SUSY GUTs and the Strong CP Problem

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Unification of Matter

16 members of a family fit into a spinor of $SO(10)$ GUT

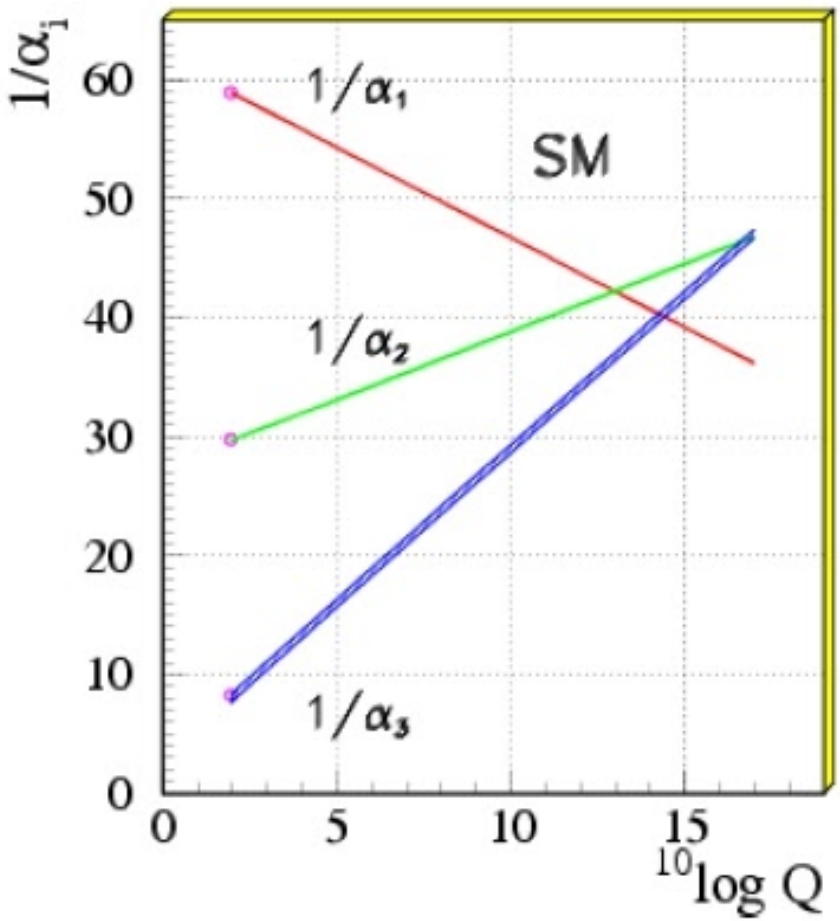
$u_r : \{-++\ +-\}$	$d_r : \{-++\ -+\}$	$u_r^c : \{+--\ ++\}$	$d_r^c : \{+--\ --\}$
$u_b : \{+-+\ +-\}$	$d_b : \{+-+\ -+\}$	$u_b^c : \{-+-\ ++\}$	$d_b^c : \{-+-\ --\}$
$u_g : \{++-\ +-\}$	$d_g : \{++-\ -+\}$	$u_g^c : \{--+\ ++\}$	$d_g^c : \{--+\ --\}$
$\nu : \{---\ +-\}$	$e : \{---\ -+\}$	$\nu^c : \{+++\ ++\}$	$e^c : \{+++\ --\}$

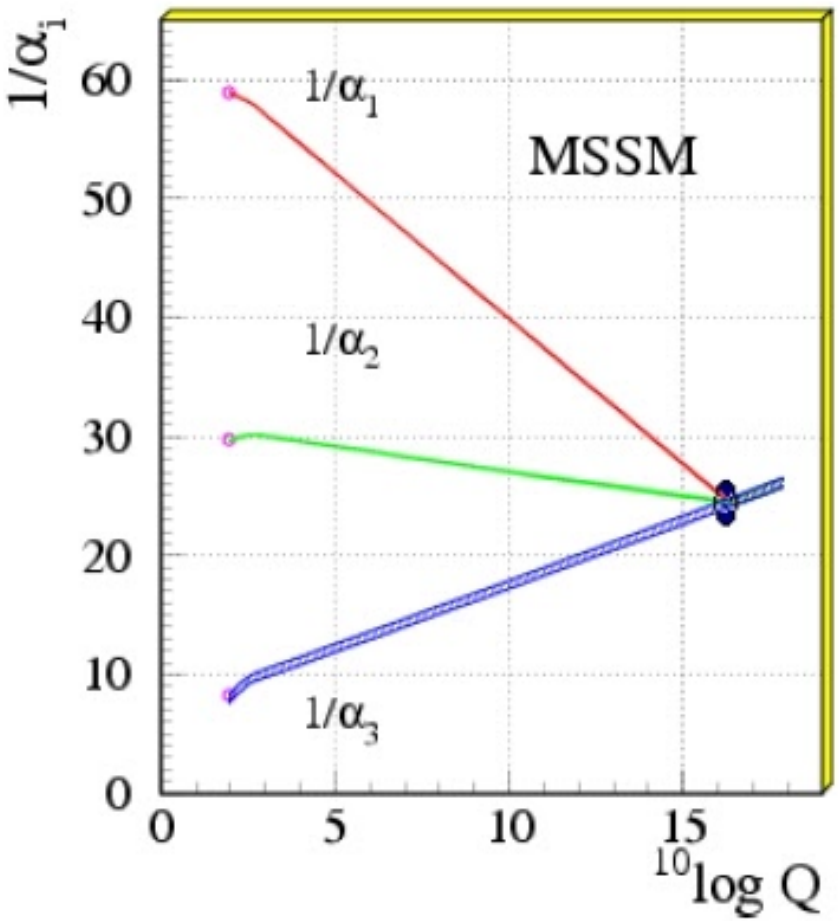
First 3 spins refer to color, last two are weak spins

