Simple PWFA Wakefield Modelling

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CLIC and transverse wakefields

- A beam with a transverse offset will interact with the acc. structure ⇒ transverse wakefield.
- The beam tail will be repelled towards the structure and the beam may become unstable.
- In CLIC, transverse wakefields limit the beam charge.
- Similar challenges also exist in PWFA.
- However, PWFA may be used to achieve gradients several o.m. larger.

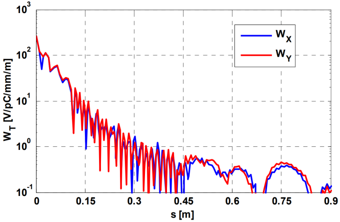

Figure: The envelope of the transverse wakefields for both planes. [2]

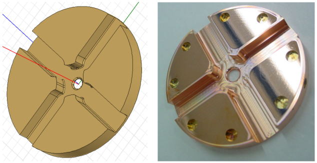

Figure: Basic cell geometry of the accelerating structure. [2]

Plasma Wakefield Acceleration[1]

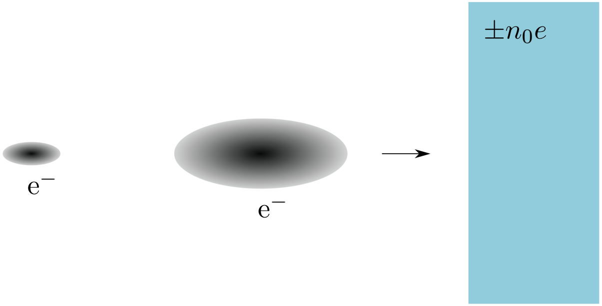

Figure: A high-energetic drive beam followed by a witness beam propagate into a homogeneous plasma.

[1] Blowout regime.

Plasma Wakefield Acceleration

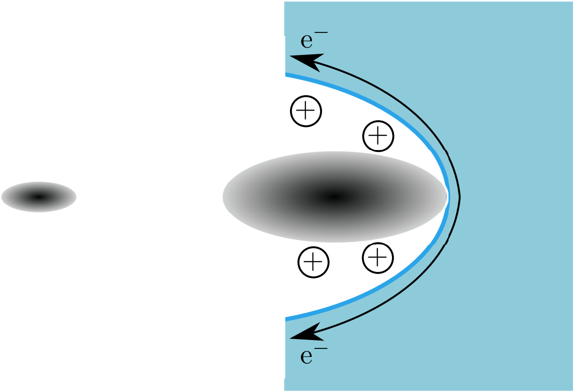

Figure: The space charge fields of the drive beam push the plasma electrons away from the axis.

Plasma Wakefield Acceleration

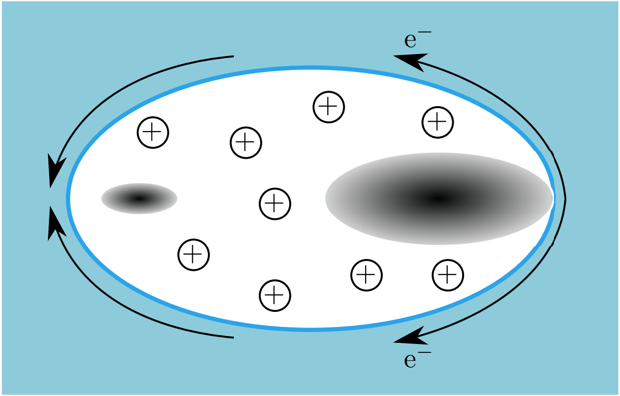

Figure: The plasma ions exert a restoring force on the plasma electrons ⇒ bubble cavity.

