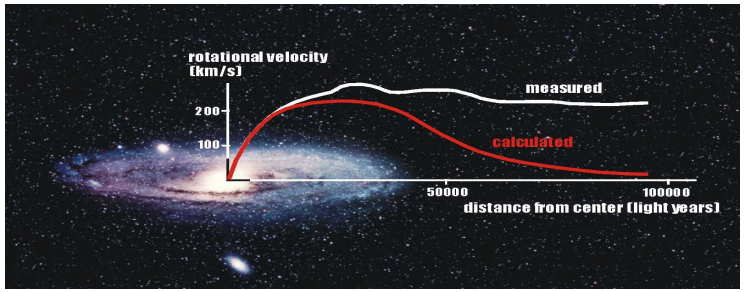
Boson Stars from Repulsive Scalar Theory and the Gravitational Wave Signature

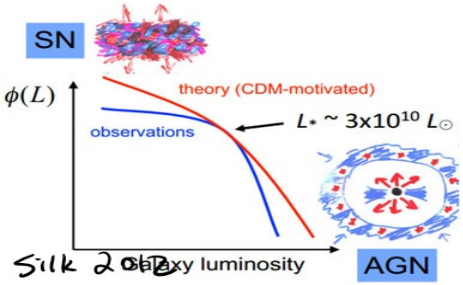
Chen Sun

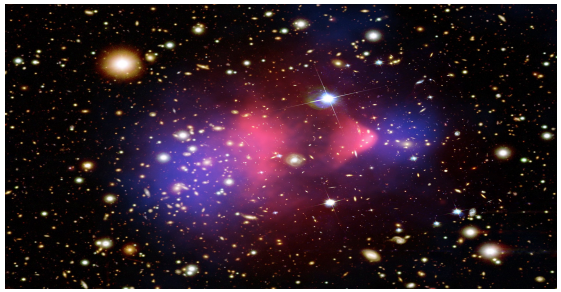
Brown University/ CAS-ITP

Croon, Gleiser, Mohapatra, CS, 1802.08259
Croon, Fan, CS, 1810.01420
Guo, Sinha, CS, 1904.07871

A Tale of Many Scales







- Large Scale:
  - Cosmic microwave background
  - Large scale structure
  - Supernovae, BAO, Bullet Cluster, etc.
- Small Scale:
  - Galactic rotation curves
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  - Missing satellites
  - Too-big-to-fail
  - Baryonic Tully-Fisher relation

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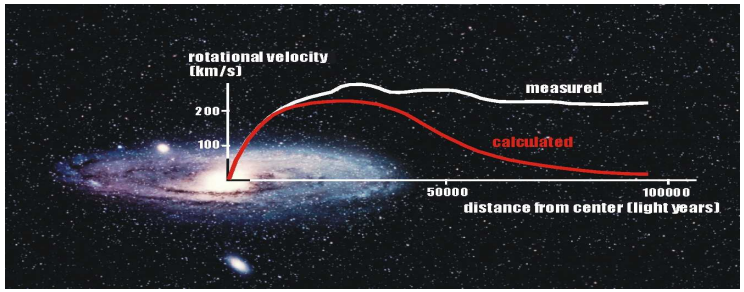

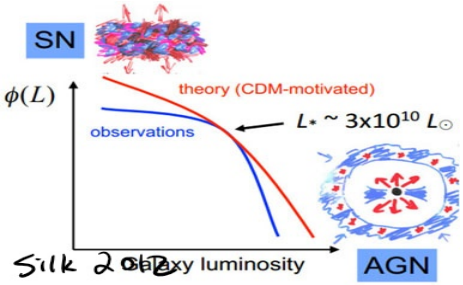

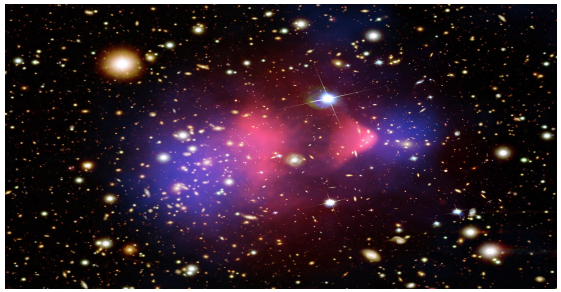

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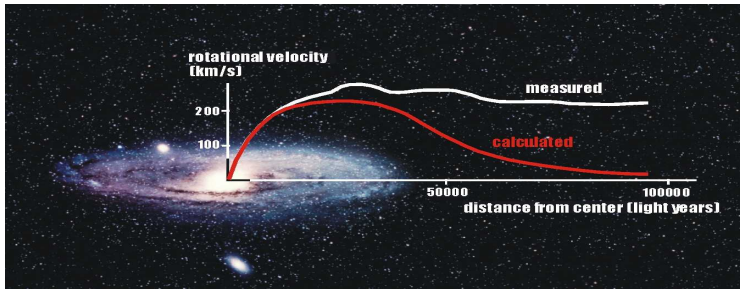

