# Kinematic Wavelet Analysis Kit (KWAK)

**Global Analyses of Kinematic Distributions with Wavelets** (1905:XXXXX)

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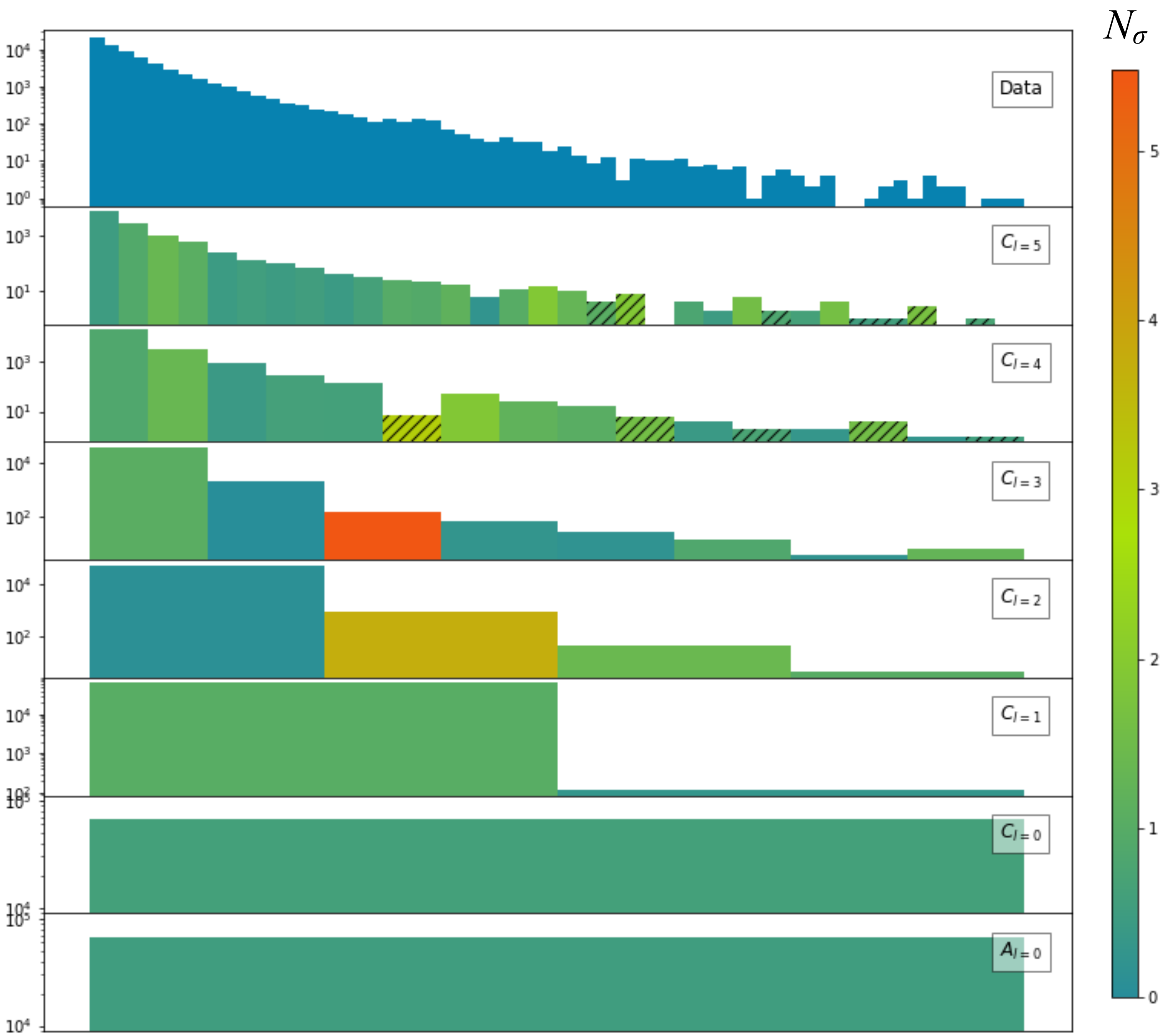
- Going beyond bump hunting in the search for signals of new physics

- A publicly-available package for statistical analysis

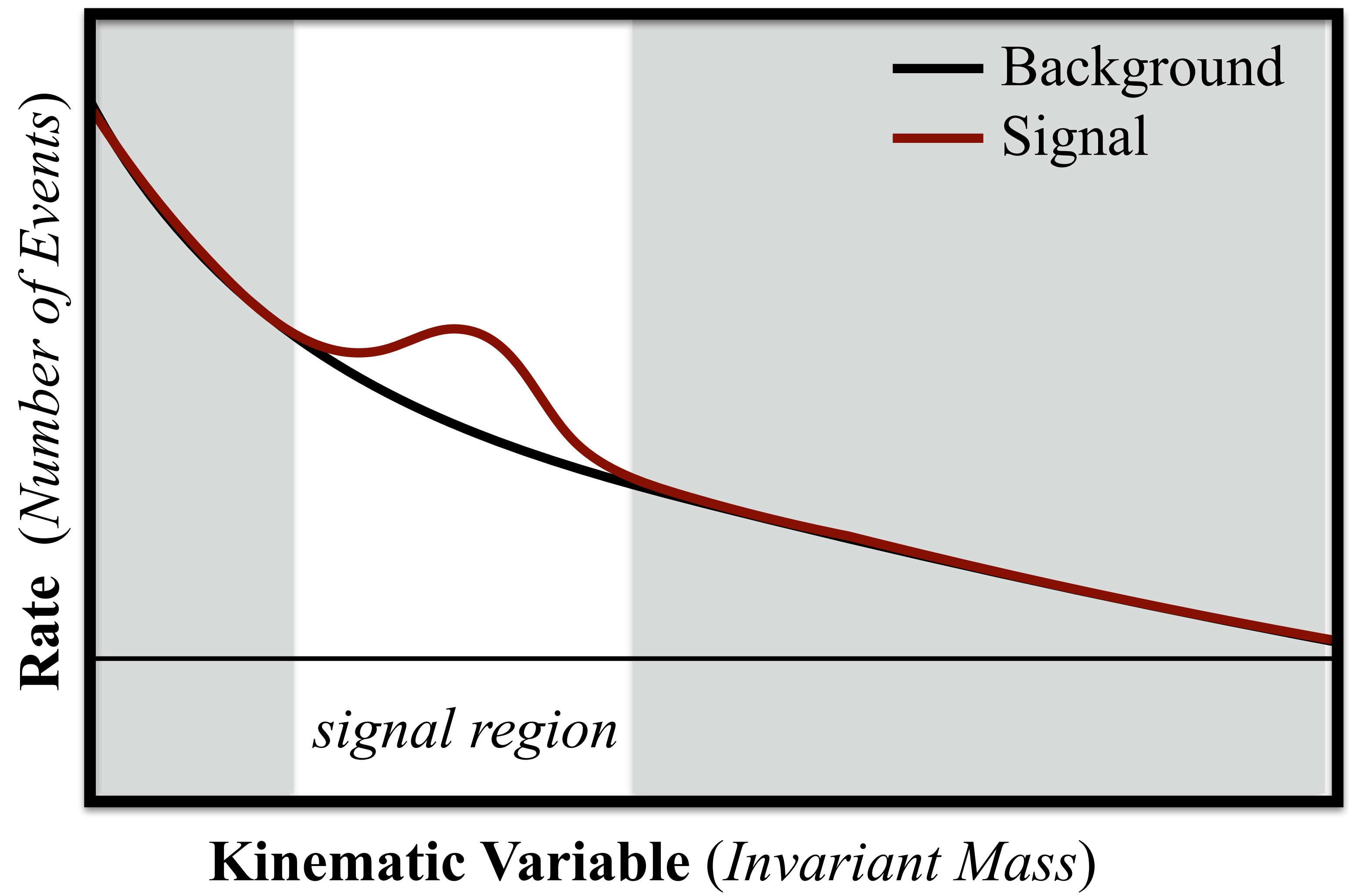


# Motivation: Benefits of a Global Analysis

A standard bump search is optimized for cases like this: here, the presence of a new particle increases the rate of some process.