Plasma Wakefield Acceleration

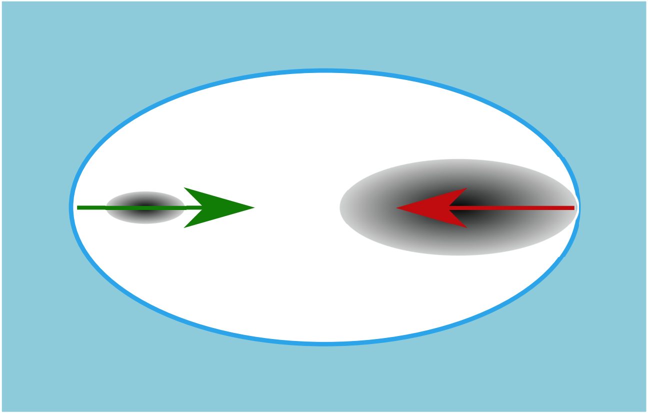

Figure: The front half of the bubble has a decelerating field - the plasma extracts energy from DB. WB accelerated at the back half of the bubble.

Plasma Wakefield Acceleration

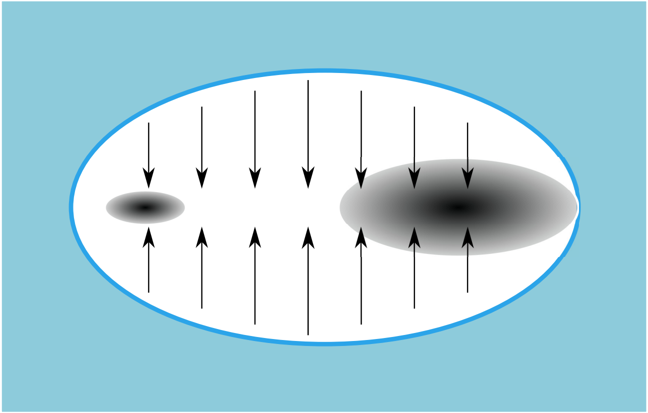

Figure: Focusing everywhere inside the bubble.

Plasma Wakefield Acceleration

Mmmmm... plasma.
Are there anything it cannot do?

Plasma Wakefield Acceleration

Benefits:

- Can sustain enormous gradients on the order of tens of GV/m.
- Focusing everywhere inside the bubble.

Challenges, for instance:

- Transverse wakefields $\Rightarrow$ instabilities and beam breakup.
  - CLIC: $7\,\mathrm{V/pC/mm/m}$.
  - PWFA simulations: $5 \cdot 10^{16}\,\mathrm{V/pC/mm/m}$.
- Crucial to understand in order to develop mitigation techniques.
- Focus on short range intrabeam wakefields.

Section 2

Modelling Wakefields

- Find an appropriate wakefunction for PWFA with FACET-II parameters (table 1).
- Compared the normalized transverse kick $\mathcal{W}_\perp(\xi) = F_\perp(\xi)/(Qq\Delta x)$ obtained using

$$\mathcal{W}_\perp(\xi) = \int_{\xi_\mathrm{H}}^{\xi} W_\perp(\xi' - \xi)\lambda(\xi')\,\mathrm{d}\xi', \tag{1}$$

and by direct calculation of $\mathcal{W}_\perp(\xi)$ using the fields from the QuickPIC simulation results.

	Drive beam	Trailing beam
Q	$-e$	$-e$
m	m_e	m_e
γ	19569	19569
N	$1.0 \cdot 10^{10}$	$3.33 \cdot 10^{9}$
Δx	0	3.7575 µm
σ_x	3.65 µm	3.65 µm
σ_y	3.65 µm	3.65 µm
σ_z	12.77 µm	6.38 µm

Table: Beam parameters used in the simulation. Q is the charge per particle, m is the mass per particle, γ is the initial Lorentz factor, N is the total number of particles, Δx is the transverse offset, from the ξ-axis, and the various σ's give the beam dimension along the x, y and z-direction.

QuickPIC

- Fully parallelized, fully relativistic, three-dimensional quasi-static PIC code.
- Quasi-static approximation.
- Reduces computational time with 2-3 order of magnitude.
- Agrees well with full PIC codes for problems of interest.