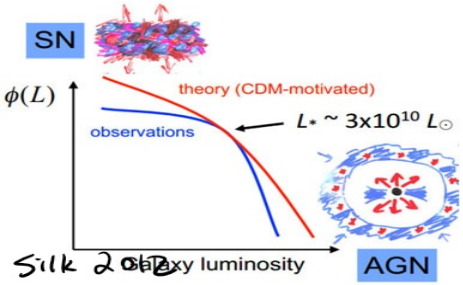

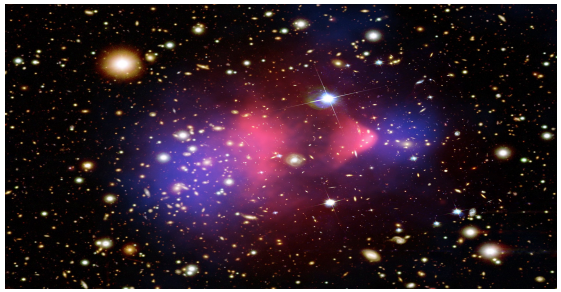

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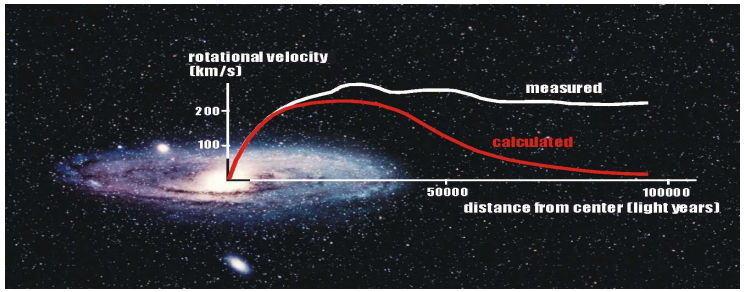

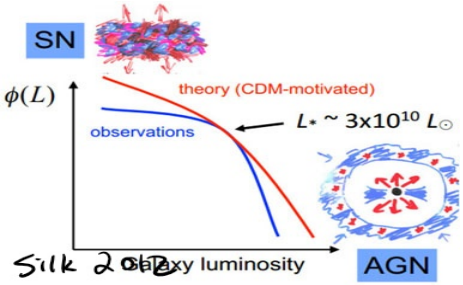

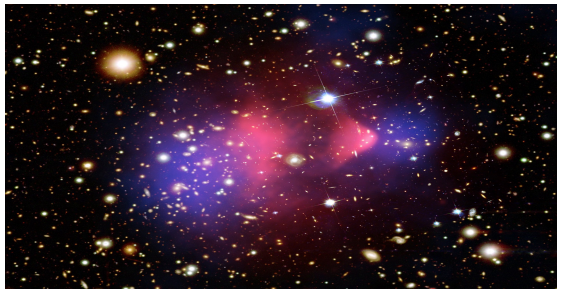

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  - New type of dark matter is required. Particle nature of DM:
    - SIDM (SIMP)
    - WDM
    - Superfluid DM
    - Axion like particles
    - BEC DM
  - Gravity needs to be modified.

When Quantum Meets Cosmology

Many body quantum system

Hartree-Fock approximation

$$\Phi(x) = \phi_1(x_1) \otimes \phi_2(x_2) \pm \phi_2(x_1) \otimes \phi_1(x_2)$$

Ground state of identical bosons
single wave function

Gross-Pitaevskii Eqn
aka nonlinear Schrödinger Eqn

$$\Phi(x) = \phi_1(x), \qquad s.t.$$
$$\mu\Phi = \left(-\frac{\nabla^2}{2m} + V(\mathbf{x}) + \lambda_{2n}|\Phi|^{2n-2}\right)\Phi$$

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Self-gravitating BEC system:

- When BEC system is large enough that gravity is no longer negligible,
- $\lambda_4 = 0, +1, -1$
- $V(\mathbf{x}) = \frac{G_N}{2}\int \frac{|\Phi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$

Superconductor:

- Switching on magnetic field, $\nabla \to \nabla - ieA$,
- $n = 4$,
- $V = 0$,
- Ginsburg-Landau theory for type I superconductor.

Soliton solutions:

- $V = 0$
- $\lambda_4 = \pm 1$
- Soliton solution.
  - $\lambda_4 > 0$, repulsive $\Phi \sim \tanh(x)$
  - $\lambda_4 < 0$, attractive $\Phi \sim \cosh(x)^{-1}$

Hydrogen atom:

- $\lambda = 0$
- $V = \frac{\alpha}{r}$

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Boson Star Mass Profile – Gravitational Bound

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^*)(\partial^\mu \phi) - \frac{1}{2} m^2 |\phi|^2 - \frac{\lambda}{4!}|\phi|^4$$

- $G_N M_* < 1/m$
  - $M_* \gtrsim M_\odot \Rightarrow m \lesssim 10^{-10}\ \mathrm{eV}$ (can be relaxed for NR case)

