Fermion Mass Hierarchy in the 5D Domain Wall Standard Model

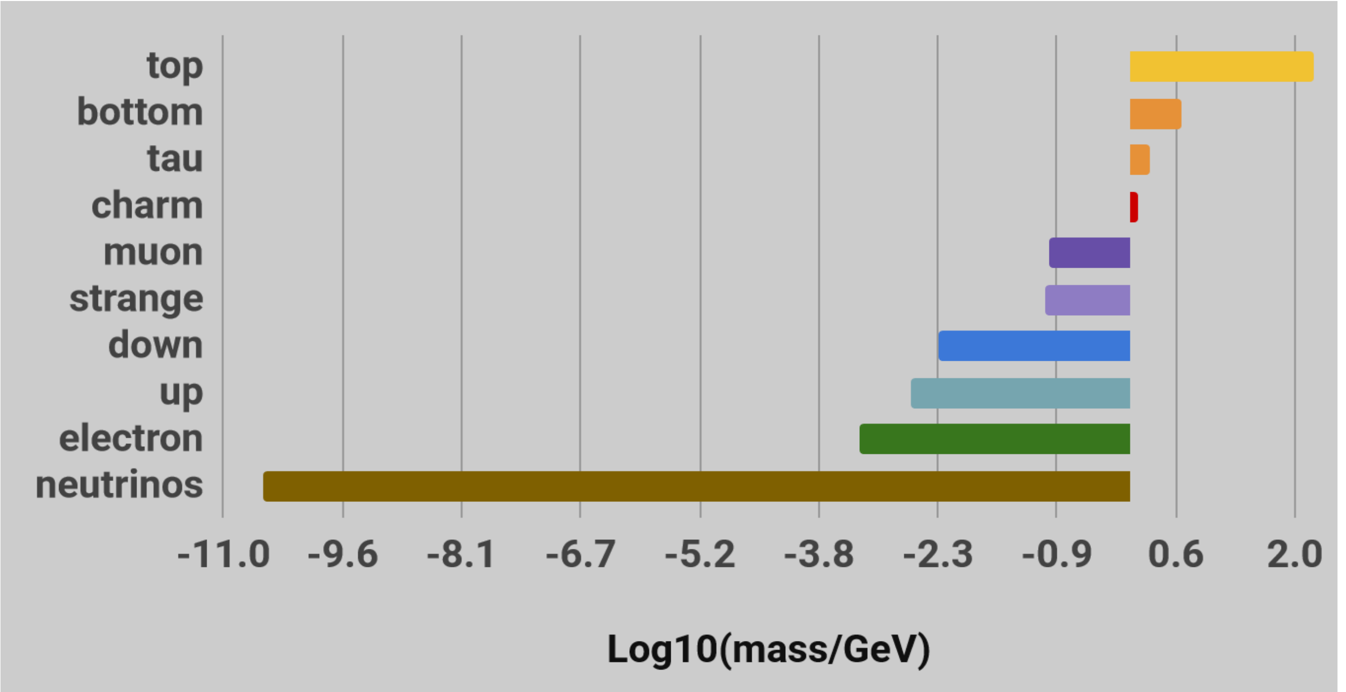
Desmond Villalba
The University of Alabama

Work in collaboration with Nobuchika Okada and Digesh Raut

1) Mod.Phys.Lett. A34 (2019) no.10, 1950080, arXiv:1712.09323
2) arXiv:1801.03007 (under review PRD)
3) arXiv:1904.10308 (under review JHEP)

Outline

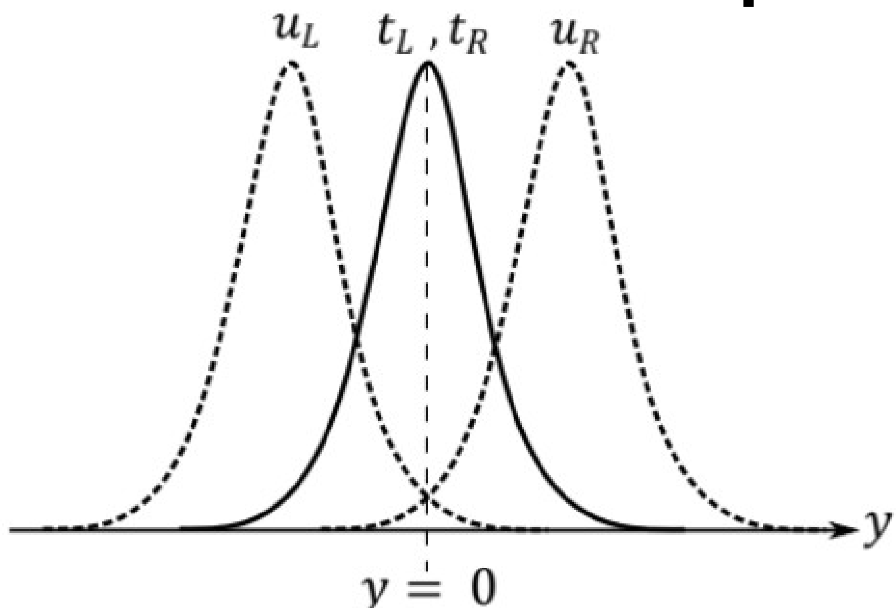
- Domain-Wall (DW) setup and brief review for localizing: gauge bosons, Higgs, fermions
- Fermion mass hierarchy
- Phenomenology