$$Y = \frac{1}{3}\Sigma(C) - \frac{1}{2}\Sigma(W)$$

Pati, Salam (1974) – Quark-lepton unification
Georgi, Glashow (1974) – $SU(5)$ unification
Georgi (1975); Fritzsch, Minkowski (1975) – $SO(10)$ unification
Georgi, Quinn, Weinberg (1974) – Gauge coupling unification

Unification of Gauge Couplings with SUSY





Yukawa Sector of SUSY $SO(10)$

$$16 \times 16 = 10_s + 120_a + 126_s$$

- At least two Higgs fields needed for family mixing
- Symmetric 10_H and $\overline{126}$ is the minimal model

$$W_{SO(10)}^{\text{Yukawa}} = 16^T \left(Y_{10}\, 10_H + Y_{126} \overline{126}_H \right) 16 \ .$$

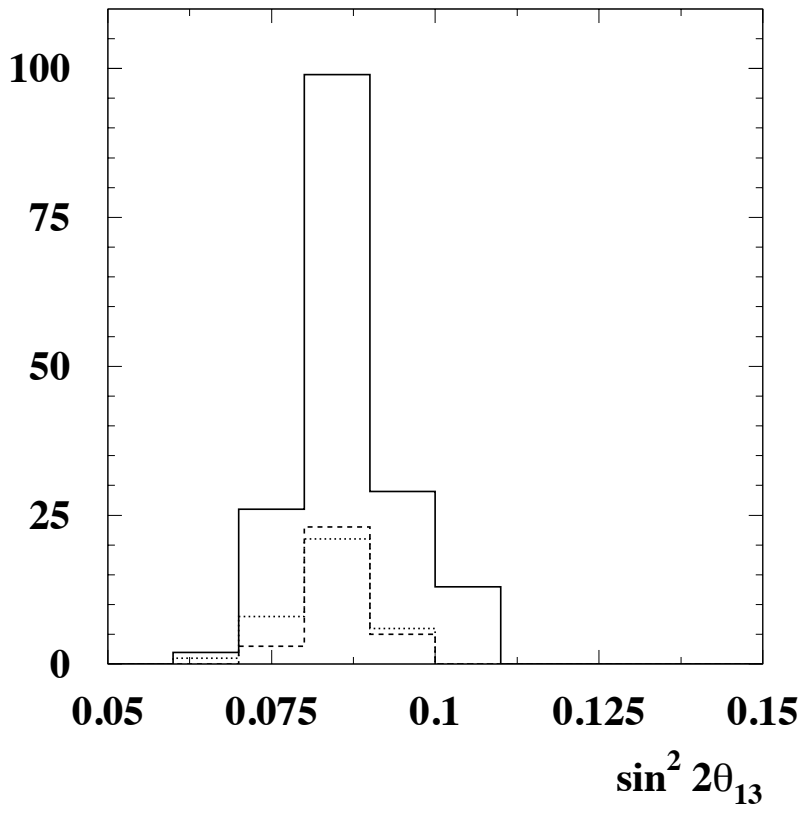
$$\begin{aligned}
M_U &= v_u^{10} Y_{10} + v_u^{126} Y_{126} \\
M_D &= v_d^{10} Y_{10} + v_d^{126} Y_{126} \\
M_E &= v_d^{10} Y_{10} - 3 v_d^{126} Y_{126} \\
M_{\nu_D} &= v_u^{10} Y_{10} - 3 v_u^{126} Y_{126} \\
M_R &= Y_{126} V_R
\end{aligned}$$

Minimal Yukawa Sector

- 12 parameters plus 7 phases to fit 18 observed quantities
- This setup fits all obsevables quite well
- Large neutrino mixings coexist with small quark mixings
- θ_{13} prediction turned out to be correct

Babu, Mohapatra (1993); Bajc, Senjanovic, Vissani (2001); (2003); Fukuyama, Okada (2002); Goh, Mohapatra, Ng (2003); Bajc, Melfo, Senjanovic, Vissani (2004); Bertolini, Malinsky, Schwetz (2006); Babu, Macesanu (2005); Dutta, Mimura, Mohapatra (2007); Aulakh et al (2004); Bajc, Dorsner, Nemevsek (2009); Joshipura, Patel (2011); Dueck, Rodejohann (2013); Babu, Bajc, Saad (2018)

Prediction for θ_{13}



KB, Macesanu (2005)