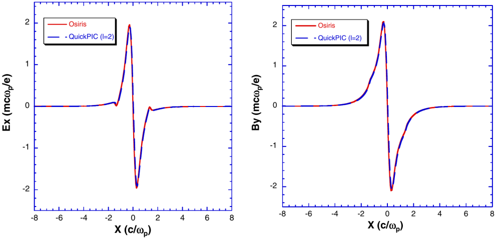

Figure: Radial electric and azimuthal magnetic fields comparisons for electron drive beam. [3]

Parametric wakefunctions

- Two wakefuctions were tested

$$W_\perp(\xi' - \xi) = \frac{2}{\pi\varepsilon_0} \frac{\xi' - \xi}{r_\mathrm{b}(\xi)^{4-\eta} r_\mathrm{b}(\xi')^\eta} \Theta(\xi' - \xi) \tag{2}$$

$$W_\perp(\xi' - \xi) = \frac{2}{\pi\varepsilon_0 a^{*4}} (\xi' - \xi)\Theta(\xi' - \xi). \tag{3}$$

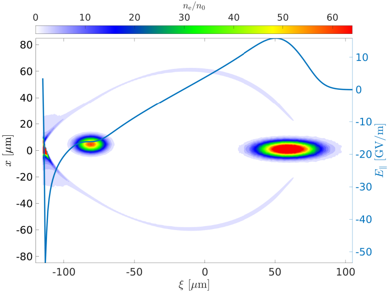

Figure: The beam and plasma electron density per unit per unit initial plasma density and the axial electric field.

- Plasma density $n_0 = 4.0 \cdot 10^{16}\,\mathrm{cm}^{-3}$.
- Area of interest: $\pm 3\sigma_{z\mathrm{WB}}$ from the center of the witness beam.

Parametric wakefunctions

- Two wakefuctions were tested

$$W_{\perp}(\xi' - \xi) = \frac{2}{\pi\varepsilon_0} \frac{\xi' - \xi}{r_{\mathrm{b}}(\xi)^{4-\eta} r_{\mathrm{b}}(\xi')^{\eta}} \Theta(\xi' - \xi) \quad (2)$$

$$W_{\perp}(\xi' - \xi) = \frac{2}{\pi\varepsilon_0 a^{*4}} (\xi' - \xi)\Theta(\xi' - \xi). \quad (3)$$

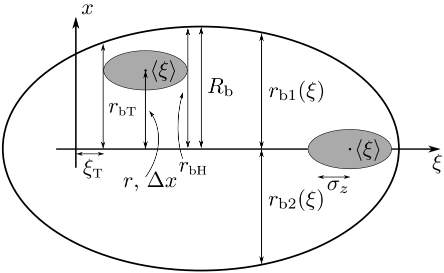

Figure: The driving and trailing beams shown together with the plasma bubble and various scales used in the calculations.

- Modified from a paper by V. Lebedev et al. [4].
- $r_{\mathrm{b}}(\xi)$ the bubble radius at ξ.
- ξ is the reference point, ξ' is the longitudinal position of preceding particles.
- η is a fitting parameter.

Parametric wakefunctions

- Two wakefuctions were tested

$$W_{\perp}(\xi' - \xi) = \frac{2}{\pi\varepsilon_0} \frac{\xi' - \xi}{r_{\mathrm{b}}(\xi)^{4-\eta} r_{\mathrm{b}}(\xi')^{\eta}} \Theta(\xi' - \xi) \quad (2)$$

$$W_{\perp}(\xi' - \xi) = \frac{2}{\pi\varepsilon_0 a^{*4}} (\xi' - \xi) \Theta(\xi' - \xi) . \quad (3)$$

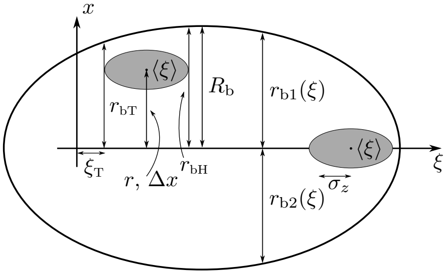

Figure: The driving and trailing beams shown together with the plasma bubble and various scales used in the calculations.

- Originally proposed for metal structures [1].
- Structure iris a in plasma?
- Best choice: $a^* = r_{\mathrm{bT}} + k_{\mathrm{p}}^{-1}$. Similar approach used by Stupakov [5].

Procedure

1. Choose a N_2 for the simulation.

The value not being changed retained the standard value shown in table 2.

