Shapes of Self-Interacting Dark Matter

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Motivations to Study SIDM

- Observationally: small scale problems with cold dark matter
  - Core vs. Cusp, Missing Satellites, Too Big to Fail, Diversity of Rotation Curves
  - Potentially resolved with baryonic feedback processes
- Theoretically: is dark matter collisional or not?
  - We have a very well posed problem with a clear prescription for how to solve it.

What is the goal?

- Compute the viscous cross section

$$\sigma_V = \int d\Omega \, \sin^2\theta \, \frac{d\sigma}{d\Omega}$$

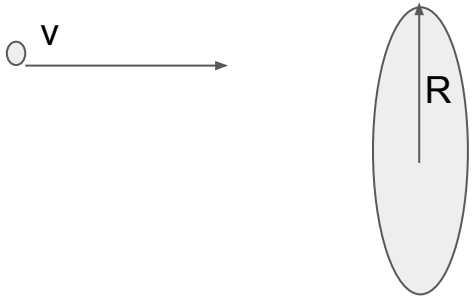
- Include non-perturbative effects - Sommerfeld enhancement
- Have a recipe to do this for arbitrary interactions
  - Compute $2 \to 2$ scattering amplitude
  - Calculate a potential
  - Renormalize
  - Compute cross section summing over angular momentum modes

Sommerfeld Enhancement

- A Classical Analogy

w/o gravity $\sigma_0 = \pi R^2$

w/ gravity $\sigma = \pi b_{max}^2 = \sigma_0 \left(1 + \frac{v_{esc}^2}{v^2}\right)$



- Non-perturbative effect that can be treated quantum mechanically
  - Match a field theory calculation onto a quantum mechanical potential
  - Solve the Schrodinger Equation

$$S = \frac{|\Psi(0)|^2}{|\Psi^0(0)|^2}$$

Arkani-Hamed, Finkbeiner, Slatyer, Weiner [0810.0713]; Lepage [9706029]

Coulomb Potential

$$V(r) = \frac{-\alpha}{2r}$$

- This potential admits an analytic solution for the Sommerfeld enhancement factor

$$S = \left| \frac{\frac{\pi}{\epsilon_v}}{1 - exp[-\frac{\pi}{\epsilon_v}]} \right| \qquad \epsilon_v \equiv \frac{v}{\alpha}$$

- As v becomes large, S starts to approach 1.
- As v approaches 0, S behaves like 1/v and starts to diverge.
- Important in the nonrelativistic limit!

EFT Approach

- Process we consider is DM scattering
- Classify all EFTs with a light mediator and fermionic dark matter
  - Study scalar, vector, pseudoscalar and axial vector interactions
  - **Dirac** and Majorana fermions and **Symmetric** vs. Asymmetric

Type	Process	Channels
Majorana	$\chi\chi \to \chi\chi$	s, t, u
Dirac, Asymmetric	$\chi\chi \to \chi\chi$	t, u
Dirac, Symmetric	$\chi\bar{\chi} \to \chi\bar{\chi}$	s, t

Various Non-Relativistic Potentials

$$V_{scalar}(r) = -\frac{\lambda^2}{4\pi r} e^{-m_\phi r}$$

$$V_{pseudoscalar}(r) = \frac{\lambda^2}{4m_\chi^2 - m_\phi^2} \frac{\delta'(r)}{2\pi r}\left(2S_1 \cdot S_2 - \frac{1}{2}\right) + \frac{\lambda^2}{4\pi} \frac{e^{-m_\phi r}}{m_\chi^2}\left[\frac{m_\phi^2}{3r} S_1 \cdot S_2 + \frac{3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2}{r^3}(1 + m_\phi r + \frac{m_\phi^2 r^2}{3})\right]$$

$$V_{vector}(r) = -\frac{\lambda^2}{4m_\chi^2 - m_A^2} \frac{\delta'(r)}{2\pi r}\left(\frac{3}{2} + 4S_1 \cdot S_2\right) - \frac{\lambda^2}{4\pi r} e^{-m_A r}$$

$$V_{axial\,vector}(r) = \frac{\lambda^2}{4m_\chi^2 - m_A^2} \frac{\delta'(r)}{2\pi r}\left(\frac{1}{2} - 2S_1 \cdot S_2\right) - \frac{\lambda^2}{\pi r} e^{-m_A r} S_1 \cdot S_2 + \frac{4m_\chi^2}{m_A^2} V_{pseudoscalar}$$

How do we renormalize?

