Double Heavy Baryons and Corrections to Heavy Quark-Diquark Symmetry

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U.S. DEPARTMENT OF ENERGY

Outline

Introduction

Formalism

Chromo-magnetic coupling of diquarks

Conclusions & Future Directions

Introduction

- **Heavy Mesons ($\bar{Q}q$):**

  -- Hadrons with single **heavy** quark (anti-quark) and a light anti-quark (quark).

  -- Described in framework of heavy-quark effective theory (HQET) where operators scale in powers of $\Lambda_{\mathrm{QCD}}/m_Q$.

  -- The light quark orbit a static source of color in $\bar{3}$ representation.

- **Double Heavy Baryons (QQq):**

  -- Hadrons with **two heavy** quarks (anti-quark) and a light quark (anti-quark).

  -- Hierarchy of energy scales characterized by the **relative velocity** $\boldsymbol{v}$ of heavy quarks:

$$m_Q > m_Q v > m_Q v^2$$

  -- Described in framework of nonrelativistic QCD (NRQCD) with power counting of operators determined by velocity.

  Bodwin, Braaten & Lepage (1994), Luke, Manohar & Rothstein (2000), Brambilla, Pineda & Soto (2000)

  -- **Attractive** Coulomb force between the heavy quarks.

  -- Ground state is a **spin-1** diquark in $\bar{\mathbf{3}}$ color representation.

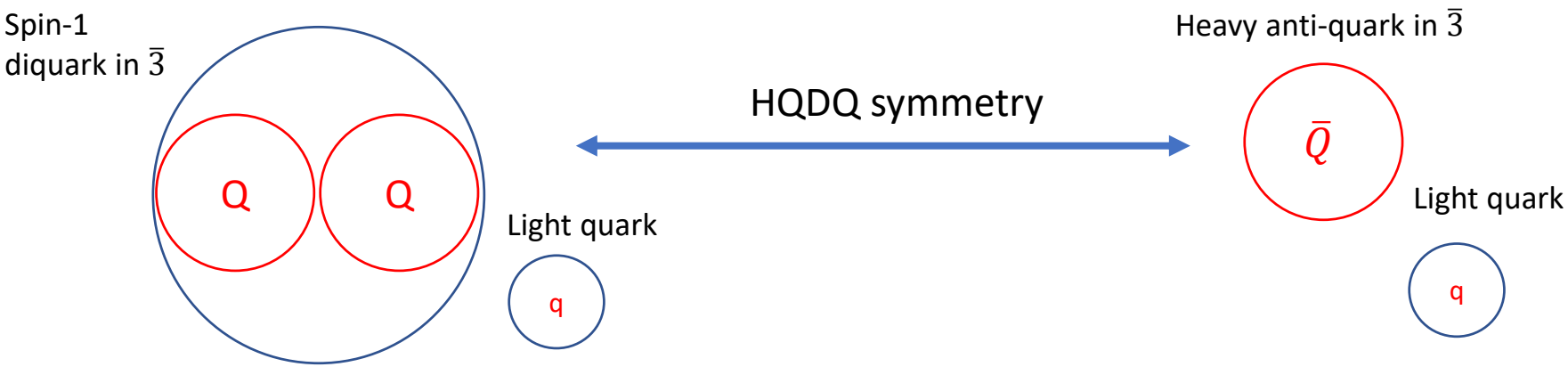
  -- Recently observed by the LHCb experiment $\Xi_{cc}^{++}(3621)$ *LHCb collaboration (2017)*

Introduction

- **Heavy quark-diquark (HQDQ) symmetry**

  -- Relates doubly heavy baryons and heavy anti-mesons.

  -- In $m_Q \to \infty$ limit, the light quark (q) in both hadrons orbit around static source of color in $\bar{3}$ representation.