	Drive beam	Trailing beam
Q	$-e$	$-e$
m	m_e	m_e
γ	19569	19569
N	$1.0 \cdot 10^{10}$	$3.33 \cdot 10^{9}$
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Table: Beam parameters used in the simulation. Q is the charge per particle, m is the mass per particle, γ is the initial Lorentz factor, N is the total number of particles, Δx is the transverse offset, from the ξ-axis, and the various σ's give the beam dimension along the x, y and z-direction.

Procedure

1. Choose a N_2 for the simulation.
2. Calculate the two theoretical transverse kicks.

Using

$$\mathcal{W}_\perp(\xi) = \int_{\xi_\mathrm{H}}^{\xi} W_\perp(\xi' - \xi)\lambda(\xi')\,\mathrm{d}\xi',$$

together with

$$W_\perp(\xi'-\xi) = \frac{2}{\pi\varepsilon_0 a^{*4}}(\xi'-\xi)\Theta(\xi'-\xi)$$

and

$$W_\perp(\xi'-\xi) = \frac{2}{\pi\varepsilon_0}\frac{(\xi'-\xi)\Theta(\xi'-\xi)}{r_\mathrm{b}(\xi)^{4-\eta} r_\mathrm{b}(\xi')^{\eta}}$$

using some chosen values for η.

Procedure

1. Choose a N_2 for the simulation.
2. Calculate the two theoretical transverse kicks.
3. Calculate the simulated transverse kick $\mathcal{W}_{\mathrm{QP}}$.

Calculated directly from the QuickPIC simulation output fields.

$$\mathcal{W}_{\mathrm{QP}}(\xi) = \mathcal{W}_{\mathrm{tot}}(\xi) - \mathcal{W}_{\mathrm{focus}} = -\frac{E_x(\xi,\Delta x) - vB_y(\xi,\Delta x)}{e\Delta x} + \frac{n_0}{2\varepsilon_0} \quad (4)$$

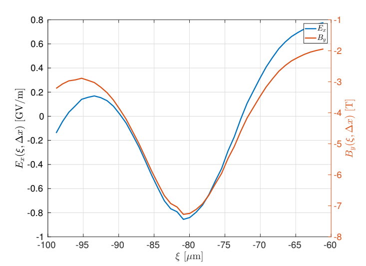


Figure: Simulation output fields evaluated at the axis $x = \Delta x$.

Procedure

1. Choose a N_2 for the simulation.
2. Calculate the two theoretical transverse kicks.
3. Calculate the simulated transverse kick $\mathcal{W}_{\mathrm{QP}}$.
4. Calculate the error + find the optimal η.

For each simulation, optimal values for η was found by minimizing the relative rms error

$$\bar{\epsilon}_{\mathrm{rms}} = \frac{\epsilon_{\mathrm{rms}}}{\overline{\mathcal{W}}_{\mathrm{QP}}},$$

- ϵ_{rms}: rms deviation between theory and simulation.
- $\overline{\mathcal{W}}_{\mathrm{QP}}$: transverse kick directly extracted from QuickPIC and averaged over the area of interest.

Procedure

1. Choose a N_2 for the simulation.
2. Calculate the two theoretical transverse kicks.
3. Calculate the simulated transverse kick $\mathcal{W}_{\text{QP}}$.
4. Calculate the error + find the optimal η.
5. For (2): recalculate $\mathcal{W}_\perp$ using the optimal value for η.

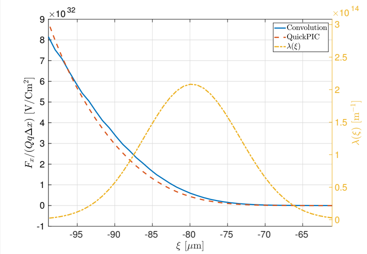


Figure: The normalized transverse kick $\mathcal{W}_\perp$ (blue) calculated by convolving the linear wakefunction (2) with the longitudinal particle distribution, and $\mathcal{W}_{\text{QP}}$ calculated directly from the output fields of the simulation (red). The longitudinal particle distribution $\lambda(\xi)$ of the witness beam is also included in the figure. $\bar{\epsilon}_{\text{rms}} = 0.17$.