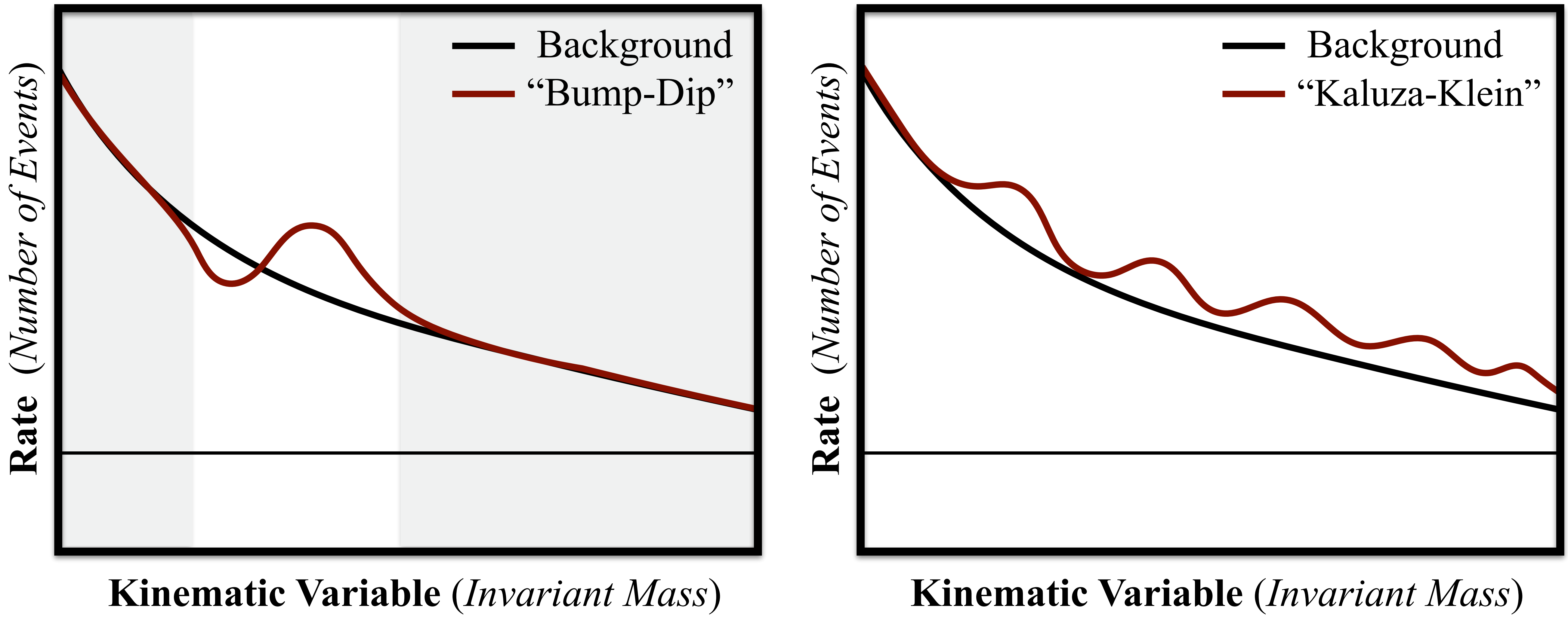
But, not all signals of new physics are so simple:

- Interference Effects
- Wide Resonances
- Multiple New States



# Goal for a Global Analysis:

How can we **systematically identify** these signs of new physics, when the signal is **not** simply a local deviation from a background model?



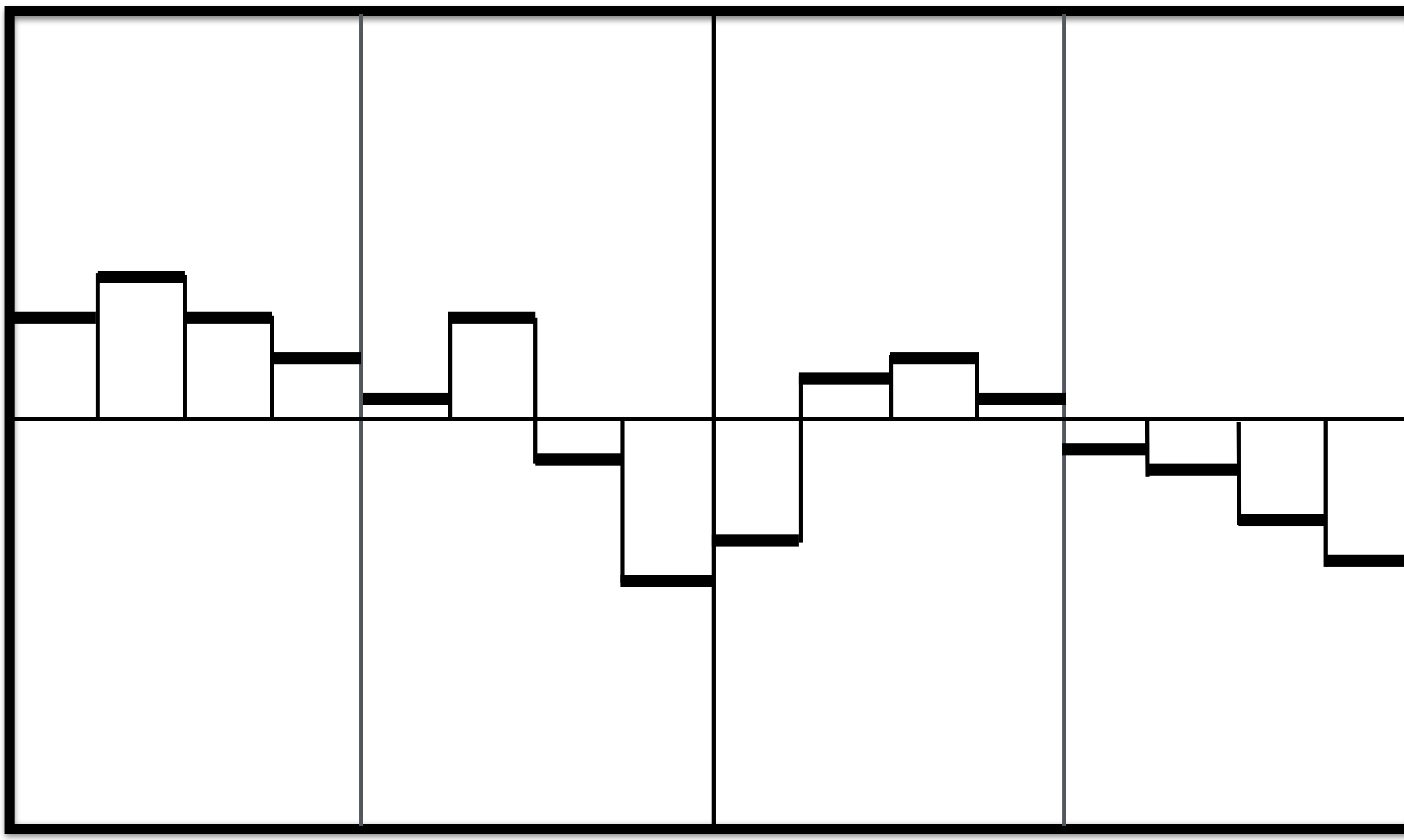

- Looking for a **basis of functions** that can pick out **global patterns**, while still identifying **local deviations**

