_5$$

N3LO



"triple-virtual" 3-loops, 2 partons

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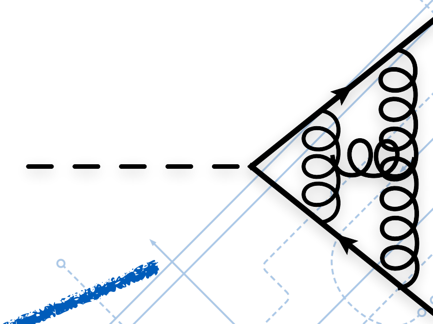

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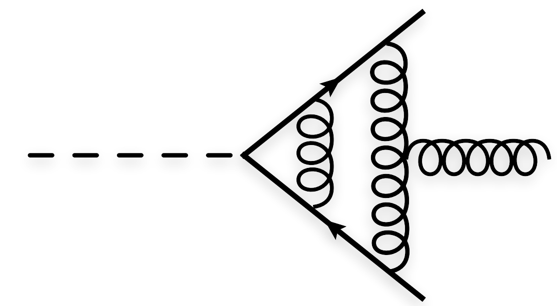

real double-virtual (2-loops 3 partons)

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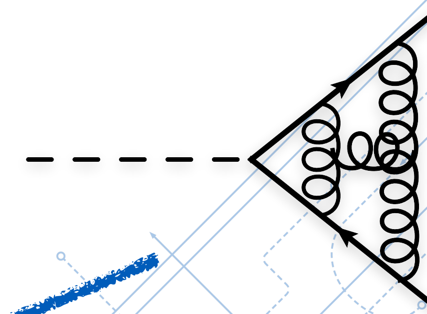

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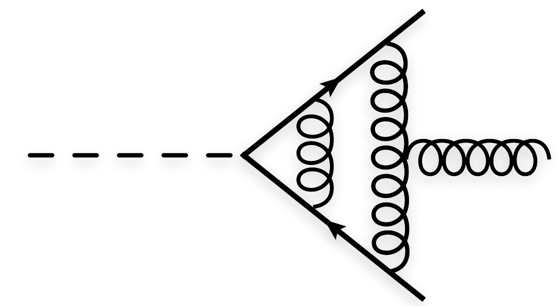

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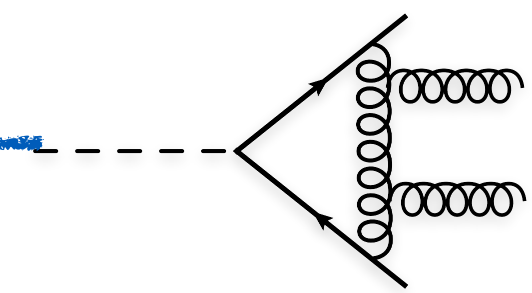

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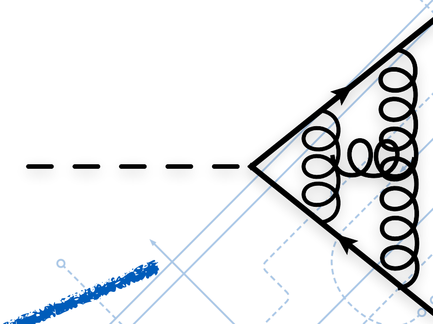

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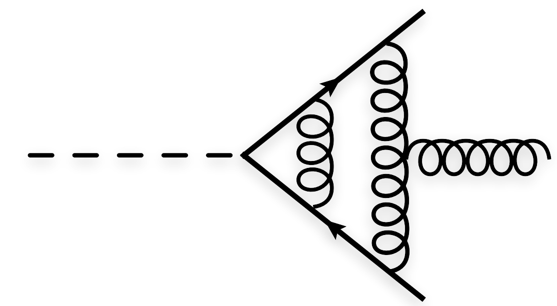

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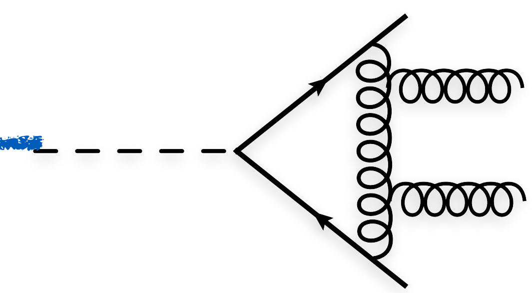

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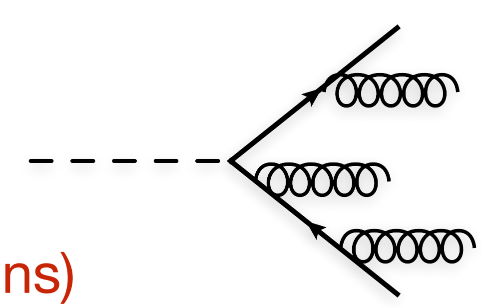

tripe real -virtual (0-loops 5 partons)

$F_i^m(\Phi_i)$ defines the measurement function and tell us how to make m-jets out of i partons.

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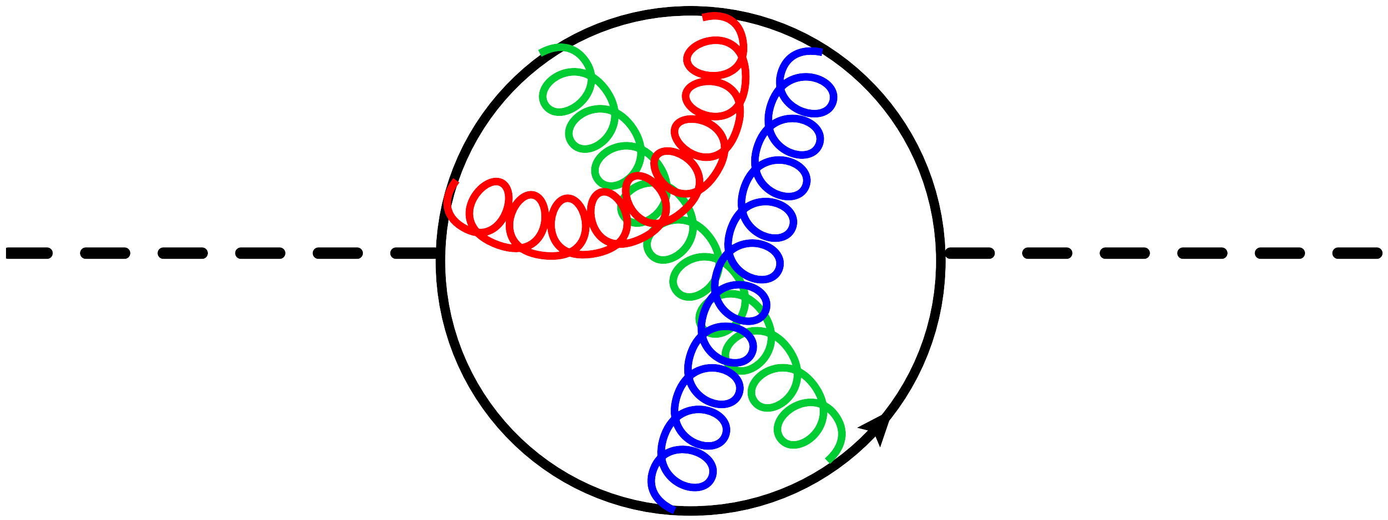
Inclusive Width

The measurement function adds nearly all of the complexity. If I'm only interested in the total decay rate I can remove it and obtain the inclusive (partial) width.

Can obtain the inclusive partial width coefficient by using the optical theorem applied to the 4-loop two-point correlator



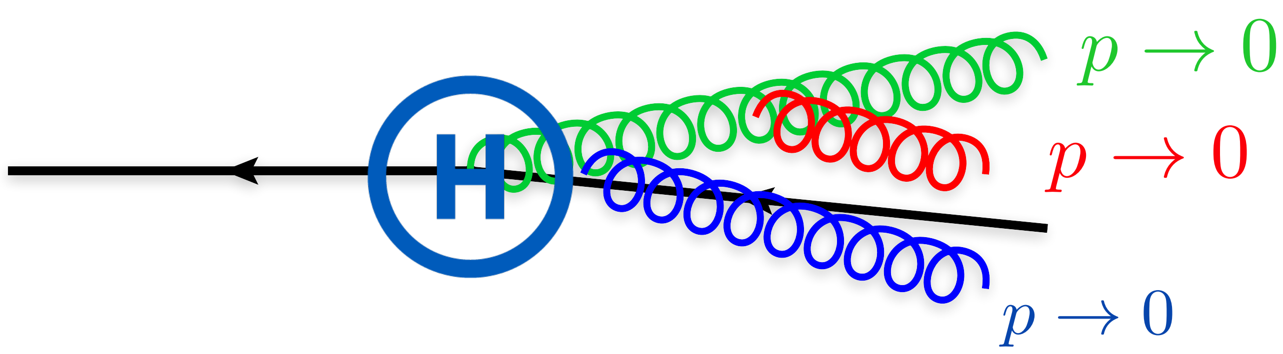
$$\Gamma^{(3)}_{H\to b\bar{b}} = s_3 + L\left(2s_2\beta_0 + s_1\beta_1 + 2s_2\gamma_m^0 + 2s_1\gamma_m^1 + 2\gamma_m^2\right)$$
$$+L^2\left(s_1\beta_0^2 + 3s_1\beta_0\gamma_m^0 + \beta_1\gamma_m^0 + 2s_1(\gamma_m^0)^2 + 2\beta_0\gamma_m^1 + 4\gamma_m^0\gamma_m^1\right)$$
$$+L^3\left(\frac{2}{3}\beta_0^2\gamma_m^0 + 2\beta_0(\gamma_m^0)^2 + \frac{4}{3}(\gamma_m^0)^3\right).$$

Chetyrkin hep-ph/9608318

Infrared issues

The measurement function makes life difficult since it exposes us to a multitude of Infrared singularities, which exist in the individual parton phase spaces, but cancel upon combination into a suitably inclusive observable.