Procedure

1. Choose a N_2 for the simulation.
2. Calculate the two theoretical transverse kicks.
3. Calculate the simulated transverse kick $\mathcal{W}_{\mathrm{QP}}$.
4. Calculate the error + find the optimal η.
5. For (2): recalculate $\mathcal{W}_{\perp}$ using the optimal value for η.
6. Repeat the process with another value for N_2.

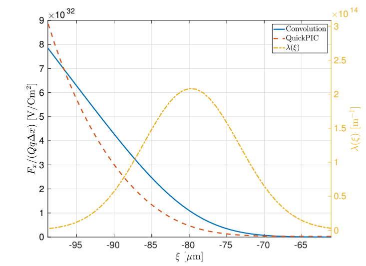


Figure: The normalized transverse kick $\mathcal{W}_{\perp}$ (blue) calculated by convolving the linear wakefunction (3) with the longitudinal particle distribution, and $\mathcal{W}_{\mathrm{QP}}$ calculated directly from the output fields of the simulation (red). The longitudinal particle distribution $\lambda(\xi)$ of the witness beam is also included in the figure. $\bar{\epsilon}_{\mathrm{rms}} = 0.396$.

Results

The fitted η-model

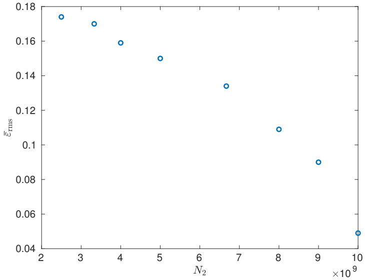


Figure: The relative rms error between model and simulation where the optimized values of η shown in figure 7 were used to calculate the transverse kick.

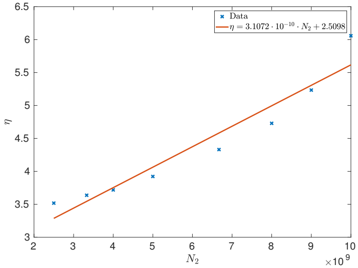


Figure: A linear fit of the parameter η as a function of N_2 obtained from optimizing (3) for the various simulations. $r = 0.962$.

Results

The predictive a^*-model

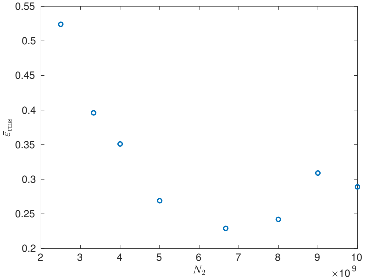


Figure: The relative rms error between model and simulation where the optimized values of a shown in figure **??** were used to calculate the transverse kick.

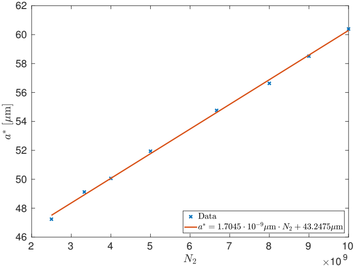


Figure: A linear fit of the parameter a^* as a function of N_2. $r = 0.999$.

Section 3

Simple Quasi-Static Numerical Model

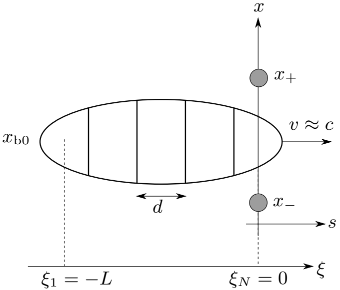
Preparations

- Inspired by Daniel Schulte's model.
- Long plasma too heavy in QuickPIC.
- Beam of length L divided into N slices with thickness d.
- Longitudinal position of slices: $\xi = [\xi_1, \xi_2, ..., \xi_N]$
- Initial offset x_{b0}.
- Offset of each beam slice: $x_{\mathrm{b}}(\xi) = x_{\mathrm{b}}(\xi_i) = [x_{\mathrm{b1}}, x_{\mathrm{b2}}, ..., x_{\mathrm{bN}}]$
- Optimal momentum spread (Lebedev et al. [4])