# The Wavelet Transformation(s)

**A compromise between "frequency" and "position"**

- A linear, invertible transformation
- Orthogonal basis of step functions, of various widths and translations

Demonstration:
A discrete signal
with **16 bins**



# The Wavelet Transformation(s)

**A compromise between "frequency" and "position"**

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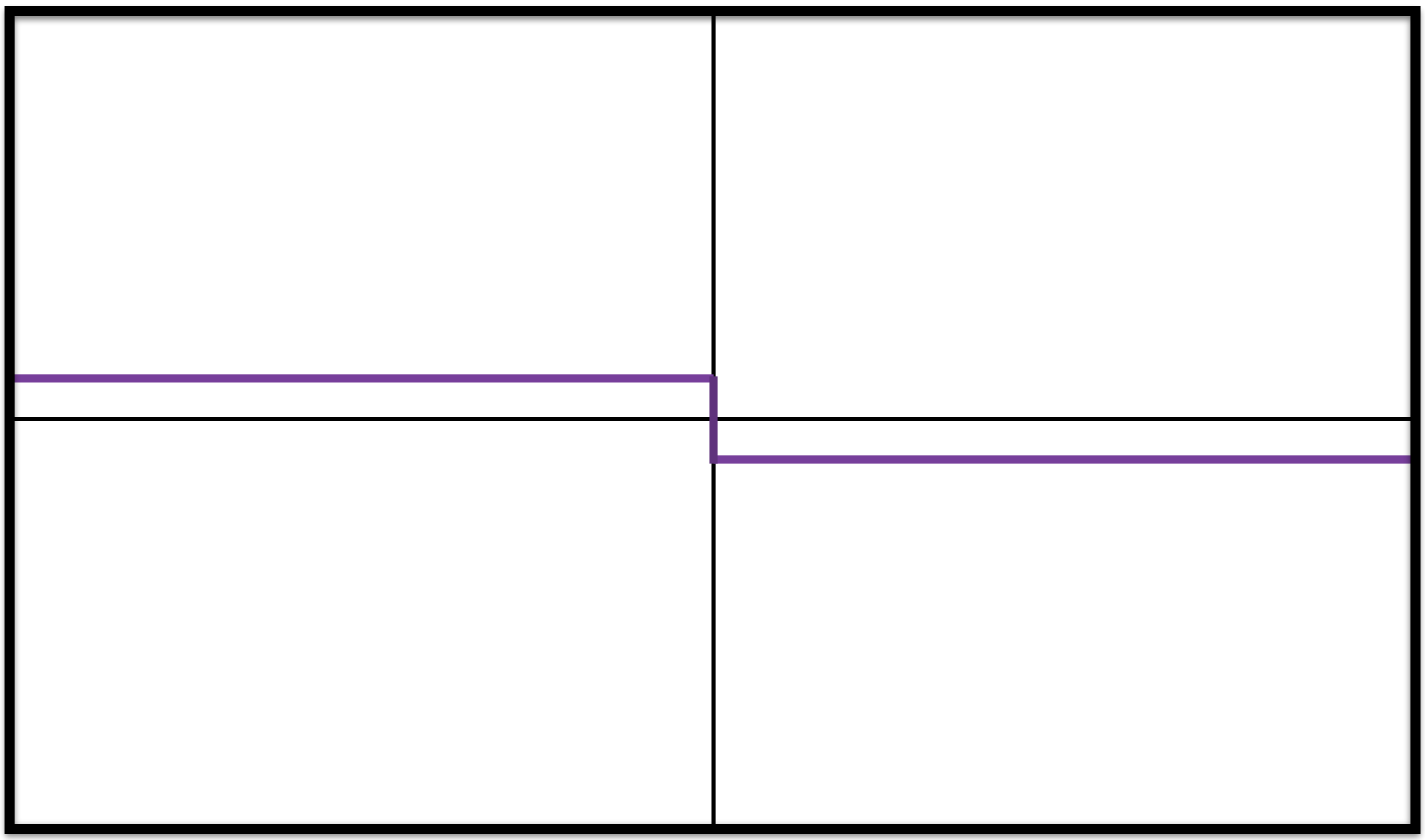
- Basis of step functions, of various widths and translations

First wavelet:
A single step function,
spanning the full width:

- **"L=1, j=1"**



# The Wavelet Transformation(s)

**A compromise between “frequency” and “position”**

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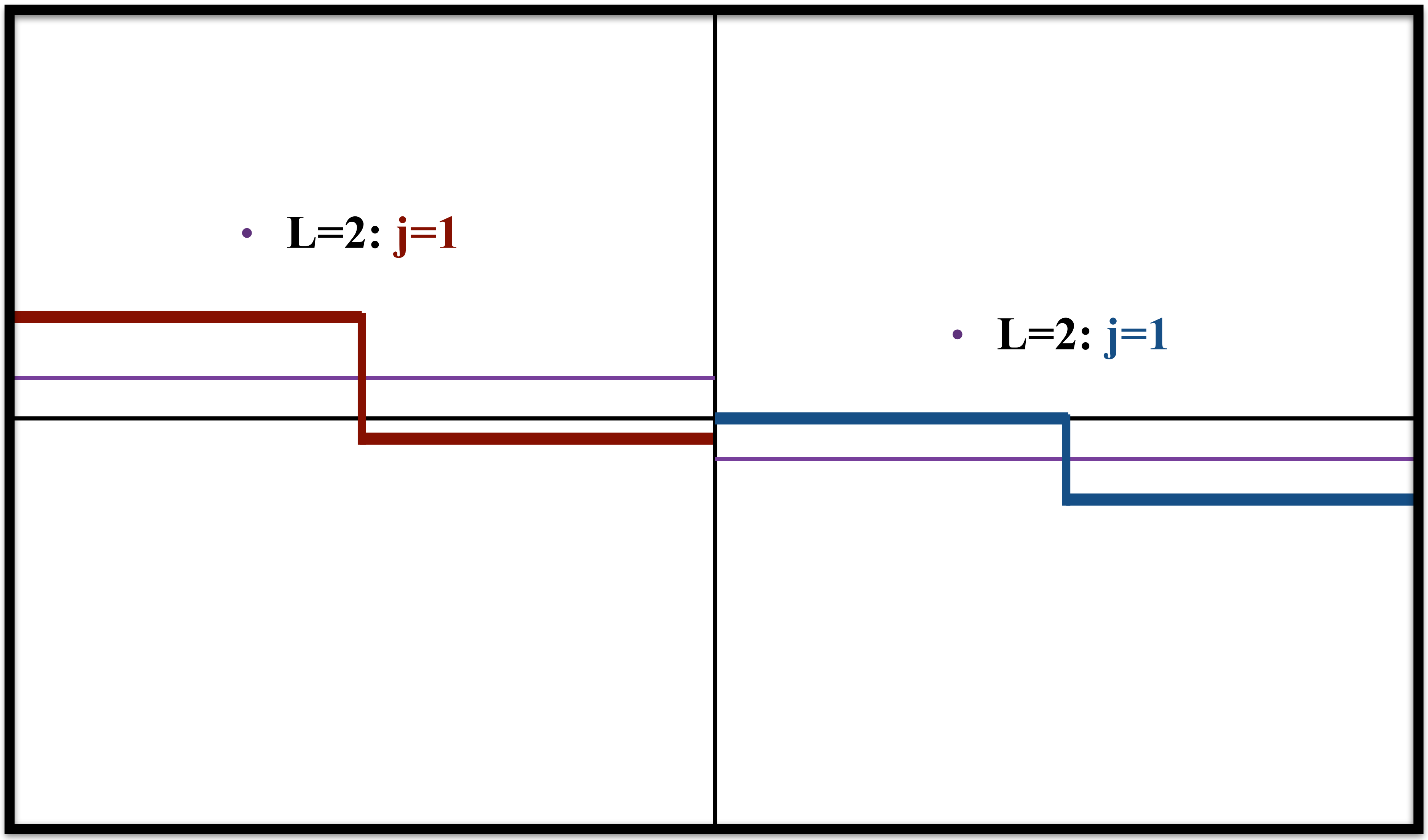
Next: subdivide each section into two:

