Cancellations in Spin-2 KK Mode Scattering Amplitudes at High Energies

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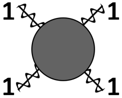
Introduction

Unitarity Violation in Massive Spin-2 State Theories

Consider tree-level longitudinal (helicity = 0) scattering of a massive spin-2 particle "1" at high energies in 4D:

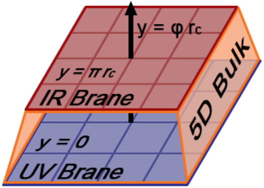
$\mathcal{M} =$  $\sim \mathcal{O}(s^{???})$ in large s regime

- **2002:** Arkani-Hamed, Georgi, and Schwartz argue that $\mathcal{M}$ diverges like $\mathcal{O}(s^5)$ in naive 4D massive gravity, and like $\mathcal{O}(s^3)$ in carefully tuned spin-2 theories. (hep-th/0210184)
- **2003:** Schwartz argues away $\mathcal{O}(s^3)$ in the 5D Torus by applying a Stueckelberg-like mechanism to the Lagrangian; does not calculate matrix elements (hep-th/0303114)
- **2019:** We explicitly calculate matrix elements, and demonstrate cancellations such that $\mathcal{M} \sim \mathcal{O}(s)$ in 5D (Orbifolded) Torus and Randall-Sundrum models.

RS Model: 5D to 4D

The Randall-Sundrum (RS) Vacuum & Lagrangian

Coordinates: Add a warped compact spatial dimension $y \in [0, \pi r_c]$ to the usual 4D spacetime (x^μ).



$$ds^2 = e^{-2k|y|}(dt^2 - d\vec{x}^{\,2}) - dy^2$$

Vac. Metric: $$\eta_{MN} = \begin{pmatrix} e^{-2k|y|}\eta_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix}$$

Orbifold Symmetry: Reflect extra-dimension about $y = 0$, demand ds^2 be invariant under $y \leftrightarrow -y$, so $y \in [-\pi r_c, +\pi r_c]$.

4D Cosmological Constant: Eliminate via CC & tensions...

$$\star \quad \mathcal{L}_{\text{5D}}^{(\text{RS,vac})} = \frac{2}{\kappa^2}\sqrt{\det \eta}\, R + \Big[\text{bulk CC} + \text{brane tensions}\Big] \quad \star$$

5D Theory: Particle Content

Many options for perturbing vacuum. The **Einstein frame parameterization** is automatically canonical in 4D:

$$G_{MN} = \begin{pmatrix} e^{-2(k|y|+\hat{u})}(\eta_{\mu\nu}+\kappa\hat{h}_{\mu\nu}) & 0 \\ 0 & -(1+2\hat{u})^2 \end{pmatrix} \qquad \hat{u} \equiv \frac{\kappa\,\hat{r}}{2\sqrt{6}} e^{+k(2|y|-\pi r_c)}$$

where κ characterizes the perturbation and $[\kappa] = [\text{Energy}]^{-3/2}$

Particles in 5D Matter-Free Randall-Sundrum Model:

- **5D Graviton** $= \hat{h}_{\mu\nu}$, a massless spin-2 5D particle
  ▷ **Origin:** local coordinate invariance of constant y sheets

- **5D Radion** $= \hat{r}$, a massless spin-0 5D particle
  ▷ **Origin:** locally perturbing distance between branes
  Caution: In realistic theories, radion requires additional external stabilization, e.g. Goldberger-Wise mechanism; radion stabilization plays no role in the present analysis.

5D Theory: Weak Field Expansion (& Code)

$$\star \quad \boxed{\mathcal{L}_{5\mathrm{D}}^{(\mathrm{RS})} = \frac{2}{\kappa^2}\sqrt{\det G}\, R + \left[\text{bulk CC} + \text{brane tensions}\right]} \quad \star$$

We obtain a useful 5D Lagrangian via **weak field expansion** = series expansion in κ. **Performing WFE and calculating the matrix elements can be computationally intensive.**

I developed a diagrammatic method to do WFE, generalized my diagrams to contain arbitrary bosonic content, and coded the whole formalism into Mathematica. **For a wide class of (4+)D metrics, my code performs WFE, IBP reduction, and calculates vertex rules in the 4D effective theory.**

I'll be releasing this code (& more) this summer!