- Introduce a UV cutoff
  - Removes high momentum states
  - Softens the short range behavior
- Add local counterterms
  - Systematically removes cutoff dependence
  - Derivative expansion
- Make sure we have the correct long range behavior

Coulomb Potential Example

- Step 1: Introduce the UV cutoff

$$\frac{1}{r} \to \frac{4\pi}{q^2} \to \frac{4\pi}{q^2} e^{-a^2 q^2/2} \to \frac{erf(r/\sqrt{2}a)}{r}$$

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- Step 2: Add a local counterterm

$$V_{eff} = -\frac{\alpha}{r} erf(r/\sqrt{2}a) + 2\pi\alpha c a^2 \delta_a^3(r)$$

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- Step 3: Perturbative matching

$$-\frac{4\pi\alpha}{q^2} e^{-a^2 q^2/2}(1 + c a^2 q^2/2) = -\frac{4\pi\alpha}{q^2}(1 + (c-1)a^2 q^2/2 + \mathcal{O}(a^4 q^4))$$

Computing σ_V

- Solve the Schrodinger Equation
  - $\phi'' + (k^2 - l(l+1)\mathrm{Ang}(r,a) - m\chi V_{reg}(r))\phi = 0$
- Match onto the asymptotic form
  - $\phi \rightarrow c_1 \sin(kr - l\pi/2) - c_2 \cos(kr - l\pi/2)$
- **Extract the phase shift (S-matrix)**
- Compute the viscous cross section

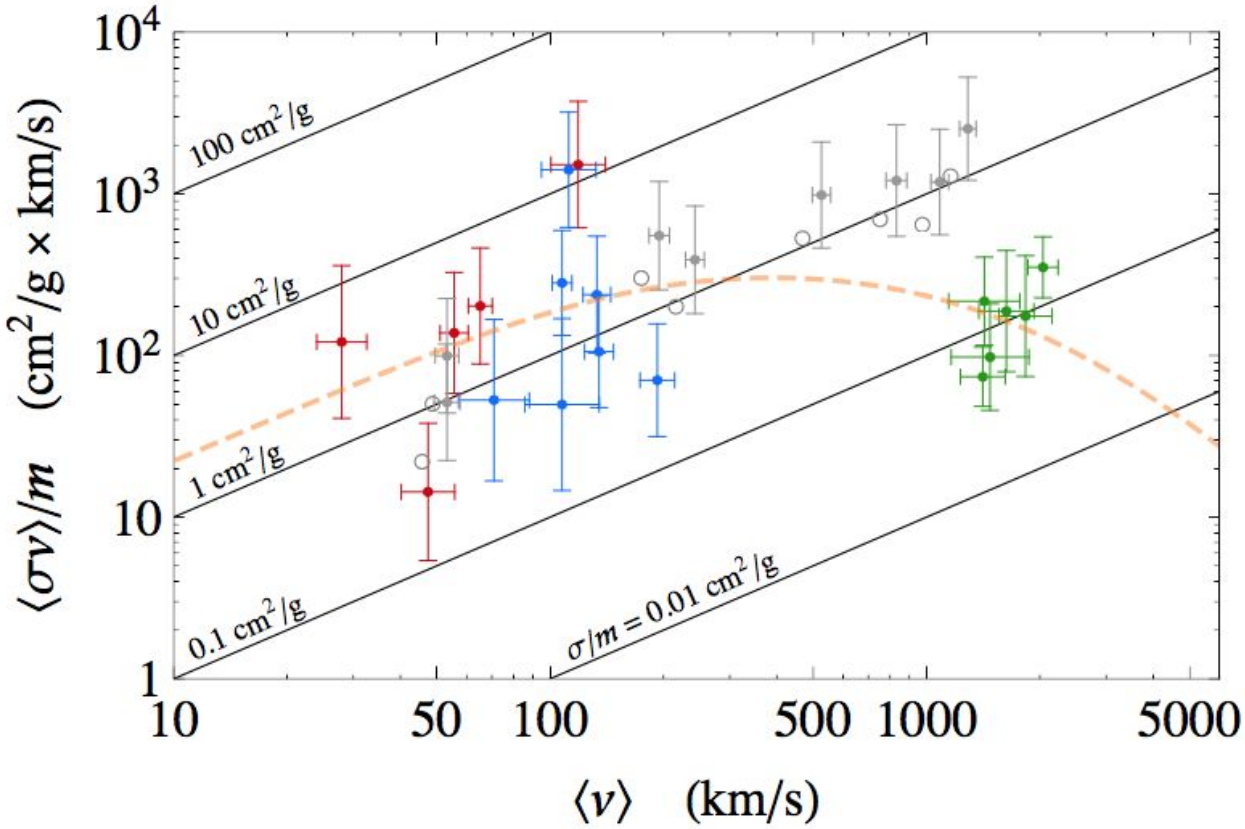
$$\sigma_V = \sum_{l=0}^{\infty} \frac{(l+1)(l+2)}{(2l+3)} sin^2(\delta_{l+2} - \delta_l)$$