DayaBay (2012): $\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005$

Best Fit Values

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{GUT}$)	Fitted values (at $\mu = M_{GUT}$)	pulls
$m_u/10^{-3}$	0.450±0.139	0.454	0.028
m_c	0.248±0.007	0.245	0.175
m_t	84.53±0.84	84.49	-0.057
$m_d/10^{-3}$	0.951±0.19	0.585	-1.92
$m_s/10^{-3}$	18.07±0.97	18.46	0.409
m_b	0.961±0.009	0.961	0.048
$m_e/10^{-3}$	0.379457	0.379468	0.002
$m_\mu/10^{-3}$	80.1068	80.0416	-0.081
m_τ	1.36781	1.36799	0.012
$\lvert V_{us}\rvert/10^{-2}$	22.54±0.06	22.54	0.057
$\lvert V_{cb}\rvert/10^{-2}$	4.14±0.06	4.14	0.013
$\lvert V_{ub}\rvert/10^{-2}$	0.358±0.012	0.358	0.020
δ_{CKM}	1.208±0.054	1.222	0.265
$\Delta m^2_{sol}/10^{-5}(\mathrm{eV}^2)$	8.679±0.218	8.683	0.019
$\Delta m^2_{atm}/10^{-3}(\mathrm{eV}^2)$	2.929±0.046	2.929	- 0.011
$\sin^2\theta_{12}^{\mathrm{PMNS}}$	0.3219±0.017	0.3204	-0.029
$\sin^2\theta_{23}^{\mathrm{PMNS}}$	0.431±0.019	0.4281	-0.0148
$\sin^2\theta_{13}^{\mathrm{PMNS}}$	0.0216±0.00082	0.02148	-0.145