- **L=2: j=1, j=2**
- **L=1: j=1**

The L=1 wavelets have a “base of support” of 8 bins each



# The Wavelet Transformation(s)

**A compromise between “frequency” and “position”**

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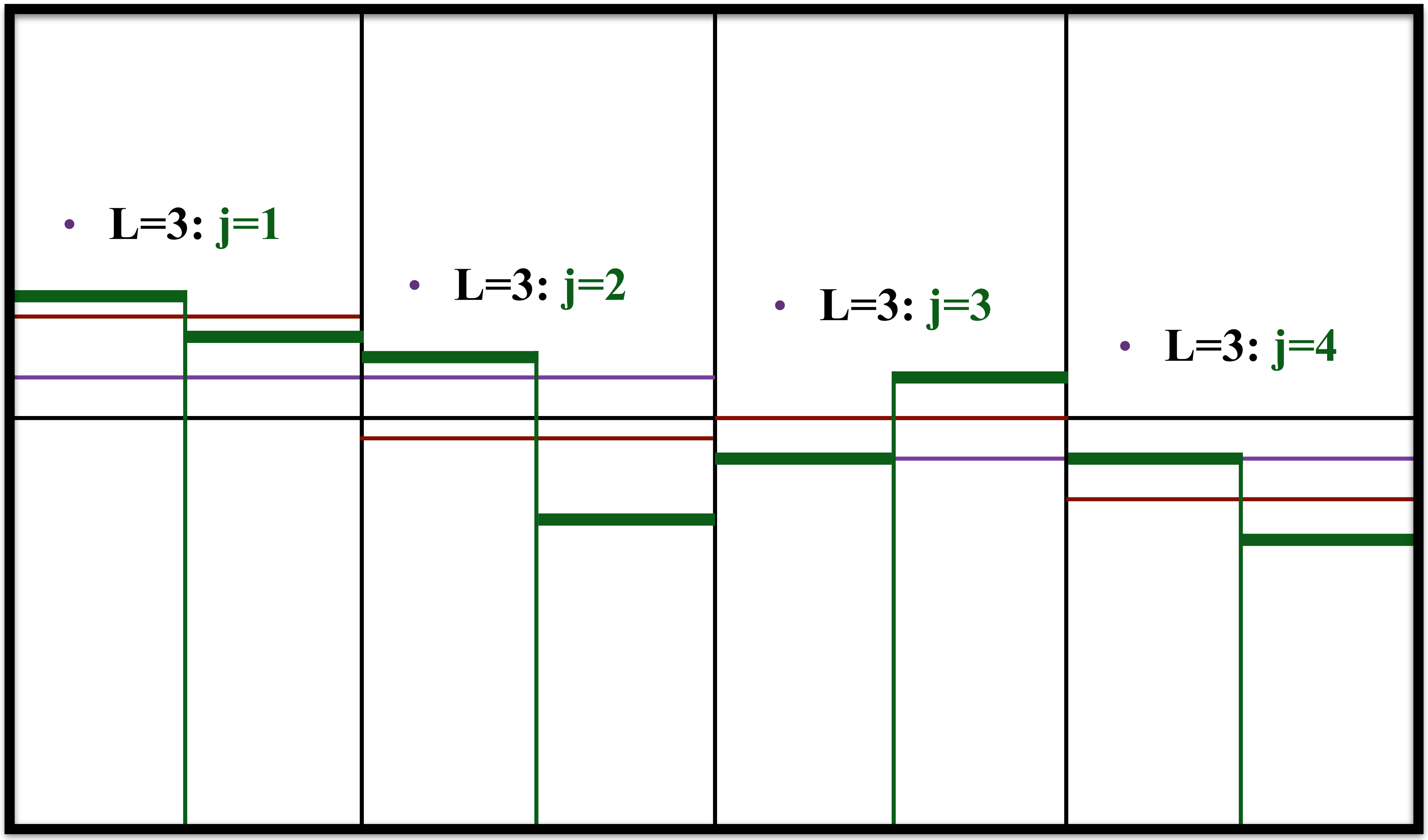
At the next-smallest level, there are four independent step functions.