5D to 4D: Organizing the 5D Lagrangian

Once we have a 5D theory, we convert it into an effective 4D theory via y-integration and **Kaluza-Klein decomposition**:

$$\mathcal{L}^{(\text{RS,eff})}_{\text{4D}} \equiv \int_{-\pi r_c}^{+\pi r_c} dy\ \mathcal{L}^{(\text{RS})}_{\text{5D}}$$

$$\underbrace{\hat{f}_{\vec{\mu}}(x,y)}_{\text{5D Field}} = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \underbrace{\hat{f}^{(n)}_{\vec{\mu}}(x)}_{\text{4D Fields}} \underbrace{\psi^{(n)}(y)}_{\text{Wfxn}}$$

Define $\mathcal{L}^{(\text{RS})}_{h^H r^R} \equiv$ all terms in $\mathcal{L}^{(\text{RS})}_{\text{5D}}$ with H graviton fields and R radion fields. By construction, each term in this set is either

- **A-Type:** has two spatial derivatives $\partial_\mu \partial_\nu$, or
- **B-Type:** has two extra-dimensional derivatives ∂_y^2

$$\begin{aligned}\mathcal{L}^{(\text{RS})}_{h^H r^R} &= \mathcal{L}^{(\text{RS})}_{A:h^H r^R} + \mathcal{L}^{(\text{RS})}_{B:h^H r^R} \\ &= \kappa^{(H+R-2)} \left[e^{k[2(R-1)|y| - R\pi r_c]}\ \overline{\mathcal{L}}^{(\text{RS})}_{A:h^H r^R} + e^{k[2(R-2)|y| - R\pi r_c]}\ \overline{\mathcal{L}}^{(\text{RS})}_{B:h^H r^R} \right]\end{aligned}$$

5D to 4D: KK Decomposition of Quadratic Spin-2

$$\mathcal{L}_{hh}^{(\mathrm{RS})} = e^{-2k|y|}\overline{\mathcal{L}}_{A:hh}^{(\mathrm{RS})} + e^{-4k|y|}\overline{\mathcal{L}}_{B:hh}^{(\mathrm{RS})}$$

$$\overline{\mathcal{L}}_{A:hh}^{(\mathrm{RS})} = -\hat{h}_{\mu\nu}(\partial^\mu\partial^\nu\hat{h}) + \hat{h}_{\mu\nu}(\partial^\mu\partial_\rho\hat{h}^{\rho\nu}) - \frac{1}{2}\hat{h}_{\mu\nu}(\Box\hat{h}^{\mu\nu}) + \frac{1}{2}\hat{h}(\Box\hat{h})$$

$$\overline{\mathcal{L}}_{B:hh}^{(\mathrm{RS})} = -\frac{1}{2}(\partial_y\hat{h}_{\mu\nu})(\partial_y\hat{h}^{\mu\nu}) + \frac{1}{2}(\partial_y\hat{h})^2$$

Extra-D wavefunctions $\psi^{(n)}(y)$ satisfy ($n \equiv$ **KK number**)

$$\frac{1}{\pi r_c}\int_{-\pi r_c}^{+\pi r_c} dy\ e^{-2k|y|}\psi^{(m)}\psi^{(n)} = \delta_{m,n}$$

$$\frac{1}{\pi r_c}\int_{-\pi r_c}^{+\pi r_c} dy\ e^{-4k|y|}(\partial_y\psi^{(m)})(\partial_y\psi^{(n)}) = m_n^2\ \delta_{m,n}$$

and thereby ensure 4D effective spin-2 Lagrangian is canonical:

$$\mathcal{L}_{hh}^{(\mathrm{RS,eff})} = \mathcal{L}_{\mathrm{Kin}}^{(S=2)}(\hat{h}^{(0)}) + \sum_{n=1}^{+\infty}\mathcal{L}_{\mathrm{FP}}(m_n, \hat{h}^{(n)})$$

5D to 4D: KK Decomposition of Quadratic Spin-0

$$\boxed{\mathcal{L}_{rr}^{(\text{RS})} = e^{+2k(|y|-\pi r_c)}\overline{\mathcal{L}}_{A:rr}^{(\text{RS})}} \qquad \text{where} \qquad \overline{\mathcal{L}}_{A:rr}^{(\text{RS})} = \frac{1}{2}(\partial_\mu \hat{r})(\partial^\mu \hat{r})$$

The radion is flat in the extra-dimension, so we may write

$$\hat{r}(x) = \frac{1}{\sqrt{\pi r_c}}\hat{r}_{\mu\nu}^{(0)}(x)\psi^{(0)} \qquad \Longrightarrow \qquad \boxed{\mathcal{L}_{rr}^{(\text{RS,eff})} = \mathcal{L}_{\text{Kin}}^{(S=0)}(\hat{r}^{(0)})}$$