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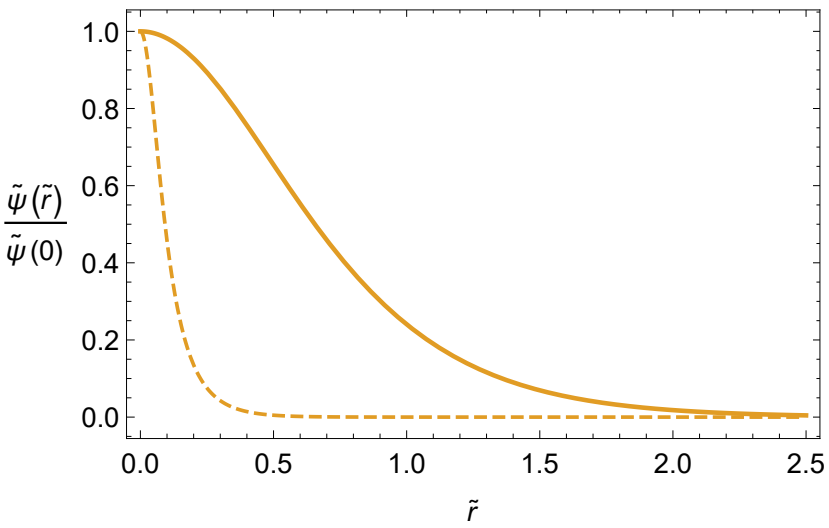
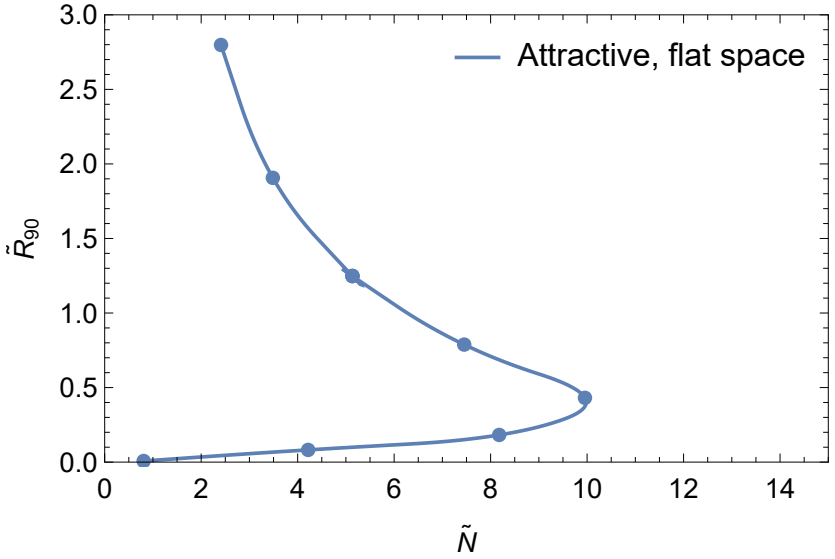
Macroscopic scale	Microscopic mechanism	Compactness $\frac{G_N M}{R}$	Mass
White dwarf	Pauli exclusion principle (degeneracy pressure of e^-)	$\sim \mathcal{O}(10^{-4})$	$\sim M_\odot$
Neutron star	Pauli exclusion principle (degeneracy pressure n)	~ 0.2	$\sim M_\odot$
Boson star	Heisenberg's uncertainty principle (kinetic energy of scalars)	?	$\sim \frac{M_{Pl}^2}{m}$

Boson Star Mass Profile – Non-linear Dispersion

$$i\dot{\phi} = -\frac{1}{2m}\nabla^2\phi - G_N m^2 \phi \int d^3\mathbf{x}' \frac{\phi^*(\mathbf{x}')\phi(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + \frac{\lambda}{8}|\phi|^2\phi,$$

Numerical solution:





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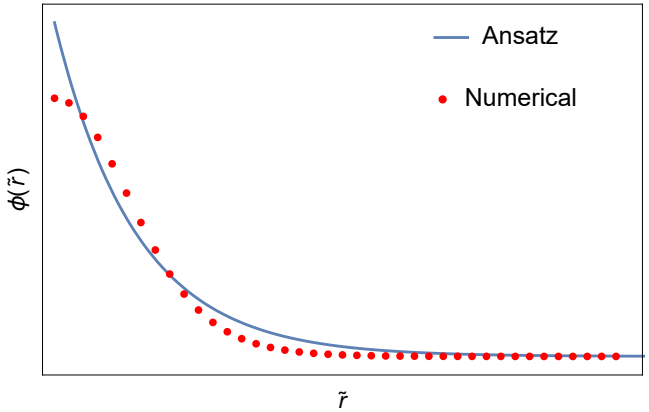

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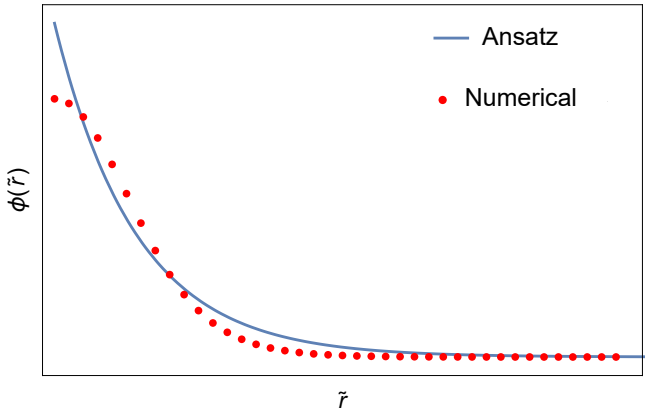

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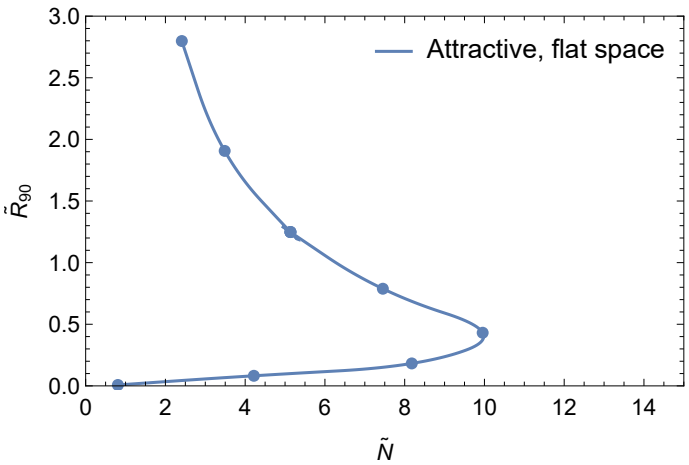