  -- Relation between hyperfine mass splitting's of lowest lying double heavy baryons and heavy anti-mesons due to $\mathcal{O}(1/m_Q)$ chromo-magnetic coupling operators.

$$m_{\Xi^*} - m_{\Xi} = \frac{3}{4}\left(m_{D^*} - m_D\right)$$

Savage & Wise (1990), Mehen & Fleming (2006)
Brambilla, Vairo & Rosch (2005)

$\Xi^*: S = 3/2$

$\Xi : S = 1/2$

$D^*: S = 1$

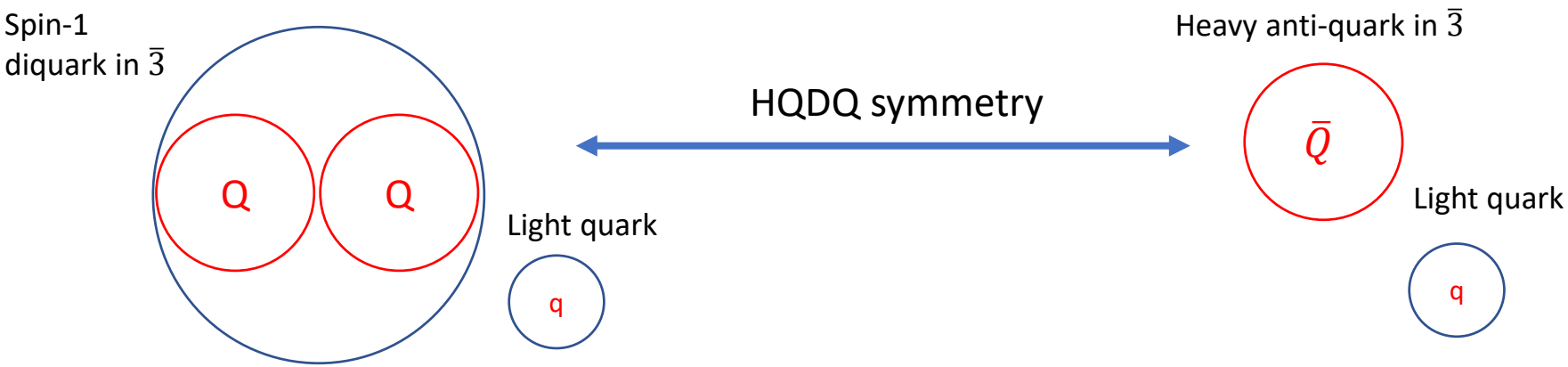
$D : S = 0$

Introduction

- **Heavy quark-diquark (HQDQ) symmetry**

  -- Doubly heavy baryons and singly heavy anti-mesons are related by HQDQ symmetry.

  -- In $m_Q \to \infty$ limit, the light quark (q) in both hadrons orbit around static source of color in $\bar{3}$ representation.



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Corrections to chromo-magnetic coupling of diquarks

Corrections to HQDQ symmetry ??

Formalism

- Interested in calculating the LO and NLO chromo-magnetic coupling of diquarks.
- Use nonrelativistic QCD (NRQCD) formalism with velocity power counting rule for operators.
- NRQCD Lagrangian :

$$\mathcal{L}_0 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left(iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m_Q} + \frac{g}{2m_Q}\,\sigma\cdot\mathbf{B} \right)\psi_{\mathbf{p}}$$
$$- \frac{1}{2}\sum_{\mathbf{p},\mathbf{q}} \frac{g_s^2}{(\mathbf{p}-\mathbf{q})^2}\psi^\dagger_{\mathbf{q}} T^A \psi_{\mathbf{p}} \psi^\dagger_{-\mathbf{q}} T^A \psi_{-\mathbf{p}} + \dots ,$$

- Use color and spin Fierz identities to group the heavy quark field ψ in spin-1, $\bar{3}$ color and spin-0, 6 color representation.
- Composite diquark operators:

Spin-1, $\bar{3}$ color

$$T^i_{\mathbf{r}} = \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{1}{2}\,\epsilon^{ijk} (\psi_{-\mathbf{p}})_j \epsilon\, \sigma (\psi_{\mathbf{p}})_{\mathbf{k}}$$

Spin-0, 6 color

$$\Sigma^{(mn)}_{\mathbf{r}} = \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{1}{\sqrt{2}}\, d^{(mn)}_{ij} (\psi_{-\mathbf{p}})_i \epsilon^T (\psi_{\mathbf{p}})_j$$

Mehen & Fleming (2006)
Brambilla, Vairo & Rosch (2005)

Formalism

- NRQCD Lagrangian in terms of composite diquark operators :