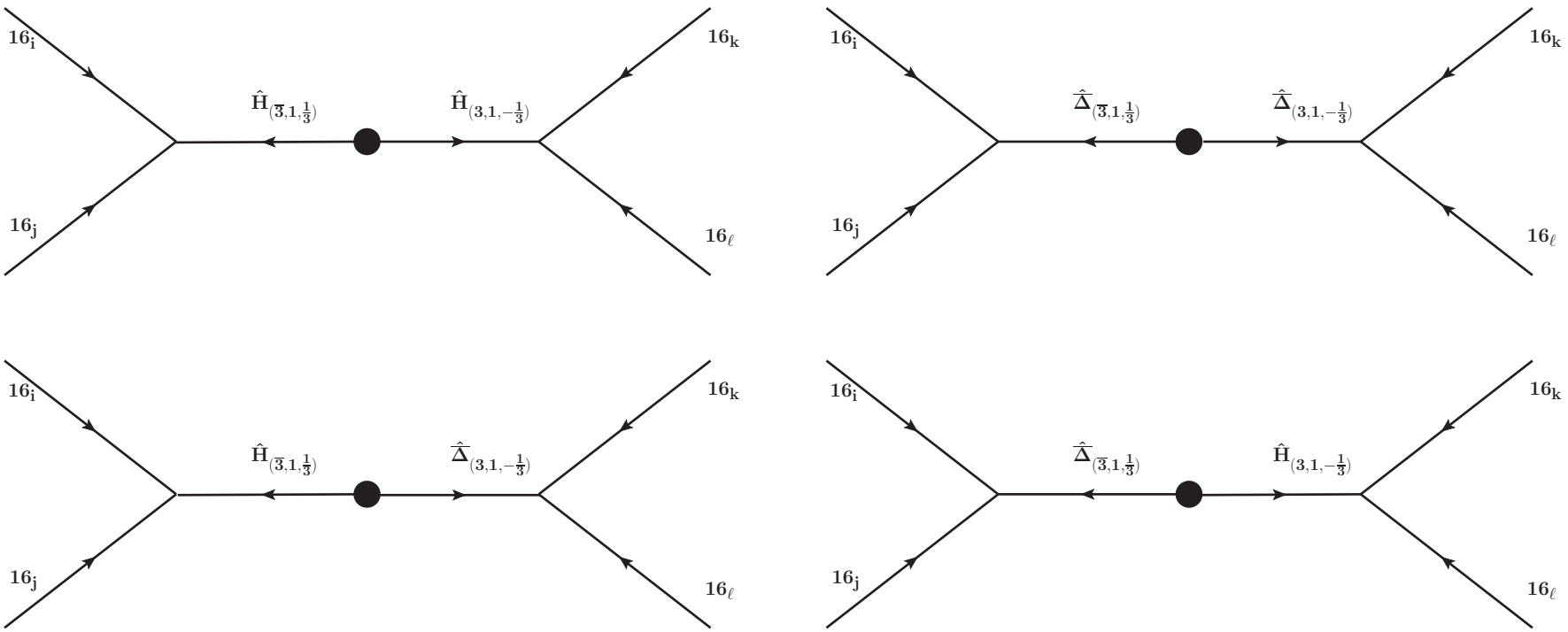
Total $\chi^2 = 4$

Symmetry Breaking Sector

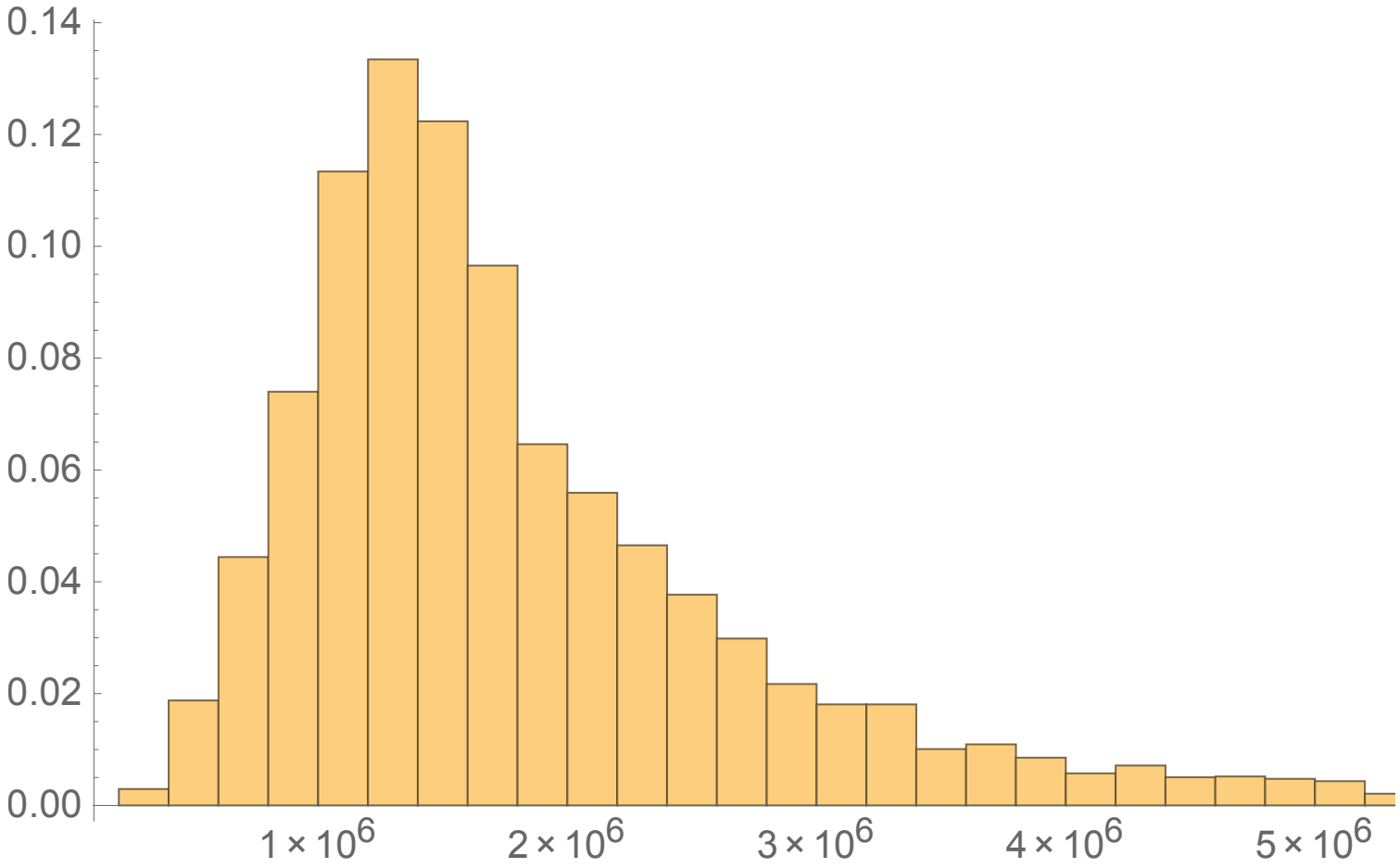
- In addition to $\{10_H + \overline{126}_H\}$, GUT symmetry breaking requires $\{126_H + 210_H + 54_H\}$
- Neutrino mass fit $\Rightarrow B - L$ scale $V_R \approx 10^{12}$ GeV
- $SO(10)$ can break down to $SM \times U(1)_{B-L}$ via $210_H + 54_H$
- $B - L = \pm 2$ singlets from $126_H + \overline{126}_H$ survive down to $V_R \sim 10^{12}$ GeV
- Gauge coupling unification is maintained; only SM singlets survive to V_R KB, Bajc, Saad arXiv: 1805.10631 [hep-ph]
- Proton lifetime requires squark and slepton masses of order 100 TeV
- Excellent fit to fermion masses is obtained

$d = 5$ Proton Decay

- $d = 5$ Baryon number violating operators are induced by color triplet Higgs fields
- For TeV scale SUSY masses, proton lifetime turns out to be too short compared to present limit of $\tau(p \to \overline{\nu} K^+) > 5.9 \times 10^{33}$ yrs.