See my Pheno 2018 talk for extensive DW review

https://indico.cern.ch/event/699148/contributions/2986240/

DW 5D setup



- Consider a **flat**, **non-compact** extra dimension where**:**
  - All fermions, gauge fields, and the Higgs are localized within certain domains along the 5th dimension
  - The mass hierarchy of the SM fermions can be resolved by the "Split Fermion Mechanism", where the chiral components of the fermion fields are localized in different locations throughout the bulk

Split fermion mechanism proposed by: Akani-Hamed, Schmaltz [hep-ph/9903417]

  - The 5D metric is given by $\eta^{MN} = \mathrm{diag}\{+,-,-,-,-\};\ M,N = 0,1,2,3,\underline{y}$

DW gauge boson

- The 5D Lagrangian for a U(1) gauge field is given by $L = L_5 + L_{GF}$, where

$$L_5 = -\frac{1}{4} s(y) F^{NM} F_{NM} = -\frac{1}{4} \frac{1}{g_5(y)^2} F^{NM} F_{NM}$$

$$L_{GF} = -\frac{s(y)}{2\xi} \left(\partial_\mu A^\mu - \frac{\xi}{s(y)} \partial_y (s(y) A_y) \right)^2$$

- The 5D vector $\mathbf{A_M}$ is comprised of the 4D **vector** and **scalar** field components $A_M(x, y) = (\underline{A_\mu(x, y)}, \underline{A_y(x, y)})$

- The y-dependent gauge coupling **confines** the gauge fields according to the geometry of the configuration and is subject to

Localization condition: $s(\pm\infty) \to 0$

Relation to 4D gauge coupling: $\int s(y) dy = \int g_5^{-2}(y) \mathrm{dy} = \frac{1}{g_4^2}$

Seminal Work in this direction:
1) G. Dvali, M. Shifman, arXiv: 9612128
2) K. Ohta, N. Sakai, arXiv: 1004.4078

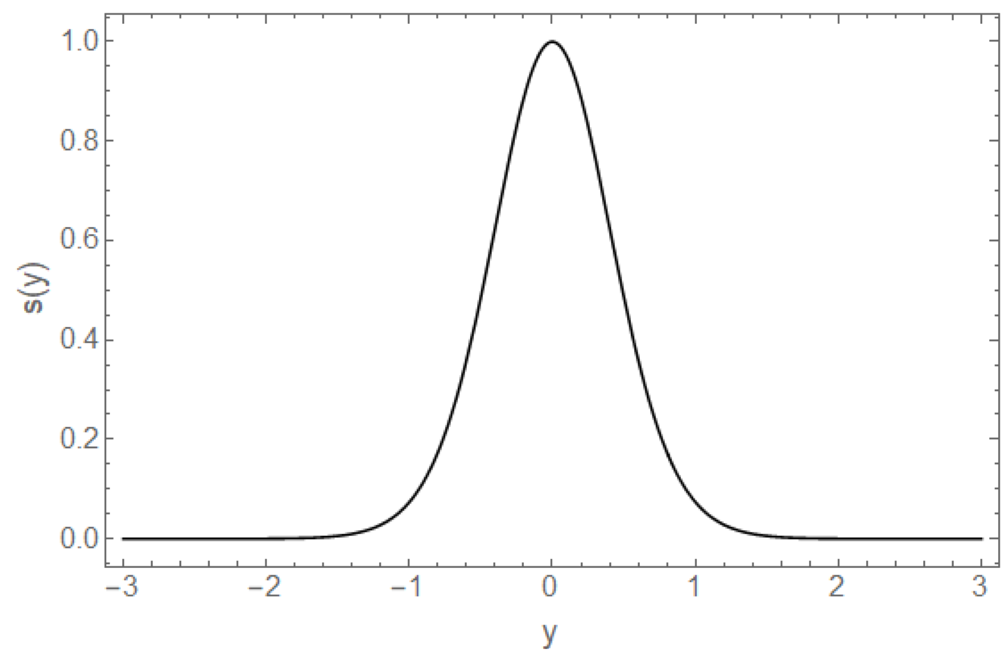
DW gauge boson

- Finding the general solution requires specification of the geometric function s(y)

- For the duration of this talk, I will use the solvable example:

$$s(y)=\frac{M}{(\cosh(m_V\, y))^{2\gamma}}$$

  - Where M, m_V and $\gamma > 0$



s(y) for M=m_V=1, γ=3

DW gauge boson

- After using this s(y) in the EOMs for the scalar and vector fields, the localized eigenfunctions and mass eigenvalues for the bound n^{th} KK-modes can be found for **γ=3**

Scalar KK-mode solution:

$$A_y(x,y)=\sqrt{\frac{4}{5}}\,g\cosh(m_V y)\,\psi^{(1)}(x)+\sqrt{\frac{8}{5}}\,g\cosh(m_V y)\sinh(m_V y)\,\psi^{(2)}(x)$$

Vector KK-mode solution:

$$A_\mu(x,y)=\underbrace{g\,A_\mu^{(0)}}_{\text{(photon)}}(x)+2g\sinh(m_V y)\,A_\mu^{(1)}(x)+\frac{g}{\sqrt{5}}\left(5-4\cosh^2(m_V y)\right)A_\mu^{(2)}(x)$$

(photon)