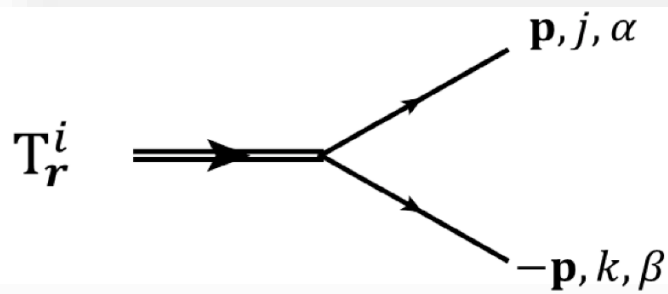
$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{2}\int d^3\mathbf{r}\, V^{(\bar{\mathbf{3}})}(r)\left(\mathbf{T}^{i\dagger}_{\mathbf{r}} - \sum_{\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{r}}\epsilon_{ijk}\frac{1}{2}(\psi^\dagger_{\mathbf{q}})_j\sigma\epsilon(\psi^\dagger_{-\mathbf{q}})_{\mathbf{k}}\right)\left(\mathbf{T}^{i}_{\mathbf{r}} - \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}}\frac{1}{2}\epsilon_{ilm}(\psi_{-\mathbf{p}})_l\epsilon\sigma(\psi_{\mathbf{p}})_{\mathbf{m}}\right)$$

$$+\frac{1}{2}\int d^3\mathbf{r}\, V^{(\mathbf{6})}(r)\left(\Sigma^{(mn)\dagger}_{\mathbf{r}} - \sum_{\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{r}}\frac{1}{\sqrt{2}}d^{(mn)}_{ij}(\psi_{\mathbf{q}})_i\epsilon(\psi_{-\mathbf{q}})_j\right)\left(\Sigma^{(mn)}_{\mathbf{r}} - \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}}\frac{1}{\sqrt{2}}d^{(mn)}_{ij}(\psi_{-\mathbf{p}})_i\epsilon^T(\psi_{\mathbf{p}})_j\right).$$

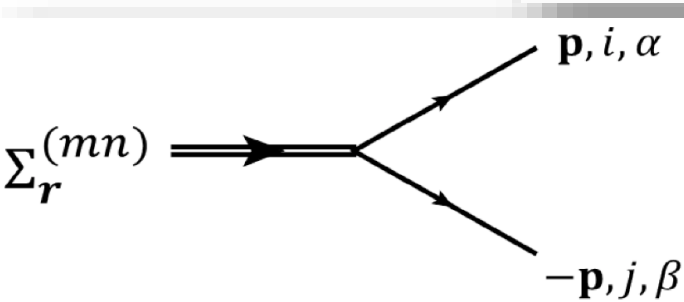
Anti-triplet potential $V^{(\bar{3})} = -\frac{2}{3}\frac{\alpha_s}{r}$

Sextet potential $V^{(6)} = \frac{1}{3}\frac{\alpha_s}{r}$

- Coupling of diquark to heavy quarks :