Minimum SUSY Mass (Wino @ 1 TeV)



Minimum SUSY scalar mass in GeV

SUSY $SO(10)$ with Peccei-Quinn Symmetry

- QCD interactions appear to conserve CP symmetry

$$\overline{\theta} = \theta_{QCD} + \mathrm{ArgDet}(M_Q)$$

  is a physical parameter

- $\overline{\theta}$ contributes to neutron EDM
- $d_n \sim 10^{-16}$ e-cm $\Rightarrow \overline{\theta} < 10^{-10}$
- Why is a dimensionless parameters of theory so small? This is the strong CP problem
- An elegant solution is the Peccei-Quinn mechanism
- $\overline{\theta}$ is promoted to a dynamical field

Peccei-Quinn Symmetry

- Assumes a gloabl $U(1)$ symmetry that has a QCD anomaly. This $U(1)$ is broken spontaneously by a scalar field at a scale $f_a \sim 10^{12}$ GeV leading to massless axion
- Explicit breaking of $U(1)$ by QCD anomaly gives axion a mass

$$\begin{aligned} E_{\rm vac} &= -\mu^4 \cos\overline{\theta}_{\rm eff} \\ \overline{\theta}_{\rm eff} &= \theta_{QCD} + {\rm ArgDet}(M_Q) + \frac{1}{f_a} a(x^\mu) \end{aligned}$$

- Minimization with respect to a sets $\overline{\theta} = 0$
- SUSY $SU(5)$ with PQ symmetry well studied; helps with suppressing $d = 5$ proton decay rate
  Hisano, Murayama, Yanagida (1993)
- We have extended this to SUSY $SO(10)$ usin DFSZ axion
  Babu, Fukuyama, Khan, Saad (2018)

SUSY $SO(10)$ with PQ Symmetry

- An additional 10_H is needed to implement PQ symmetry, as in SUSY $SU(5)$

Fields	16_{F_i}	210_H	54_H	126_H	$\overline{126}_H$	10_H	$10'_H$	S_1	S_2	S_3
$U(1)_{\rm PQ}$	-1	0	0	-2	+2	+2	-2	-8	+8	+4

Table: PQ charge assignment.

- Singlet sector breaks PQ symmetry in SUSY limit:

$$W_S = M_S S_1 S_2 + \kappa S_1 S_3^2$$

In SUSY limit, S_3 is undetermined, but it gets fixed once SUSY breaking is included

- $X(S\overline{S} - M^2)$ also works

Superpotential

Symmetry breaking sector:

$$
\begin{aligned}
W^{PQ}_{SO(10)} &= \frac{1}{2} m_1 210_H^2 + m_2 \overline{126}_H 126_H + m_3' 10_H 10_H' + \frac{1}{2} m_5 54_H^2 \\
&+ \lambda_1 210_H^3 + \lambda_2 210_H \overline{126}_H 126_H + \lambda_3 126_H 10_H 210_H \\
&+ \lambda_4' \overline{126}_H 10_H' 210_H + \lambda_8 54_H^3 + \lambda_{10} 54_H 210_H^2 + \lambda_{13}' 54_H 10_H 10_H' \\
&+ \frac{1}{2} \lambda_5' 10_H'^2 S_3
\end{aligned}
$$

Minimal Yukawa sector preserved under PQ:

$$W^{Yuk\ PQ}_{SO(10)} = 16^T (Y_{10} 10_H + Y_{\overline{126}} \overline{126}_H) 16$$

Color-triplet and $SU(2)$ doublet scalar mass matrices determine $d =$ proton decay rates

$SU(2)_L$ doublet Higgs mass matrix

The doublet mass matrix:

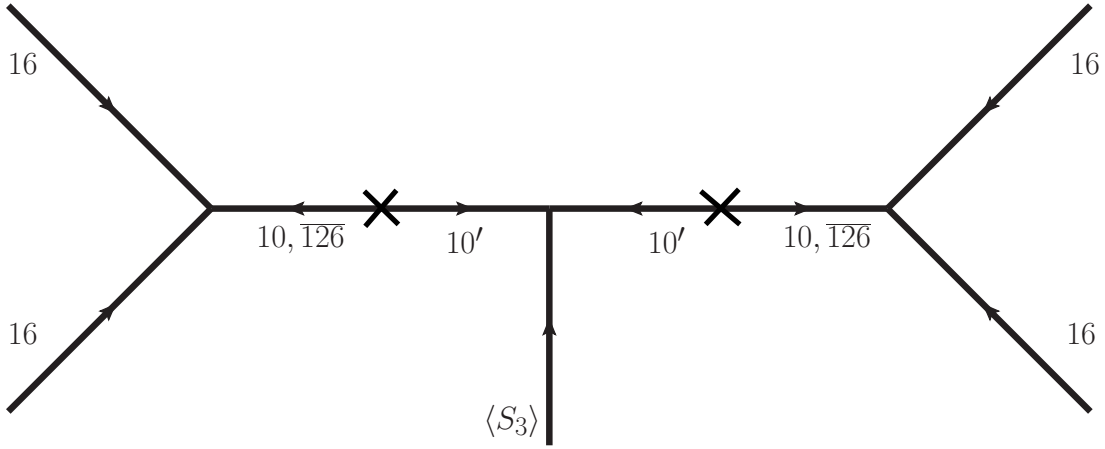
$$\begin{pmatrix} H_d & \overline{\Delta}_d & \Delta_d & \Phi_d & H'_d \end{pmatrix} \mathcal{M}_D \begin{pmatrix} H_u \\ \Delta_u \\ \overline{\Delta}_u \\ \Phi_u \\ H'_u \end{pmatrix}$$