Particles in 4D Effective Randall-Sundrum Model:

- **5D Graviton** $\hat{h}_{\mu\nu}$ becomes many 4D particles:
  - **4D Graviton:** a massless spin-2 4D particle $\hat{h}_{\mu\nu}^{(0)}$
  - **KK Modes:** massive spin-2 4D particles $\hat{h}_{\mu\nu}^{(n)}$ for $n > 0$
- **5D Radion** $\hat{r}$ becomes a single 4D particle:
  - **4D Radion:** a massless spin-0 particle $\hat{r}^{(0)}$

4D Theory: Effective Lagrangian

Per field content, the 4D effective RS Lagrangian equals...

$$\mathcal{L}^{(\text{RS,eff})}_{h^H r^R} = \left[\frac{\kappa}{\sqrt{\pi r_c}}\right]^{(H+R-2)} \sum_{\vec{n}=\vec{0}}^{+\infty} \left\{ \overline{a}_{(R|\vec{n})} \cdot \mathcal{K}_{(\vec{n})}\left[\overline{\mathcal{L}}^{(\text{RS})}_{A:h^H r^R} \right] + \overline{b}_{(R|\vec{n})} \cdot \mathcal{K}_{(\vec{n})}\left[\overline{\mathcal{L}}^{(\text{RS})}_{B:h^H r^R} \right] \right\}$$

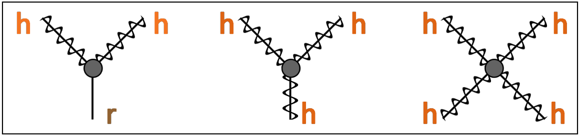
where $\mathcal{K}$ is an operator that maps 5D fields to 4D fields, and

$$\overline{a}_{(R|n_1 \cdots n_H)} \equiv \frac{1}{\pi r_c} \int_{-\pi r_c}^{+\pi r_c} dy \; e^{k[2(R-1)|y| - R\pi r_c]} \times \psi^{(n_1)} \dots \psi^{(n_H)} \left[\psi^{(0)}\right]^R$$

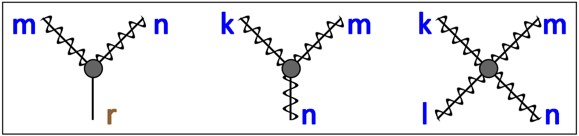
$$\overline{b}_{(R|n_3 \cdots n_H|n_1 n_2)} \equiv \frac{r_c}{\pi} \int_{-\pi r_c}^{+\pi r_c} dy \; e^{k[2(R-2)|y| - R\pi r_c]} \times (\partial_y \psi^{(n_1)})(\partial_y \psi^{(n_2)}) \psi^{(n_3)} \dots \psi^{(n_H)} \left[\psi^{(0)}\right]^R$$

4D Theory: Vertices & Explicit Parameters

$\mathcal{L}_{5D}^{(RS)}$ contains the following important vertices:



which then imply 4D effective vertices via KK decomposition:



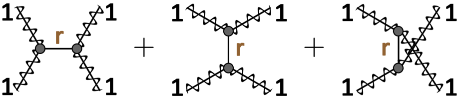
Lastly, there are 3 free parameters (κ,k,r_c). For this talk, set..

Lightest Spin-2 Mass:	$m_1 = 1$ TeV
Unitless Parameter:	$kr_c = 9.5$
4D Planck Mass:	$M_{\text{Pl}} = 2.435 \times 10^{15}$ TeV

RS Model Results

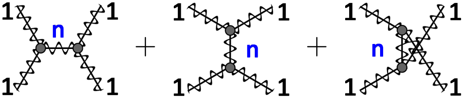
High-Energy Behavior of Longitudinal Matrix Elements

$\mathcal{M}_r =$



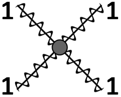
$\sim \mathcal{O}(s^3)$

$\mathcal{M}_h(n) =$



$\sim \mathcal{O}(s^5)$

$\mathcal{M}_{sg} =$



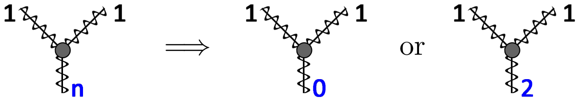
$\sim \mathcal{O}(s^5)$

We've directly confirmed these behaviors, which are consistent with the arguments of Arkani-Hamed, et. al. Define...