Mass eigenvalues obey:

$$m_n^2=n(2\gamma-n)\,m_V^2\,;\,n=0,1,..<\gamma$$

γ=3 →

$$m_1^2=5m_V^2 \qquad m_2^2=8m_V^2$$

$$\boxed{m_1=\sqrt{5/8}\,m_2}$$

Gauge coupling defined as:

$$g=\sqrt{\frac{15\,m_V}{16\,M}}$$

DW Higgs field

- To localize the Higgs field and the Higgs VEV, the same procedure is employed as in the gauge scenario

$$L_5^H = g^2 s\left[(D^M H)^\dagger (D_M H) - \frac{1}{2}\left(H^\dagger H - \frac{v^2}{2}\right)^2\right]$$

- For $s(y) = \dfrac{M}{(\cosh(m_V y))^{2\gamma}}$, and **γ=3**, the KK-mode solution

for the **real** Higgs field component after expanding about the vacuum $H = (v + h + i\varphi)/\sqrt{2}$, is

Higgs KK-mode solution:

$$h(x, y) = h^{(0)}(x) + 2\sinh(m_V y)\, h^{(1)}(x) + \frac{1}{\sqrt{5}}\left(5 - 4\cosh(m_V y)^2\right) h^{(2)}(x)$$

(SM Higgs)

DW and the Higgs mechanism

- Extending the 5D Abelian Higgs model to the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ is accomplished by using the same s(y) for each gauge group, up to a normalization factor

- After electroweak symmetry breaking, the masses are found to be

SM gauge boson zero-mode masses:

$$m_\gamma = m_{gluons} = 0 \qquad m_W = \frac{1}{2} g_2 v \qquad m_Z = \frac{1}{2}\sqrt{g_2^2 + g_Y^2}\, v$$

SM gauge boson KK-mode masses:

$$m_\gamma^{(n)} = m_{gluons}^{(n)} = m_n \qquad m_W^{(n)} = \sqrt{m_n^2 + m_W^2} \qquad m_Z^{(n)} = \sqrt{m_n^2 + m_Z^2}$$

DW fermions

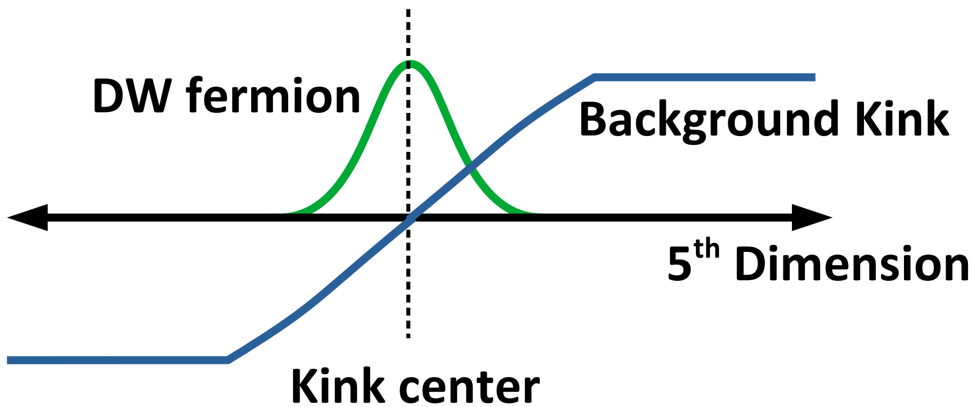
- The 5D Lagrangian for a Dirac fermion decomposed into its $\psi_{L/R}$ components and is **coupled** to the "kink-solution" ϕ_0 is given by

$$L_5^F \supset i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R - \bar{\psi}_L\partial_y\psi_R + \bar{\psi}_R\partial_y\psi_L + \underline{Y\phi_0(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)}$$

and

$$\phi_0 = \frac{m_\phi}{\sqrt{\lambda}}\tanh(m_\phi y) \approx \frac{m_\phi^2 y}{\sqrt{\lambda}}$$

(near y=0)



- After using the KK expansion for the L/R handed fermion modes in $\boldsymbol{L^F_5}$, the zero mode solutions to the EOM are found

Left-handed zero mode: $\psi_L^{(0)}(x,y) = \psi_L(x)\chi_L^{(0)}(y) \approx \psi_L(x)\left[\underline{C_L e^{\frac{-m_F^2 y^2}{2}}}\right]$

Right-handed zero mode: $\psi_R^{(0)}(x,y) = \psi_R(x)\chi_R^{(0)}(y) \approx \psi_R(x)\left[C_R e^{\frac{m_F^2 y^2}{2}}\right] = \underline{0}$

zero mode projected out!

DW fermion masses

- Extending to the SM case, the 5D Yukawa interaction for the quarks is

$$L_Y = -Y_u^{ij}\bar{Q}^i \tilde{H} u^j - Y_d^{ij}\bar{Q}^i H d^j + h.c. = -Y_u^{ij}\bar{Q}_L^i \tilde{H} u_R^j - Y_d^{ij}\bar{Q}_L^i H d_R^j + h.c$$

$i, j = 1, 2, 3$ is the generation index

$$\tilde{H} = i\sigma_2 H$$

- The fields are decomposed into their chiral components

Quark doublet zero mode: $Q = \underline{Q_L} + Q_R$

Up quark singlet zero mode: $u = u_L + \underline{u_R}$

Down quark singlet zero mode: $d = d_L + \underline{d_R}$

DW fermion masses

- The 4D effective Yukawa is found by integrating out the extra dimension
  - The up-type quarks effective Yukawa couplings’ are