Each has a base of support of 4 bins

- **L=3: j=1, 2, 3, 4**
- **L=2: j=1, 2**
- **L=1: j=1**



# The Wavelet Transformation(s)

**A compromise between "frequency" and "position"**

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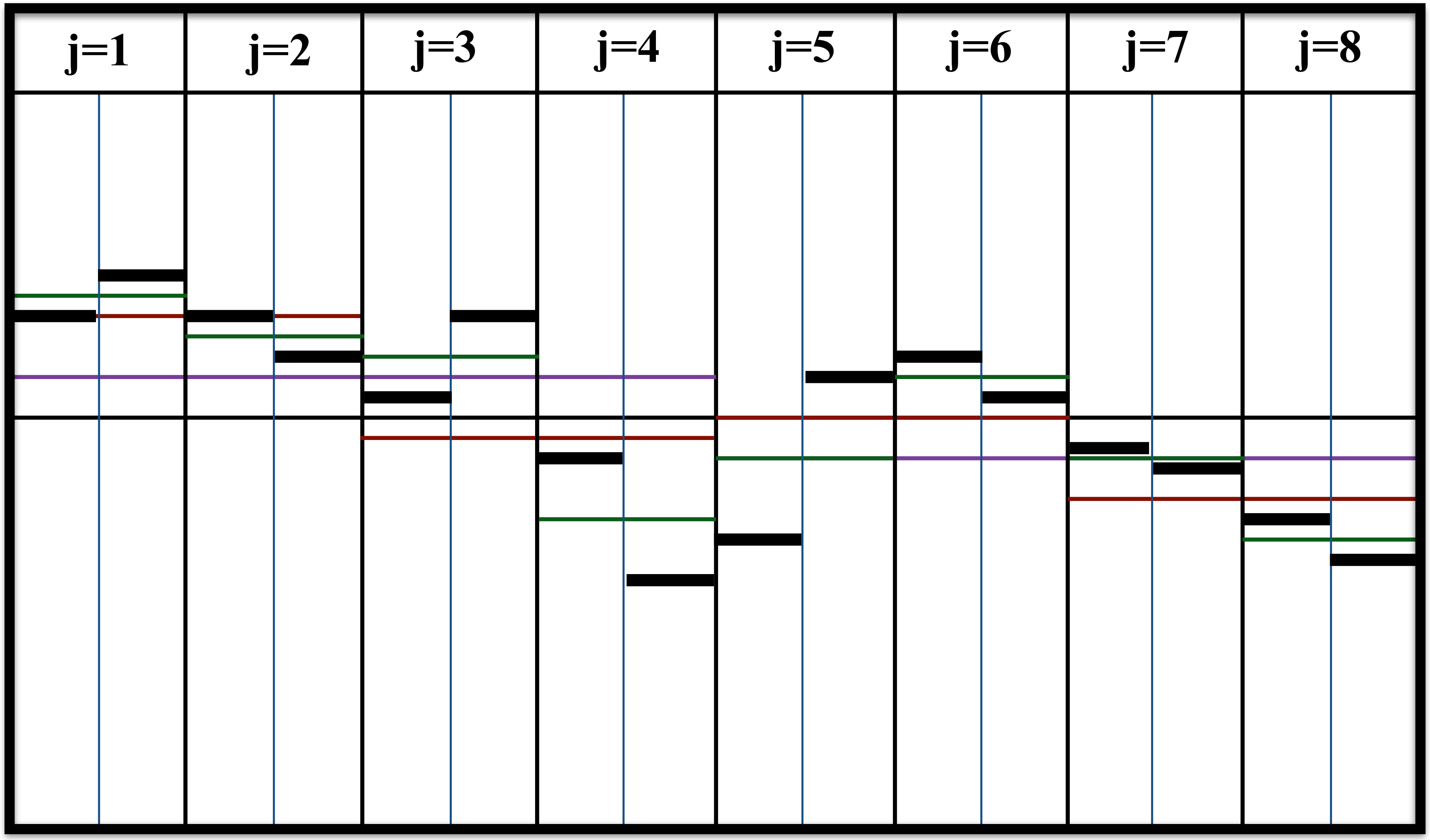
- Basis of step functions, of various widths and translations

The final set of wavelets includes 8 step functions, each with a base of support of 2 bins.

- **L=4: j=1, 2, 3, 4, 5, 6, 7, 8**
- **L=3: j=1, 2, 3, 4**
- **L=2: j=1, 2**
- **L=1: j=1**



# The Wavelet Transformation(s)

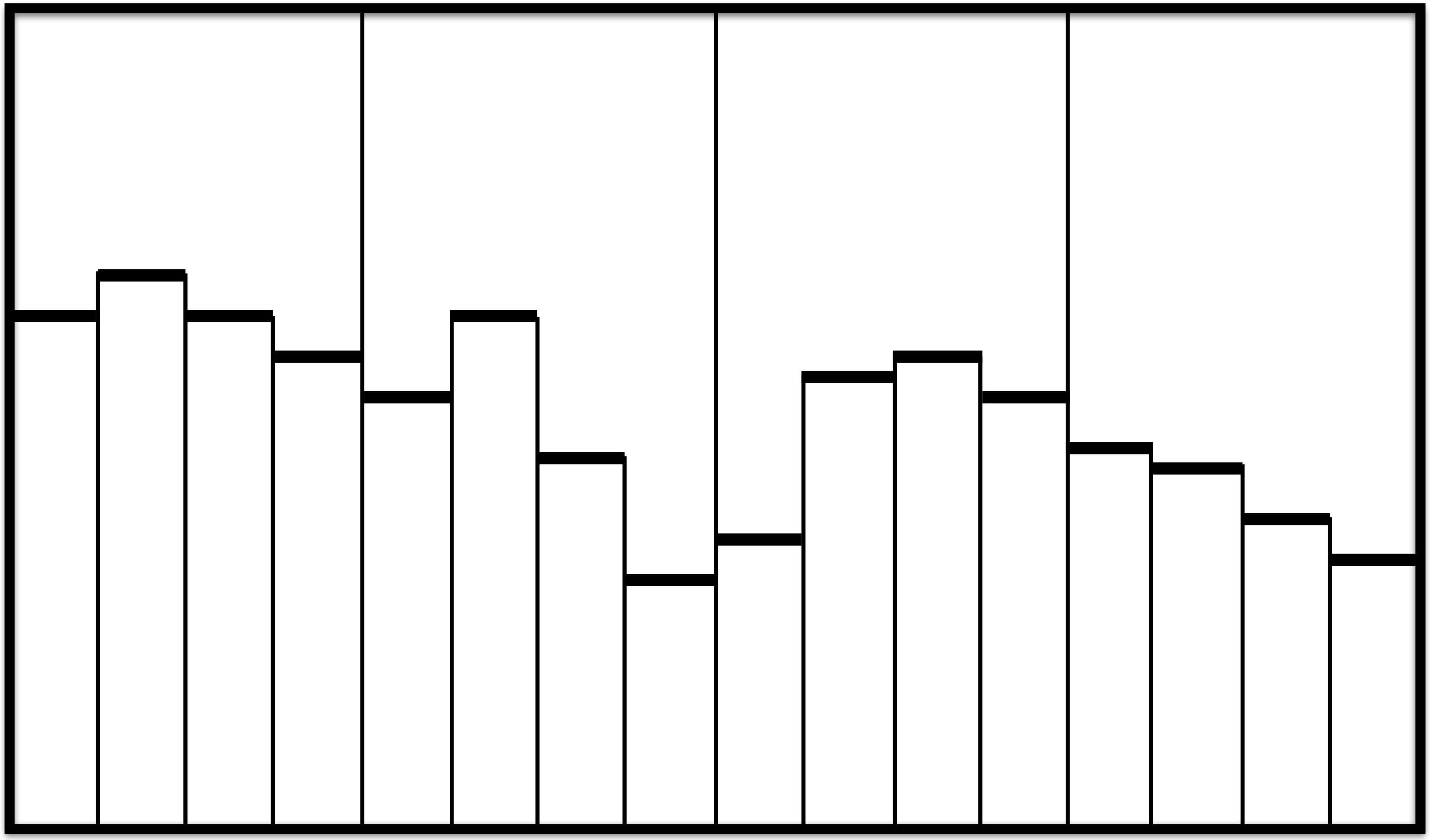
**A compromise between "frequency" and "position"**

- A linear, invertible transformation
- Basis of step functions, of various widths and translations

The wavelet transformation converts the 16 bin signal into:

- **L=4: 8**
- **L=3: 4**
- **L=2: 2**
- **L=1: 1**
- **L=0: 1**

***16 wavelet coefficients***



# Building A Statistical Analysis Tool

Given a hypothesis for each bin, the **probability distribution** for each **wavelet coefficient** can be calculated…