$$\frac{\Delta p}{p}(\xi) = -\eta_{\mathrm{t}} \frac{\xi^2}{L^2}, \qquad (4)$$

- Initial transverse distance between beam slice and plasma:

$$x_{\pm} = x_{\mathrm{b0}} \pm n_0^{1/3} \qquad (5)$$



Preparations

- Gaussian longitudinal particle density $\lambda(\xi)$.
- Number of electrons of a slice located at ξ:

$$N_{\mathrm{eb}}(\xi) = \int_{\xi-d/2}^{\xi+d/2} \lambda(\xi')\,\mathrm{d}\xi'. \tag{6}$$

- Gradient

$$E_{\parallel} = c\sqrt{\frac{m_{\mathrm{e}} n_0}{\varepsilon_0}} \tag{7}$$

- Time steps
  - Time step used to propagate the plasma slices along the beam:

$$\Delta t = \frac{d}{\max(v_{\mathrm{b}\xi})} \approx \frac{d}{c} \tag{8}$$

  - Time step used to propagate the beam forward:

$$\Delta T = \frac{1}{200}\frac{\lambda_\beta}{c} = \frac{\pi}{100e}\sqrt{\frac{2\gamma_0\varepsilon_0 m_{\mathrm{e}}}{n_0}} \tag{9}$$

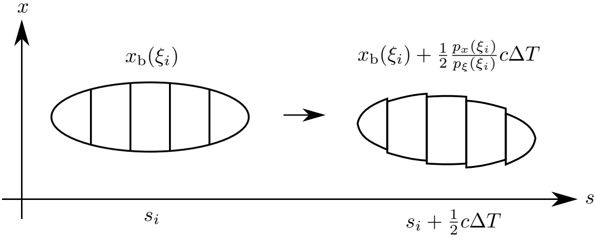
- Initial Lorentz factor $\gamma_0 \gg 1$.

Propagate the beam

- Leapfrog integration.
- Drift-kick-drift.
- Propagate the beam half a time step:

$$x_{\mathrm{b}}(\xi) \to x_{\mathrm{b}}(\xi) + \frac{1}{2}\frac{p_x(\xi)}{p_\xi(\xi)}c\Delta T, \qquad s_i \to s_i + \frac{1}{2}c\Delta T \tag{10}$$

- $p_x(\xi)$, $p_\xi(\xi)$ transverse and longitudinal momenta of slice at ξ.



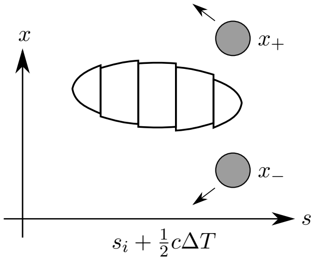
Plasma-beam interaction

- Calculate the intra-beam wakefields using (3). For each beam slice at ξ_i:

$$a^* = x_+ + k_\mathrm{p}^{-1} \quad (11)$$

$$E_x(\xi_i) = -e \int_{\xi_\mathrm{h}}^{\xi_i} W_\perp(\xi' - \xi_i, a^*) \lambda(\xi') x_\mathrm{b}(\xi') \,\mathrm{d}\xi' \quad (12)$$

$$F_x(\xi_i) = -eN_\mathrm{eb} \left(E_x(\xi_i) + \frac{n_0 e}{2\varepsilon_0} x_\mathrm{b}(\xi_i) \right) \quad (13)$$



Plasma-beam interaction

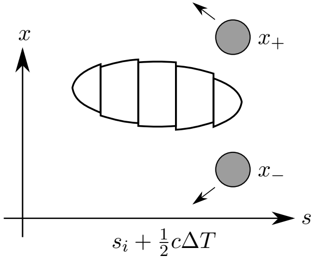
- Propagate plasma particles backwards along the beam:

$$x_\pm \to x_\pm + \frac{1}{2} v_\pm \Delta t. \qquad (11)$$

- Apply transverse force on the plasma particle:

$$p_\pm \to p_\pm + \frac{e^2}{2\varepsilon_0} \cdot \left(\frac{2N_{\mathrm{eb}}(\xi_i)}{\pi d(x_\pm - x_{\mathrm{b}}(\xi_i))} - n_0 x_\pm \right) \Delta t \qquad (12)$$

$$v_\pm = \mathrm{sign}(p_\pm) \left(\frac{m_{\mathrm{e}}}{p_\pm^2} + \frac{1}{c^2} \right)^{-1/2}. \qquad (13)$$