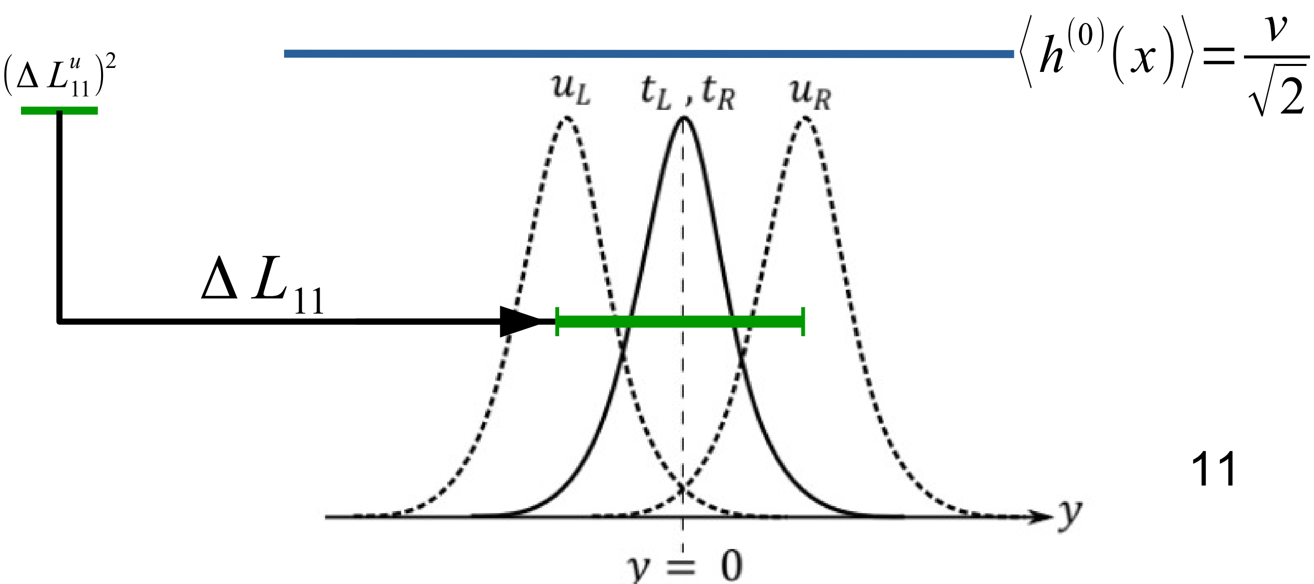
$$Y_{\text{eff}}^{\text{ij}} = Y_u^{\text{ij}} \int \chi_L^{\text{i}(0)}(y - y_L)\,\chi_R^{\text{j}(0)}(y - y_R)\,d\,y$$

- Therefore, the mass matrices for the up-type and down-type quarks are

Up-type mass matrix: $M_u^{\text{ij}} = \left(\frac{v}{\sqrt{2}}\right) Y_u^{\text{ij}} e^{-m_F^2 (\Delta L_{\text{ij}}^u)^2}$ **Down-type mass matrix:** $M_d^{\text{ij}} = \left(\frac{v}{\sqrt{2}}\right) Y_d^{\text{ij}} e^{-m_F^2 (\Delta L_{\text{ij}}^d)^2}$

- Comparing to measured fermion masses, the separation distance between the chiral fields can be found $\Delta L = y_L - y_R$, for example

$$m_u \approx M_u^{11} = \left(\frac{v}{\sqrt{2}}\right) Y_u^{11} e^{-m_F^2 (\Delta L_{11}^u)^2}$$



Realistic fermion mass matrices

- In order to compare M_u^{ij} and M_d^{ij} to the measured masses of the SM fermions, they are diagonalized by the unitary rotations: $u_L^i \rightarrow V_L^{ij} u_L^j$, $d_L^i \rightarrow W_L^{ij} d_L^j$ and similarly for L $\leftrightarrow$ R $\quad (i, j = 1, 2, 3)$

- Since the up-type quark masses are relatively hierarchical, we make the assumption that $V_L^{ij} = V_R^{ij} \approx \delta^{ij}$, which means that $W_L^{ij} = W_R^{ij} \approx V_{CKM}^{ij}$,

Up-type : $\quad \mathrm{diag}\{m_u, m_c, m_t\} = V_L^\dagger M_u V_R \approx M_u$

Down-type: $\quad \mathrm{diag}\{m_d, m_s, m_b\} = W_L^\dagger M_d W_R \approx V_{CKM}^\dagger M_d V_{CKM}$

- After finding ΔL for all SM fermions, the localization position for chiral fields is defined as $y_L = \Delta L/2$ and $y_R = -(\Delta L/2)$

