# Evolution of Primordial Magnetic Fields from their Generation till Recombination

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# Collaborators

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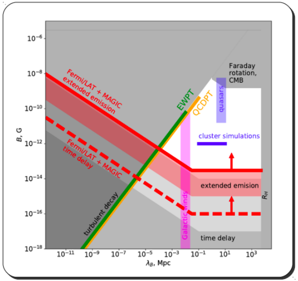
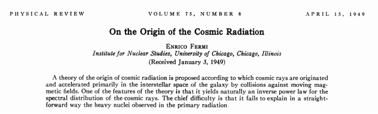
- **Tanmay Vachaspati** (*Arizona State University*)

# Introduction

- Magnetic fields ($\sim \mu$G) are detected at different scales in the universe.
- Small seed (primordial) fields can be amplified by various mechanisms. (*Picture from I. Vovk's Presentation.*)




- What is the origin of these primordial fields?
- Generation mechanism affects the statistical properties.



PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

**On the Origin of the Cosmic Radiation**

ENRICO FERMI
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(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

# Generation Mechanisms

## Inflationary Magnetogenesis

- Seed fields arise from vacuum fluctuations[a] - very large correlation lengths.
- Involves the breaking of conformal symmetry.
- Scale invariant (or nearly) power spectrum.
- Typically involves couplings like $R^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ or $f(\phi)F_{\mu\nu}F^{\mu\nu}$.

[a] Michael S. Turner and Lawrence M. Widrow. "Inflation-produced, large-scale magnetic fields". In: *Phys. Rev. D* 37 (10 1988), pp. 2743–2754. DOI: 10.1103/PhysRevD.37.2743. URL: https://link.aps.org/doi/10.1103/PhysRevD.37.2743; B. Ratra. "Cosmological 'seed' magnetic field from inflation". In: *Astrophysical Journal Letters* 391 (May 1992), pp. L1–L4. DOI: 10.1086/186384.

# Generation Mechanisms (Contd.)

## Phase Transition Magnetogenesis

- An out of equilibrium, first-order transition is typically needed.
- The turbulence is coupled to the magnetic fields, affecting its evolution.
- Violent bubble nucleation generates significant turbulence[a].
- *Causal* processes – limited correlation lengths ($H_\star^{-1}$).
- Two main phase transitions are:
  1. Electroweak Phase Transition ($T \sim 100\,\text{GeV}$)
  2. QCD Phase Transition ($T \sim 150\,\text{MeV}$)

[a] Edward Witten. “Cosmic separation of phases”. In: *Phys. Rev. D* 30 (2 1984), pp. 272–285. DOI: 10.1103/PhysRevD.30.272. URL: https://link.aps.org/doi/10.1103/PhysRevD.30.272.

# Phase Transition Magnetogenesis

PHYSICAL REVIEW D **96**, 123528 (2017)

**Evolution of hydromagnetic turbulence from the electroweak phase transition**

Axel Brandenburg,[1,2,3,4] Tina Kahniashvili,[5,6,7,*] Sayan Mandal,[5] Alberto Roper Pol,[1,8]
Alexander G. Tevzadze,[9,7] and Tanmay Vachaspati[10]

PHYSICAL REVIEW FLUIDS **4**, 024608 (2019)

**Dynamo effect in decaying helical turbulence**

Axel Brandenburg,[1,2,3,4,5,*] Tina Kahniashvili,[5,6,7] Sayan Mandal,[5] Alberto Roper Pol,[1,8]
Alexander G. Tevzadze,[5,7,9] and Tanmay Vachaspati[10]

# 1. Modeling Magnetic Fields

**Stochastic**, and statistically **isotropic**, **homogeneous**, and **Gaussian** magnetic fields. We work with the correlation function,

$$\mathcal{B}_{ij}(r) \equiv \langle B_i(\mathbf{x}) B_j(\mathbf{x}+\mathbf{r}) \rangle = M_{\rm N}(r)\delta_{ij} + \left[M_{\rm L}(r) - M_{\rm N}(r)\right]\hat{r}_i\hat{r}_j + M_{\rm H}(r)\epsilon_{ijl}r_l$$

In Fourier space,

$$\mathcal{F}^{(B)}_{ij}(\mathbf{k}) = \int d^3\mathbf{r}\, e^{i\mathbf{k}\cdot\mathbf{r}}\, \mathcal{B}_{ij}(r)$$

This gives the *symmetric* and *helical* parts,

$$\frac{\mathcal{F}^{(B)}_{ij}(\mathbf{k})}{(2\pi)^3} = P_{ij}(\hat{\mathbf{k}})\frac{E_M(k)}{4\pi k^2} + i\epsilon_{ijl}k_l\frac{H_M(k)}{8\pi k^2}$$

Here $P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i\hat{k}_j$.