For example consider the triple-soft limit in which all of the gluons in this diagram have vanishing momentum.



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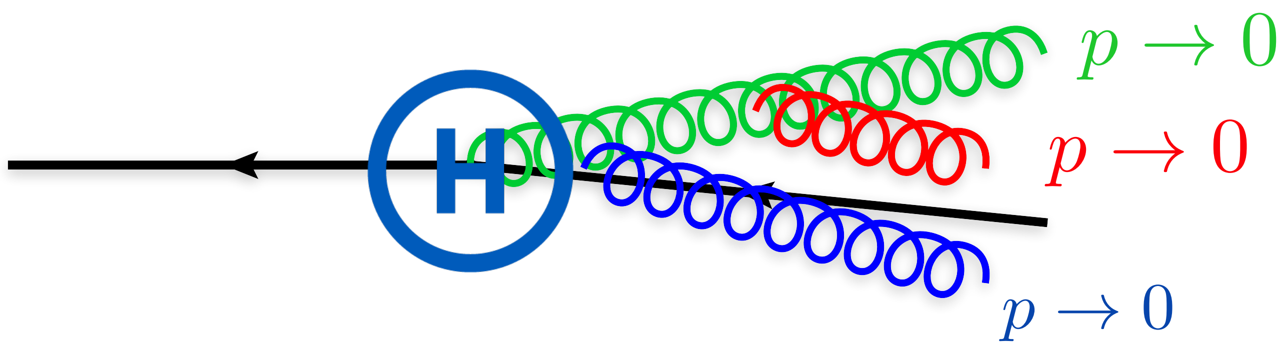

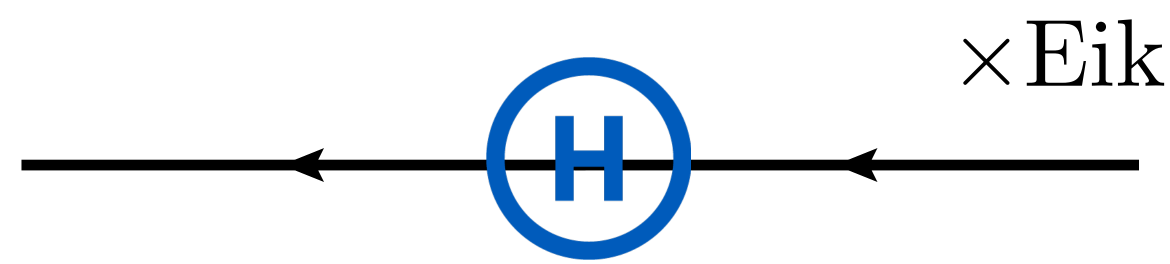

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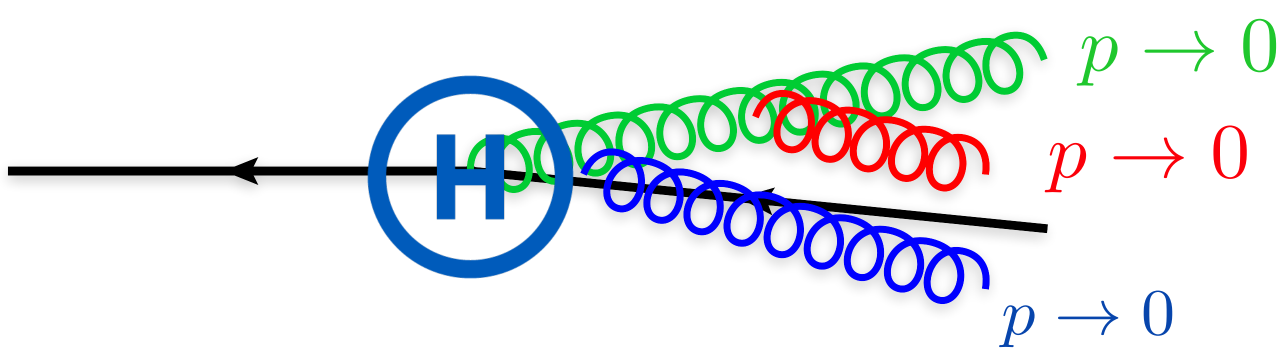

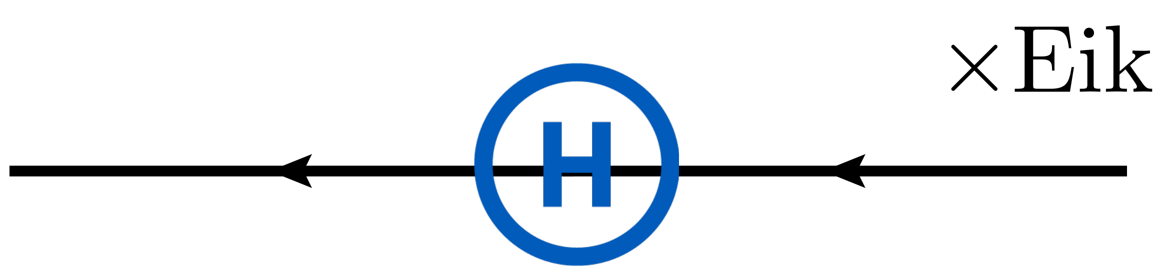

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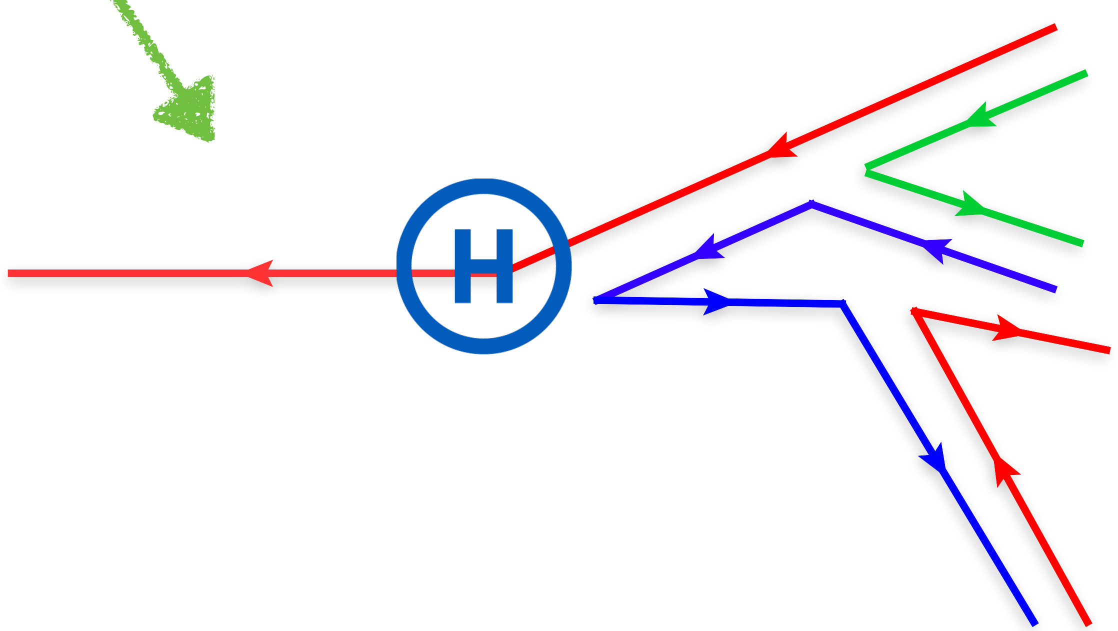
But the color is still present!



Projection to Born

There is no one set technique for dealing with IR issues at either NLO, NNLO or N3LO.