$$\mathcal{M}(n_{\max}) = \mathcal{M}_{sg} + \mathcal{M}_r + \sum_{n=0}^{n_{\max}} \mathcal{M}_h(n) = \sum_k \overline{\mathcal{M}}^{(k)}(n_{\max}) \cdot s^k$$

Torus: Fully-Analytic Result

The torus limit ($kr_c = 0$) has **KK number conservation:**



Therefore, we may calculate $\mathcal{M}$ exactly with a finite sum:

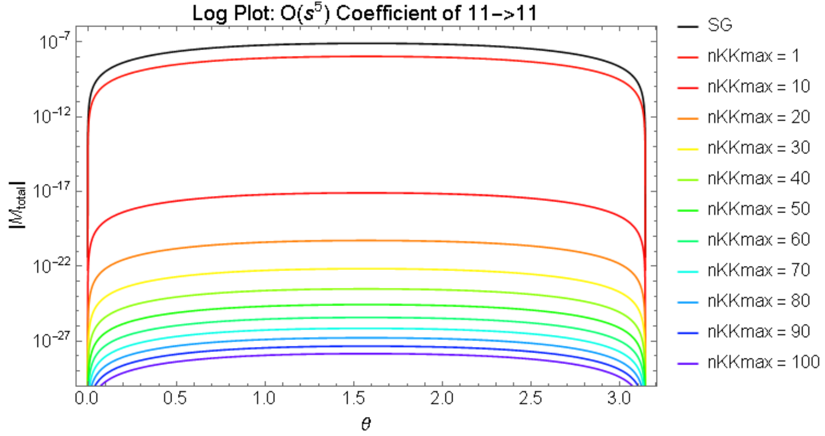
$$\mathcal{M} = \mathcal{M}_{sg} + \mathcal{M}_r + \mathcal{M}_h(0) + \mathcal{M}_h(2) = \sum_{k=-\infty}^{+5} \overline{\mathcal{M}}^{(k)} \cdot s^k$$

$$\overline{\mathcal{M}}^{(5)} = \overline{\mathcal{M}}^{(4)} = \overline{\mathcal{M}}^{(3)} = \overline{\mathcal{M}}^{(2)} = 0$$

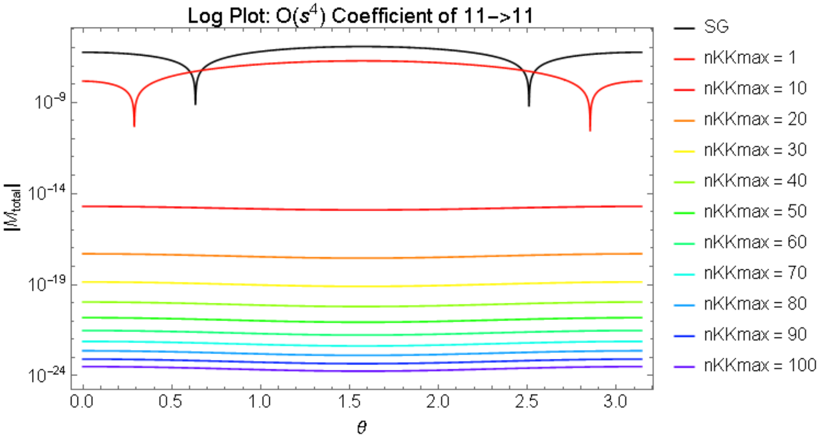
$$\overline{\mathcal{M}}^{(1)} = \frac{3\kappa^2}{256\pi r_c} \left[7 + \cos(2\theta)\right] \csc^2 \theta$$

When $kr_c \neq 0$ instead, we have an infinite sum...

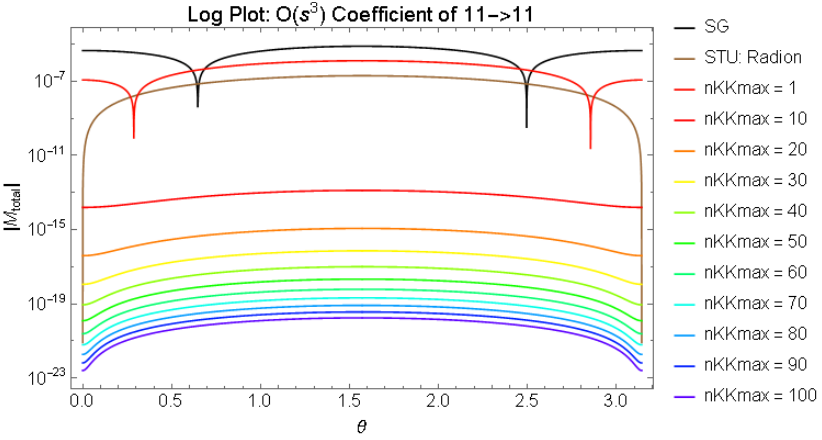
RS: Cancellations, $\mathcal{O}(s^5)$ :: $kr_c = 9.5$ & $m_1 = 1$ TeV