Conclusions

- Understanding the space of theories of SIDM is an interesting theoretical problem to solve
- The viscous cross section is the relevant quantity of interest
- There is a well-defined procedure for computing the viscous cross section
- Sommerfeld enhancement is included in a consistent manner

Thank You!

Backup



Red: Dwarf galaxy data

Blue: Low Surface Brightness galaxy data

Green: Cluster data

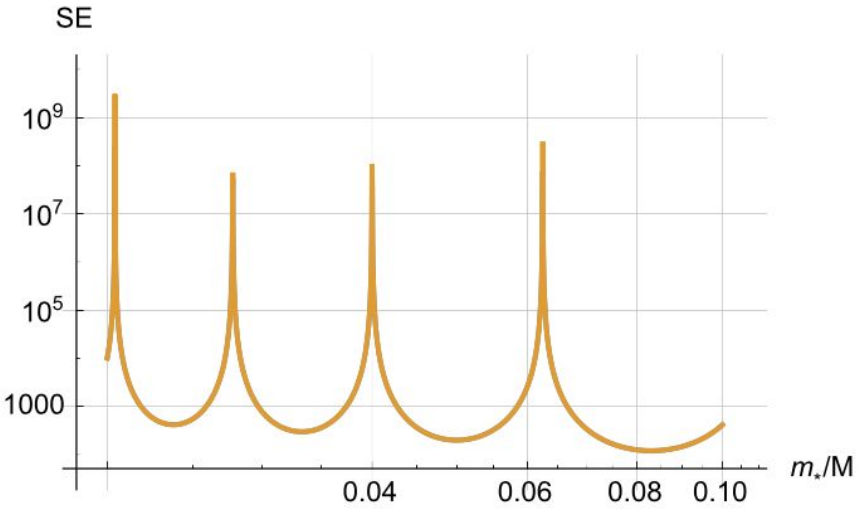
Gray: SIDM N-body simulation halos

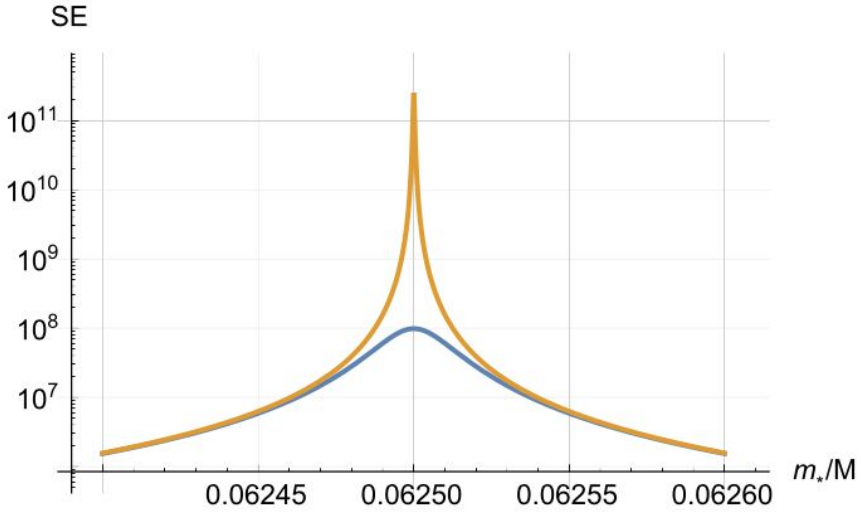
Best fit dark photon model curve shown

Kaplinghat, Tulin, Yu [1508.03339]

Hulthen Potential

$$V(r) = -\frac{\alpha m_* e^{-m_* r}}{1 - e^{-m_* r}}.$$





Blum, Sato, Slatyer [1603.01383]

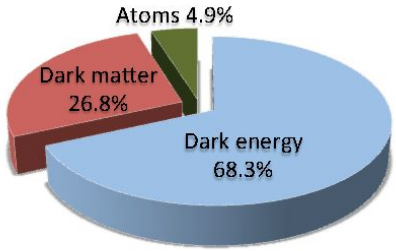
Various Non-Relativistic Potentials