# 1. Modeling Magnetic Fields (Contd.)

*Mean magnetic energy density:* $\mathcal{E}_{\mathrm{M}} = \int dk\, E_{\mathrm{M}}(k)$.

*Magnetic integral scale:* $\xi_{\mathrm{M}}(t) = \dfrac{\int_0^\infty dk\, k^{-1} E_{\mathrm{M}}(k)}{\mathcal{E}_{\mathrm{M}}}$.

*Magnetic Helicity:* $\mathcal{H}_{\mathrm{M}} = \dfrac{1}{V}\int_V \mathbf{A}\cdot\mathbf{B}\, d^3\mathbf{r} = \int dk\, H_{\mathrm{M}}(k)$.

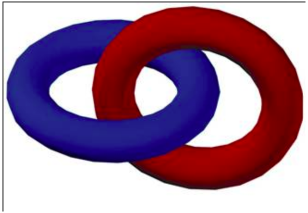

Figure: From aa.washington.edu

We can relate the symmetric and helical components,

$$|\mathcal{H}_M| \leq 2\xi_M \mathcal{E}_M \qquad \Rightarrow \qquad |H_M(k)| \leq 2k^{-1} E_M(k)$$

# 2. Helicity and Parity Violation

- Helical magnetic fields are produced by mechanisms that involve ($P$) violation.
- $P$ (and $CP$) violation can be related to processes giving rise to baryogenesis.
- This is one of the Sakharov conditions.

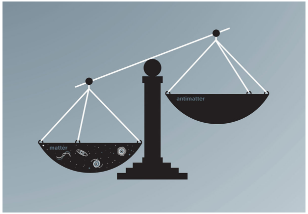


Figure: From fnal.gov

This has been studied (examples[1]) by several authors.

[1] Tanmay Vachaspati. "Estimate of the primordial magnetic field helicity". In: *Phys. Rev. Lett.* 87 (2001), p. 251302. DOI: 10.1103/PhysRevLett.87.251302. arXiv: astro-ph/0101261 [astro-ph]; Kohei Kamada and Andrew J. Long. "Evolution of the Baryon Asymmetry through the Electroweak Crossover in the Presence of a Helical Magnetic Field". In: *Phys. Rev.* D94.12 (2016), p. 123509. DOI: 10.1103/PhysRevD.94.123509. arXiv: 1610.03074 [hep-ph].

# 3. Methods

Our free parameters:

- Initial correlation length ($\xi_{M\star}$) (ratio to $H_\star^{-1}$).
- Initial energy density ($\rho_{M\star}$) (ratio to $\rho_{R\star}$).
- Initial fractional helicity ($\sigma_\star$).
- Initial velocity of the plasma, $u_\star$.

We assume (also for velocity) the initial spectra $E_M(k, t_\star)$ and $H_M(k, t_\star)$ where:

$$\frac{F_{ij}(\mathbf{k}, t)}{(2\pi)^3} = P_{ij}(\hat{\mathbf{k}})\frac{E_M(k, t)}{4\pi k^2} + i\epsilon_{ijl}k_l\frac{H_M(k, t)}{8\pi k^2}$$

Direct numerical simulations (DNS) using the Pencil Code – study the evolution of $\mathcal{E}_M(t)$ and $\xi_M(t)$.

## Case I: The Batchelor Spectrum, No Helicity

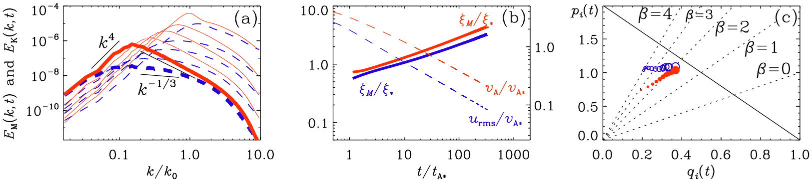


Figure: $Q_\star = 10$.