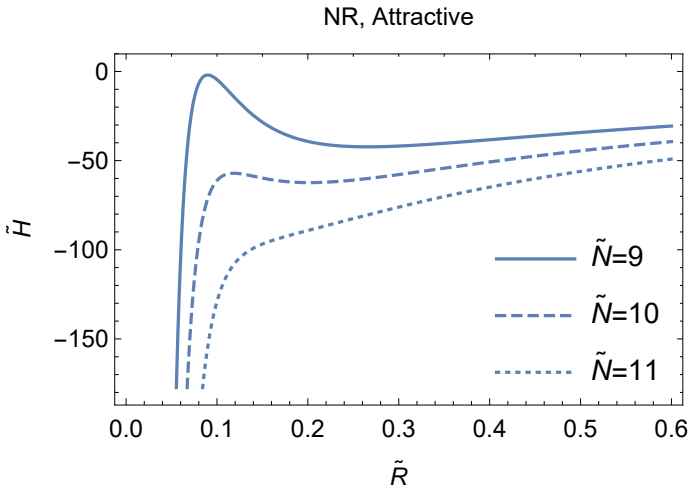

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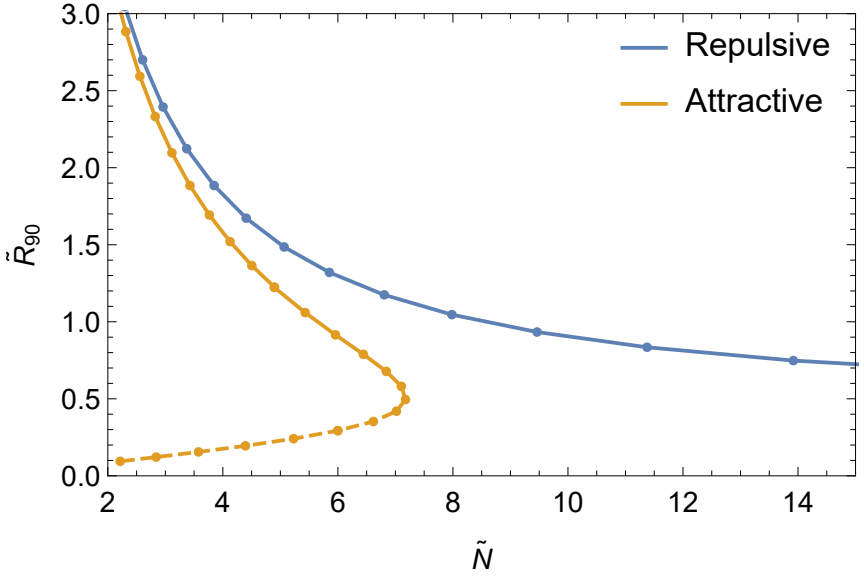

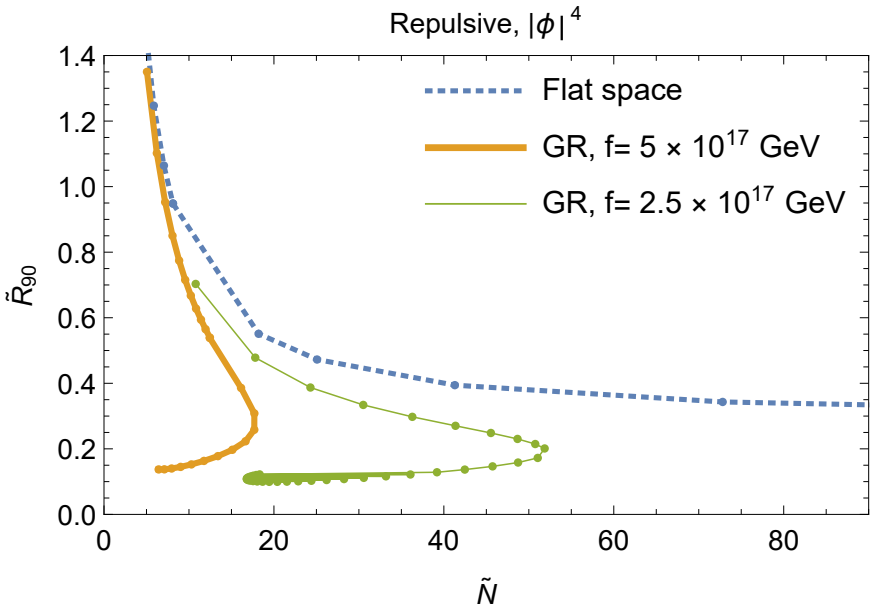

Boson Star Mass Profile – Space-Time Backreaction

Keep the exponential ansatz for the scalar wave function, but also account for the back reaction of the spacetime:

$$ds^2 = -(1+2V(r))dt^2 + (1-2V(r))dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2,$$

$$\Phi(r) = \sqrt{\frac{N}{\pi m R^3}}\, \mathrm{e}^{-r/R}, \qquad V(r) = -\frac{G_N M(r)}{r}.$$

Integrate the energy density

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Integrate the energy density by plugging the ansatz into:

$$H = \int_0^\infty dr\, 4\pi r^4 \left(\frac{\mu^2}{2(1+2V(r))}\Phi^2 + \frac{1}{2}m^2\Phi^2 + \frac{1}{2(1-2V(r))}(\partial_r\Phi)^2 + \frac{1}{4!}\left(\frac{m^2}{f^2}\right)\Phi^4 \right)$$

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We recover all the NR limit, with an extra term:

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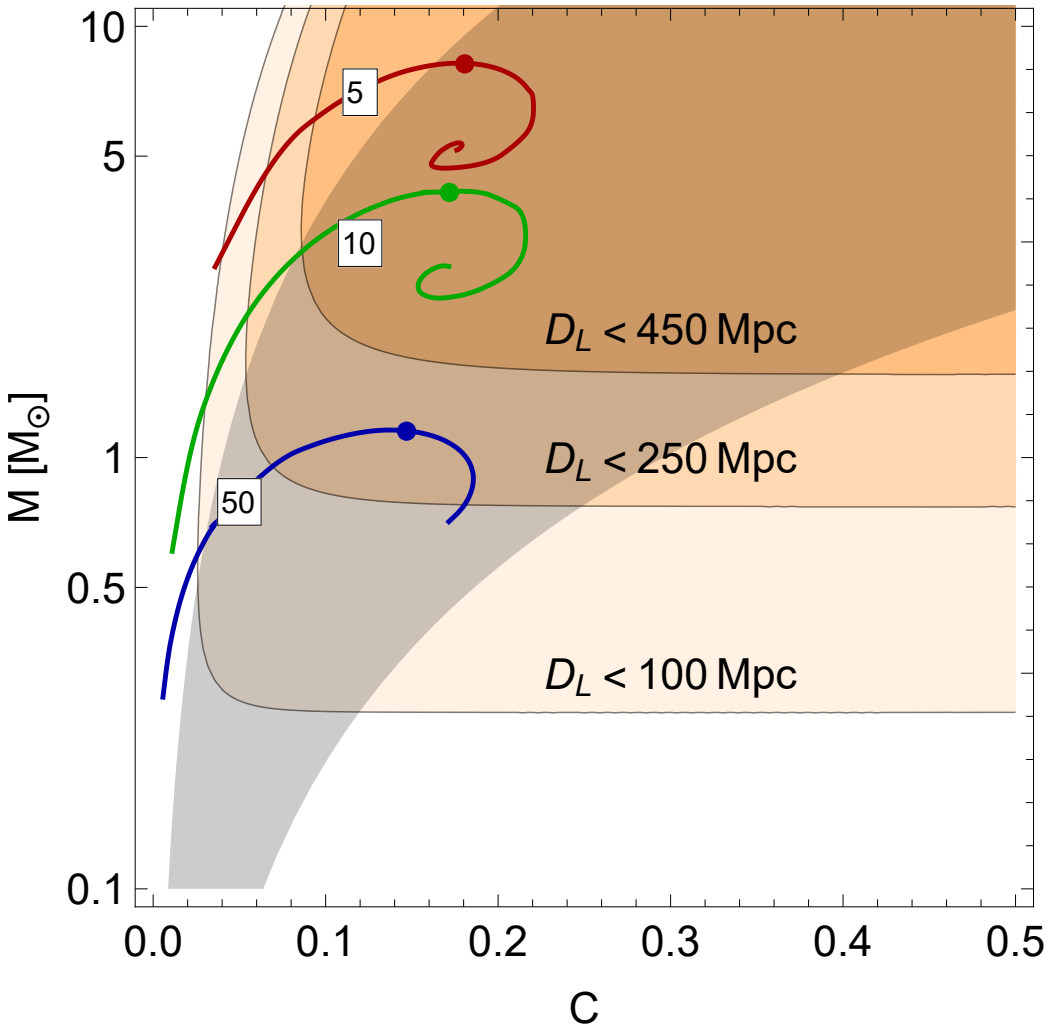
Boson Star at LIGO

In the context of binary mergers,



From top to bottom, $f = 5 \times 10^{16}$ GeV, $f = 10^{17}$ GeV, $f = 5 \times 10^{17}$ GeV.

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  - shape of potential

Boson Star – Toward More Realistic Models

- Light scalars are very dangerous in EFT...
- ...unless it is a pseudo Nambu-Goldstone boson with an approximate shift symmetry

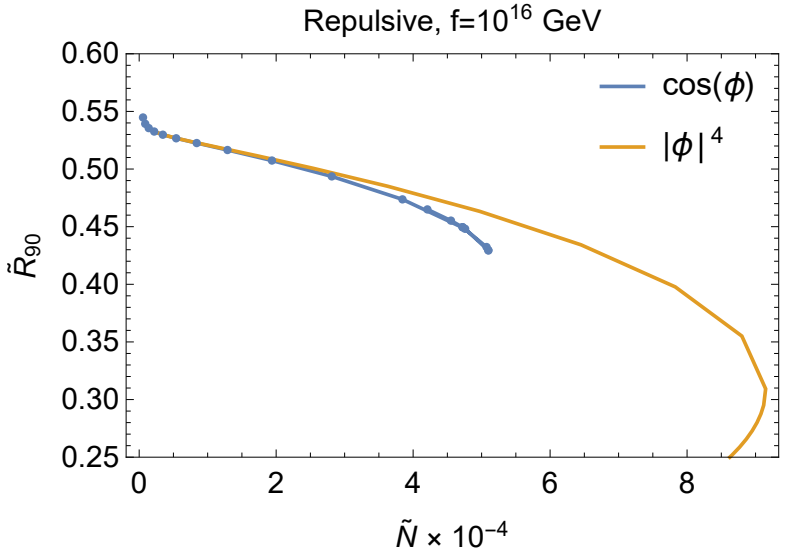
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) \underbrace{-\frac{1}{2}m^2\phi^2 - \frac{\lambda_{eff}}{4!}\frac{m^2}{f^2}\phi^4 + \cdots}_{V(\phi)},$$

- It is known that QCD axions are attractive. That leads to small compactness, even if it is stable. $\Lambda(1-\cos(\phi/f)) = m^2\phi^2 - \lambda\phi^4 + \ldots$.
- Take the pheno model:

$$V(|\phi|) = \frac{1}{2}m^2|\phi|^2 + \frac{1}{2Q^2}m^2 f^2\left[1-\cos\left(\frac{Q|\phi|^2}{f^2}\right)\right],$$