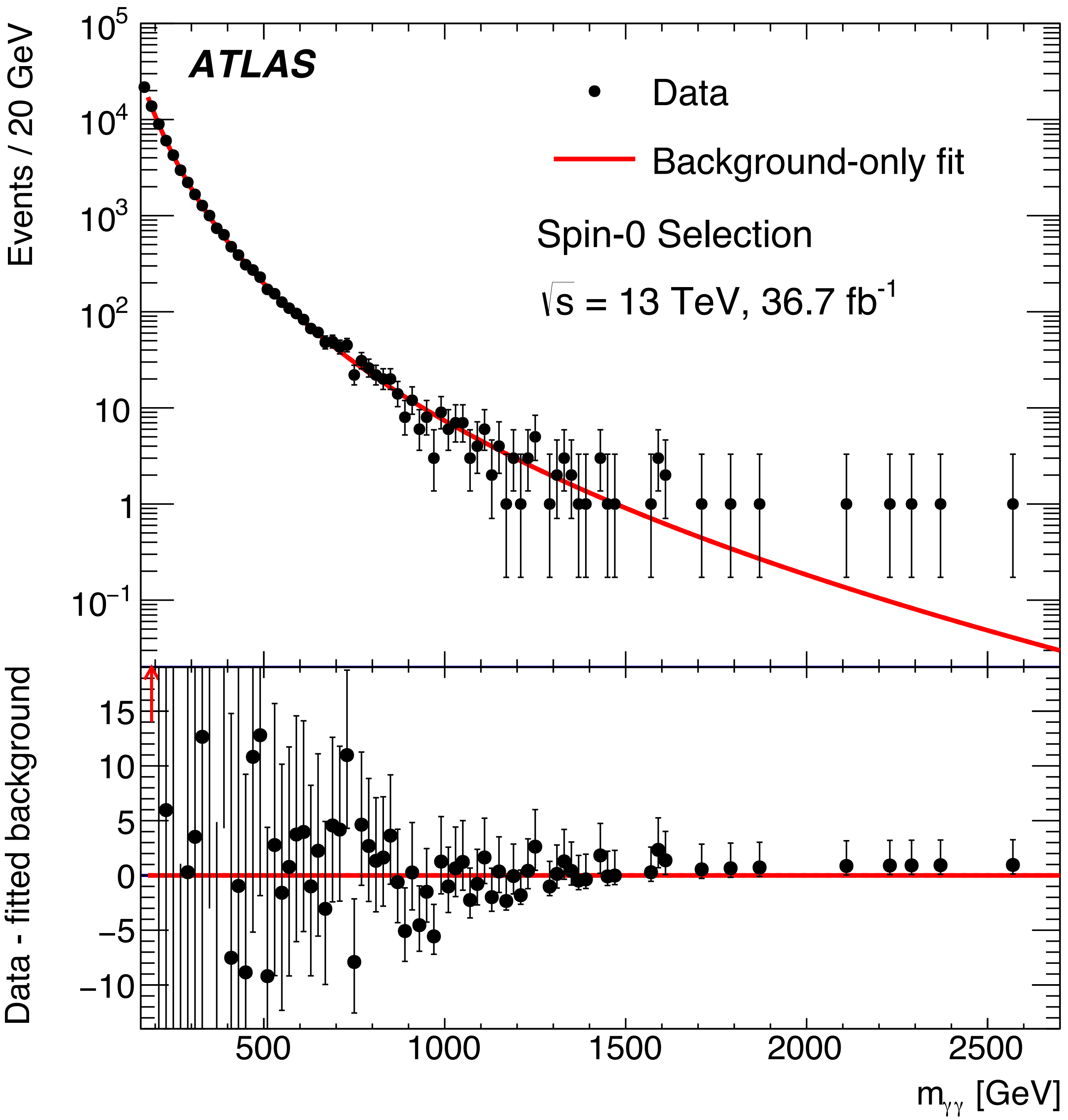
- Exactly (with Poisson statistics):

$$p(\tilde{f}|\mu_1,\mu_2) = \sum_{f_1,f_2} \frac{e^{-\mu_1-\mu_2}\,\mu_1^{f_1}\mu_2^{f_2}}{f_1!f_2!}\Bigg|_{\tilde{f}=f_1-f_2}$$

$$= e^{-\mu_1-\mu_2}\left(\frac{\mu_1}{\mu_2}\right)^{\tilde{f}/2}\mathcal{I}_{\tilde{f}}(2\sqrt{\mu_1\mu_2})$$

- Or approximately, by generating a large number of pseudoexperiments.

Then, calculate a likelihood ($p$-value) for each wavelet coefficient to identify the significant deviations.