$$\mathcal{M}_D = \begin{pmatrix} 0 & \frac{\lambda_3\Phi_2}{\sqrt{10}} - \frac{\lambda_3\Phi_3}{2\sqrt{5}} & 0 & 0 & m'_3 + \sqrt{3/5}\lambda'_{13}E \\ 0 & m_2 + \frac{\lambda_2\Phi_2}{15\sqrt{2}} - \frac{\lambda_2\Phi_3}{30} & 0 & 0 & \frac{\lambda'_4\Phi_2}{\sqrt{10}} - \frac{\lambda'_4\Phi_3}{2\sqrt{5}} \\ -\frac{\lambda_3\Phi_2}{\sqrt{10}} - \frac{\lambda_3\Phi_3}{2\sqrt{5}} & 0 & m_2 + \frac{\lambda_2\Phi_2}{15\sqrt{2}} + \frac{\lambda_2\Phi_3}{30} & \frac{\lambda_2\overline{v}_R}{10} & 0 \\ -\frac{\lambda_3 v_R}{\sqrt{5}} & 0 & \frac{\lambda_2 v_R}{10} & m_1 + \frac{\lambda_1\Phi_2}{\sqrt{2}} + \frac{\lambda_1\Phi_3}{\sqrt{2}} - \frac{\sqrt{3}}{4\sqrt{5}}\lambda_{10}E & 0 \\ m'_3 + \sqrt{3/5}\lambda'_{13}E & 0 & -\frac{\lambda'_4\Phi_2}{\sqrt{10}} - \frac{\lambda'_4\Phi_3}{2\sqrt{5}} & \lambda'_4\frac{\overline{v}_R}{\sqrt{5}} & \lambda'_5 S_3 \end{pmatrix}$$

An extra pair of doublet has mass of order PQ scale $\sim 10^{12}$ GeV

Suppression of Proton Decay Rate

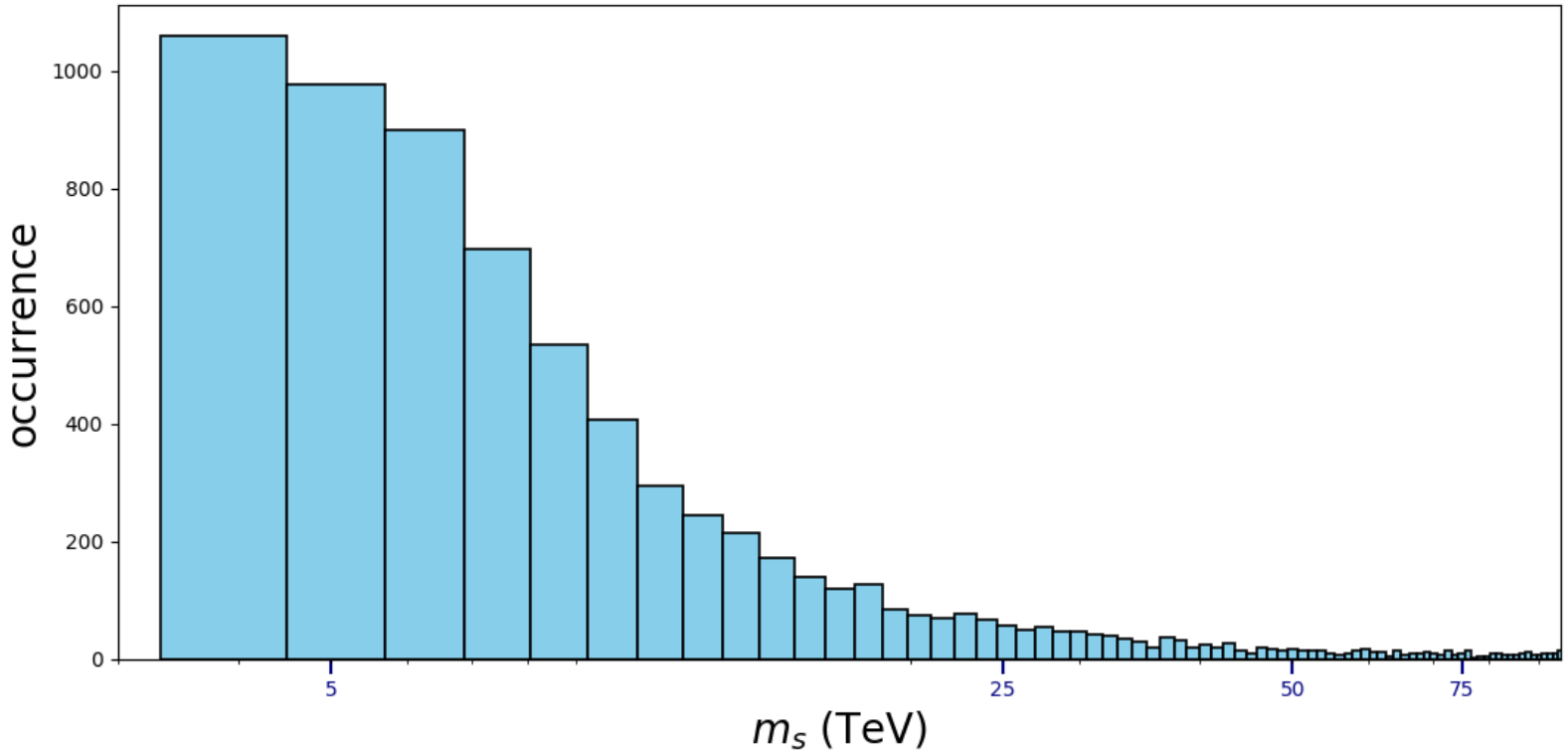
- $d = 5$ proton decay amplitude can only proceed with a suppression of $M_{PQ} \sim 10^{12}$ GeV