Boson Star – Toward More Realistic Models

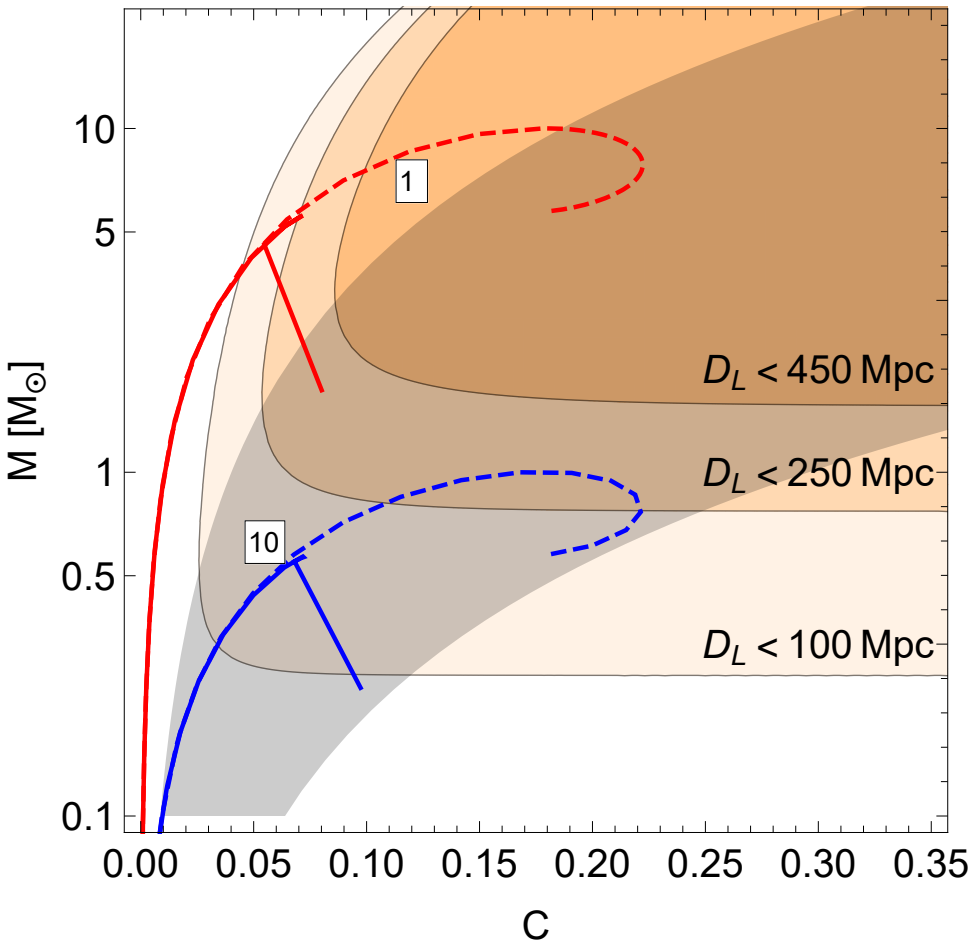
The subleading terms can be significant



- Subleading ϕ^8 is attractive
- the potential is bounded from both below and above

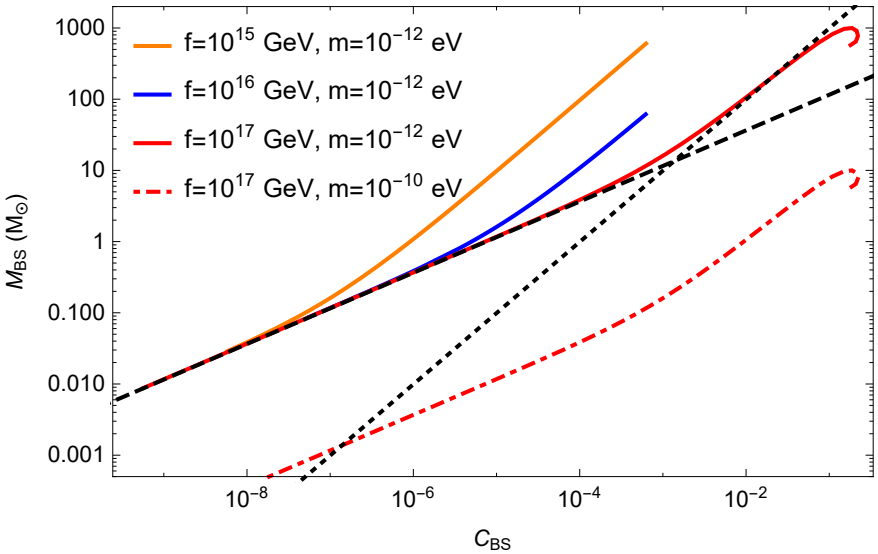
Boson Star – Toward More Realistic Models

The subleading terms can be significant:



Boson Star Linear VS Non-linear Regime

$$H(N,R) \approx \frac{N}{2mR^2} + \frac{\lambda N^2}{32\pi f^2 R^3} - \frac{5 G_N m^2 N^2}{16R}$$