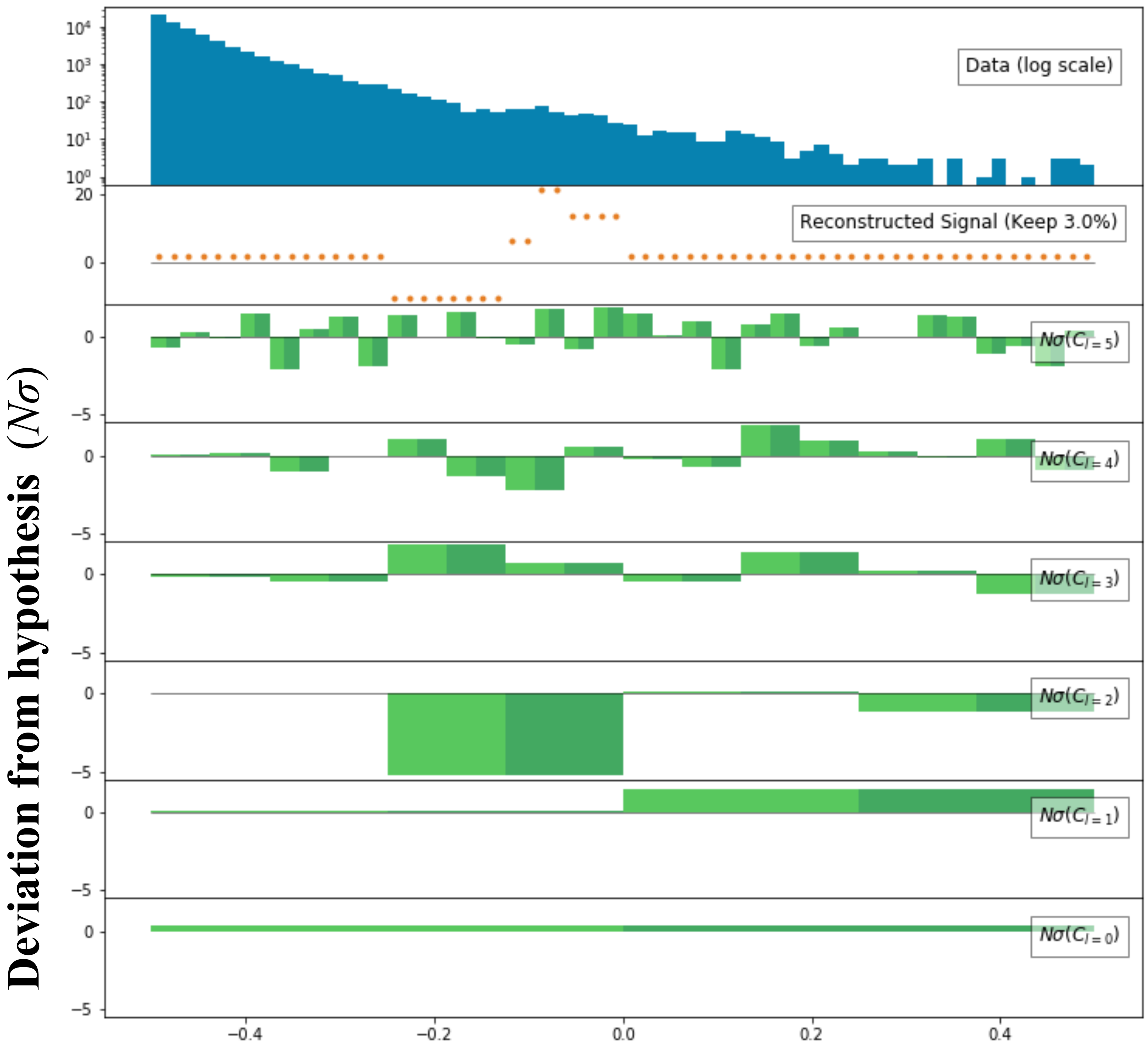
# Demonstration: A Bump-Dip Signal

As a simple example, we add a large dip-bump signal on top of a falling background, modeled from the ATLAS 37 $fb^{-1}$ diphoton data.

One wavelet coefficient shows a 5σ deviation from the background-only hypothesis.

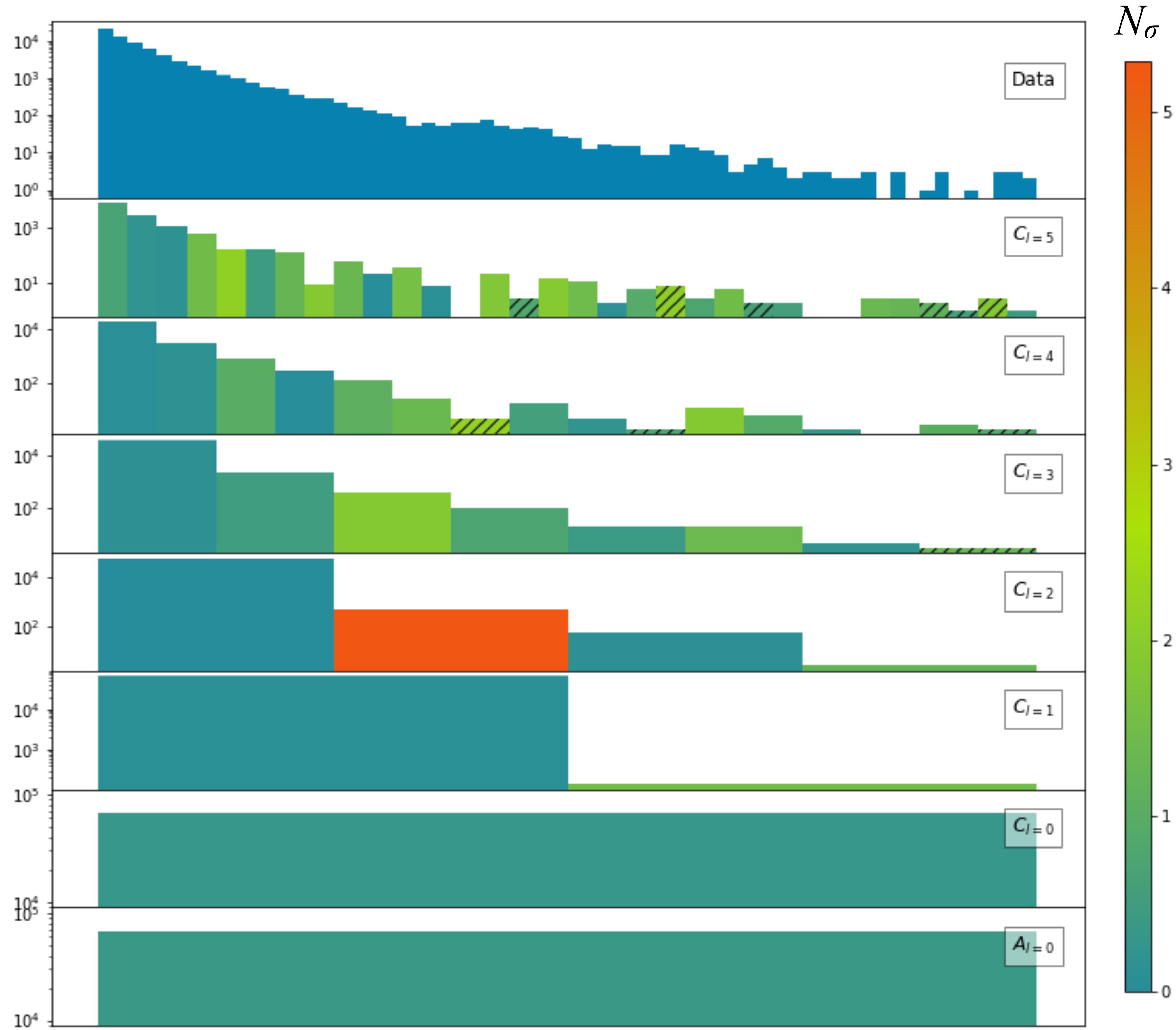
The injected signal can be recovered by performing the inverse wavelet transformation on just the 3% most deviant coefficients.

- **Localized signal is still localized in "wavelet space"**



# Demonstration: A Bump-Dip Signal

Alternative: plot the value of each wavelet coefficient, with a color coding based on the deviation from the background hypothesis