We’re going to use Projection to Born Method **(Cacciari, Dreyer, Karlberg, Salam, Zanderighi 1506.02660)**

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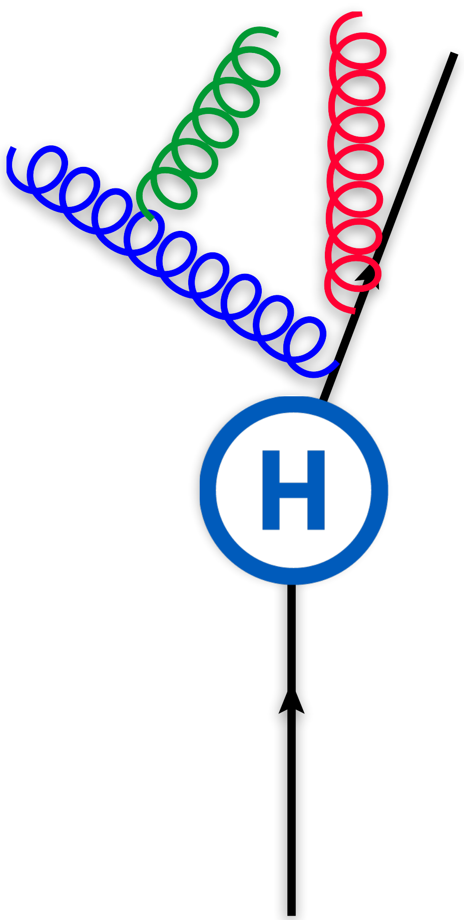

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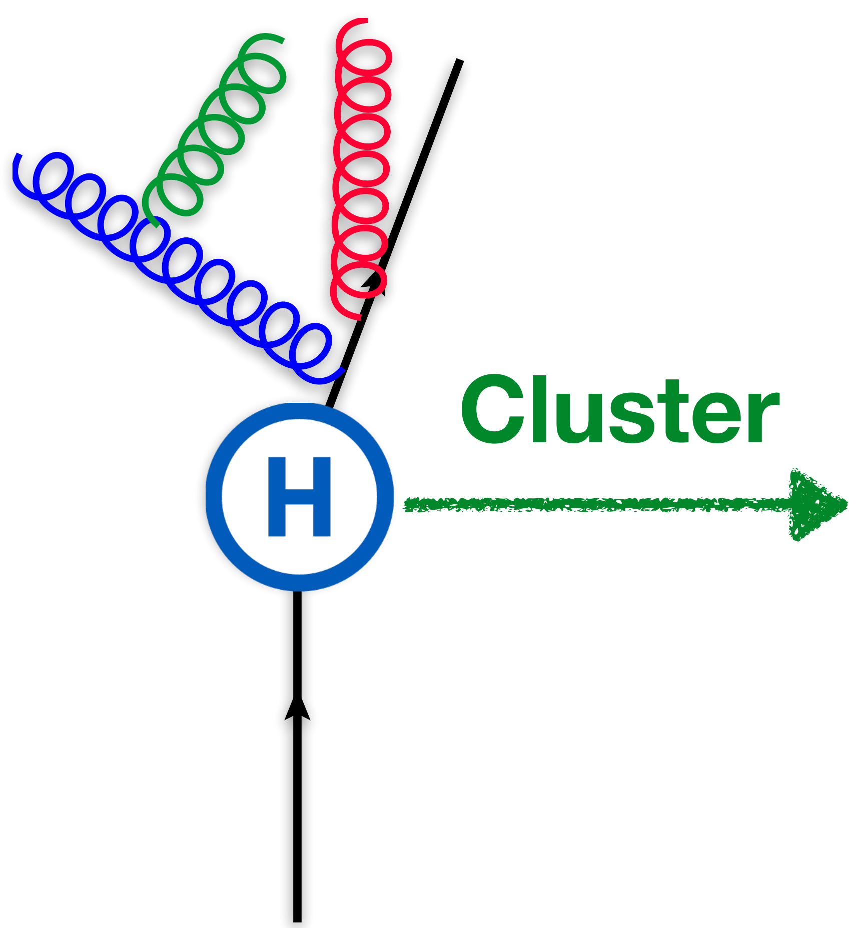

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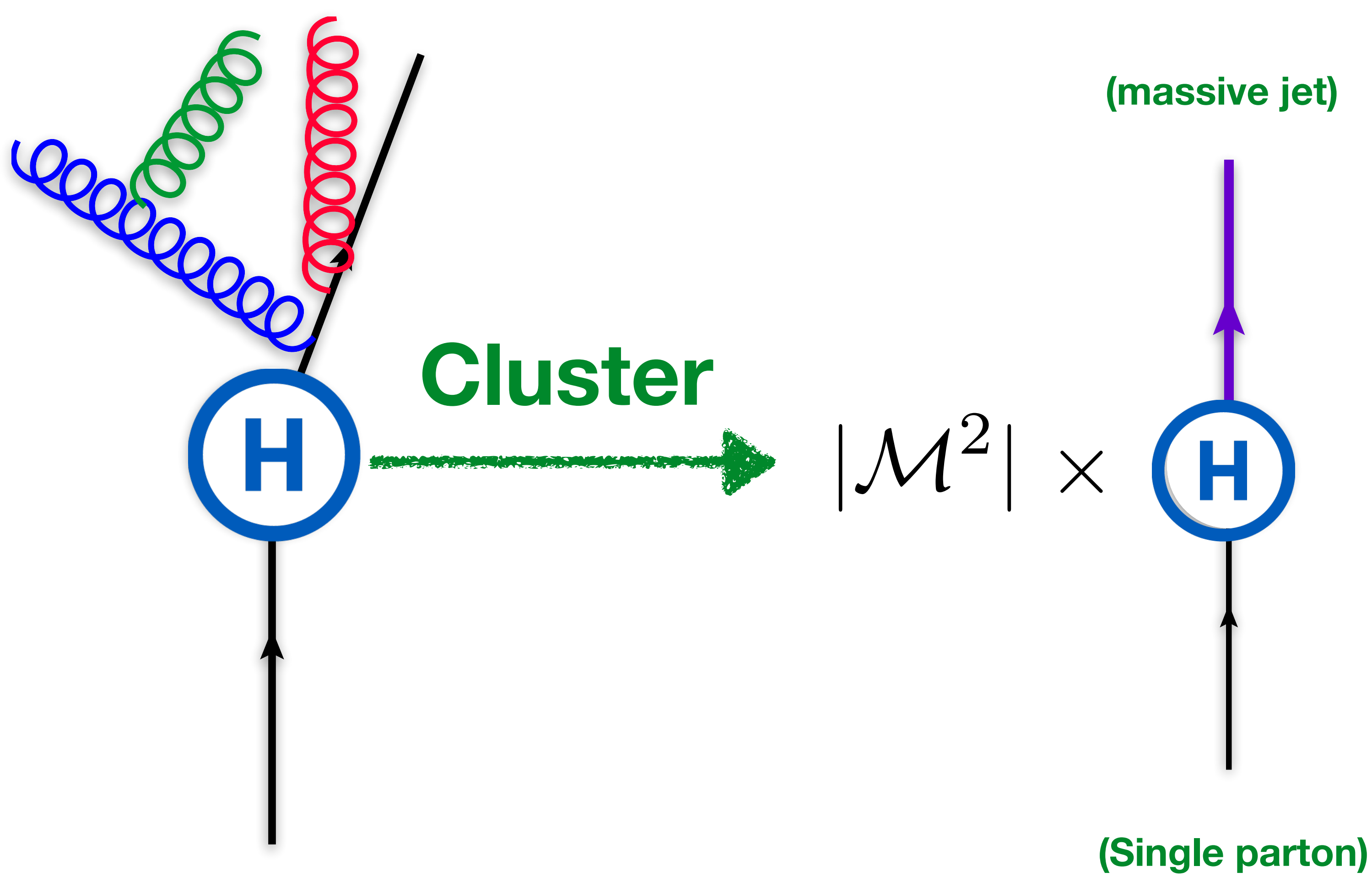