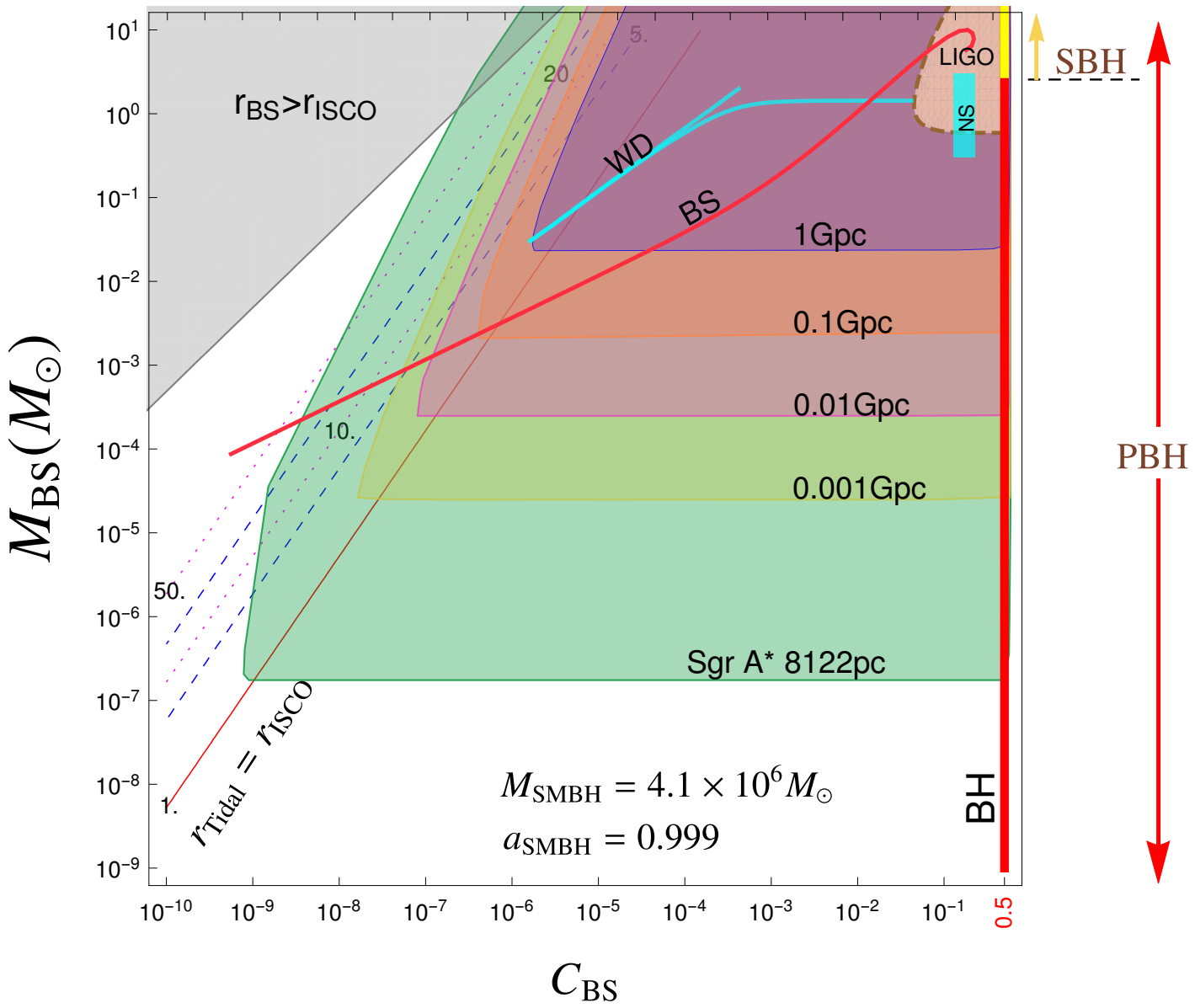


- Linear regime: kinetic energy vs gravity $N \sim 1/(m^3 R)$

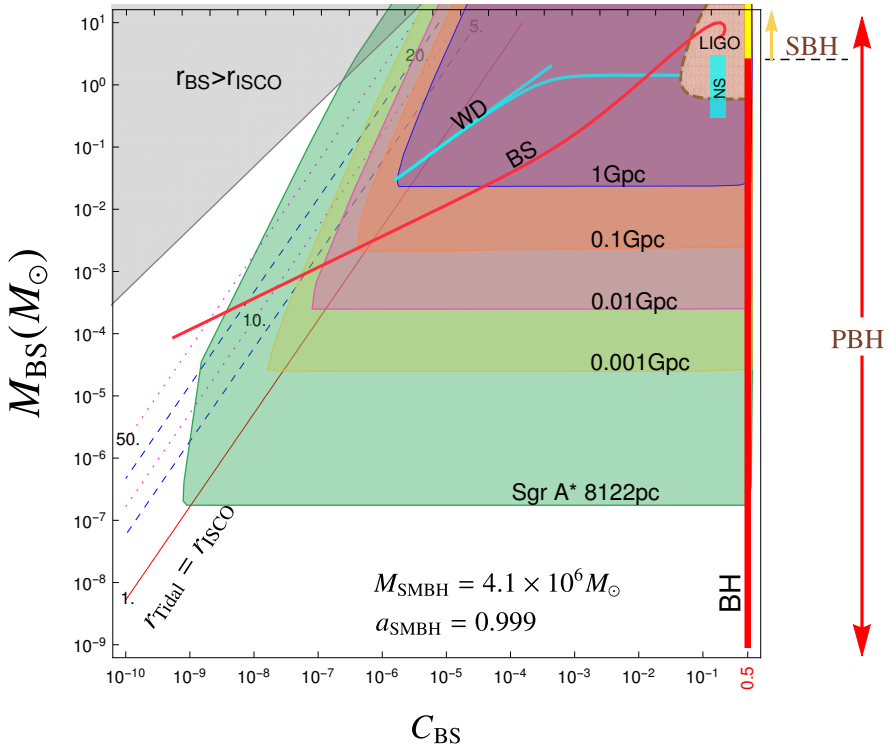
$$C_{\rm BS} \sim \frac{mN}{R} \sim m^2 M_{\rm BS}^2$$