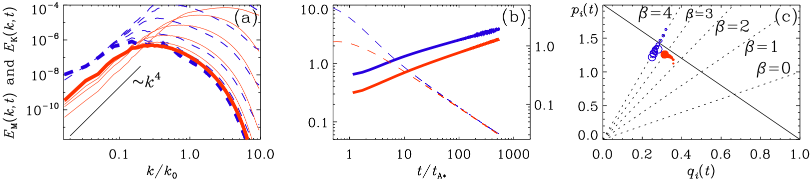


Figure: $Q_\star = 0.1$.

# 4. Results (Contd.)

## Case II: White Noise Spectrum, No Helicity

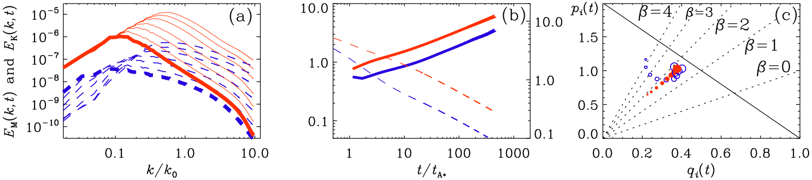


Figure: $Q_\star = 1$.

No inverse cascade.

## Case III: White Noise Spectrum, With Helicity

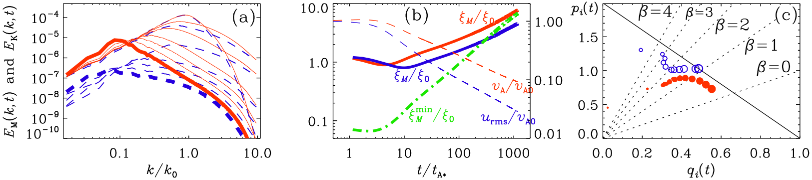


Figure: $Q_\star = 1$.

At late times: (i) Some inverse transfer, (ii) Turnover from $k^2$ to $k^4$, (iii) Partial to fully helical.

## Case IV: Batchelor Spectrum, With *Kinetic* Helicity

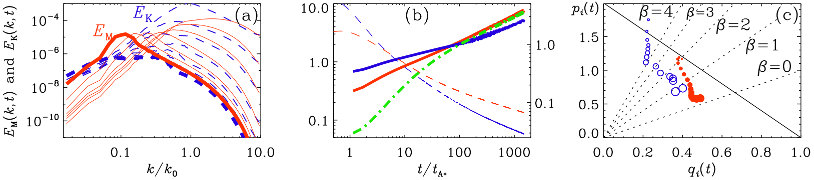


Figure: $Q_\star = 1$.

Kinetic helicity transferred to magnetic helicity.

$\mathbf{P}_i$ goes towards $\beta = 0$, away from equilibrium.

# 4. What We Learn

- Initial helicity leads to maximal helicity at later times. Helicity conserving evolution ($\beta = 0$).

- **No initial helicity**: Decay along $\beta = 2$ - conserving[2] the *Saffman Integral*.

- **Kinetically dominant**: Decay along $\beta = 4$ - conserving the *Loitsiansky Integral*.

- We can predict the field characteristics at recombination.

[2] P. A. DAVIDSON. "On the decay of Saffman turbulence subject to rotation, stratification or an imposed magnetic field". In: *Journal of Fluid Mechanics* 663 (2010), 268292. DOI: 10.1017/S0022112010003496.

# 5. What We Learn (Contd.)

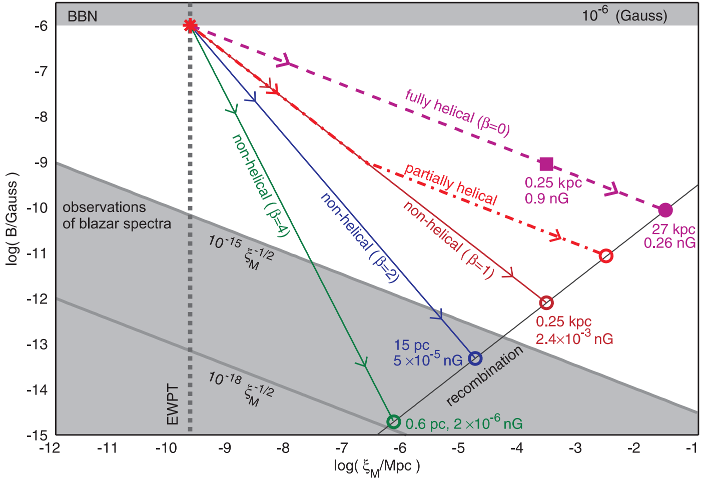


Figure: Comparing existing observational constraints to our analysis.