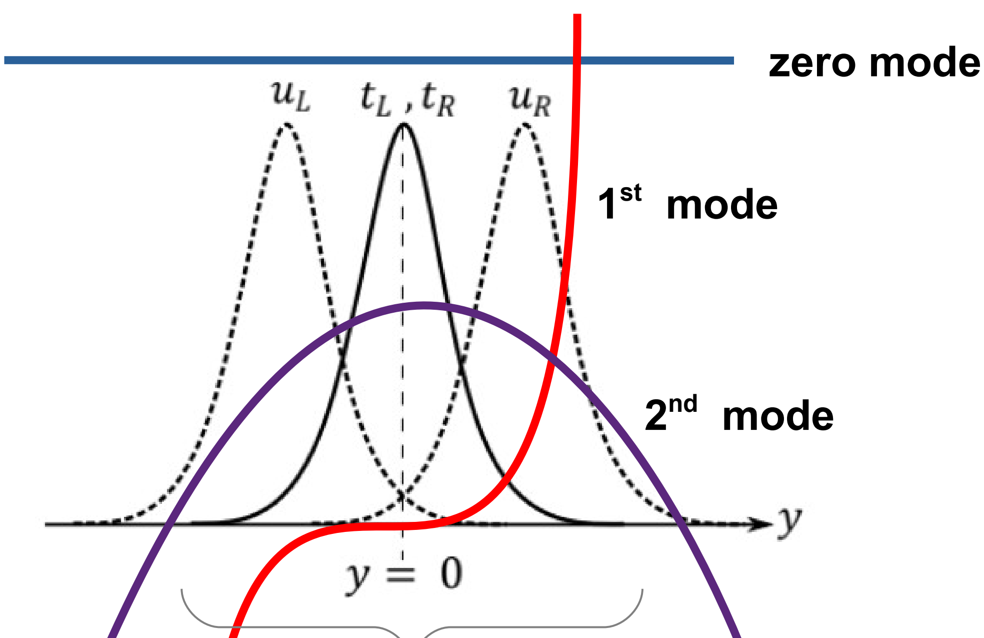
- This means the fermion mass hierarchy that extends over **13** orders of magnitude difference can be reinterpreted as a milder hierarchy among the separation distances ΔL, which are all of the **same order**!

Effective gauge interactions

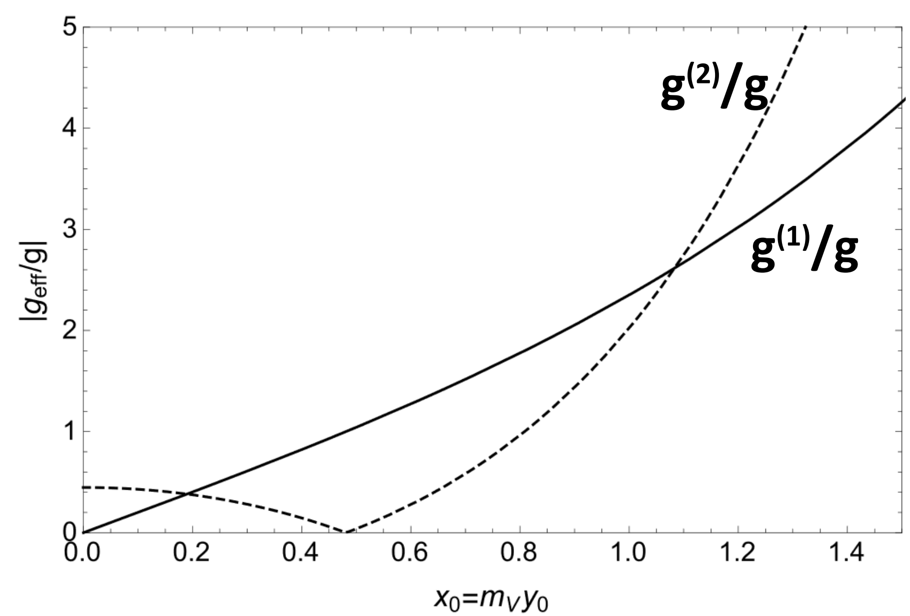
- Interactions with the KK-modes of the gauge boson are described by

$$L_4 \supset Q\, g\, \bar{\psi}_L \gamma^\mu A_\mu^{(0)} \psi_L + \sum_{n=1}^{\infty} Q\, g_{eff}^{(n)} A_\mu^{(n)} \bar{\psi}_L \gamma^\mu \psi_L$$

- Where the effective coupling is determined by the overlap between the **fermion zero mode** and the **gauge n^th KK-mode** after integrating out the extra dimension: $g_{\text{eff}}^{(n)} = g \int \left(\chi_L^{(0)}(y)\right)^2 \chi^{(n)}(y)\, dy$