Mediator	Interaction	$\frac{1}{r}$	$\frac{s_1 \cdot s_2}{r}$	$\frac{3(s_1\cdot\hat{r})(s_2\cdot\hat{r})-s_1\cdot s_2}{r^3}$
Scalar	$\lambda_s \overline{\chi}\chi\phi$	$-\lambda_s^2$	0	0
Vector	$\lambda_v \overline{\chi}\gamma^\mu\chi A_\mu$	$\pm\left(1+\frac{m_A^2}{4m_\chi^2}\right)$	$\pm\frac{2\lambda_v^2 m_A^2}{3m_\chi^2}$	$\mp\frac{\lambda_v^2}{m_\chi^2}\left(1+m_A r+\frac{m_A^2 r^2}{3}\right)$
Pseudoscalar	$i\lambda_p \overline{\chi}\gamma^5\chi\phi$	0	$\frac{\lambda_p^2 m_\phi^2}{3m_\chi^2}$	$\frac{\lambda_p^2}{m_\chi^2}\left(1+m_\phi r+\frac{m_\phi^2 r^2}{3}\right)$
Axial Vector	$\lambda_a \overline{\chi}\gamma^5\gamma^\mu\chi A_\mu$	0	$\frac{-8\lambda_a^2}{3}\left(1-\frac{m_A^2}{8m_\chi^2}\right)$	$\lambda_a^2\left(\frac{1}{m_\chi^2}+\frac{4}{m_A^2}\right)\left(1+m_A r+\frac{m_A^2 r^2}{3}\right)$

Bellazzini, Cliche, Tanedo [1307.1129]