- Proton decay rate suppressed by $(M_{PQ}/M_{\rm GUT})^2 \sim 10^{-8}$ relative to non-PQ $SO(10)$ model

- $\tau_p \sim 10^{25} - 10^{26}$ yrs. (with TeV scalars) becomes $\tau_P \sim 10^{33} - 10^{34}$ yrs.

Allowed SUSY Scalar Masses

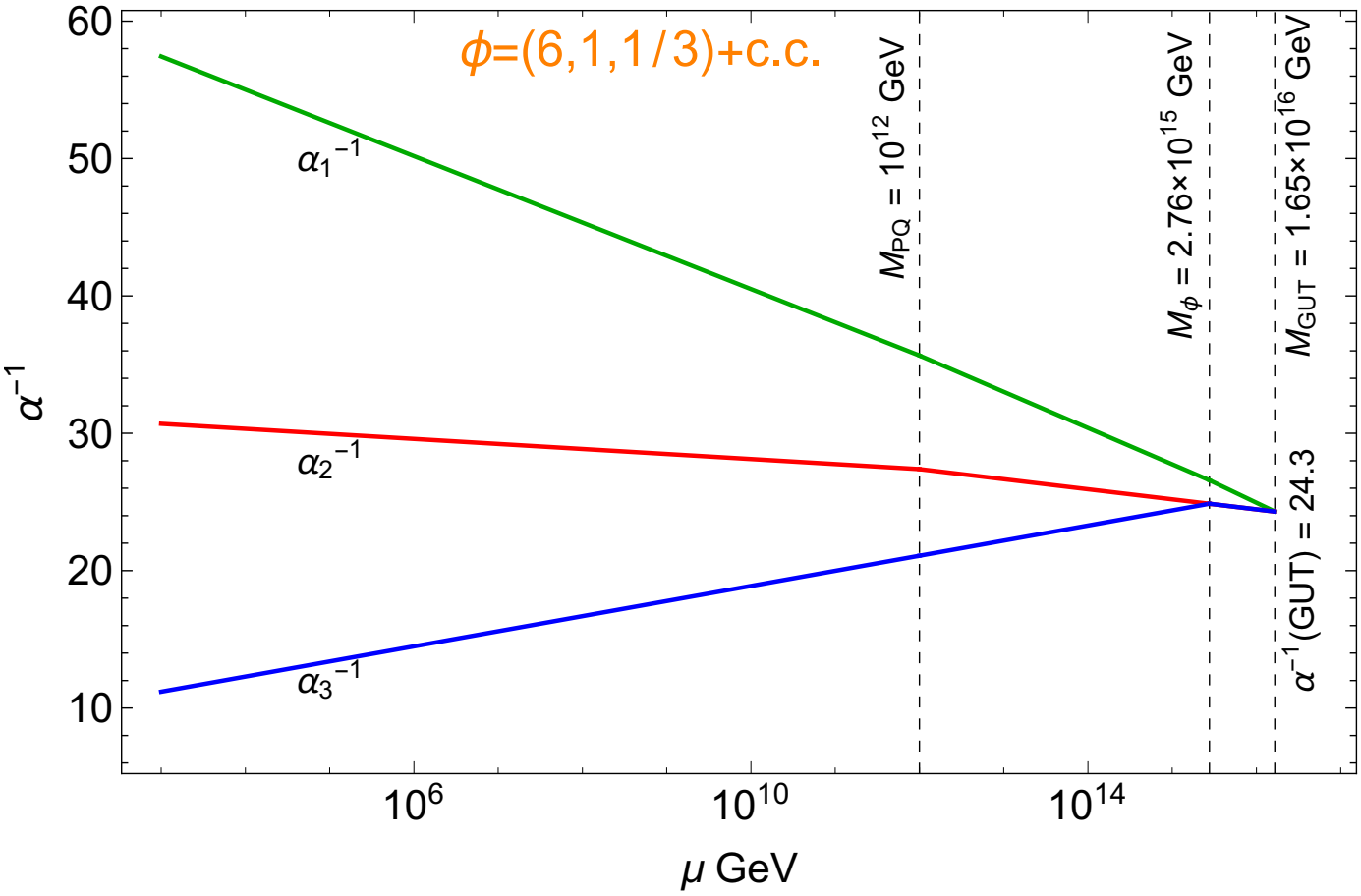


Minimum SUSY scalar mass allowed by proton decay (with Wino mass fixed at 1 TeV)

Gauge Coupling Unification

- Extra pair of doublets at PQ scale necessary for fermion mass generation
- These fields affect unification of gauge couplings
- To compensate for their effects, certain colored particles should have masses below GUT scale
- Color sextet scalars slightly below GUT scale can do the job