- Nonlinear regime: repulsion vs gravity $R \sim 1/(mf)$

$$C_{\rm BS} \sim \frac{M_{\rm BS}}{R} \sim m \cdot f \cdot M_{\rm BS}$$

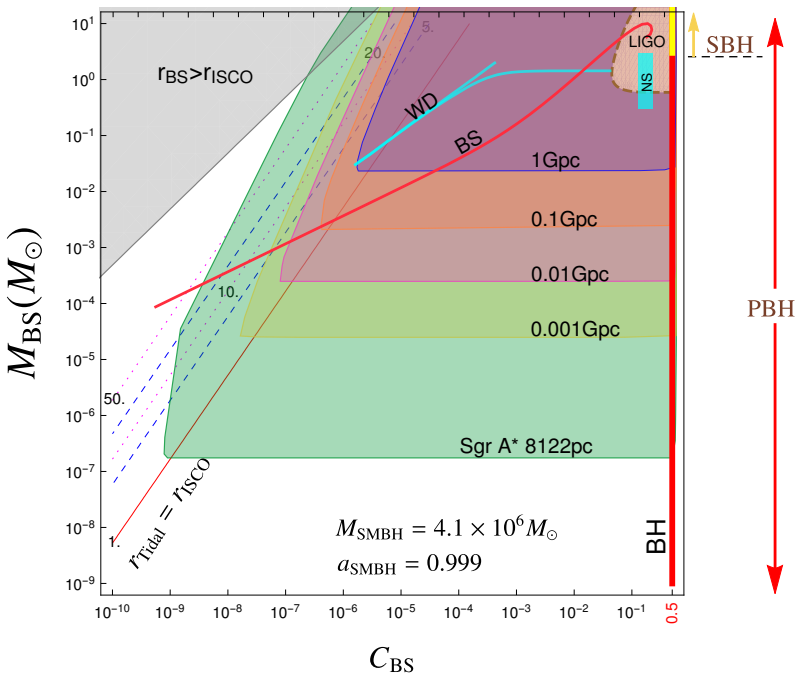


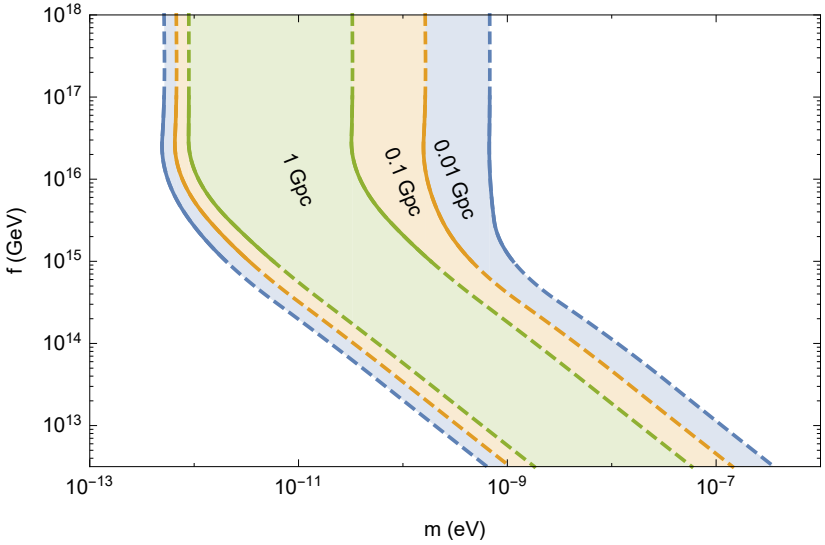
c.f. Huai-Ke Guo's talk right after this one.



- Can probe the boson star in both linear and nonlinear regime
- Small boson star mass makes it distinguishable from all known sources
- Tidal force disruption further breaks the degeneracy with Primordial BH's.

Scalar Parameter Space





Outlook

- Cosmological scale
  DM self-interaction $\Leftrightarrow H_0$, power spectrum $\Leftrightarrow$ CMB, LSS, 21cm
  BEC of light scalars $\Leftrightarrow$ phase transition $\Leftrightarrow$ CMB, BBN

- Galactic scale
  DM self-interaction $\Leftrightarrow$ shape of halo $\Leftrightarrow$ lensing, survey $(\mathbf{x}, \mathbf{v})$
  dissipative DM $\Leftrightarrow$ DM substructure $\Leftrightarrow$ lensing, survey $(\mathbf{x}, \mathbf{v})$
  BEC light scalars $\Leftrightarrow$ size of halo $\Leftrightarrow$ lensing, survey $(\mathbf{x}, \mathbf{v})$

- Stellar scale
  axions $\Leftrightarrow$ gap of BH (m, a_*) $\Leftrightarrow$ GW waveform
  ADM bound state $\Leftrightarrow$ dark star $\Leftrightarrow$ gravitational wave
  ADM $\Leftrightarrow$ neutron star $\Leftrightarrow$ gravitational wave
  BEC of light scalars $\Leftrightarrow$ boson star $\Leftrightarrow$ gravitational wave

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- Lifetime of the compact real scalar stars
- Excited states
- Extended potential structure, e.g. double well, nonminimal coupling, etc.
- Momentum corrections
- Merger process

Thank you!

Dilute Boson Star – Tidal Force

(not to scale)

- When tidal force $\sim$ self-gravity, the star is severely deformed.

$$\ell = \left(\frac{C_{BH}}{C_*}\right)\left(\frac{M_*}{M_{BH}}\right)^{2/3} r_{BH}$$

- Want this condition met not too early before the boson star falls into the BH, *i.e.* $\ell \sim r_{BH}$.
- For $M_* \sim M_\odot$, this is true for dilute boson stars $C \sim 10^{-4}$.