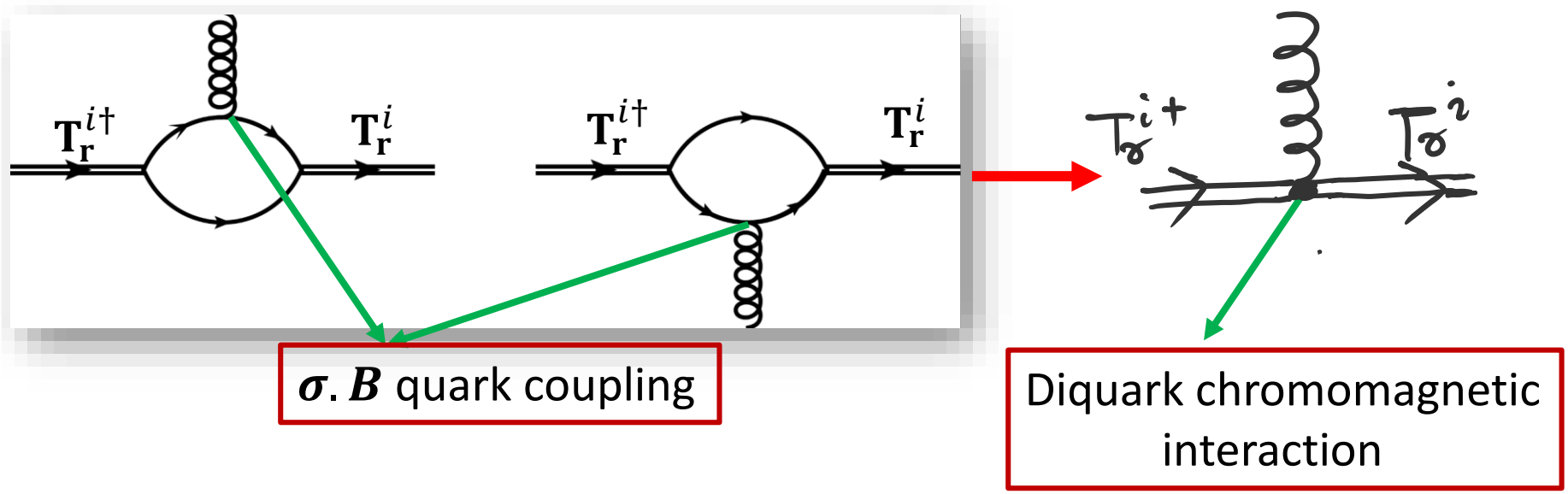
$$-ie^{-i\mathbf{p}.\mathbf{r}}\, V^{(\bar{3})}(r)\frac{\epsilon_{ijk}}{2}\,(\boldsymbol{\sigma\epsilon})_{\alpha\beta}$$



$$-ie^{-i\mathbf{p}.\mathbf{r}}\, V^{(6)}(r)\frac{d^{(mn)}_{ijk}}{\sqrt{2}}\,\epsilon_{\alpha\beta}$$

Chromo-magnetic coupling

- $\mathcal{O}(v^2)$ diagrams contributing to the chromo-magnetic coupling of diquarks:



- Leading order chromo-magnetic Lagrangian of diquarks :

$$\mathcal{L}_{\sigma.\mathbf{B}} = \frac{g}{2m_Q} \int d^3\mathbf{r}\, i\, \mathbf{T}_{\mathbf{r}}^{i\,\dagger} \cdot \mathbf{B}^c\, \bar{T}_{ij}^c \times \mathbf{T}_{\mathbf{r}}^j.$$

Mehen & Fleming (2006)
Brambilla, Vairo & Rosch (2005)

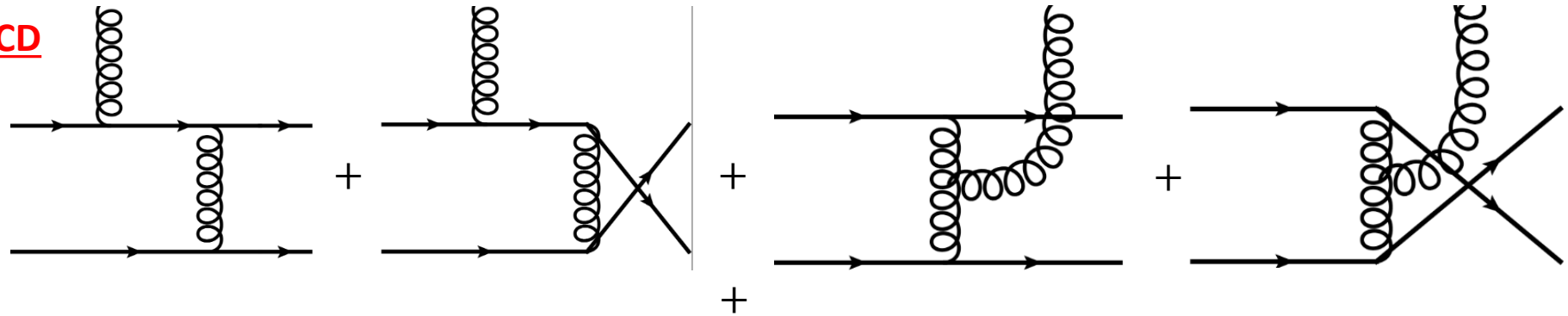
- $\mathcal{O}(1/m_Q)$ correction to heavy spin symmetry.
- Responsible for hyperfine mass splittings in the ground state of doubly heavy baryons.