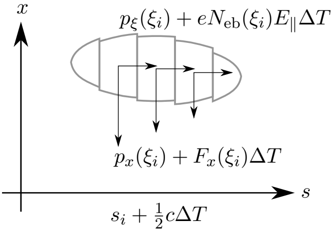


- Propagate again:

$$x_\pm \to x_\pm + \frac{1}{2} v_\pm \Delta t, \quad \xi_{\mathrm{p}} \to \xi_{\mathrm{p}} - d \qquad (14)$$

- Repeat process until the end of the beam.

Apply the Force on the beam



Change the transverse and longitudinal momentum:

$$p_x(\xi) \to p_x(\xi) + F_x(\xi)\Delta T \qquad (15)$$

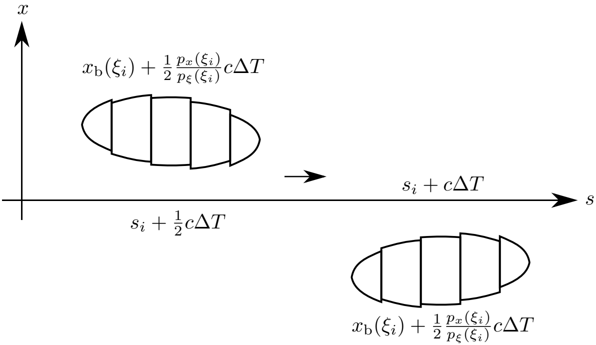
$$p_\xi(\xi) \to p_\xi(\xi) + eN_{\mathrm{eb}}(\xi)E_\parallel \Delta T \qquad (16)$$

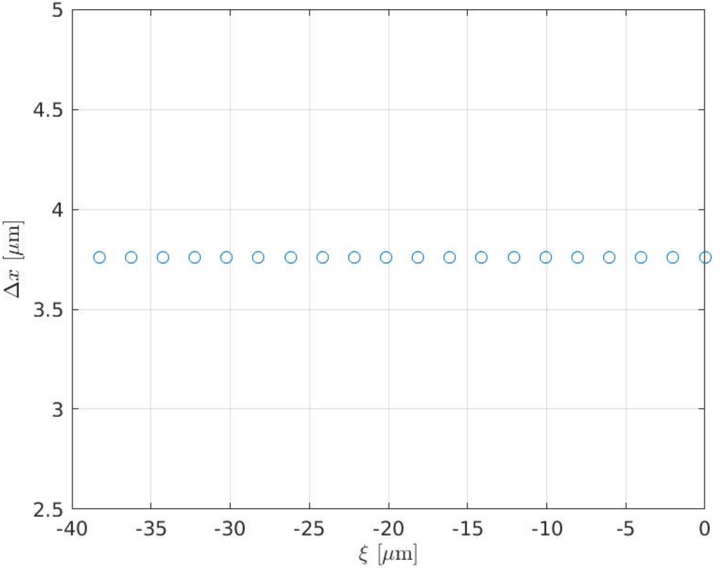
Propagate the beam

Propagate the beam another half a time step:

$$x_{\mathrm{b}}(\xi) \to x_{\mathrm{b}}(\xi) + \frac{1}{2}\frac{p_x(\xi)}{p_\xi(\xi)}c\Delta T \quad (17)$$

$$s_i \to s_i + \frac{1}{2}c\Delta T \quad (18)$$




5
4.5
4
3.5
3
2.5
-40
-35
-30
-25
-20
-15
-10
-5
0
Δx [μm]
ξ [μm]