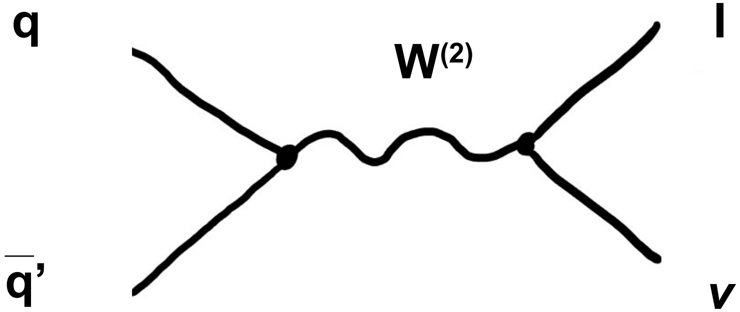


different KK-mode gauge couplings

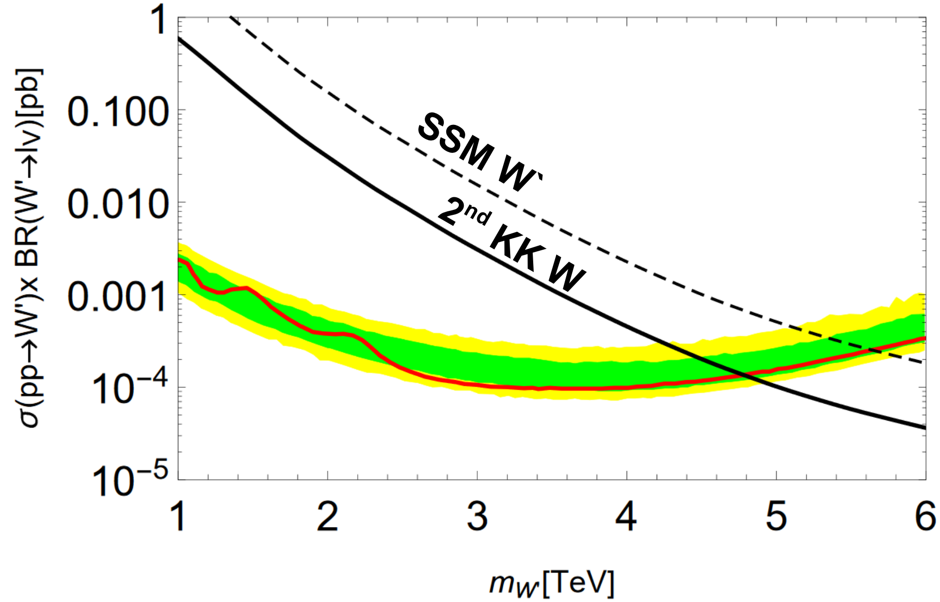


localization dependent coupling position!

Phenomenology of KK $W^{(2)}$ boson



W (2) KK-mode propagating

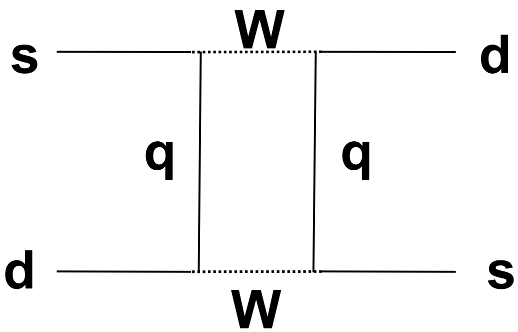


ATLAS-CONF-2018-017

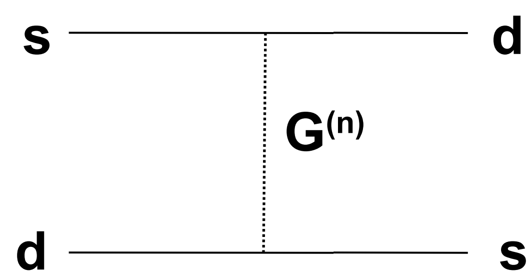
- The DW SM can be constrained by comparing the KK gauge boson $W^{(2)}$ to the LHC Run-2 ATLAS & CMS constraint on W'_{SSM}
- Latest ATLAS Data(red line) at 13 TeV and 79.8 fb^{-1} puts a constraint at $m_{W'}[\mathrm{TeV}]\geq 5.6$ for SSM theory prediction(dotted line)
- SSM process in the narrow width approximation is $\sigma(q\bar{q}'\to W')\propto g^2$, interpretation to ours for $g_{\mathrm{eff}}^{(2)}/g\approx 1/\sqrt{5}$ (solid line) places a bound of $m_{(2)}[\mathrm{TeV}]\geq 4.8$ and is found from

$$\sigma(q\bar{q}'\to W^{(2)}\to l\nu)=\sigma(q\bar{q}'\to W'\to l\nu)\left(\frac{g_{\mathrm{eff}}^{(2)}}{g}\right)^2\approx 0.2\times\sigma(q\bar{q}'\to W'\to l\nu)$$

Flavor Changing Neutral Currents (FCNC)



SM K-$\bar{K}$ Oscillation



Tree level flavor violation via KK Gluon

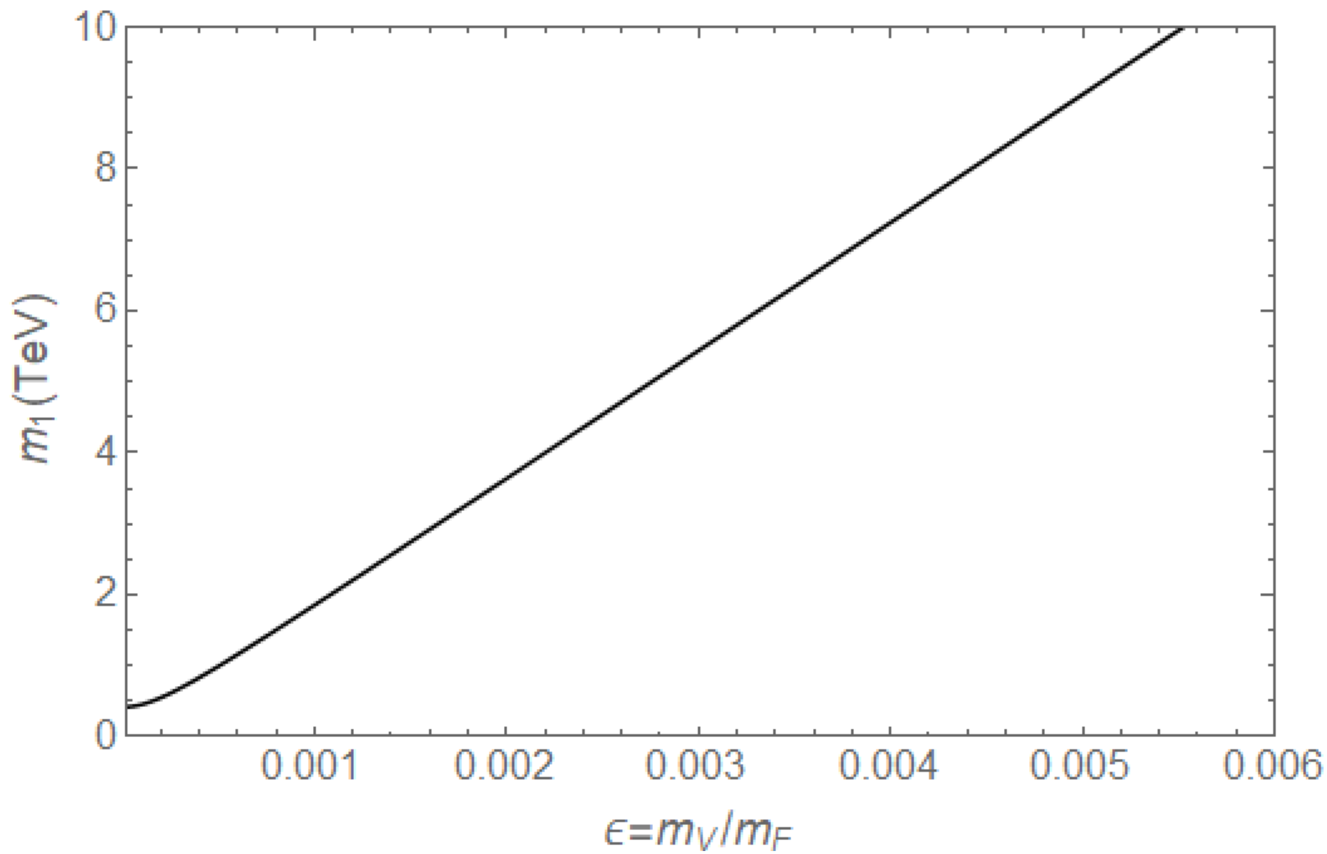
- In the SM FCNC effects are reduced by GIM mechanism and occur at **loop level**
- In the DW SM FCNC effects occur at **tree level** due to gauge coupling non-universality
- Largest contribution is due to KK Gluon interaction