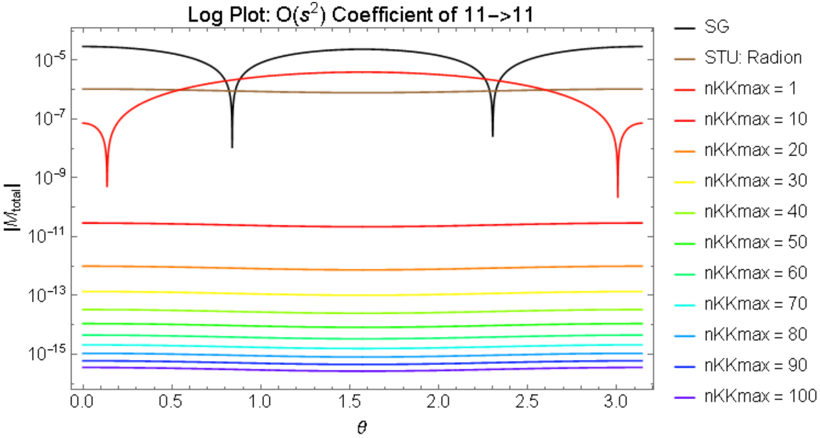
RS: Cancellations, $\mathcal{O}(s^4)$ :: $kr_c = 9.5$ & $m_1 = 1$ TeV



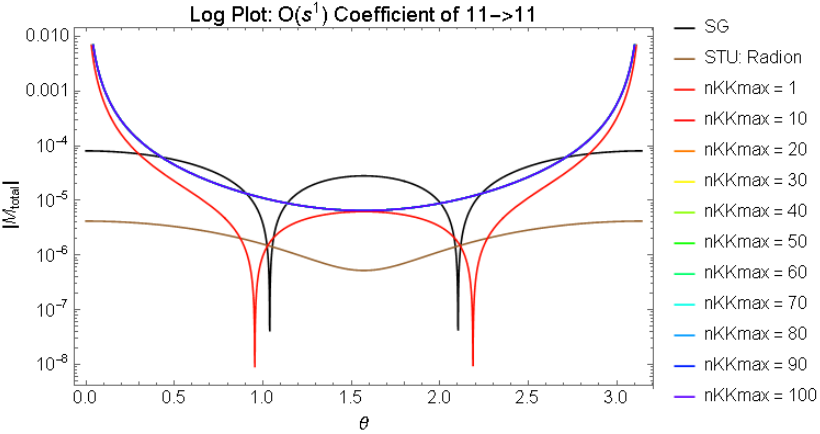
RS: Cancellations, $\mathcal{O}(s^3)$ :: $kr_c = 9.5$ & $m_1 = 1$ TeV



RS: Cancellations, $\mathcal{O}(s^2)$ :: $kr_c = 9.5$ & $m_1 = 1$ TeV



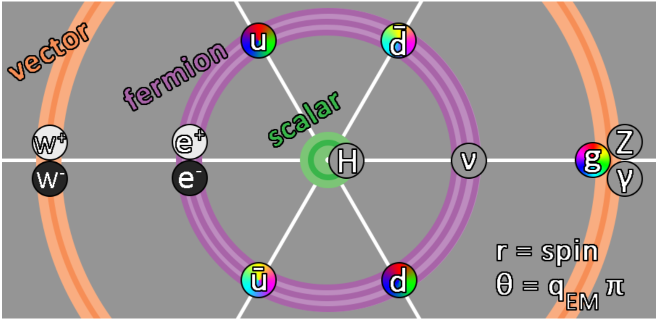
RS: Cancellations, $\mathcal{O}(s)$ :: $kr_c = 9.5$ & $m_1 = 1$ TeV



Conclusion

We're wrapping up our analysis now (including unitarity bounds & implications of the RS model as an EFT). Both the analysis and my code will be available in Summer 2019.

Thank you for attending! Questions?



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