A little benchmarking

Consider the first and second beam slice and compare them to

$$x_2(s) = \sqrt{\frac{\beta_x(s)\varepsilon_{\mathrm{N}x}}{\gamma(s)}}\left[\cos(\mu(s)) + \frac{A(s)}{2} q_1 q_2 W_\perp(\xi_1 - \xi_2)\sin(\mu(s))\right] \tag{19}$$

with

$$\beta_x(s) = \frac{c}{e}\sqrt{\frac{2\gamma(s)\varepsilon_0 m_\mathrm{e}}{n_0}} \tag{20}$$

$$\mu(s) = \int_0^s \frac{\mathrm{d}s'}{\beta_x(s')} \tag{21}$$

$$A(s) = \int_0^s \frac{\beta_x(s')}{E_2(s')}\,\mathrm{d}s' \tag{22}$$

$$\varepsilon_{\mathrm{N}x} = \frac{\gamma_0}{\beta_x(0)} x_{\mathrm{b}0}^2 \tag{23}$$

$$q_1 = N_\mathrm{eb}(\xi_N)e \tag{24}$$

$$q_2 = N_\mathrm{eb}(\xi_{N-1})e \tag{25}$$

$$p_2 = p_\xi(\xi_{N-1}) \tag{26}$$

$$M_2 = N_\mathrm{eb}(\xi_{N-1})m_\mathrm{e} \tag{27}$$

$$E_2 = \sqrt{(p_2 c)^2 + (M_2 c^2)^2} \tag{28}$$

$$\gamma \approx \frac{p_2}{M_2 c}. \tag{29}$$

A little benchmarking

Consider the first and second beam slice and compare them to

$$x_2(s) = \sqrt{\frac{\beta_x(s)\varepsilon_{\mathrm{N}x}}{\gamma(s)}}\left[\cos(\mu(s)) + \frac{A(s)}{2}q_1 q_2 W_\perp(\xi_1 - \xi_2)\sin(\mu(s))\right] \tag{19}$$

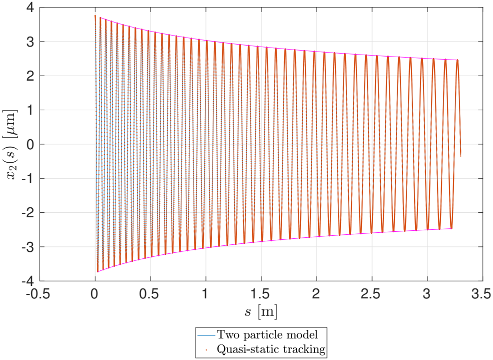


Figure: Plot of the transverse motion of beam slice 2 and an analytical two-particle model.

Optimization procedure for PWFA parameters

1. Identify how to choose beam length for a given charge.
2. Loop through different beam charges, adjust the length and check the stability.

- Start with an offset and check the amplification of this offset.
  - Translate to normalized coordinates, since the beam size decreases.
  - Normalize to the beam size as a function of energy.
- If the RMS amplitude

$$\sum_i \left[\left(\frac{x_i}{\sigma_x} \right)^2 + \left(\frac{x_i'}{\sigma_{x'}} \right)^2 \right] \tag{20}$$

  is a factor 2 larger than the initial value, the beam will be unstable.
- Plot RMS amplitude VS. beam charge.

Summary

- Acc. gradient several o.m. larger than conventional NC acc. structures may be achieved with PWFA.
- Transverse wakefields have to be understood and mitigated.
- Proposed two wakefunctions:

  1. $W_\perp(\xi' - \xi) = \frac{2}{\pi\varepsilon_0} \frac{\xi' - \xi}{r_\mathrm{b}(\xi)^{4-\eta} r_\mathrm{b}(\xi')^\eta} \Theta(\xi' - \xi)$
  2. $W_\perp(\xi' - \xi) = \frac{2}{\pi\varepsilon_0 a^{*4}} (\xi' - \xi)\Theta(\xi' - \xi)$

- Compared against QuickPIC simulations.
- Simple quasi-static model.
- May be used to optimize PWFA parameters.

Make accelerators small again!

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PhD thesis, University of California, Los Angeles, Dec 2018.

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G. Stupakov.
Short-range wakefields generated in the blowout regime of plasma-wakefield acceleration.
Phys. Rev. Accel. Beams, 21:041301, Apr 2018.