$$L_4 \supset \sum_{n=1}^{\infty} G_{\mu}^{(n)} (\bar{d}_L \;\; \bar{s}_L \;\; \bar{b}_L)\left(\underline{V^{\dagger}_{CKM}\, g^{(n)}_{s_{eff}}\, V_{CKM}}\right)\gamma^{\mu}\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

(non-diagonal)

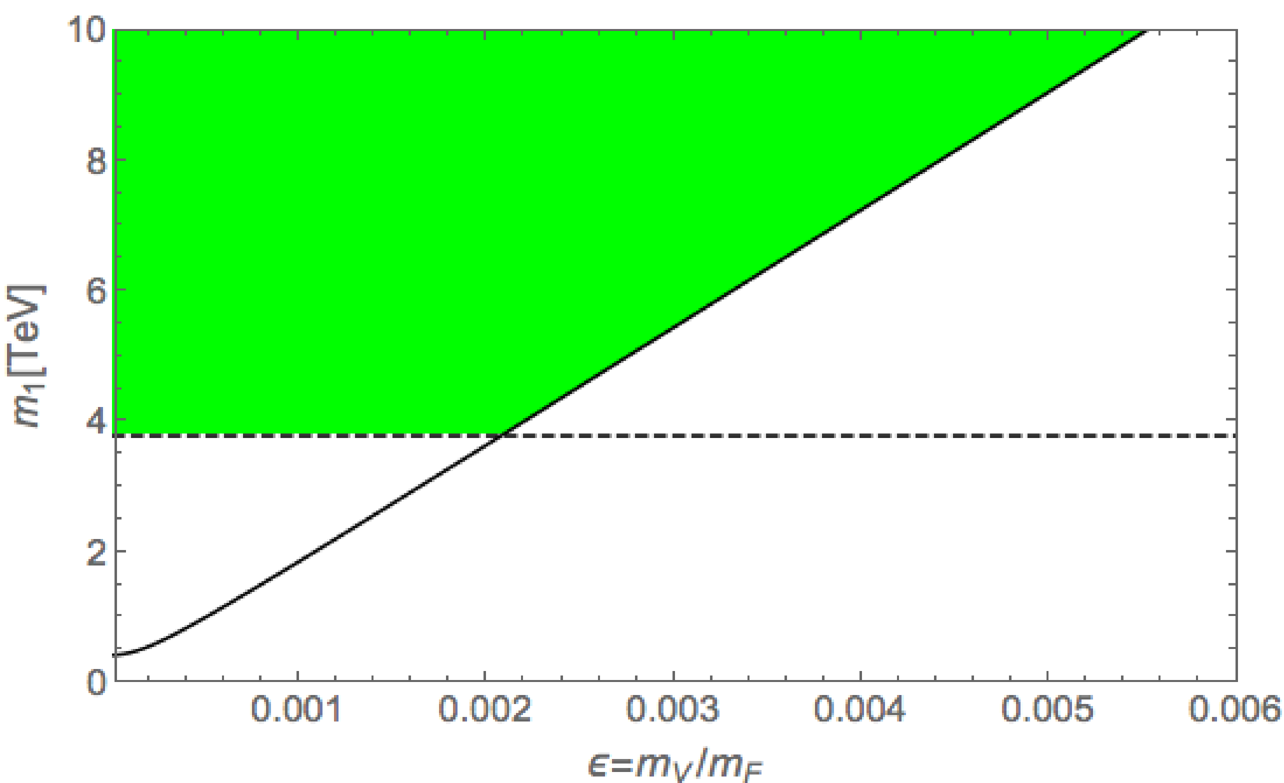
FCNC constraints

- Following the procedure from **B. Lillie, J. Hewett, PRD 68, 116002 (2003)** with updated **K-$\overline{\mathbf{K}}$** meson oscillation data, constraints can be placed on the first KK gauge boson mass $\mathbf{m_1}$ (solid black line)
- The ε parameter is a ratio of the **gauge boson mass** scale to the **DW fermion width**, this ratio determines the strength of the effective KK gauge coupling $\mathbf{g_{eff}^{(n)}}$