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$F_2^5(\Phi_5)$

University at Buffalo

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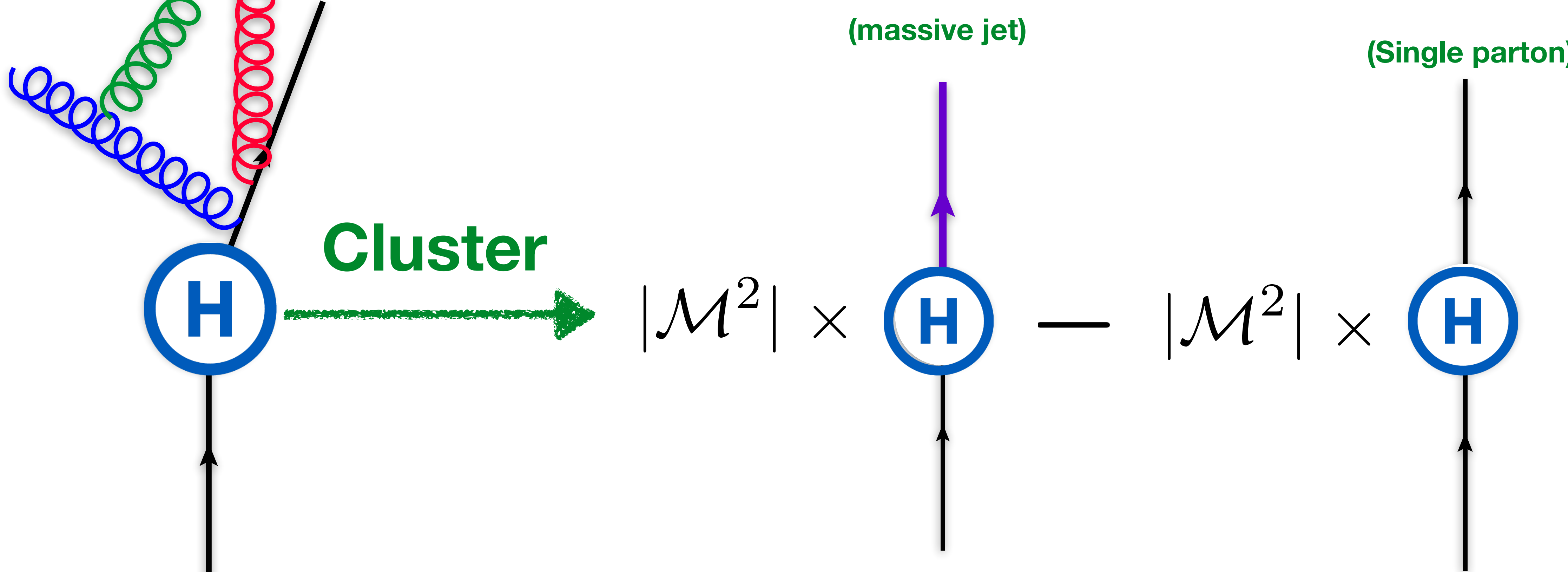
Subtract Counterterm



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$F_2^5(\Phi_5)$ $F_2^2(\Phi_B)$

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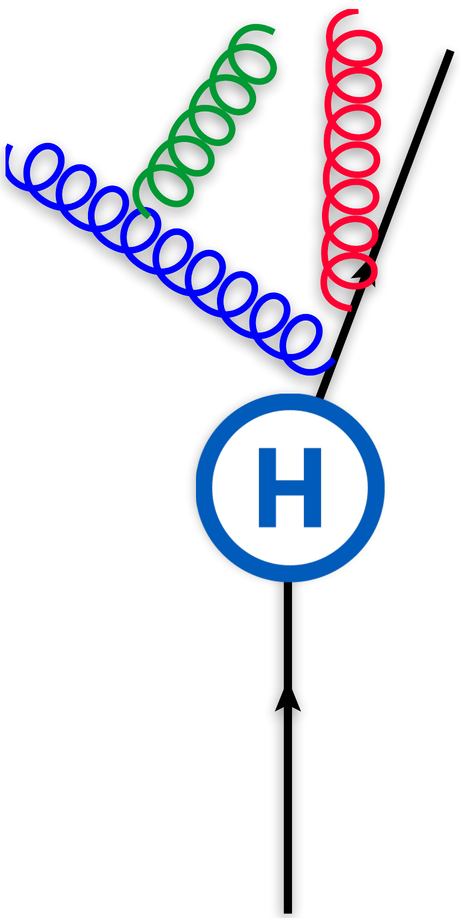
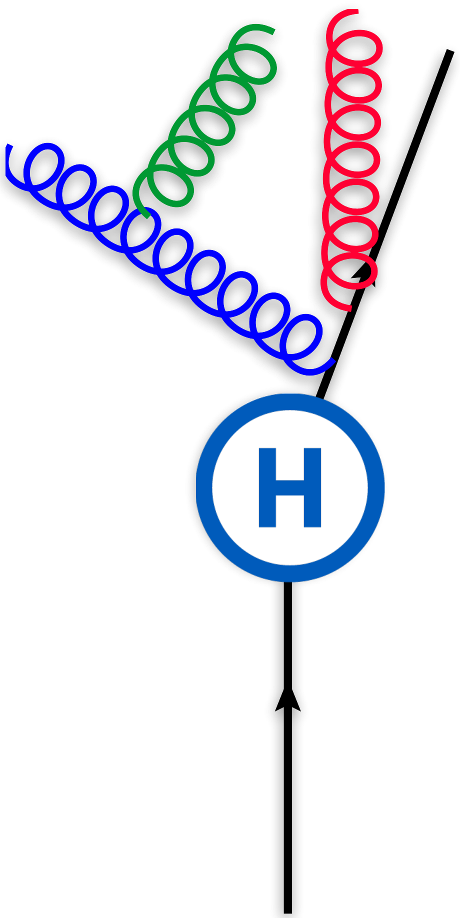


$$|\mathcal{M}^2| \times \left(F_2^5(\Phi_5) - F_2^2(\Phi_B) \right)$$

Cancels exactly when the full phase space **matches** the projected born one.

Projection to Born



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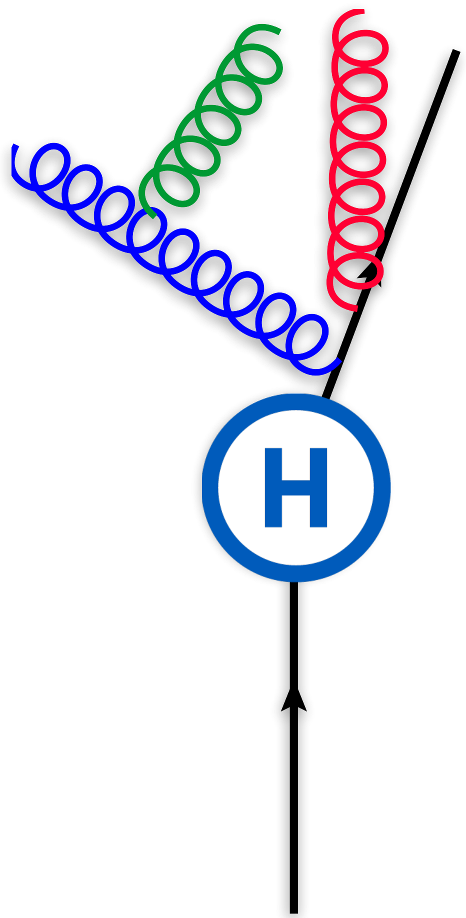

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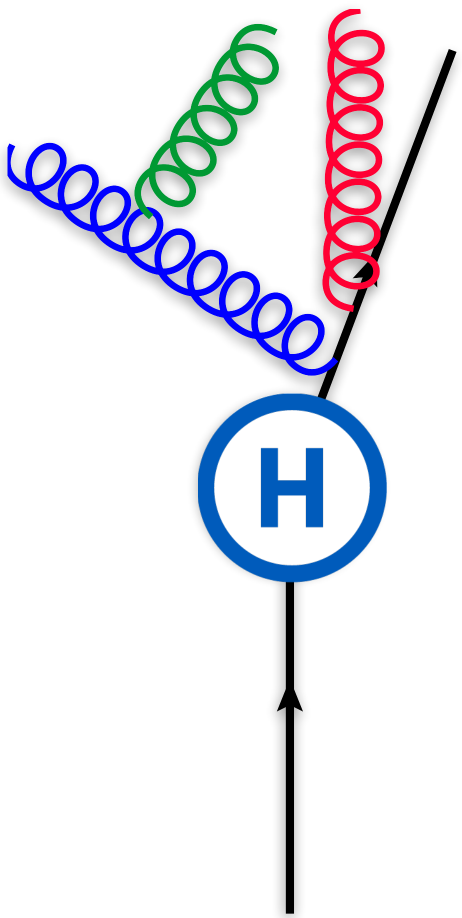

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This is an **NNLO** calculation.

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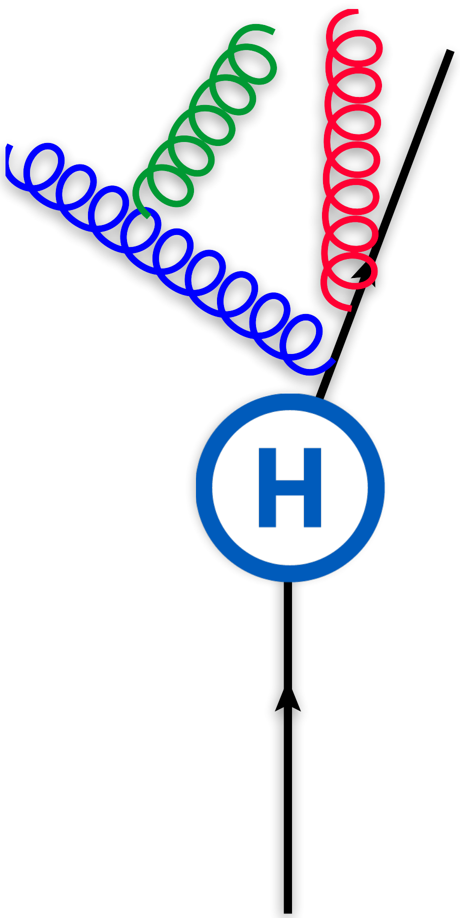

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This is an **NNLO** calculation.