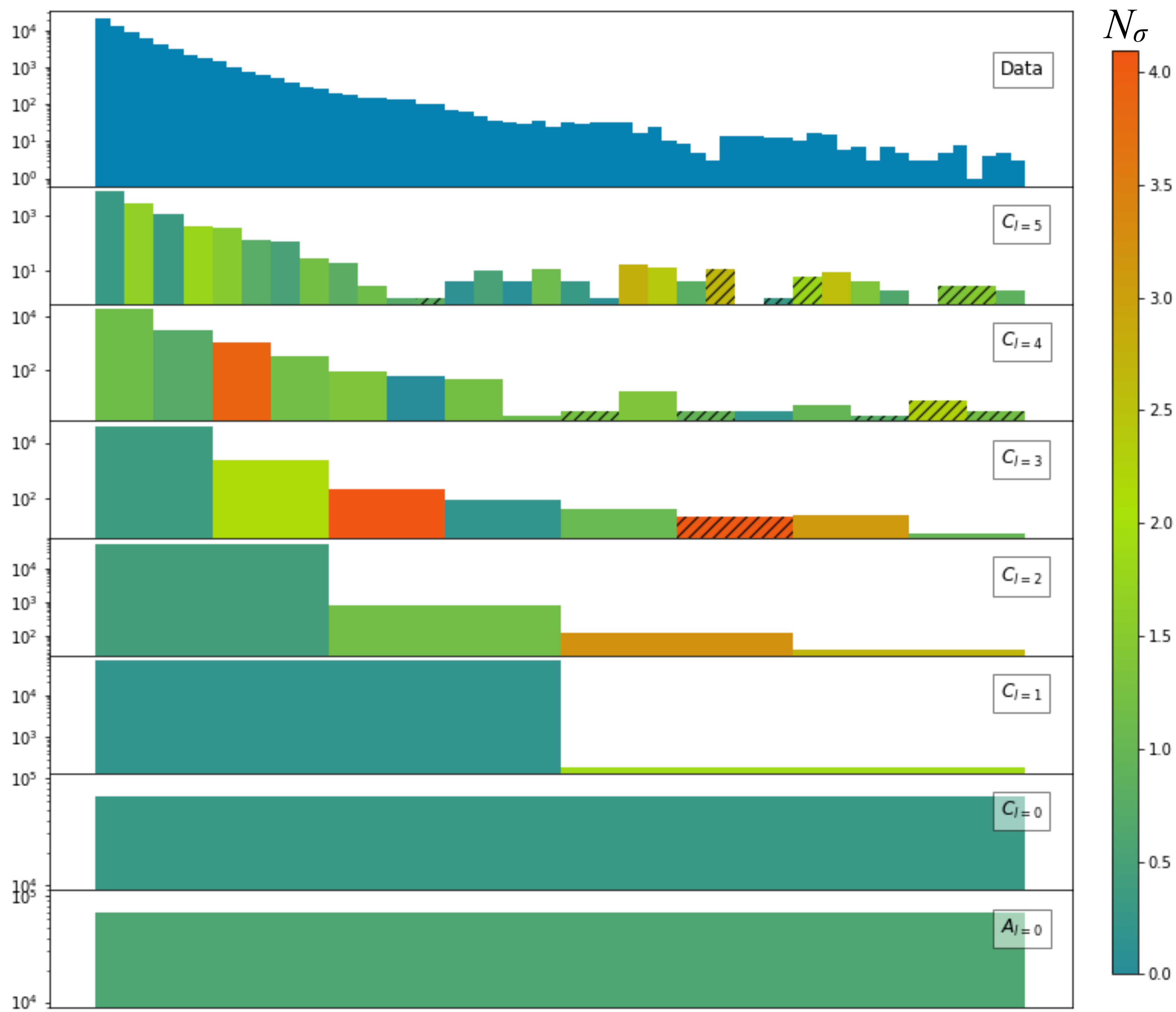



- **Localized signal is still localized in "wavelet space"**

# Demonstration II: A "Kaluza-Klein" Inspired Model

**Fixed-Resolution Global Significance:**

- Global patterns spanning multiple wavelet coefficients can be detected by combining the significances of non-overlapping (uncorrelated) bins
- Collecting all wavelet coefficients of the same "L":
  - L=1: 1.28$\sigma$
  - L=2: 3.37$\sigma$
  - **L=3: 5.39$\sigma$**
  - L=4: 2.95$\sigma$
  - L=5: 2.52$\sigma$

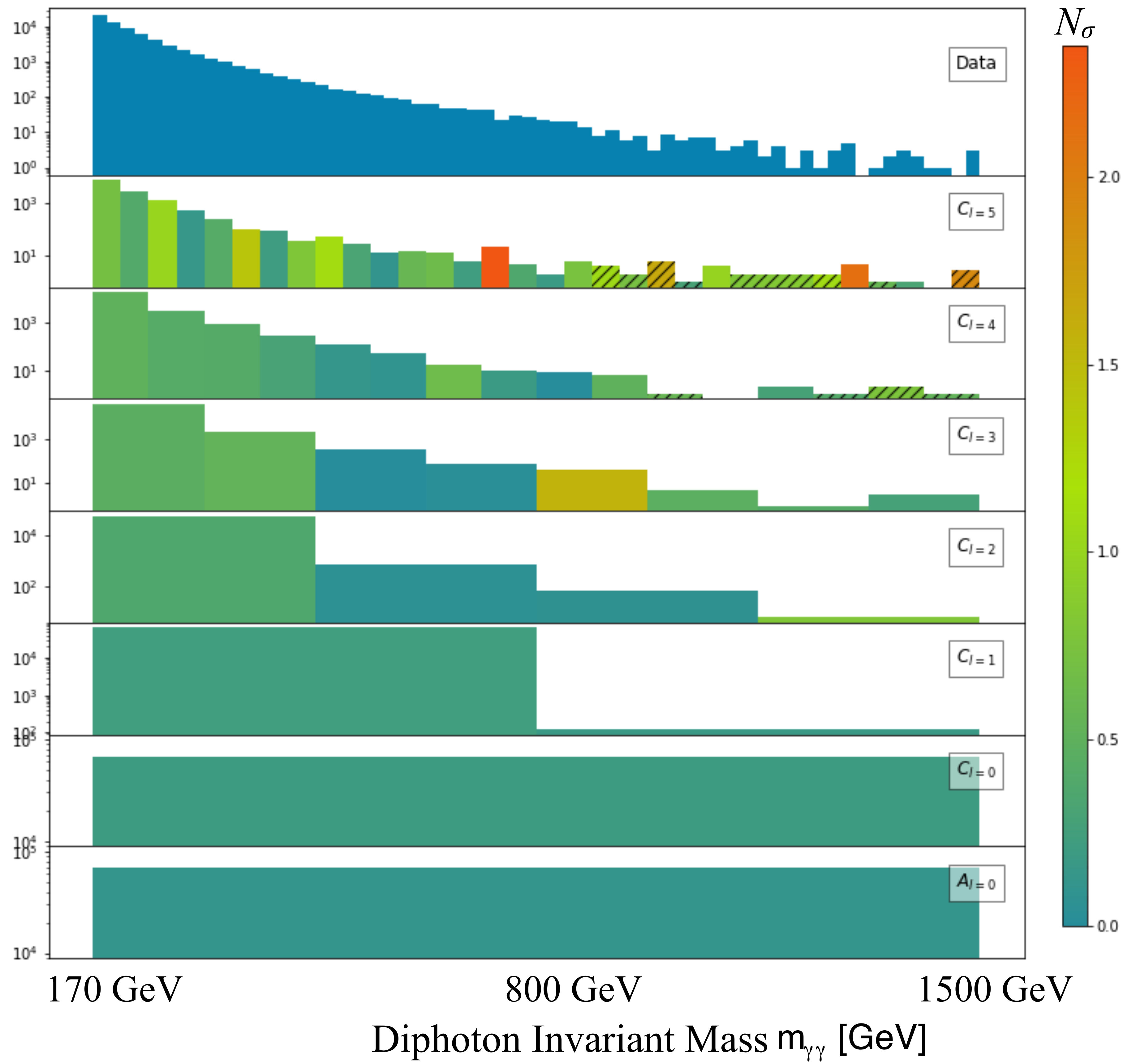


# Demonstration III: ATLAS 37 fb$^{-1}$ Diphoton

**Fixed Resolution Global Significance:**

- L=1: 0.062σ
- L=2: 0.055σ
- L=3: 0.083σ
- L=4: 0.00053σ
- L=5: 0.59σ

The ATLAS diphoton data shows some 2σ deviations at L=5, but nothing at the lower levels



**Global Analyses of Kinematic Distributions with Wavelets** (1905:XXXXX)

**BL**, T. Plehn, A. Romero, and T. Tait

Kinematic Wavelet Analysis Kit:

- A **publicly-available** package for global analysis
- Identifies **local and global** deviations
- **Reconstructs signal** with inverse transformation
- Flexible: fast approximation of probability distributions, or an alternative arbitrary-precision calculation (assuming purely statistical fluctuations)
- Easily adapted to other families of wavelet transformations

https://github.com/alexxromero/kwak_wavelets