$^{1}/_{m_Q} \times {}^{\alpha_s}/_{r}$ correction

- Corrections to the chromo-magnetic coupling of diquarks from **effective five-point** contact operator that couples 4 heavy quark with a $\boldsymbol{\sigma}.\boldsymbol{B}$ vertex.

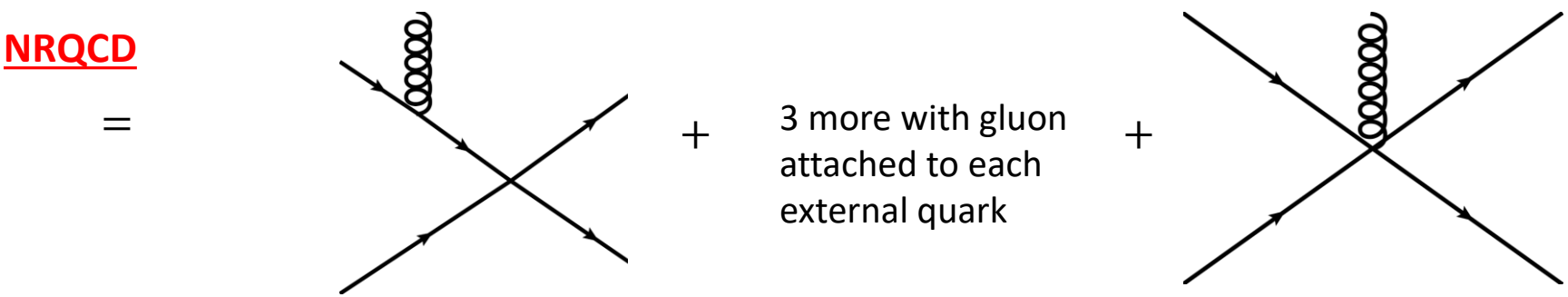
- Effective 5-point operator obtained from **matching** diagrams in QCD and NRQCD.

QCD



6 more with gluon attached to each external quark

NRQCD



AM & Mehen (2019)

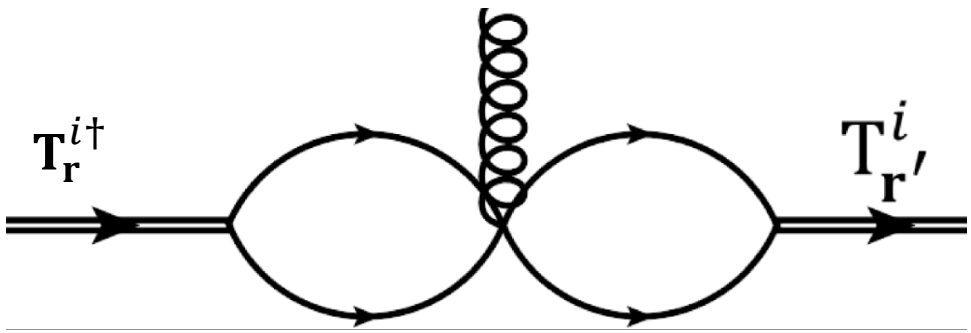
Effective 5-point operator

$^{1}/_{m_Q} \times {}^{\alpha_s}/_{r}$ correction

- Effective Lagrnagian for 5-point contact operator:

$$\mathcal{L}_{eff} = -\frac{g^3}{2}\frac{1}{2m_Q}\sum_{\mathbf{P_1},\mathbf{P_2},\mathbf{P_3},\mathbf{P_4}} \psi^\dagger_{\mathbf{P_4}} T^a \psi_{\mathbf{P_2}} \psi^\dagger_{\mathbf{P_3}} \left(T^a T^c + T^c T^a\right) \frac{\sigma\cdot\mathbf{B}^c}{2m_Q} \psi_{\mathbf{P_1}} \frac{1}{(\mathbf{P_4}-\mathbf{P_2})^2}$$

- 5-point contact operator contributing to the chromo-magnetic coupling of diquarks:



$\mathcal{O}(v^4)$ diagram in velocity power counting of NRQCD

- Corrections to chromo-magnetic coupling of diquarks from 2-loop diagram:

$$\mathcal{L}'_{\sigma.\mathbf{B}} = \frac{ig\alpha_s}{6m_Q^2}\int d^3\mathbf{r}\ \mathbf{T}^{i\,\dagger}_{\mathbf{r}} \cdot \frac{1}{r}\,\mathbf{B}^c\,\bar{T}^c_{ij} \times \mathbf{T}^j_{\mathbf{r}}.$$

- Next-to-leading order (NLO) Lagrangian: $\boldsymbol{\mathcal{O}(^{1}/_{m_Q^2} \times \alpha_s/r)}$ corrections to heavy spin-symmetry.