The Universal Energy Budget

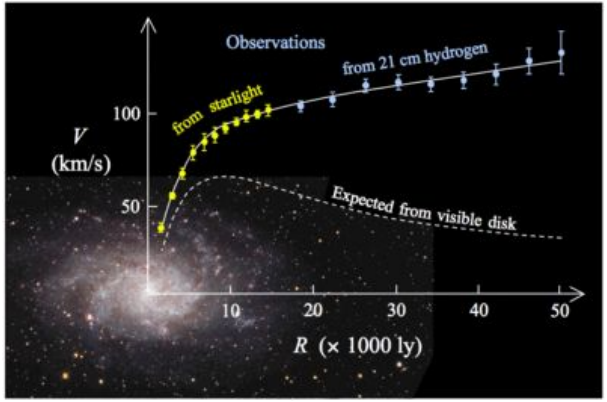
- What makes up our universe?
  - Radiation
  - Ordinary Matter
  - Dark Matter (CDM)
  - Cosmological Constant (Λ)
- ΛCDM is successful on the largest scales



Mitsou [1310.1072]

CDM Signatures & Evidences

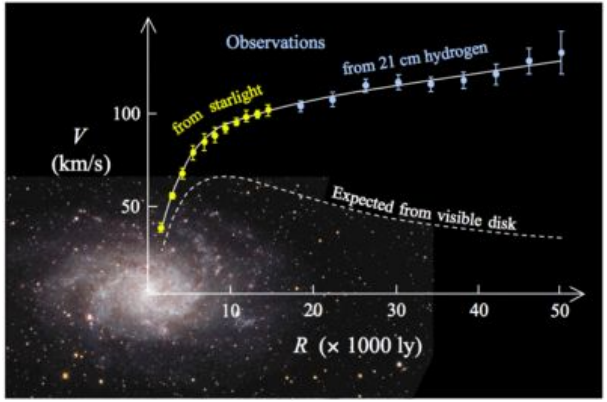
- Rotation Curves



M33 Rotation Curve

CDM Signatures & Evidences

- Rotation Curves
- Gravitational Lensing - Bullet Cluster



M33 Rotation Curve

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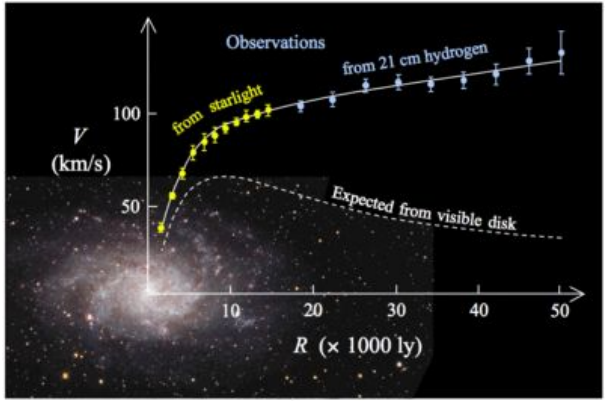
- Rotation Curves
- Gravitational Lensing - Bullet Cluster
- Cosmic Microwave Background



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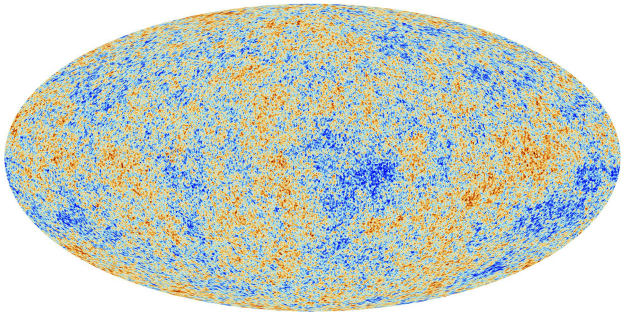

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- Diversity Problem

Baryonic Feedback vs. Self-Interacting Dark Matter

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  - Flatten the dark matter cusp into a core
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Baryonic Feedback vs. Self-Interacting Dark Matter

- Supernova driven outflows can help:
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  - Deplete baryons and render low mass halos incapable of forming satellites
- SIDM is an interesting alternative
  - Alleviate core vs cusp problem and too big to fail problem by scattering
  - Can give rather interesting signals in experiments depending on how it interacts with the Standard Model
  - Theoretically well motivated question to ask whether dark matter is collisional or not, even if it is just within the dark sector