So ingredient 1 for a projection to Born Method is an NNLO calculation of H=>3j (Mondini's talk)

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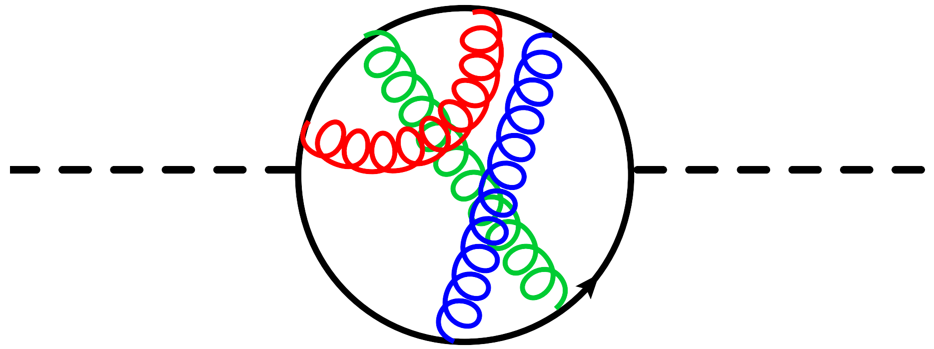
So ingredient 2 for a projection to Born Method is the inclusive partial width (cross section) as a function of the LO kinematics.



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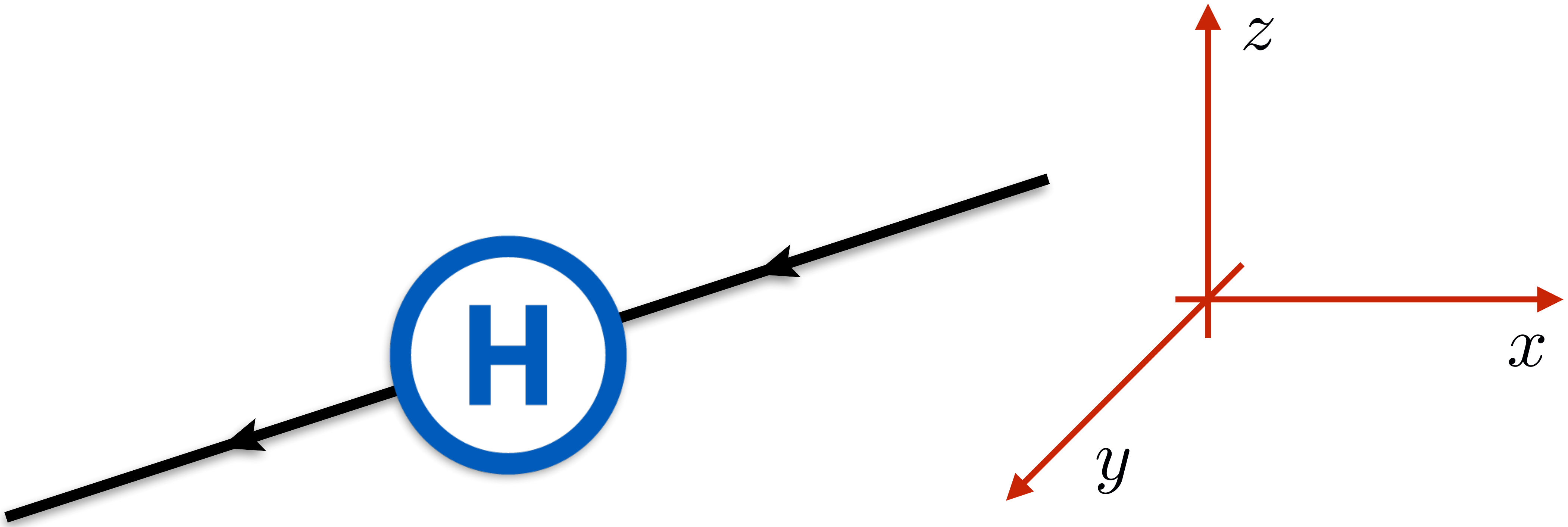
LO Phase space

H=>bb at LO is super trivial, two massless partons back to back.

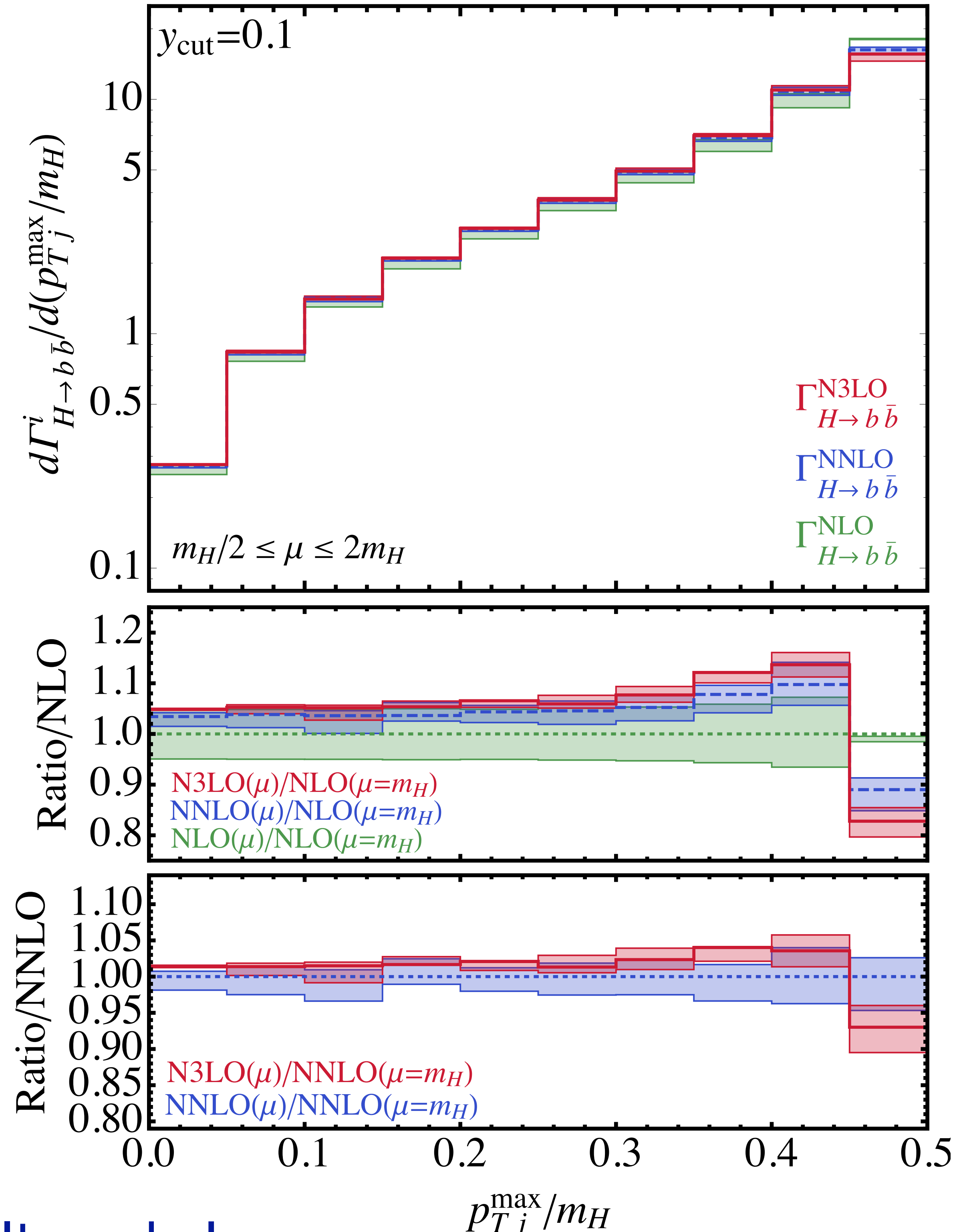
We want some interesting observables at LO to test the IR cancellations in our code, so we introduce a fictitious collision axis, and measure relative to that.