AM & Mehen (2019)

$^{1}/_{m_Q} \times {}^{\alpha_s}/_{r}$ correction

- In $m_Q \to \infty$, ground state spatial wavefunction of diquarks in doubly heavy baryons:

$$\phi(\mathbf{r}) = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0}$$

Bohr Radius:
$\boldsymbol{a_0 = 3/\alpha_s m_Q}$

- Chromo-magnetic coupling of diquarks from 2-loop diagram:

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Suppression from Heavy quark mass

Enhancement from the Coloumb exchange

$$\left\langle \frac{1}{r} \right\rangle = \frac{\alpha_s m_Q}{3}$$

AM & Mehen (2019)

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Suppression from Heavy quark mass

Enhancement from Coloumb exchange

Overall $\boldsymbol{\mathcal{O}(\alpha_s^2/m_Q)}$ order chromo-magnetic coupling of diquarks !!!

AM & Mehen (2019)

Correction to HQDQ symmetry

- HQDQ symmetry relates mass hyperfine splittings of ground state double heavy baryons and heavy anti-mesons.
- Correction to HQDQ symmetry due to the $\mathcal{O}(\alpha_s^2/m_Q)$ order chromo-magnetic coupling of diquark from effective 5-point operator.
- Relation between mass hyperfine splittings:

$$\underbrace{m_{P^*} - m_P}_{\text{Hyperfine splitting in heavy meson}} = \frac{4}{3}\left(m_{\Sigma^*} - m_\Sigma\right)\left(1 + \frac{\alpha_s}{12 m_Q}\left\langle \frac{1}{r} \right\rangle\right),$$

$$= \frac{4}{3}\left(m_{\Sigma^*} - m_\Sigma\right)\left(1 + \frac{\alpha_s^2}{9}\right)$$

- Correction to hyperfine splitting of diquarks from $\mathcal{O}(\alpha_s^2/m_Q)$ coupling :

Bottom quark $\approx 1.4 \times 10^{-2}$
Charm quark $\approx 3.1 \times 10^{-2}$

AM & Mehen (2019)

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- Correction to hyperfine splitting of diquarks from $\mathcal{O}(\alpha_s^2/m_Q)$ coupling :

Bottom quark $\approx 1.4 \times 10^{-2}$
Charm quark $\approx 3.1 \times 10^{-2}$

Small correction to hyperfine splittings of doubly charm / bottom quark systems !!

AM & Mehen (2019)

Small correction to HQDQ symmetry!!

Conclusions

- Corrections to HQDQ symmetry from effective **5-point contact operator.**
- Chromo-magnetic coupling of diquark from 5-point contact operator:

$$\mathcal{L}'_{\sigma.\mathbf{B}} = \frac{ig\alpha_s}{6m_Q^2} \int d^3\mathbf{r}\ \mathbf{T}_{\mathbf{r}}^{i\,\dagger} \cdot \frac{1}{r}\, \mathbf{B}^c\, \bar{T}_{ij}^c \times \mathbf{T}_{\mathbf{r}}^j.$$

Naïve expectation: $\boldsymbol{\mathcal{O}(1/m_Q^2)}$ correction

Actually: $\boldsymbol{\mathcal{O}(\alpha_s^2/m_Q)}$ correction due to $\frac{1}{r}$ factor.

- **<u>Small</u>** corrections to HQDQ symmetry from $\mathcal{O}(\alpha_s^2/m_Q)$ coupling :

Bottom quark $\approx 1.4 \times 10^{-2}$ Charm quark $\approx 3.1 \times 10^{-2}$

Future Directions

- RG evolution coupling constant of effective 5–point operator due to one loop ?? .

Thank You!!