FCNC and LHC Run-2

- Combining these two constraints further narrows the allowed parameter region (green area) for $\mathbf{m_1}$, which is confined by both FCNC constraints (solid black line) and LHC Run-2 data (dashed black line) using $m_1[\text{TeV}]=\sqrt{5/8}\,m_2=3.78$
- Interestingly, these combined constraints show two distinct regions
  - For **ε < 0.002** the LHC Run-2 constraint is more severe
  - For **ε > 0.002** the FCNC constraint is more severe



Conclusions

- Localize Gauge fields in 5D non-compactified flat space
- Reproduced SM mass hierarchy
- FCNC/LHC data provided interesting KK gauge boson phenomenology possibilities
  - 1) The KK-mode of the SM gauge bosons are extremely heavy and unlikely to be produced at the LHC, while future FCNC measurements can reveal the existence of these heavy modes **($\varepsilon > 0.002$)**
  - 2) The width of the localized SM fermions is very narrow, leading to almost universal 4D KK-mode gauge couplings **($\varepsilon < 0.002$)**

Thanks to

Graduate School

Department of Physics & Astronomy College of Arts & Sciences

Back up slides

DW Future directions

- Higgs KK phenomenology
- DW gravity sector

DW gauge boson

- The 5D Lagrangian for the Abelian gauge field can be decomposed into

5D scalar Lagrangian: $$L_{scalar} = -\frac{1}{2} s A_y \Box_4 A_y + \frac{1}{2} s \xi A_y \partial_y \left(\frac{1}{s} \partial_y (s A_y)\right)$$

5D vector Lagrangian: $$L_{gauge} = \frac{1}{2} s A^{\mu} \left(\eta_{\mu\nu} \Box_4 - \left(1 - \frac{1}{\xi}\right) \partial_{\mu} \partial_{\nu}\right) A^{\nu} - \frac{1}{2} A_{\mu} \partial_y (s \partial_y A^{\mu})$$

- Use the Kaluza-Klein (KK) mode decomposition for both fields in order to find the solutions to the y-dependent equations of motion (EOM)

Scalar KK-modes: $$A_y(x, y) = \sum_{n=0}^{\infty} \eta^{(n)}(x) \psi^{(n)}(y)$$

Vector KK-modes: $$A_{\mu}(x, y) = \sum_{n=0}^{\infty} A_{\mu}^{(n)}(x) \chi^{(n)}(y)$$

DW fermions

- A real scalar field is introduced in order to localize the fermions throughout the 5D bulk

$$L_5 = \frac{1}{2}(\partial_M \phi)(\partial^M \phi) - V(\phi), \quad V(\phi) = \frac{m_\phi^4}{2\lambda} - m_\phi^2 \phi^2 + \frac{\lambda}{2}\phi^4$$

- The "kink-solution," which breaks translational invariance along the 5th dimension is given by

$$\phi_0 = \frac{m_\phi}{\sqrt{\lambda}} \tanh(m_\phi y) \approx \frac{m_\phi^2 y}{\sqrt{\lambda}}$$

(near y=0)

- The 5D Lagrangian for a Dirac fermion decomposed into its $\psi_{L/R}$ components and is **coupled** to ϕ is given by

$$L_5^F \supset i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + i\bar{\psi}_R \gamma^\mu \partial_\mu \psi_R - \bar{\psi}_L \partial_y \psi_R + \bar{\psi}_R \partial_y \psi_L + \underline{Y\phi_0(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)}$$

DW fermion masses and parameter choice

- The top quark Yukawa is fixed to $Y_u^{33}=Y_t=0.995$ according to its mass $m_t[\mathrm{GeV}]=173$, and a mild hierarchy choice is assigned to all remaining Yukawa elements
- The CKM (and PMNS) matrix elements are from the latest PDG values
- We employ the same procedure for the leptons, where the normal neutrino hierarchy is assumed and the rotation matrices are defined to be

Charged lepton rot. matrix: $\widetilde{V}_L^{ij}=\widetilde{V}_R^{ij}\approx\delta^{ij}$,

Down-type rot. matrix: $\widetilde{W}_L^{ij}=\widetilde{W}_R^{ij}\approx V_{\mathrm{PMNS}}^{ij}$,

Example ΔL

Quarks	ΔL_{ii}
$u_L,\, u_R$	3.597
$c_L,\, c_R$	1.938
$t_L,\, t_R$	≈ 0
$d_L,\, d_R$	3.294
$s_L,\, s_R$	1.902
$b_L,\, b_R$	0.993

Leptons	ΔL_{ii}
$e_L,\, e_R$	3.879
$\mu_L,\, \mu_R$	2.712
$\tau_L,\, \tau_R$	1.814
${\nu^e}_L,\, {\nu^e}_R$	5.638
${\nu^\mu}_L,\, {\nu^\mu}_R$	5.412
${\nu^\tau}_L,\, {\nu^\tau}_R$	5.390