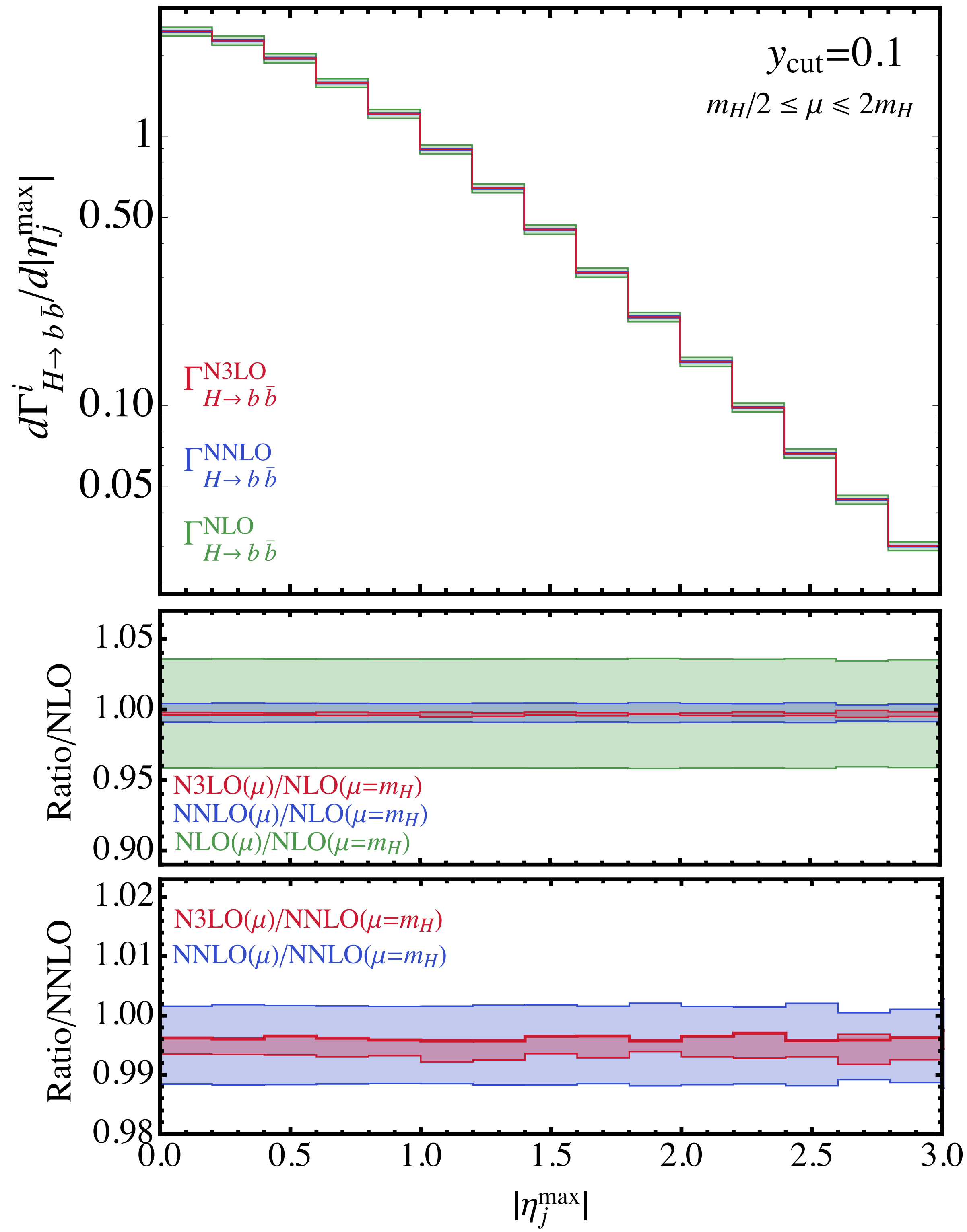


Now we can define pt and rapidity like at the LHC.

We cluster with the Durham jet algorithm
(ask your academic grandparents)

Results

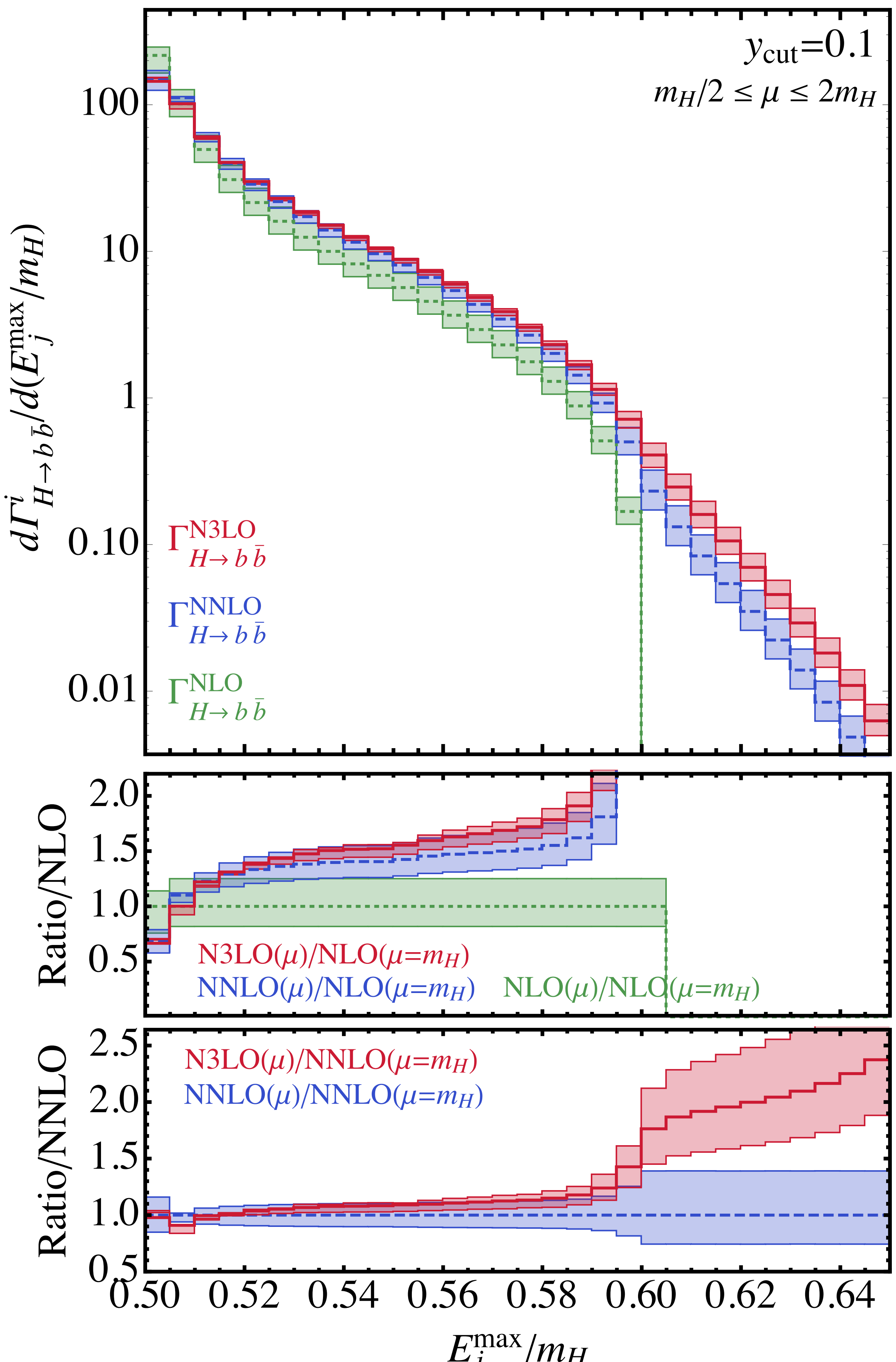




It works!

The size of corrections are observable dependent, scale variation is tiny

Results



Corrections to “real” observables are a little different.

Three regions :

