Shapes of Self-Interacting Dark Matter

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In collaboration with M. Reece and P. Agrawal
Motivations to Study SIDM

- **Observationally:** small scale problems with cold dark matter
  - Core vs. Cusp, Missing Satellites, Too Big to Fail, Diversity of Rotation Curves
  - Potentially resolved with baryonic feedback processes

- **Theoretically:** is dark matter collisional or not?
  - We have a very well posed problem with a clear prescription for how to solve it.
What is the goal?

- Compute the viscous cross section
  \[ \sigma_V = \int d\Omega \sin^2 \theta \frac{d\sigma}{d\Omega} \]

- Include non-perturbative effects - Sommerfeld enhancement

- Have a recipe to do this for arbitrary interactions
  - Compute 2 → 2 scattering amplitude
  - Calculate a potential
  - Renormalize
  - Compute cross section summing over angular momentum modes
Sommerfeld Enhancement

● A Classical Analogy

\[ \sigma_0 = \pi R^2 \]

\[ \sigma = \pi b_{\text{max}}^2 = \sigma_0 \left( 1 + \frac{v_{\text{esc}}^2}{v^2} \right) \]

● Non-perturbative effect that can be treated quantum mechanically
  ○ Match a field theory calculation onto a quantum mechanical potential
  ○ Solve the Schrödinger Equation

\[ S = \frac{\left| \Psi(0) \right|^2}{\left| \Psi(0) \right|^2} \]

Arkani-Hamed, Finkbeiner, Slatyer, Weiner [0810.0713]; Lepage [9706029]
Coulomb Potential

- This potential admits an analytic solution for the Sommerfeld enhancement factor

\[ S = \left| \frac{\frac{\pi}{\epsilon_v}}{1 - \exp\left(-\frac{\pi}{\epsilon_v}\right)} \right| \]

\[ \epsilon_v \equiv \frac{v}{\alpha} \]

- As \( v \) becomes large, \( S \) starts to approach 1.
- As \( v \) approaches 0, \( S \) behaves like \( 1/v \) and starts to diverge.
- Important in the nonrelativistic limit!
EFT Approach

- Process we consider is DM scattering
- Classify all EFTs with a light mediator and fermionic dark matter
  - Study scalar, vector, pseudoscalar and axial vector interactions
  - Dirac and Majorana fermions and Symmetric vs. Asymmetric

<table>
<thead>
<tr>
<th>Type</th>
<th>Process</th>
<th>Channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Majorana</td>
<td>$\chi\chi \rightarrow \chi\chi$</td>
<td>s, t, u</td>
</tr>
<tr>
<td>Dirac, Asymmetric</td>
<td>$\chi\chi \rightarrow \chi\chi$</td>
<td>t, u</td>
</tr>
<tr>
<td>Dirac, Symmetric</td>
<td>$\bar{\chi}\chi \rightarrow \bar{\chi}\chi$</td>
<td>s, t</td>
</tr>
</tbody>
</table>
Various Non-Relativistic Potentials

\[ V_{\text{scalar}}(r) = -\frac{\lambda^2}{4\pi r} e^{-m_\phi r} \]

\[ V_{\text{pseudoscalar}}(r) = \frac{\lambda^2}{4m_\chi^2 - m_\phi^2} \frac{\delta'(r)}{2\pi r} \left( 2S_1 \cdot S_2 - \frac{1}{2} \right) + \frac{\lambda^2}{4\pi} \frac{e^{-m_\phi r}}{m_\chi^2} \left[ \frac{m_\phi^2}{3r} S_1 \cdot S_2 + \frac{3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2}{r^3} (1 + m_\phi r + \frac{m_\phi^2 r^2}{3}) \right] \]

\[ V_{\text{vector}}(r) = -\frac{\lambda^2}{4m_\chi^2 - m_A^2} \frac{\delta'(r)}{2\pi r} \left( \frac{3}{2} + 4S_1 \cdot S_2 \right) - \frac{\lambda^2}{4\pi r} e^{-m_A r} \]

\[ V_{\text{axial vector}}(r) = \frac{\lambda^2}{4m_\chi^2 - m_A^2} \frac{\delta'(r)}{2\pi r} \left( \frac{1}{2} - 2S_1 \cdot S_2 \right) - \frac{\lambda^2}{\pi r} e^{-m_A r} S_1 \cdot S_2 + \frac{4m_\chi^2}{m_A^2} V_{\text{pseudoscalar}} \]
How do we renormalize?

- Introduce a UV cutoff
  - Removes high momentum states
  - Softens the short range behavior

- Add local counterterms
  - Systematically removes cutoff dependence
  - Derivative expansion

- Make sure we have the correct long range behavior
Coulomb Potential Example

- Step 1: Introduce the UV cutoff

\[
\frac{1}{r} \rightarrow \frac{4\pi}{q^2} \rightarrow \frac{4\pi}{q^2} e^{-a^2 q^2 / 2} \rightarrow \frac{erf(r / \sqrt{2a})}{r}
\]
Coulomb Potential Example

- **Step 1**: Introduce the UV cutoff
  \[
  \frac{1}{r} \rightarrow \frac{4\pi}{q^2} \rightarrow \frac{4\pi}{q^2} e^{-a^2 q^2 / 2} \rightarrow \frac{erf(\sqrt{2a} r)}{r}
  \]

- **Step 2**: Add a local counterterm
  \[
  V_{eff} = -\frac{\alpha}{r} erf(\sqrt{2a} r) + 2\pi \alpha ea^2 \delta^3_a(r)
  \]
Coulomb Potential Example