# 6. Gravitational Waves

- GWs can be generated by bubble collisions during the electroweak phase transition.
- The resulting magnetic field, and its coupling to the turbulence needs to be modeled.
- These **B** can also source turbulence, and hence more GWs.
- See **Tina Kahniashvili's talk** for more details.

*Thank You!*

# Supplementary Slides

# Turbulence, MHD, and the $pq$ Diagram

$$\mathcal{L} = \int r^2 \langle \mathbf{u}(\mathbf{x}) \cdot \mathbf{u}(\mathbf{x}+\mathbf{r}) \rangle \, d\mathbf{r} \propto \ell^5 u_\ell^2$$

$$\mathcal{S} = \int \langle \mathbf{u}(\mathbf{x}) \cdot \mathbf{u}(\mathbf{x}+\mathbf{r}) \rangle \, d\mathbf{r} \propto \ell^3 u_\ell^2$$

$$\mathrm{Re} = \frac{u_{rms}\xi_M}{\nu}$$

$$p_i(t) = \frac{d \ln \mathcal{E}_i}{dt}, \qquad q_i(t) = \frac{d \ln \xi_i}{dt}$$

$$p_i = (\beta_i + 1) q_i$$

Equilibrium line: $p_i = 2(1 - q_i)$.

# Decay Laws

We take the maximum *comoving* correlation length at the epoch of EW Phase transition,

$$\xi_\star \equiv \xi_{\max} = H_\star^{-1}\left(\frac{a_0}{a_\star}\right) \sim 6 \times 10^{-11}\,\mathrm{Mpc}$$

and the maximum *mean* energy density as,

$$\mathcal{E}_\star = 0.1 \times \frac{\pi^2}{30} g_\star T_\star^4 \sim 4 \times 10^{58}\,\mathrm{eV\,cm^{-3}}$$

*Non-helical case*: $\frac{\xi}{\xi_\star} = \left(\frac{\eta}{\eta_\star}\right)^{\frac{1}{2}}, \qquad \frac{\mathcal{E}}{\mathcal{E}_\star} = \left(\frac{\eta}{\eta_\star}\right)^{-1}.$

*Helical case*: $\frac{\xi}{\xi_\star} = \left(\frac{\eta}{\eta_\star}\right)^{\frac{2}{3}}, \qquad \frac{\mathcal{E}}{\mathcal{E}_\star} = \left(\frac{\eta}{\eta_\star}\right)^{\frac{-2}{3}}.$

*Partial*: Turnover when $\left(\frac{\eta_{\frac{1}{2}}}{\eta_\star}\right) = \exp\left(\frac{1}{2\sigma}\right).$

# Pencil Code

We solve the hydromagnetic equations for an isothermal relativistic gas with pressure $p = \rho/3$

$$\frac{\partial \ln \rho}{\partial t} = -\frac{4}{3}(\nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \ln \rho) + \frac{1}{\rho}\left[\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \eta \mathbf{J}^2\right], \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} + \frac{\mathbf{u}}{3}(\nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \ln \rho) - \frac{\mathbf{u}}{\rho}\left[\mathbf{u} \cdot (\mathbf{J} \times \mathbf{B}) + \eta \mathbf{J}^2\right]$$
$$-\frac{1}{4}\nabla \ln \rho + \frac{3}{4\rho}\mathbf{J} \times \mathbf{B} + \frac{2}{\rho}\nabla \cdot (\rho \nu \mathsf{S}), \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mathbf{J}), \quad (3)$$

where $\mathsf{S}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij}\nabla \cdot \mathbf{u}$ is the rate-of-strain tensor, $\nu$ is the viscosity, and $\eta$ is the magnetic diffusivity.