Gauge Coupling Unification



Unification with charge 1/3 color sextet below GUT scale

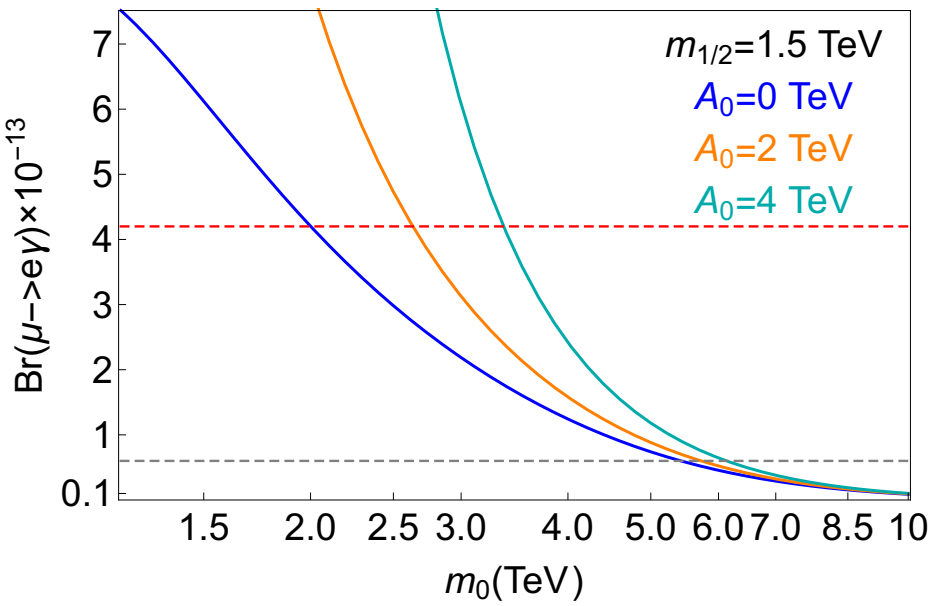
Gauge boson mediated proton decay

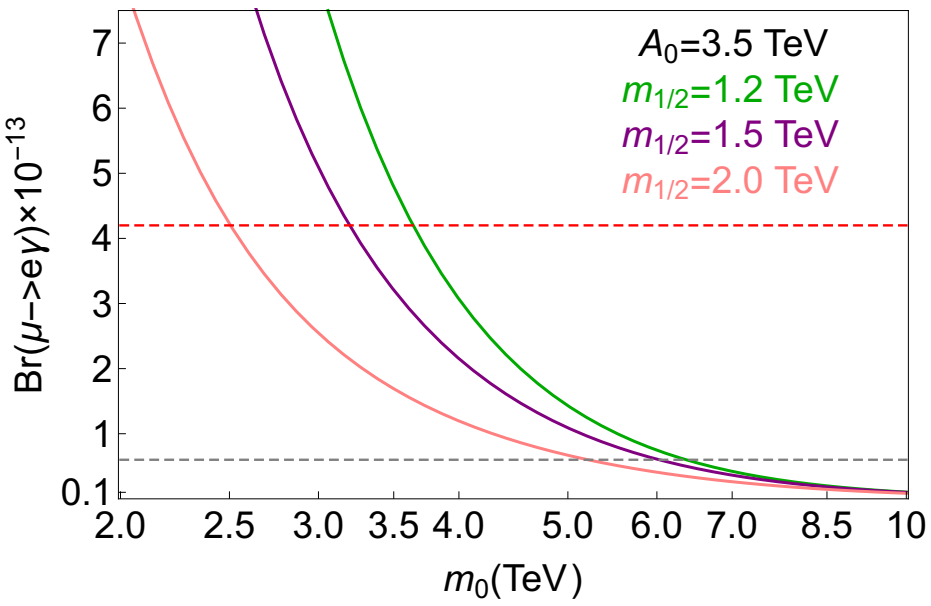
- Since GUT scale is now lowered, $d = 6$ gauge boson mediated proton decay becomes important
- Including threshold corrections, we believe that absolute lifetime can be $> 1.6 \times 10^{34}$ yrs for $p \to e^+\pi^0$
- Branching ratios can be predicted with $d = 6$ operatros dominant:

p decay modes	Branching ratio
$p \to \overline{\nu}\pi^+$	52.5%
$p \to e^+\pi^0$	40.9%
$p \to \mu^+K^0$	4.42%
$p \to \mu^+\pi^0$	1.14%
$p \to \overline{\nu}K^+$	0.69%
$p \to e^+K^0$	0.23%
$p \to e^+\eta$	0.04%
$p \to \mu^+\eta$	0.001%

Lepton Flavor Violation

- RGE running from GUT scale to ν_R scale induces LFV
- Dirac and Majorana masses of neutrinos are all determined
- $\mu \to e\gamma$ is the most prominent process





Conclusions

- Minimal Yukawa sector of $SO(10)$ is rather predictive and works quite well
- PQ symmetry successfully implemented
- SUSY scalar masses allowed to be in the multi-TeV range with PQ symmetry
- Both $d = 5$ and $d = 6$ proton decay rates near current limits
- Proton decay branching ratios may test such high scale theories
- $\mu \to e\gamma$ is close to current experimental limit