- Step 1: Introduce the UV cutoff
  \[ \frac{1}{r} \rightarrow \frac{4\pi}{q^2} \rightarrow \frac{4\pi}{q^2} e^{-a^2 q^2 / 2} \rightarrow \frac{e r f(r / \sqrt{2a})}{r} \]

- Step 2: Add a local counterterm
  \[ V_{\text{eff}} = -\frac{\alpha}{r} e r f(r / \sqrt{2a}) + 2\pi\alpha c a^2 \delta_a^3(r) \]

- Step 3: Perturbative matching
  \[ -\frac{4\pi\alpha}{q^2} e^{-a^2 q^2 / 2} (1 + c a^2 q^2 / 2) = -\frac{4\pi\alpha}{q^2} (1 + (c - 1)a^2 q^2 / 2 + \mathcal{O}(a^4 q^4)) \]
Computing $\sigma_V$

- Solve the Schrodinger Equation
  - $\phi'' + (k^2 - l(l+1)\text{Ang}(r,a) - m \chi V_{\text{reg}}(r))\phi = 0$
- Match onto the asymptotic form
  - $\phi \rightarrow c_1 \sin(kr-l\pi/2) - c_2 \cos(kr-l\pi/2)$
- Extract the phase shift (S-matrix)
- Compute the viscous cross section

$$\sigma_V = \sum_{l=0}^{\infty} \frac{(l+1)(l+2)}{(2l+3)} \sin^2(\delta_{l+2} - \delta_l)$$
Conclusions

● Understanding the space of theories of SIDM is an interesting theoretical problem to solve
● The viscous cross section is the relevant quantity of interest
● There is a well-defined procedure for computing the viscous cross section
● Sommerfeld enhancement is included in a consistent manner
Thank You!
Backup
Red: Dwarf galaxy data

Blue: Low Surface Brightness galaxy data

Green: Cluster data

Gray: SIDM N-body simulation halos

Best fit dark photon model curve shown

Kaplinghat, Tulin, Yu [1508.03339]
Hulthen Potential

\[ V(r) = -\frac{\alpha m_* e^{-m_* r}}{1 - e^{-m_* r}}. \]

Blum, Sato, Slatyer [1603.01383]
## Various Non-Relativistic Potentials

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<tr>
<th>Mediator</th>
<th>Interaction</th>
<th>$\frac{1}{r}$</th>
<th>$\frac{s_{1} \cdot s_{2}}{r}$</th>
<th>$\frac{3(s_{1} \cdot \hat{r})(s_{2} \cdot \hat{r}) - s_{1} \cdot s_{2}}{r^3}$</th>
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<td>Scalar</td>
<td>$\lambda_s \overline{\chi} \chi \phi$</td>
<td>$-\lambda_s^2$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Vector</td>
<td>$\lambda_v \overline{\chi} \gamma^\mu \chi A_\mu$</td>
<td>$\pm \left(1 + \frac{m_A^2}{4m_\chi^2}\right)$</td>
<td>$\pm \frac{2\lambda_v^2 m_A^2}{3m_\chi^2}$</td>
<td>$\mp \frac{\lambda_v^2}{m_\chi^2}\left(1 + m_A r + \frac{m_A^2 r^2}{3}\right)$</td>
</tr>
<tr>
<td>Pseudoscalar</td>
<td>$i\lambda_p \overline{\chi} \gamma^5 \chi \phi$</td>
<td>0</td>
<td>$\frac{\lambda_p^2 m_\phi^2}{3m_\chi^2}$</td>
<td>$\frac{\lambda_p^2}{m_\chi^2}\left(1 + m_\phi r + \frac{m_\phi^2 r^2}{3}\right)$</td>
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<tr>
<td>Axial Vector</td>
<td>$\lambda_a \overline{\chi} \gamma^5 \gamma^\mu \chi A_\mu$</td>
<td>0</td>
<td>$-\frac{8\lambda_a^2}{3}\left(1 - \frac{m_A^2}{8m_\chi^2}\right)$</td>
<td>$\lambda_a^2\left(\frac{1}{m_\chi^2} + \frac{4}{m_A^2}\right)\left(1 + m_A r + \frac{m_A^2 r^2}{3}\right)$</td>
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Bellazzini, Cliche, Tanedo [1307.1129]
The Universal Energy Budget

- What makes up our universe?
  - Radiation
  - Ordinary Matter
  - Dark Matter (CDM)
  - Cosmological Constant ($\Lambda$)

- $\Lambda$CDM is successful on the largest scales
CDM Signatures & Evidences

- Rotation Curves

M33 Rotation Curve
CDM Signatures & Evidences

- Rotation Curves
- Gravitational Lensing - Bullet Cluster
CDM Signatures & Evidences

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- Gravitational Lensing - Bullet Cluster
- Cosmic Microwave Background
Small Scale Problems with CDM

- Core vs. Cusp Problem
  - Simulations show cuspy profiles whereas rotation curve observations show cored profiles
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  - Most luminous galaxies predicted to inhabit the most massive subhalos.
  - Massive subhalos are expected to form stars and should host observable galaxies.
  - Low mass galaxies have observed velocities too small to be consistent with the mass of the subhalos they are expected to inhabit.
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- **Diversity Problem**
Baryonic Feedback vs. Self-Interacting Dark Matter

- Supernova driven outflows can help:
  - Flatten the dark matter cusp into a core
  - Deplete baryons and render low mass halos incapable of forming satellites
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- SIDM is an interesting alternative
  - Alleviate core vs cusp problem and too big to fail problem by scattering
  - Can give rather interesting signals in experiments depending on how it interacts with the Standard Model
  - Theoretically well motivated question to ask whether dark matter is collisional or not, even if it is just within the dark sector