Effective Field Theory from On-shell Amplitudes

Minglei Xiao

ITP, Chinese Academy of Science

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T. Ma, J. Shu, MX, [1902.06752]

Motivation

Two ways to compute scattering amplitudes:

1. QFT approach: operators + Feynman diagrams.
   - Good at computing off-shell correlations.
   - Built-in locality.
2. On-shell approach: building blocks + recursion relation.
   - Encode E.O.M.
   - Manifest gauge invariance.

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In the non-renormalizable regime:

QFT approach $\rightarrow$ Effective Field Theory: $\Delta\mathcal{L} = \sum_i c_i \mathcal{O}_i^{(d>D)}$.

- Theory Input: Wilson Coefficients $\{c_i\}$.
- Redundancies: EOM, IBP (independent counting via Hilbert Series by H. Murayama *etc.*).

On-shell approach $\rightarrow$ Some equivalent construction of EFT.

A Brief Review of On-shell Method

Using analyticity of scattering amplitude, promoting

$$\mathcal{A}(p_i) \to \mathcal{A}(\hat{p}_i) = \mathcal{A}(p_i; z).$$

$$\begin{aligned}
\text{Cauchy}: \quad \mathcal{A}_{\text{phy}} \equiv \mathcal{A}(0) &= \frac{1}{2\pi i} \oint_{z=0} \frac{\mathcal{A}(z)}{z} \\
&= -\sum_i \text{Res}_{z=z_i} \frac{\mathcal{A}(z)}{z} + \frac{1}{2\pi i} \oint_{z=\infty} \frac{\mathcal{A}(z)}{z} \\
&= \sum_i \frac{\text{Res}_{s=m_i^2} \mathcal{A}(s)}{s - m_i^2} + \mathcal{A}_0(z \to \infty),
\end{aligned}$$

$$\text{Unitarity}: \quad \text{Res}_{s=m_i^2} \mathcal{A}(s) = \mathcal{A}_L^{(i)} \mathcal{A}_R^{(i)}.$$

$\mathcal{A}_0(z \to \infty)$ is the unfactorizable part of $\mathcal{A}_{\text{phy}}$!

Amplitude-Operator Correspondence

Lesson: all unfactorizable amplitudes are needed as theory input to complete the recursion relation.

$$\text{Unfactorizable Amplitudes} \Leftrightarrow \text{Wilson Coefficients}$$

Thus we propose

$$\text{Amplitude Basis} \Leftrightarrow \text{Operator Basis}$$

The amplitude basis consists of

- 3-pt amplitudes with *spurious poles* (gauge couplings)
- Local amplitudes

$$\text{High Dimensional Local Amplitudes} \Leftrightarrow \text{Irrelevant Operators}$$

Constructing Amplitude Basis

Local amplitudes are polynomials of helicity spinors $\{|i\rangle^{\alpha}, |i]_{\dot{\alpha}}\}_{i=1\ldots n}$.

$$p_{i\mu}\sigma^{\mu}_{\alpha\dot{\alpha}} = |i]_{\dot{\alpha}}\langle i|_{\alpha}$$

(can be generalized to massive case by N. Arkani-Hamed, Y.-T. Huang and S.-H. Shao)

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- Lorentz invariance: spinor contraction $[ij], \langle ij\rangle$; little group weights – helicity.
- Gauge invariance: incorporated in Lorentz invariance.
- EOM: automatically satisfied.
- Momentum Conservation (IBP): $\sum_i p_i = -\sum_i |i]\langle i| = 0$.
- Spin-Statistics: need to be checked.

Examples & Subtleties

Example 1: $\mathcal{A}(V^+\psi^+\psi^-\phi) \sim [12]^2\langle 23\rangle g(s)$

$$s_{ij} = (p_i + p_j)^2 = 2p_i \cdot p_j = [ij]\langle ji\rangle$$

Examples & Subtleties

Example 1: $\mathcal{A}(V^+\psi^+\psi^-\phi) \sim [12][1|p|3\rangle g(s)$

- Independent number of $[i|p|j\rangle = n-3$.
- Dimension of amplitude $d = n + [\mathcal{A}] \geq 2n_V + \frac{3}{2}n_f + n_s$.

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Example 2: $\mathcal{A}(\psi^+\psi^+\psi^+\psi^+) \sim ([12][34] \text{ or } [13][42])g(s)$

- Schouten Identity: $|i][jk] + |j][ki] + |k][ij] = 0$.

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Example 3: $\mathcal{A}(V^+V^+V^+) \sim [12][23][31]$

- No Mandelstam variables for $n=3$.
- Totally anti-symmetric for the vectors:
  1. If they are identicle particles, such amplitude should vanish;
  2. Non-Abelian gauge bosons may have this amplitude with the structure constant

$$\mathcal{A}(V^{a+}V^{b+}V^{c+}) \sim f^{abc}[12][23][31]$$

Some Results

The general form of amplitude basis for real world:

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State of the art:

- We obtained (quite easily) all the 84 operators at dim-6 for SMEFT [1902,06752].
- We also obtained 30 operators at dim-7, with some efforts on subtleties.
- We listed all Lorentz factors at dim-8 using new technique, but still no systematic way to generate all operators.

Summary & Outlook

- Not only counting the number of operator basis, but also writing down their amplitudes.
- Using on-shell methods, amplitude basis can be directly used in computations.
- Systematic way to list all amplitudes under the constraint of momentum conservation and Schouten identity using Grassmiannian Harmonics.
  (Young Tableau representation introduced by B. Henning and T. Melia)
- Spin-statistics for identical particles still hard to impose.
- Expect to develop techniques for all possible EFT computations (just borrow from scattering amplitude community).