1) $\delta(\mathcal{O})$ all phases spaces contribute, small uncertainty

2) Bulk, 3 parton+ phase spaces contribute, NNLO style scale variation

3) Tail 4 parton+ phase spaces contribute, looks like NLO scale variation

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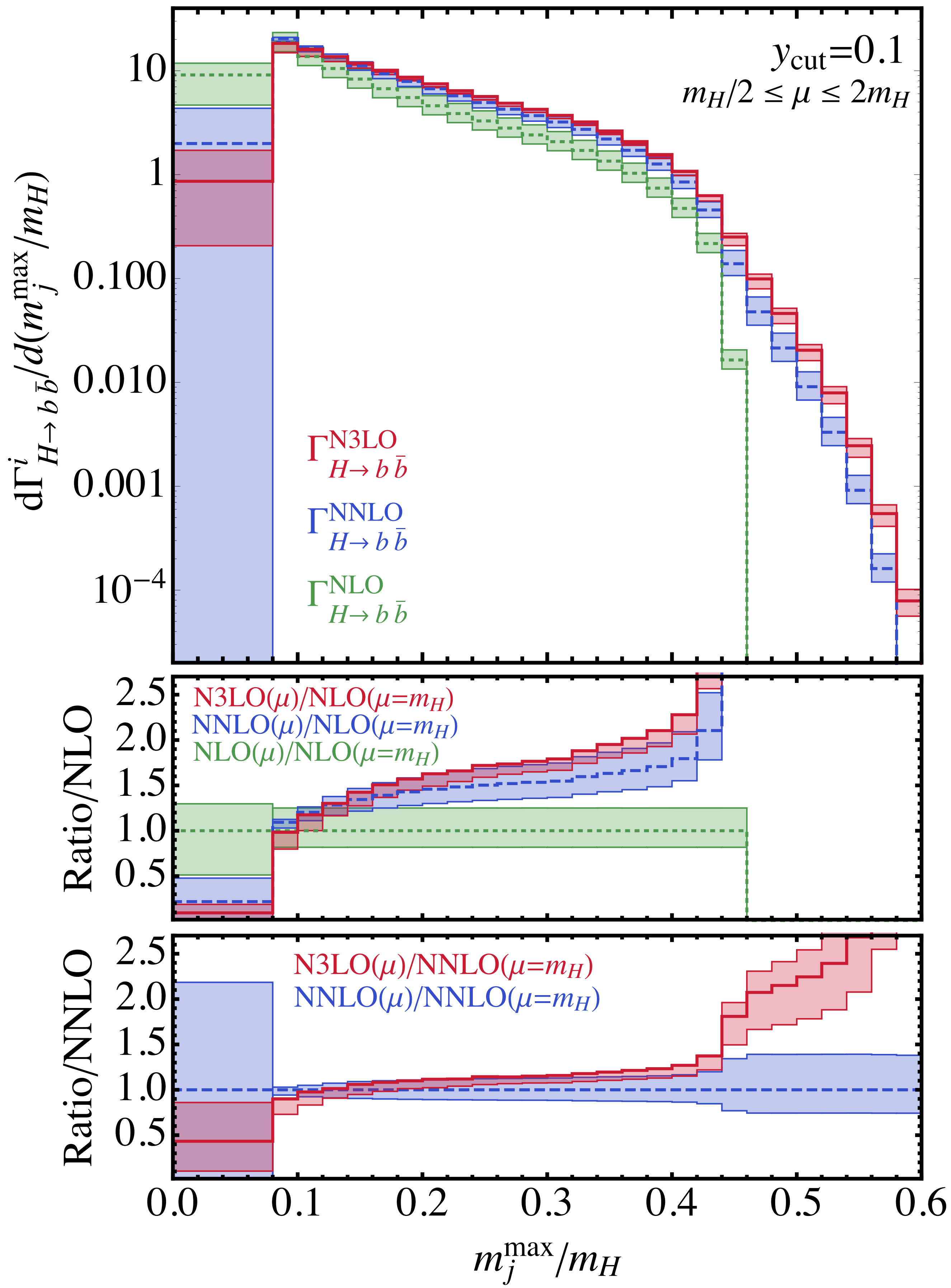

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Conclusions

- We computed N3LO corrections to H=>bb
- We used the projection to Born method + N-jettiness slicing to deal with the IR singular structure
- Our calculation is fully differential and could be deployed out of the rest frame for LHC/ FCC applications.
- Differential effects at N3LO are larger than small inclusive ones.

BACKUP

Durham Jet

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})}{Q^2},$$

P2B Full

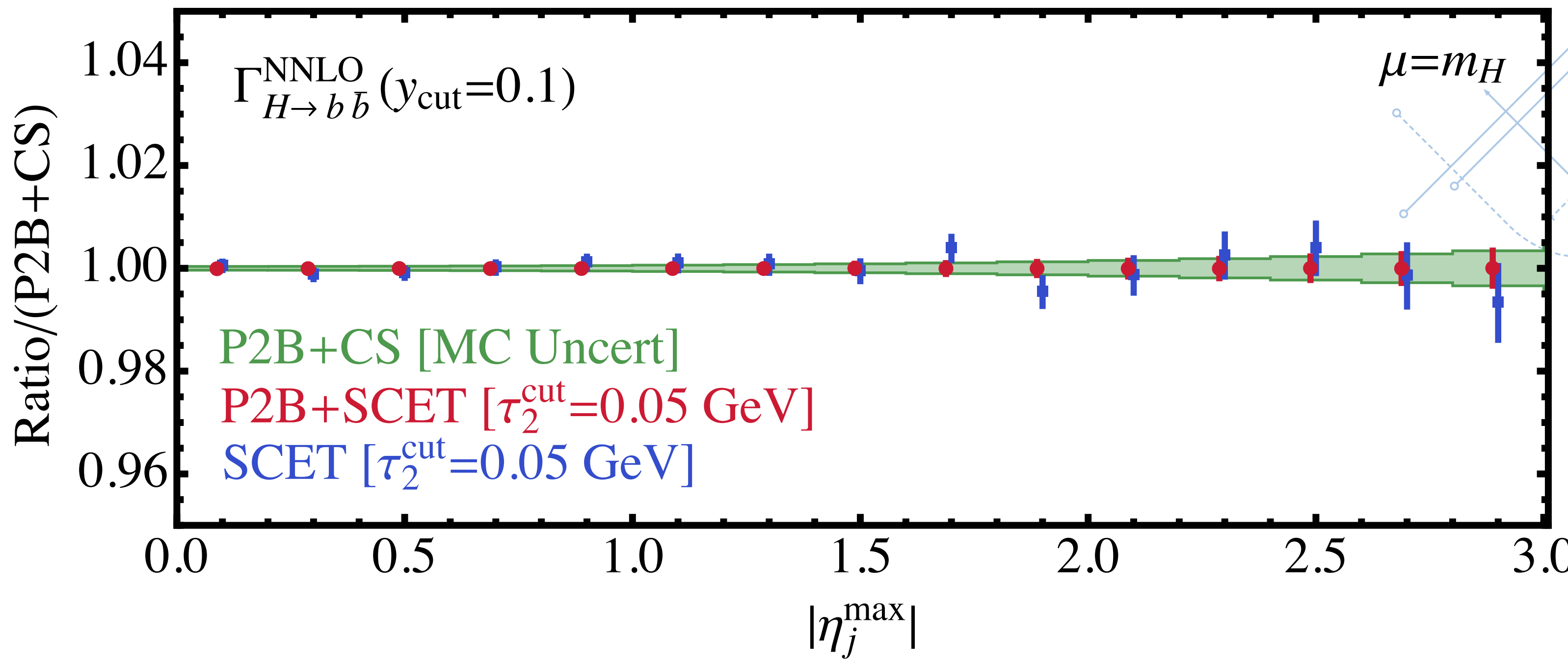
Inclusive

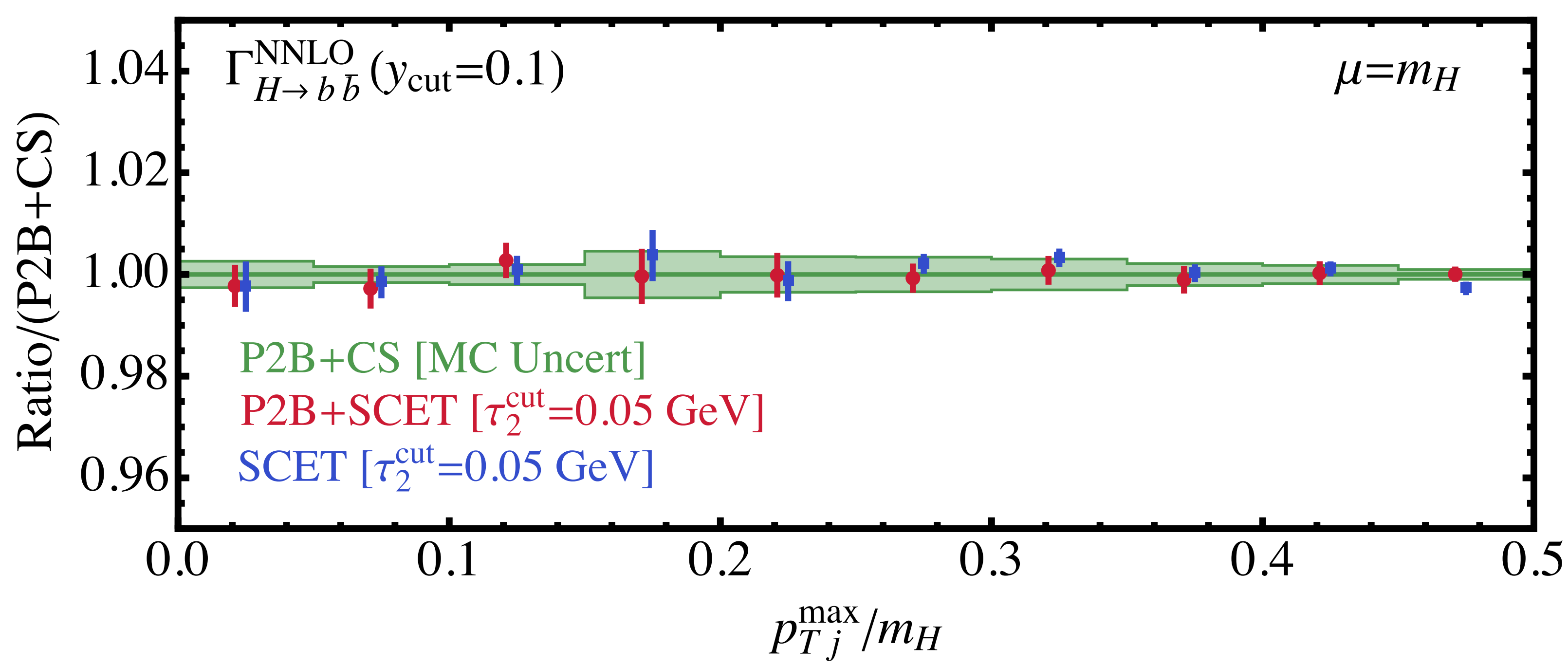
$$\Delta\Gamma^{\rm N3LO}_{H\to b\bar{b}} = \left(\frac{\alpha_s}{\pi}\right)^3 \int 8\pi\, \Gamma^{\rm LO}_{H\to b\bar{b}} \Gamma^{(3)}_{H\to b\bar{b}}\, d\Phi_2 = \int \Delta\hat{\Gamma}^{\rm N3LO}_{H\to b\bar{b}}\, d\Phi_2\,.$$

Differential

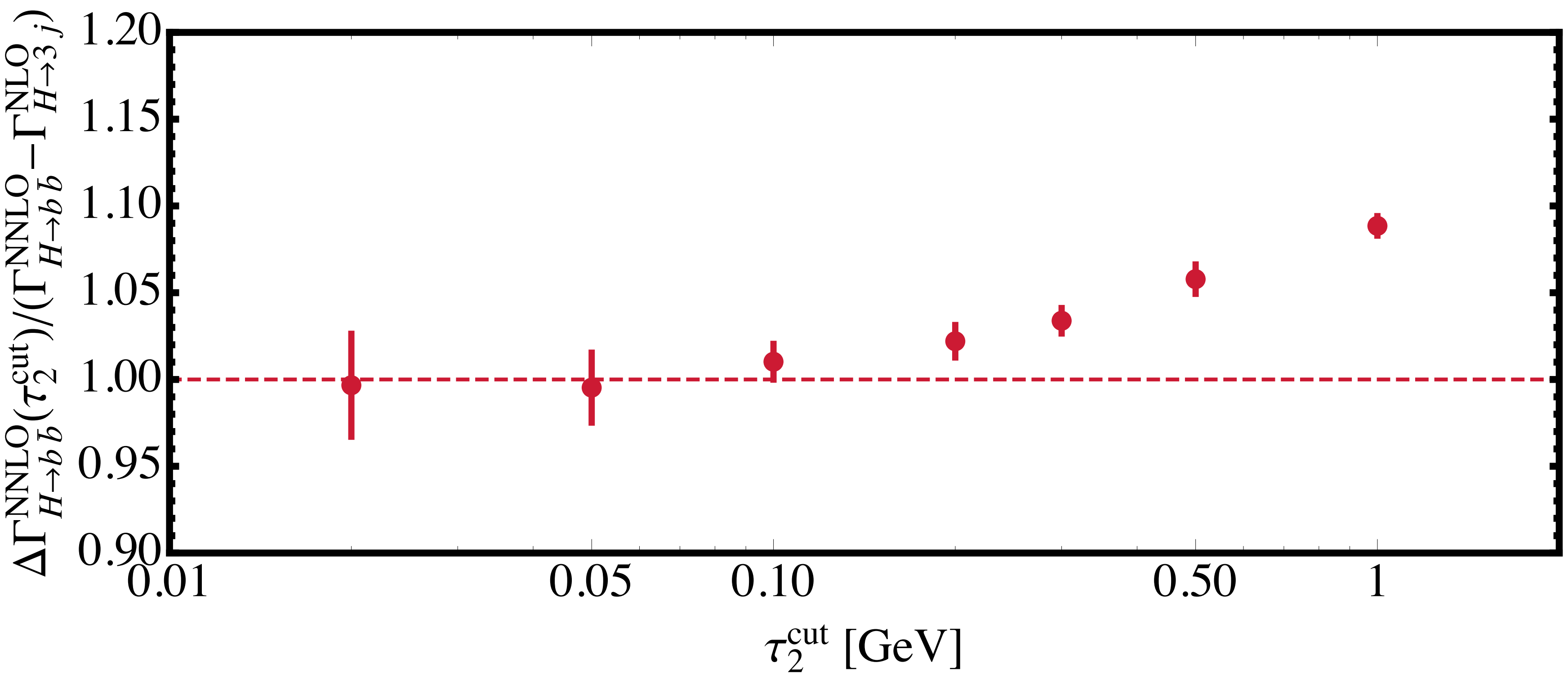
$$\frac{d\,\Delta\Gamma^{\rm N3LO}_{H\to b\bar{b}}}{d\,\mathcal{O}_m} = \int \Delta\hat{\Gamma}^{\rm N3LO}_{H\to b\bar{b}} F_2^m(\Phi_B) d\Phi_B + \int d\Gamma^{RVV}_{H\to b\bar{b}} \left[F_3^m(\Phi_3) - F_2^m(\Phi_B)\right] d\Phi_3 + \int d\Gamma^{RRV}_{H\to b\bar{b}} \left[F_4^m(\Phi_4) - F_2^m(\Phi_B)\right] d\Phi_4 + \int d\Gamma^{RRR}_{H\to b\bar{b}} \left[F_5^m(\Phi_5) - F_2^m(\Phi_B)\right] d\Phi_5\,.$$

NNLO Validation



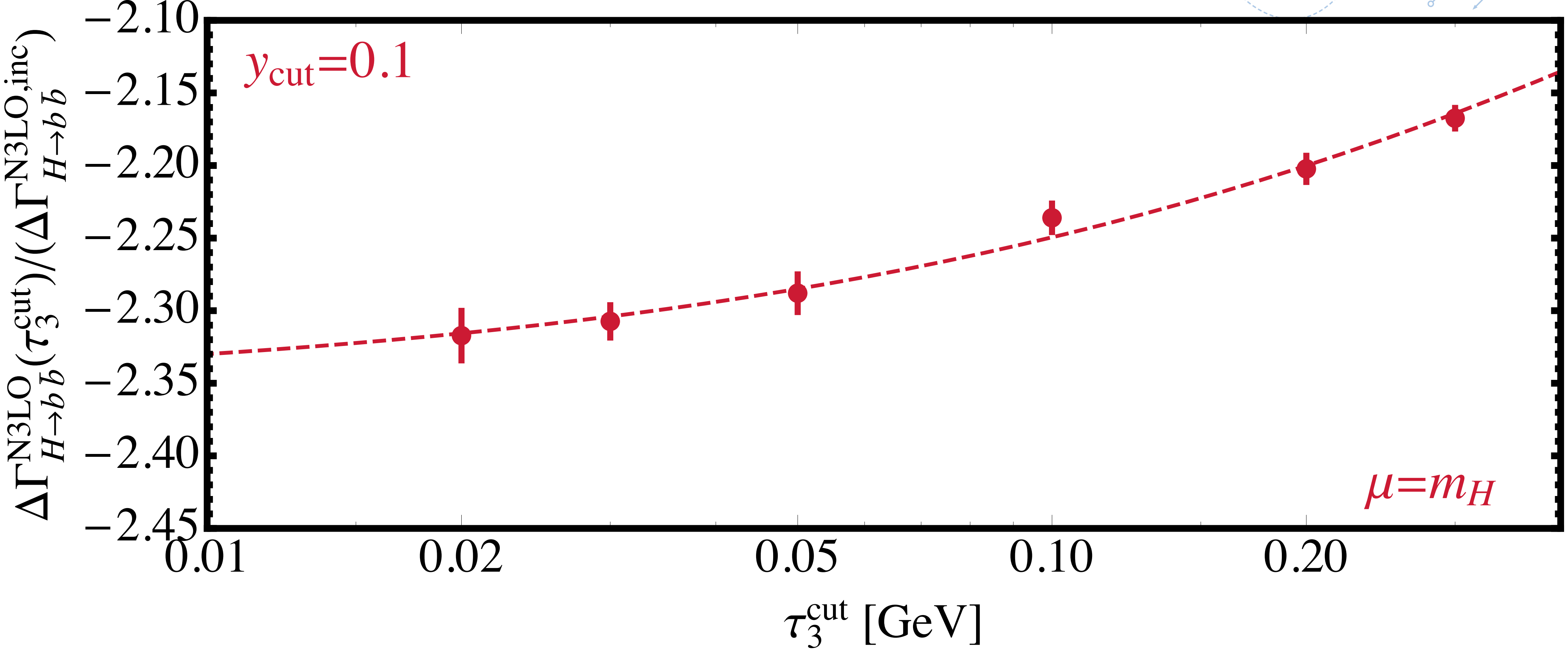


NNLO Validation



$$\tau_N = \sum_{j=1,n} \min_{i=1,2,N} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\}$$

N3LO Validation



$$\tau_N = \sum_{j=1,n} \min_{i=1,2,N} \left\{ \frac{2q_i \cdot p_j}{Q_i} \right\}$$